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A damage model predicting moderate temperature and size effects on concrete in compression

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Abstract. Experimental isotherm compressive tests show that concrete behaviour is dependent on temperature. The aim of such tests is to reproduce how concrete will behave under environmental changes within a moderate range of temperature. In this paper, a novel constitutive elastic damage behaviour law is proposed based on a free energy with an apparent damage depending on temperature. The proposed constitutive behaviour leads to classical theory of thermo-elasticity at small strains. Fixed elastic mechanical characteristics and fixed evolution law of damage independent of temperature and the material volume element size are considered. This approach is applied to compressive tests. The model predicts compressive strength and secant modulus of elasticity decrease as temperature increases. A power scaling law is assumed for specific entropy as function of the specimen size which leads to a volume size effect on the stress-strain compressive behaviour. The proposed model reproduces theoretical and experimental results from literature for tempertaures ranging between 20℃ and 70℃. The effect of the difference in the coefficient of thermal expansion between the mortar and coarse aggregates is also considered which gives a better agreement with FIB recommendations. It is shown that this effect is of a second order in the considered moderate range of temperature.

Keywords: concrete; scalar damage; compressive strength; Young modulus; temperature; volume size effect

1. Introduction

It was experimentally confirmed that mechanical properties of concrete like the compressive and tensile strengths and the apparent modulus of elasticity, exhibit a temperature effect at low and moderate high temperatures between 20℃ and 70℃ (Yu *et al.* 1989, Miura 1989, Shoukry *et al.* 2011, Nandan and Singh. 2014, Kallel *et al.* 2018, Wang *et al.* 2018). The aim of such tests done generally in compression at low and moderate high temperatures is to reproduce after curing how concrete and reinforced concrete will behave in service under environmental changes with new moderate temperature conditions. Effect of temperature on behaviour of concrete in compression remains the most important input when modelling reinforced concrete columns, beams and slabs according to design codes which neglect contribution of concrete in tension for example in steel reinforced concrete sections.

In literature several studies are related to residual mechanical properties after exposure to high temperatures, for example under fire, where concrete hydration takes place at temperatures of up to 100℃.

After exposure to high temperatures, tests are done at room temperatures and show that mechanical properties may be affected in a non reversible way (see for example Abdulhaleem *et al.* 2018, Ashteyat *et al.* 2018, Eren Gulsan *et al.* 2018, Liang *et al.* 2017). Due to a higher thermal expansion difference and water evaporation, the compressive strength dropped abruptly after exposure to temperatures higher than 500℃ (Eren Gulsan *et al.* 2018).

However, fewer studies are related to moderate temperature effect on concrete which is a phenomenon independent of concrete damage under fire and very elevated temperature. Mechanical properties of unloaded concrete structures may be change in a reversible manner with temperature changing in a moderate range between 20℃ and 70℃. Furthermore new temperature conditions have an effect on structural health monitoring methods based on dynamic analysis and which often use changes in the modal parameters to identify damage. In fact, the vibration parameters are not only influenced by damage but also by temperature. For example, Nandan and Singh (2014), Wang *et al.* (2018) introduce in a dynamic analysis of concrete beams and slabs, the thermal effect on the secant Young modulus with a reversible linear decreasing as temperature increases and increasing as temperature decreases. Such phenomenon should be predictable by a damage constitutive behavior which exhibits a temperature effect coupled with mechanical damage.

