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The effect of ball size on the hollow center cracked disc (HCCD) in Brazilian test

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Abstract. Hollow center cracked disc (HCCD) in Brazilian test was modelled numerically to study the crack propagation in the pre-cracked disc. The pre-existing edge cracks in the disc models were considered to investigate the crack propagation and coalescence paths within the modelled samples. The effect of particle size on the hollow center cracked disc (HCCD) in Brazilian test were considered too. The results shows that Failure pattern is constant by increasing the ball diameter. Tensile cracks are dominant mode of failure. These crack initiates from notch tip, propagate parallel to loading axis and coalescence with upper model boundary. Number of cracks increase by decreasing the ball diameter. Also, tensile fracture toughness was decreased with increasing the particle size. In this research, it is tried to improve the understanding of the crack propagation and crack coalescence phenomena in brittle materials which is of paramount importance in the stability analyses of rock and concrete structures, such as the underground openings, rock slopes and tunnel construction.

Keywords: SENRBB test; pre-existing edge cracks; PFC3D

1. Introduction

The propagation and coalescence of internal natural cracks under different loading conditions may considerably reduce the strength and durability of concrete structures. The mechanism of failure and fracture of concretes can be affected by the mechanical behavior of pre-existing internal cracks. Concrete beams containing internal cracks can be used to study the mechanism of cracks propagation and control of cracking in concretes. The beam specimens under three and four-points bending can be used to perform the flexure tests in concrete beams. These experimental samples are specially prepared to investigate the fracture mechanics based design of engineering structures by determining the Mode I and Mode II fracture toughness and the tensile strength of the concretes. Therefore, several researches used the concrete beam tests to study the failure and fracturing process of various concretes and rock like materials under various loading conditions (Dai *et al*. 2011, Wang *et al*. 2011, Wang *et al*. 2012, Yoshihara 2013, Lancaster *et al*. 2013, Jiang *et al*. 2014, Noel and Soudki 2014). The fracture pattern and failure process of asphalt mixtures predicted by Zeng *et al*. (2014) by using the three point beam specimens and a damage model mechanism. In their study they considered the effects of cracks location and the coarse aggregates distribution on the damage mechanism and cracks propagation paths of asphalt mixtures. It has also been shown that the simulation results are in a good agreement with the corresponding experimental ones. The reinforced concrete beam {RCB) specimens used by Wang *et al* (2015) to study the shear deformation considering the low span-effective depth ratios. They predicted the diagonal crack angles by studying the multi-angle truss models. They also analyzed the effects of the diagonal crack angles on the bending moment variations along the span and showed that their experimental results are comparable to the corresponding numerical predictions of the diagonal cracks angles. Several experimental and numerical studies were accomplished on many brittle specimens to study the cracks initiation and cracks propagation of the pre-existing cracks (Belytschko *et al*. 1999, Silling 2000, Yang *et al*. 2009, Janeiro and Einstein 2010, Janeiro and Einstein 2010, Wu *et al*. 2010, Yang 2011, Cheng-Zhi and Ping 2012, Ameen *et al*. 2011, Leonel *et al*. 2012, Yoshihara 2013, Lancaster *et al*. 2013, Ramadoss 2013, Jiang *et al*. 2014, Pan *et al*. 2014; Mobasher *et al*. 2014, Zhou *et al*. 2012, Zeng *et al*. 2014, Noel and Soudki 2014, Oliveira and Leonel 2014, Haeri *et al*. 2014, Kim and Taha 2014, Sarfarazi *et al*. 2014, Zhou *et al*. 2015, Zhou *et al*. 2015, Tiang *et al*. 2015, Wan Ibrahim *et al*. 2015, Silva *et al*. 2015, Gerges *et al*. 2015, Yang 2015, Wasantha *et al*. 2015, Lee and Chang 2015, Kequan and Zhoudao 2015, Haeri 2015a, b, c, Haeri *et al*. 2015a, b, c, Li *et al*. 2015, Liu *et al*. 2015, Li *et al*. 2016, Fan *et al*. 2016, Li *et al*. 2016, Akbas 2016, Rajabi 2016, Haeri *et al*. 2016a, b, c, Haeri and Sarfarazi 2016, Sardemir 2016, Mohammad 2016, Shuraim 2016, Shaowei *et al*. 2016, Yaylac 2016, Bi *et al*. 2016, Zhou *et al*. 2016, Wang *et al*. 2016, 2017, Wang *et al*. 2017, Silling 2017, Bi *et al*. 2017). In this paper, the hollow center cracked disc (HCCD) in Brazilian test was modeled numerically to study the crack propagation in the pre-cracked disc.