Damage mechanics was firstly developed by (Kachanov 1958) and has been applied to model creep damage. In the context of Portland cement concrete, the term damage is related to irreversible changes of microstructures and the damage variable describes micro-cracks and voids created in the material and traduced by a loss of the apparent or sequent stiffness. For very small strains, changes are reversible and classical theory of thermo-elasticity is

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generally assumed. Application of thermo-elasticity to concrete structures is valid within a moderate range of temperature variation considering constant values of the elastic stiffness. In classical theory of thermo-elasticity the stress tensor $\overline{\overline{\sigma}}$ is expressed by the Hooke's law in three dimensions by $\overline{\overline{\sigma}} = \overline{\overline{\overline{\overline{\Lambda}}}} : \overline{\overline{\varepsilon}}$, where $\overline{\overline{\overline{\overline{\Lambda}}}}$ is the elastic stiffness tensor, independent of temperature and strain, and $\overline{\overline{\varepsilon}}$ is the elastic strain tensor equal to the difference between total strain and thermal dilatation. In the case of isotropic behavior, elastic stiffness is defined as function of the Young modulus E and the Poisson's ratio υ, assumed intrinsic characteristics of the material. Intrinsic should be understood as a property of material which remains constant within a moderate range of temperature and independent of the structure size. Continuum damage behavior can be exemplified by simple isotherm isotropic damage model with one scalar mechanical damage variable *D* to represent distributed micro-cracks considering the effective stress concept and hypothesis of strain and energy equivalence (Lemaître and Chaboche 1978, Mazars 1986 and 1991, Ju 1989). It's given by Eqs. (1)-(4).

The apparent or sequent stiffness $\overline{\overline{\overline{\overline{\Lambda}}}}_D = (1-D)\overline{\overline{\overline{\overline{\Lambda}}}}$ should be understood as the reduced stiffness of a representative volume element (RVE) due to damage and where the stiffness of the non-damaged material at the microscopic level in the RVE remains equal to $\overline{\overline{\overline{\overline{\Lambda}}}}$.

In order to build a damage constitutive behavior in the framework of thermodynamics of generalized standard materials, the most common choice of state variables is the elastic strain second order tensor $\overline{\overline{\varepsilon}}$, the absolute temperature *T* and the damage variable *D*.

$$\overline{\overline{\sigma}} = \overline{\overline{\overline{\overline{\Lambda}}}}_D : \overline{\overline{\varepsilon}} = (1-D)\overline{\overline{\overline{\overline{\Lambda}}}} : \overline{\overline{\varepsilon}} \tag{1}$$

$$D = \omega(Y_{max}) \tag{2}$$

$$Y_{max}(t) = \max Y(\tau) \tag{3}$$

$$Y(t) = \frac{1}{2}(\overline{\overline{\varepsilon}} : \overline{\overline{\overline{\overline{\Lambda}}}} : \overline{\overline{\varepsilon}}) \tag{4}$$

In the context of generalized standard behaviours, verifying Clausius Duhem inequality, stress tensor, entropy *S* and the associated variable to damage *Y* are respectively given by $\overline{\overline{\sigma}} = \frac{\partial \psi}{\partial \overline{\overline{\varepsilon}}}$, $S = -\frac{\partial \psi}{\partial T}$, $Y(t) = -\frac{\partial \psi}{\partial D}$ as function of the specific free energy of material defined per unit volume and given by Eq. (5).

$$\psi = \frac{1}{2}(1-D)\overline{\overline{\varepsilon}} : \overline{\overline{\overline{\overline{\Lambda}}}} : \overline{\overline{\varepsilon}} + \varphi(T) \tag{5}$$

Where φ is a function of *T*. The scalar mechanical damage variable *D* varies from 0 for reversible states to 1 for a fully damaged state depending on Y_{max} which is the maximum of *Y* reached in the previous history of the material.

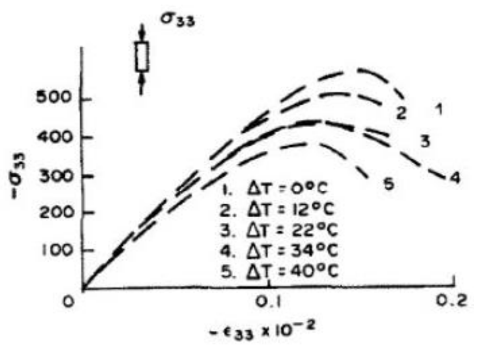


Fig. 1 Experimental strain-stress uniaxial compressive behavior of concrete under different temperatures tested by Yu *et al.* (1989) (stress in10^5Pa)

Fig. 1 presents an example of experimental isotherm compressive behaviour of ordinary concrete from tests done at different temperatures ranging from 0° to 40° and conducted by Yu *et al*. (1989). Yu *et al*. (1989) proposed continuum temperature-dependent elastic damage behaviour asymptotic to isotropic thermo-elasticity at small strains when material is undamaged and with an orthotropic damage tensor.

In the case of compression tests presented in Fig. 1, Young modulus *E* is independent of temperature and corresponds to the same initial slope of the experimental curves at different temperatures.

Furthermore, it was observed through experimental tests that there is a volume size effect of concrete specimens on their compressive strength and behavior (see for example Del Viso *et al.* 2008, Vu *et al.* 2018, Miled *et al.* 2012, Eren Gulsan *et al.* 2018).

Continuum damage theory was unable to describe such phenomena. It was extended later to non-local damage approach which gives a connection between a considered volume element size *L* and the damage parameter *D* in the constitutive behavior through the damage process (Pijaudier-Cabot and Bazant 1987).

In non-local damage approaches, the macroscopic stress-strain response of a damaged volume element subject to homogenous strain $\overline{\overline{\varepsilon}}$ applied at its boundary can be expressed by Eq. (6). A scaling law can be introduced through a *D* dependency on *L*. This can be explained, for example, by a random character of the heterogeneous material micro structure, or by a fractal dimension of the damaged surface (Ostoja-Starzewski 1998, Bazant and Jirasek 2002, Kale and Ostoja-Starzewski 2017, Rinaldi and Mastilovic 2014, Mazars *et al*. 1991).

$$\overline{\overline{\sigma}}(L) = (1-D(L))\overline{\overline{\overline{\overline{\Lambda}}}} : \overline{\overline{\varepsilon}} \tag{6}$$

In Eq. (6), *D* is not an intrinsic property of material but a property also of the considered volume element size *L*.

2. A temperature dependent scalar damage model

In the case of uncoupled damage and temperature effects, free energy defined by Eq. (5) leads to an entropy

$S=-\frac{\partial\varphi(T)}{\partial T}$ depending only on temperature.

In order to describe disorder observed through a damage process under isotherm quasi-static loading, entropy should be expressed as function of D, $\overline{\overline{\varepsilon}}$ and T. For example in previous studies, damage evolution functions linked to entropy and statistical mechanics was introduced (Ostoja-Starzewski 1998, Basaran and Nie 2004, Limam *et al*. 2014). Basaran and Nie (2004) proposed a continuous damage theory based on statistical mechanics and define damage variable D directly as function of entropy. Ostoja-Starzewski (1998) shows that the specific free energy should be written as given by Eq. (7) where in a non-local damage approach a scaling law should be associated to specific entropy depending on L. In fact, internal energy is extensive and specific internal energy should not exhibit a scaling effect. Furthermore, based on an extensive internal energy and non-additive entropy, Limam *et al.* (2014) proposed a specific free energy given by Eq. (8) in agreement with Eq. (7).

The proposed free energy is a quadratic function of D. The associated variable to damage is $Y_D=-\frac{\partial\psi}{\partial D}=\frac{1}{2}[1+T\ \ (1-D)\kappa(L)\ ]\ \overline{\overline{\varepsilon}}:\overline{\overline{\overline{\overline{\Lambda}}}}:\overline{\overline{\varepsilon}}\geq 0\cdot$

The specific internal energy U is independent of L and given by Eq. (9). The specific entropy is a function of damage and given by Eq. (10).