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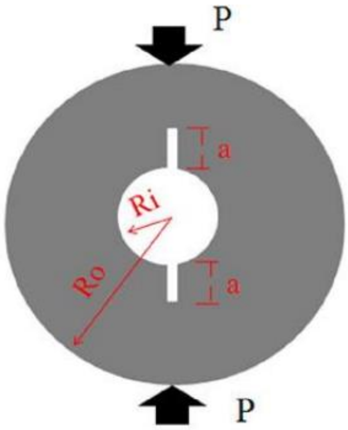


Fig. 1 HCCD geometry

Table 1 Micro properties used to represent the intact rock

Parameter	Value	Parameter	Value
Type of particle	disc	Parallel bond radius multiplier	1
Density (kg/m^3)	3000	Young modulus of parallel bond (GPa)	40
Minimum radius (mm)	0.27	Parallel bond stiffness ratio	1.7
Size ratio	1.56	Particle friction coefficient	0.4
Porosity ratio	0.08	Parallel bond normal strength, mean (MPa)	70
Damping coefficient	0.7	Parallel bond normal strength, SD (MPa)	2
Contact young modulus (GPa)	40	Parallel bond shear strength, mean (MPa)	70
Stiffness ratio	1.7	Parallel bond shear strength, SD (MPa)	2

A discrete element simulation of HCCD test

Fig. 1 illustrate the schematic view and geometrical dimensions of HCCD specimens (Shiryaev and Kotkis 1982). As shown in this figure, HCCD specimen contains a disc with radius R_O (mm) with an internal hole of radius R_i (mm) drilled at the center of the disc. Two straight central cracks of length a (mm), are symmetrically created at the surface of the internal hole. This kind of vertical notch type cracks are more suitable for measuring the Mode I critical stress intensity factors of the HCCD specimen in which the analytical formula can be written as

$$KI = \frac{P}{t(R_O - R_i)}\sqrt{\pi a}YI \quad (1)$$

$$YI\text{=}0.62 \quad (2)$$

where P is applied compressive load (KN) and t is the thickness of specimen (mm); YI is two dimensionless stress intensity factors.

2. Numerical simulation:

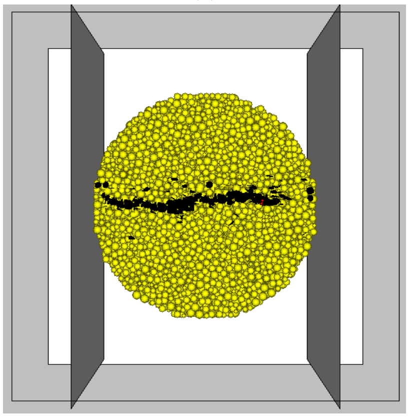
2.1 Bonded particle model and Particle Flow Code 3D (PFC3D)

A three dimensional particle flow code (PFC3D) provided by Itasca (version 3.1) used by Potyondy and Cundall 2004, to represent the material model as an assembly of rigid particles. These particles can move independently and interact one another at the distinct contact points. A central finite difference scheme is provided in PFC3D to calculate the relative movements and internal forces of the particles within the particle assembly. Both linear and non-linear contact models considering the frictional sliding along the particles can be used for modeling the contact mechanism within the assembly. The linear contact model adopted in PFC3D and used in the present study, provides a linear elastic relationship in between the relative displacements and contact forces of the particles. A parallel-bond particle model is generated by the routines provided by Itasca, 1999, version 3.1. However, the following micro properties of the particles and particle assembly should be defined in the numerical simulation of the geo-mechanical problems: the contact modulus of ball-to-ball, the stiffness ratio Kn over Ks, the frictional coefficient of the ball, the parallel normal and shear bond strengths, the minimum ball radius, the radius multiplier of parallel-bond, the modulus and the stiffness ratio of the parallel-bond. The appropriate micro properties of the particle assembly can be stablished by conducting a suitable calibration procedure as proposed by Itasca, 1999. The laboratory tests experiments can provide the macro mechanical properties based on the continuum behavior of the material samples but they cannot be used directly as the particles contact properties and their bonding characteristics. Therefore, an inverse modeling technic is used to predict the suitable micro-mechanical properties for the numerical models from the macro-mechanical properties measured in the laboratory. The mostly adopted method is the trial-and-error approach used to relate these two sets of material properties (Itasca 1999). This approach involves the assumption of initial micro mechanical values and compares the deformation characteristic the s and strength of the particles assembly with those of the measured laboratory values. The process is repeated till obtaining the micro-mechanical properties matching well with those of the macro mechanical ones. Therefore, the suitable micro mechanical properties of the particle assembly are determined for modeling the discontinuous jointed rocks and geo-materials.