$$\psi(L)=U-T\ S(L) \tag{7}$$

$$\psi=\frac{1}{2}[(1-D)-T\ \ D(1-\frac{D}{2})\kappa(L)\ ]\ \overline{\overline{\varepsilon}}:\overline{\overline{\overline{\overline{\Lambda}}}}:\overline{\overline{\varepsilon}}+\varphi(T) \tag{8}$$

$$U=\frac{1}{2}(1-D)\ \ \overline{\overline{\varepsilon}}:\overline{\overline{\overline{\overline{\Lambda}}}}:\overline{\overline{\varepsilon}}+\varphi(T)-T\frac{\partial\varphi(T)}{\partial T} \tag{9}$$

$$S(L)=\ \frac{1}{2}D(1-\frac{D}{2})\kappa(L)\ \ \overline{\overline{\varepsilon}}:\overline{\overline{\overline{\overline{\Lambda}}}}:\overline{\overline{\varepsilon}}-\frac{\partial\varphi(T)}{\partial T} \tag{10}$$

For a thermodynamic process in equilibrium, it is shown that specific entropy $S(L)$ increases as function of L and tends to a constant to infinity (Tsallis 2009). For volume elements larger than a RVE of a size L_0, $\kappa(L)$ tends to a constant κ and entropy become extensive and specific entropy S tends to a constant. For $L\leq L_0$ we assume a power scaling law for specific entropy given by Eq. (11) with $\alpha>0$ a parameter considered as function of loading configuration. This assumption was inspired from works related to the fractal concepts and strength of concrete (Carpinteri *et al*. 1991). For $L\geq L_0$ we assume that entropy is extensive which means that $\kappa(L)=\kappa=\kappa(L_0)$ and $\alpha=0$.

$$\kappa(L)\ =\kappa(L_0)(\frac{L}{L_0})^{\alpha} \tag{11}$$

The next assumptions are also made.

1) In the particular case of reversible changes, the classical theory of thermo-elasticity is recovered. It means that, the Young modulus E and the Poisson's ratio of concrete v are independent of temperature and size L.
2) The evolution law of damage is considered as a fixed material characteristic independent of temperature and size L.
3) The effect of moisture content is not considered.

3. The proposed damage model applied to the compression test

In order to model the compression test, consider an isotropic damageable elastic material subject to a monotonic strain control denoted ε_c applied in a uniform temperature T. Consider the free energy given by Eq. (8). The uniaxial compressive behaviour is given by Eq. (12).

$$\sigma_c=\frac{\partial\psi}{\partial\varepsilon_c}=(1-D_c-T\kappa(L)D_c(1-\frac{D_c}{2}))E\varepsilon_c \tag{12}$$

In Eq. (12), stress is the sum of two components the first one is derived from the internal energy and given by Eq. (13).

$$\frac{\partial U}{\partial\varepsilon_c}=(1-D_c)E\varepsilon_c \tag{13}$$

The second component is an entropic stress given by Eq. (14).

$$-T\frac{\partial S}{\partial\varepsilon_c}=(-T\kappa(L)D_c(1-\frac{D_c}{2}))E\varepsilon_c \tag{14}$$

This entropic stress is opposite to the first one Eq. (13) and points to the direction of higher entropy and disorder induced by damage. It becomes important when temperature (and/or) L increase which explains firstly the volume size effect phenomenon on the compressive behavior and secondly the observed phenomenon related to strength and apparent Young modulus decreasing when temperature increases. The entropy per unit volume of the material is a sum of damage entropy denoted S_D and thermal entropy denoted S_T as follows

$$S=-\frac{\partial\psi}{\partial T}=S_D+S_T\ =\kappa(L)D_c(1-\frac{D_c}{2})\frac{1}{2}E{\varepsilon_c}^2-\frac{\partial\varphi}{\partial T} \tag{15}$$

The damage entropy is null before damage initiation in reversible states (D_c=0).