2.2 Preparing and calibrating the numerical model

The Brazilian test was used to calibrate the tensile strength of specimen in PFC3D model. The standard process of generation of a PFC3D assembly to represent a test model involves four steps: (a) particle generation and packing the particles, (b) isotropic stress installation, (c) floating particle elimination, and (d) bond installation. Adopting the micro-properties listed in Table 1 and the standard calibration procedures (Potyondy and Cundall 2003), a calibrated PFC particle assembly was created. The diameter of the Brazilian disk considered in the numerical tests was 54 mm. The specimen was made of 15,615 particles. The disk was crushed by the lateral walls moved

(a)

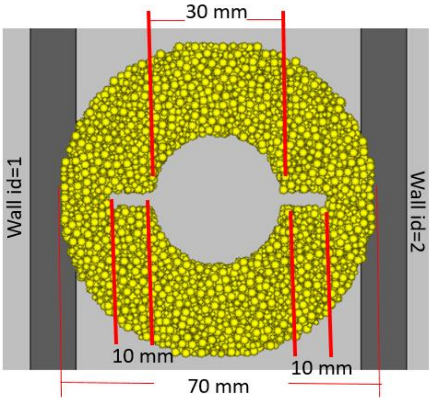


(b)

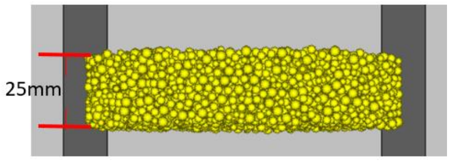
Fig. 2 failure pattern in (a) physical sample, (b) PFC2D model

Table 2 Brazilian tensile strength of physical and numerical samples

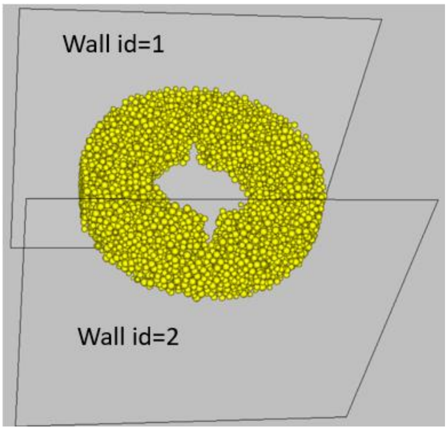
Physical tensile strength (MPa)	4.5 and 4.7
Numerical tensile strength (MPa)	4.5



(a)



(b)



(c)

Fig. 3(a) HCCD model in PFC3D

toward each other with a low speed of 0.016 m/s. Figs. 2(a), (b) illustrate the failure patterns of the numerical and experimental tested samples, respectively. Also displacement vector of particle and bond force distribution was shown in Fig. 2(b). The failure planes experienced in numerical and laboratory tests are well matching. The numerical tensile strength and a comparison of its experimental measurements were presented in Table 2. This table shows a good accordance between numerical and experimental results.

2.3 Model preparation using Particle Flow Code

After calibration of PFC3D, HCCD test were simulated by creating a disc model in PFC2D (by using the calibrated micro-parameters) (Fig. 3). The PFC specimen have external diameter of 70 mm and internal diameter of 30 mm (Fig. 3(a)). Also notch length was 10 mm (Fig. 3(a)). The model thickness was 25 mm (Fig. 3(b)). minimum diameter of balls changes in 18 different models i.e., 0.85 mm (Fig. 4(a)), 0.95 mm (Fig. 4(b)), 1.05 mm (Fig. 4(c)), 1.15 mm (Fig. 4(d)), 1.25 mm (Fig. 4(e)), 1.35 mm (Fig. 4(f)), 1.45 mm (Fig. 4(g)), 1.55 mm (Fig. 4(h)), 1.65 mm (Fig. 4(i)), 1.75 mm (Fig. 4(j)), 1.85 mm (Fig. 4(k)), 1.95 mm (Fig. 4(l)), 2.05 mm (Fig. 4(m)), 2.15 mm (Fig. 4n), 2.25 mm (Fig. 4(o)), 2.35 mm (Fig. 4(p)), 2.45 mm (Fig. 4(q)) and 2.55 mm (Fig. 4®). totally, 18 models consisting various ball number has been built, i.e., 53274 balls, 38159 balls, 28262 balls, 21512 balls, 16751 balls, 13297 balls, 10731 balls, 8785 balls, 7283 balls, 6104 balls, 5167 balls, 4412 balls, 3797 balls, 3292 balls, 2872 balls, 2521 balls, 2224 balls and 1973 balls. These models are loaded by two loading walls (Fig. 3(c)). The Tensile force was registered by taking the reaction forces on the wall *id*=1.

3. Numerical results

3.1 The effect of particle size on the failure pattern of specimens

Figs. 5(a)-(r) shows the effect of particle size on the failure pattern of models with number of balls of 53274 balls, 38159 balls, 28262 balls, 21512 balls, 16751 balls, 13297 balls, 10731 balls, 8785 balls, 7283 balls, 6104 balls, 5167 balls, 4412 balls, 3797 balls, 3292 balls, 2872 balls,