In Eq. (12), the parameter $D_c+T\kappa(L)D_c(1-\frac{D_c}{2})$ can be interpreted as the apparent damage in order to recall the classical definition of the non local scalar damage defined by Eq. (6). It is noticed also from Eq. (12) that a linear decreasing of the apparent or secant Young modulus $(1-D_c-T\kappa D_c(1-\frac{D_c}{2}))E$ emerges without a need to introduce any dependence of temperature neither on E nor on the evolution law of D_c. We consider the evolution law of damage given by Eq. (16) (Mazars *et al*. 1991, Torrenti *et al*. 2013), in which, the gradual degradation of the material under a uni-axial compression test is governed by the lateral extension due to the Poisson's effect and in agreement with the splitting failure mode (Nemat-Nasser and Horii 1982). The damage variable D is defined by the linear combination of two scalar variables noted D_t and D_c, corresponding respectively to the damage in tension and compression. In the multiaxial loading case we have $D=\alpha_t D_t+\alpha_c D_c$, where α_t and α_c are defined as a function of the signs of the principal stresses and where, in uniaxial tension, α_t=1 and α_c=0, and in uniaxial compression, α_t=0 and α_c=1. Coupling between elastic strain and damage is introduced through the evolution law of damage and the history of loading.

The damage evolution law given by Eq. (16) is governed by an equivalent strain $\tilde{\varepsilon}$ related to the local measure of extensions and defined by Eq. (17) as the maximum of all the equivalent extension elastic stains reached in the history of loading.

$$D_{c,t} = 0 \text{ if } (\tilde{\varepsilon} < \varepsilon_{D0})$$

$$D_{c,t} = 1 - \frac{\varepsilon_{D0}(1-A_{c,t})}{\tilde{\varepsilon}} - \frac{A_{c,t}}{\exp(B_{c,t}(\tilde{\varepsilon}-\varepsilon_{D0}))} \text{ if } (\tilde{\varepsilon} \geq \varepsilon_{D0}) \quad (16)$$

Where

$$\tilde{\varepsilon} = \sup(\sqrt{\sum_i \langle \varepsilon_i \rangle_+^2}) \quad (17)$$

$$\langle \varepsilon_i \rangle_+ = \varepsilon_i \quad if \quad \varepsilon_i \geq 0$$

$$\langle \varepsilon_i \rangle_+ = 0 \quad if \quad \varepsilon_i < 0$$

Where ε_i are the principal strains, ε_{D0} is the elastic equivalent extension elastic strain threshold and A_c, A_t, B_c and B_t are fixed characteristics of the material identified from tensile and compression experimental tests.

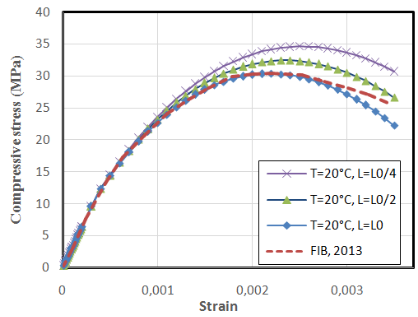
For the compressive test, the equivalent strain extension defined by Eq. (17) is given by $\tilde{\varepsilon} = -\sqrt{2}\nu\varepsilon_c$, where the principal strains are the axial contraction $\varepsilon_1=\varepsilon_c<0$ and the two lateral extensions due to the Poisson effect are $\varepsilon_2=\varepsilon_3=-\nu\varepsilon_c>0$. The compression damage evolution under the monotonic increasing of ε_c is therefore expressed by Eq. (18).

$$D_c = 0 \text{ if } (|\varepsilon_c| \leq \frac{\varepsilon_{D0}}{\sqrt{2}\nu})$$

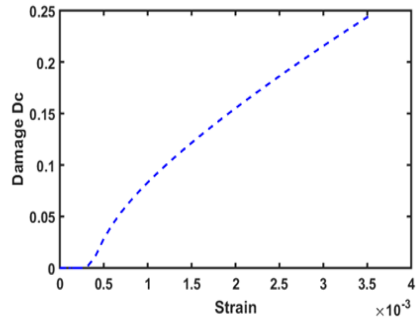
$$D_c = 1 - \frac{\varepsilon_{D0}(1-A_c)}{-\sqrt{2}\nu\varepsilon_c} - \frac{A_c}{\exp(B_c(-\sqrt{2}\nu\varepsilon_c - \varepsilon_{D0}))} \text{ if } (|\varepsilon_c| > \frac{\varepsilon_{D0}}{\sqrt{2}\nu}) \quad (18)$$

4. The T-dependant damage model applied to C30/37 concrete compression test at standard conditions

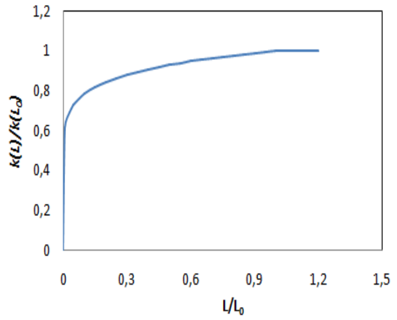
In this section the compressive behaviour of a regular concrete C30/37 at standard conditions is presented in agreement with FIB recommendations. Fig. 2(a) presents, an example of the compressive stress-strain curve given by Eqs. (12) and (18), for a C30/37 concrete at a temperature T=293K using the Young modulus E and Poisson's ratio ν and parameters of the evolution law of damage identified and defined in Table 1. As shown in Fig. 2(a), using these identified parameters, the compressive behavior of a standard specimen ($L=L_0$) obtained by the model reproduces FIB recommendations. Table 2 recapitulates some characteristics. According to the FIB recommendations (Müller *et al*. 2013), for a C30/37, the tangent modulus of elasticity at the origin of the stress-strain diagram is estimated by $E_{ci} = E_{c0}\alpha_E(\frac{f_{cm}}{10})^{1/3}$ where f_{cm} is the compressive strength, α_E is a coefficient depending on aggregates and E_{c0}=21.5 GPA. The tangent modulus of elasticity at the origin is E_{ci}=33.6 GPA while the secant modulus from the origin to 0.4 f_{cm} is E_c=29.7 GPA. The strain corresponding to the peak compressive strain is ε_{c1}=2.3‰. Furthermore, as showing in Fig. 2(a), when considering $L=L_0/2$ and $L=L_0/4$ and using the same fixed



(a) Damage model and FIB results



(b) Evolution law of damage



(c) Scaling law of entropy

Fig. 2 Uniaxial compressive behavior of a C30/37 concrete at standard conditions

Table 1 The fixed material characteristics and parameters

ν	E (MPa)	ε_{D0}	κ	A_t	B_t
0.2	32000	10^{-4}	0.0089	0.936	249

Table 2 Comparison of the *T*-dependant model and the FIB model at ambient temperature *T*=293K

Peak compressive strain	$\varepsilon_{c1}^{FIB} = 2.3°\%$	$\varepsilon_{c1}^{T-model} = 2.3°\%$
Tangent modulus of elasticity at the origin	$E_{ci}^{FIB} = 33.6\text{GPa}$	$E_{ci}^{T-model} = 32.0\text{GPa}$
Secant modulus from the origin to 0.4 f_{cm}	$E_{c}^{FIB} = 29.7\text{GPa}$	$E_{c}^{T-model} = 30.8\text{GPa}$

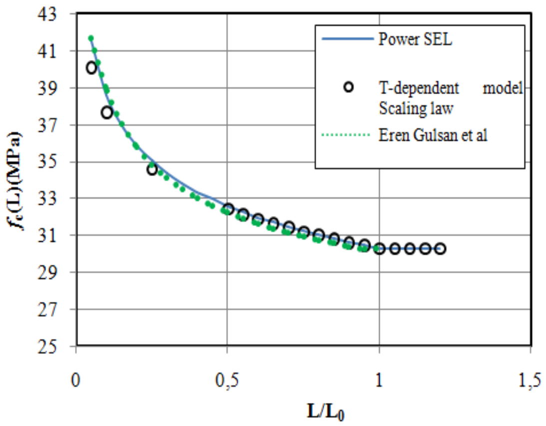


Fig. 3 Comparison of the model size effect prediction and size effect laws from literature

parameters of Table 1, a size effect on compressive strength is highlighted. The power scaling law on specific entropy given by Eq. (11) and presented in Fig. 2(c) was applied with α=0.105. The evolution law of damage is fixed and presented in Fig. 2(b).

Fig. 3 presents a comparison between the compressive strength $f_c(L)$ predicted by the damage model for specimens with different sizes *L* and a power size effect law (Miled *et al.* 2012) given by $f_c(L)=f_c(L_0)(L/L_0)^{-\alpha}$ considering α=0.105. In Fig. 3 is presented also a size effect law proposed by (Eren Gulsan *et al.* 2018) for cubic specimens in compression. This size effect law given by Eq. (19) was derived taking in to account fracture mechanics theory (Bažant and Xiang 1997).

It was applied, here, considering a standard size L_0=32 cm and a characteristic materiel length l_0=2 cm. A good agreement is observed between the different approaches.

$$f_c(L) = \frac{0.76\, f_c(L_0)}{\sqrt{1+\dfrac{L}{l_0}}} + 0.81\, f_c(L_0) \tag{19}$$

5. The compressive behavior of concrete at different temperatures

The objective of this section is to apply the T-dependant damage model to the uniaxial compression test at various temperatures using the fixed characteristics identified at *T*=293K in section 4 (Table 1).

In order to illustrate results of the *T*-dependent damage model, the same typical C30/37 concrete of section 4 is considered with the same characteristics indicated in Table 1. Fig. 4 presents the compressive stress-strain curves given by the model expressed by Eqs. (12) and (18) with $L=L_0$ at different temperatures from 293K to 343K using the fixed parameters for the evolution law of damage (Table 1).

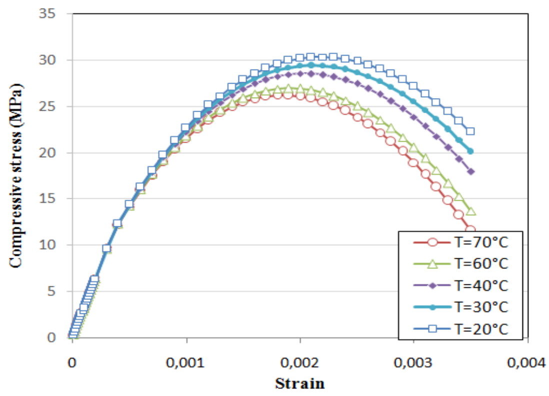


Fig. 4 Compressive behavior at different temperatures obtained by the *T*-dependant model

Table 3 Comparison of the compressive strength of a C30/37 concrete for different temperatures (the *T*-dependant model and FIB results)

T (°C)	20	30	40	60	70
f_{cm}^{FIB} (MPa)	30	29.1	28.2	26.4	25.5
$f_{cm}^{T-model}$ (MPa)	30	29.4	28.6	27	26.2

$$f_{cm}(T) = f_{cm}(1.06 - 0.003T)\ (\text{T in °C}) \tag{20}$$

The compressive behavior obtained by the model at different temperatures is in agreement with the trend emphasized by experimental results, obviously, the decreasing of the secant elastic modulus and the compressive strength when temperature increases is highlighted.

This decreasing of the secant elastic modulus and the compressive strength when temperature increases is predicted by the FIB model code. Effect of temperature in the range of 0°C≤T≤80°C on the compressive strength of a regular concrete with moisture content m=2% (f_{cm}=30 MPa) is given by Eq. (20). Table 3 summarizes results of the model and the FIB recommendations Eq. (20).

6. Effect of the difference in the coefficient of thermal expansion

The effect of the difference in the coefficient of thermal expansion (CET) between the mortar and coarse aggregates on the compressive behavior was also reported in literature (Masad *et al.* 2013). We consider in this section, at the microscopic level, concrete as a 2-phase composite material, composed of the mortar and aggregates. In order to illustrate this effect, a phenomenological simplified model is considered. The same axial strain is considered in aggregates and mortar. The elastic strain induced in the mortar phase by a thermal variation (*T*-*Tc*) is given by Eq. (21).

Table 4 Comparison of the compressive strength of a C30/37 concrete for different temperatures (the *T*-dependant model and the FIB results)

T (℃)	20	30	40	60	70
f_{cm}^{FIB} (MPa)	30	29.1	28.2	26.4	25.5
$f_{cm}^{T-\text{model}}$ (MPa)	30	29.4	28.6	27	26.2
σ_p (MPa)	0	0.17	0.35	0.67	0.86
$f_{cm}^{T-\text{model}} - \sigma_p$	30	29.2	28.4	26.3	25.3

This hypothesis gives an upper bound of stresses induced in aggregates and mortar given by Eqs. (22) and (23) with a total stress induced by temperature equal to 0, given by Eq. (24) and in agreement with a free boundary condition. Table 4 presents an actualization of Table 3 considering concrete as prestressed by compression induced in mortar by the difference in CTE. Typical values of CTE (Masad *et al*. 2013) are considered and presented in Table 5. A better agreement with FIB recommendations is highlighted as shown in Table 4. The proposed model remains valuable for a moderate range of temperatures.

However, after exposure to high temperatures, residual mechanical properties may be affected in a nonreversible way due to a higher effect of the difference in the CET and water evaporation.

$$\varepsilon_p^e = \frac{\phi E_{ag}}{\phi E_{ag} + (1-\phi)E_p}\left(\lambda_{ag} - \lambda_p\right)(T - Tc) \tag{21}$$

$$\sigma_p = \frac{\phi E_{ag} E_p}{\phi E_{ag} + (1-\phi)E_p}\left(\lambda_{ag} - \lambda_p\right)(T - Tc) \tag{22}$$

$$\sigma_{ag} = \frac{(1-\phi) E_{ag} E_p}{\phi E_{ag} + (1-\phi)E_p}\left(\lambda_p - \lambda_{ag}\right)(T - Tc) \tag{23}$$

$$\sigma^T = \phi\sigma_{ag} + (1-\phi)\sigma_p = 0 \tag{24}$$

Where,

−E_{ag}, E_p are respectively the aggregate, and mortar Young’s Modulus.

−λ_{ag}, λ_p are respectively the aggregate, and paste thermal expansion coefficients.

−ϕ is the aggregate volume fraction.

−T is the actual temperature.

−T_C is the temperature at which concrete is cured, assumed in this example equal to 20℃.

7. Conclusions

• The proposed constitutive elastic damage behavior is expressed as a sum of a stress derived from internal energy and an opposite entropic stress pointing to the direction of damage, disorder and higher entropy.

• A fixed evolution law of damage is considered and the model leads asymptotically to Hooke’s law at reversible states.

• The proposed model was applied to predict compressive behavior of concrete tested at different temperatures and predicts a secant Young modulus, a compressive strength decreasing as temperature increases and a size effect on the compressive strength induced by entropy scaling as function of specimens sizes.

• The comparison of model predictions with a size effect law derived by fracture mechanics and a volume power size effect law shows a good agreement.

• The comparison of model predictions of temperature effect with the FIB-recommendations shows a good agreement. Taking in to account the difference between CTE gives a better agreement.

Table 5 Characteristics of aggregates and mortar

	Young’s Modulus E (GPa)	CTE (C^{-1})	Volume fraction
Aggregates properties	36	8.10^{-6}	0.25
Mortar properties	31	10.10^{-6}	0.75

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