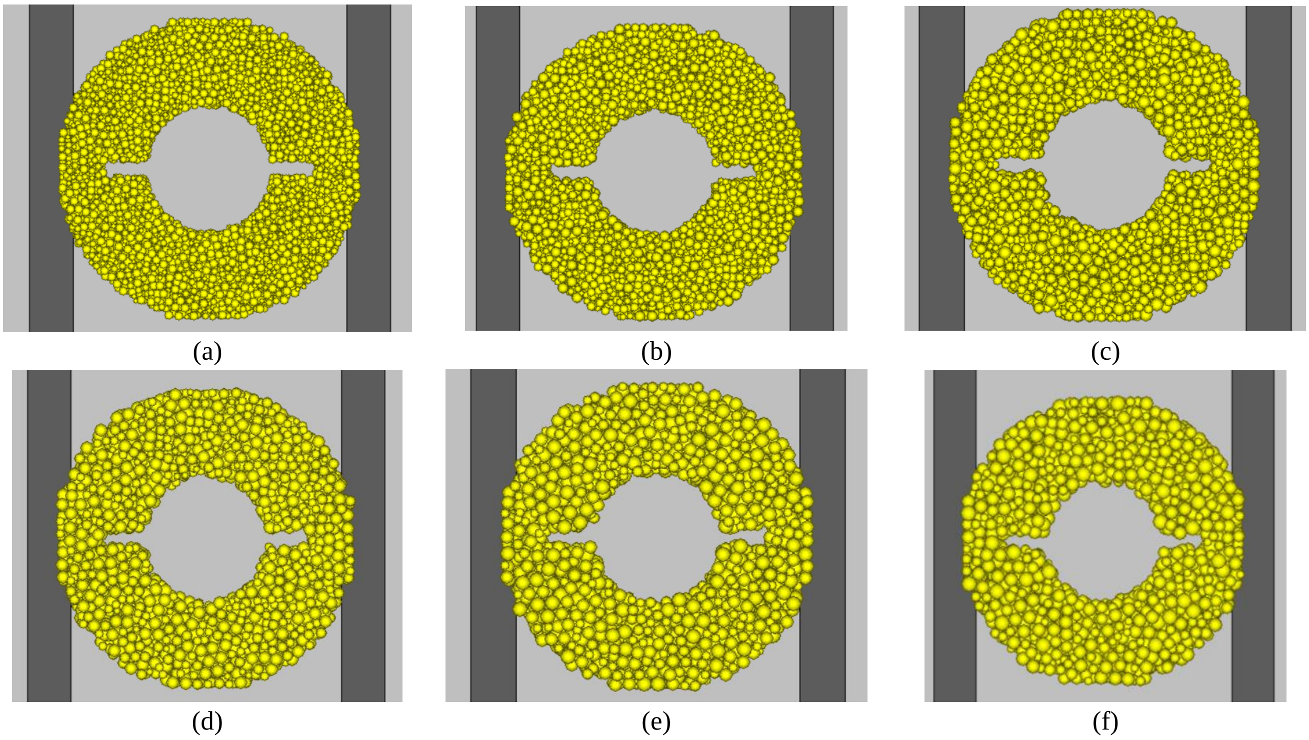


Fig. 4 The HCCD specimen with different ball radius of, (a) 0.85 mm, (b) 0.95 mm, (c) 1.05 mm, (d) 1.15 mm, (e) 1.25 mm, (f) 1.35 mm

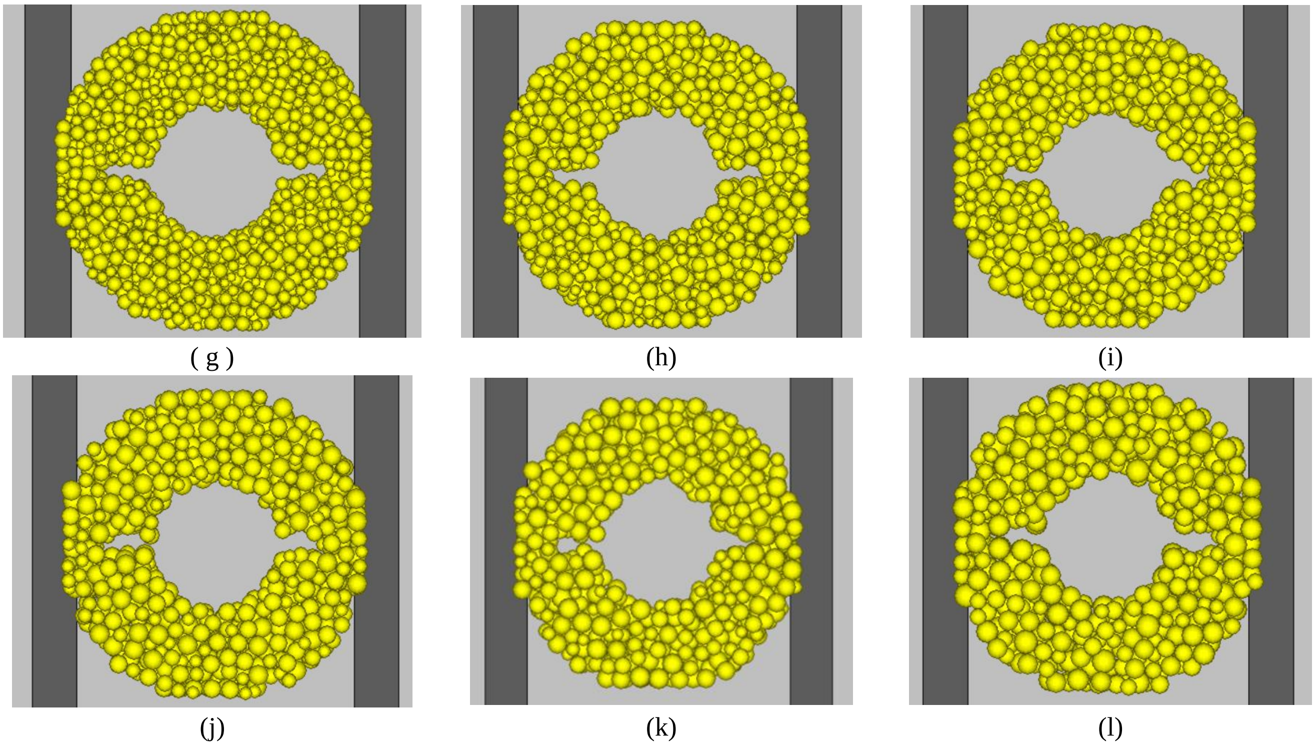
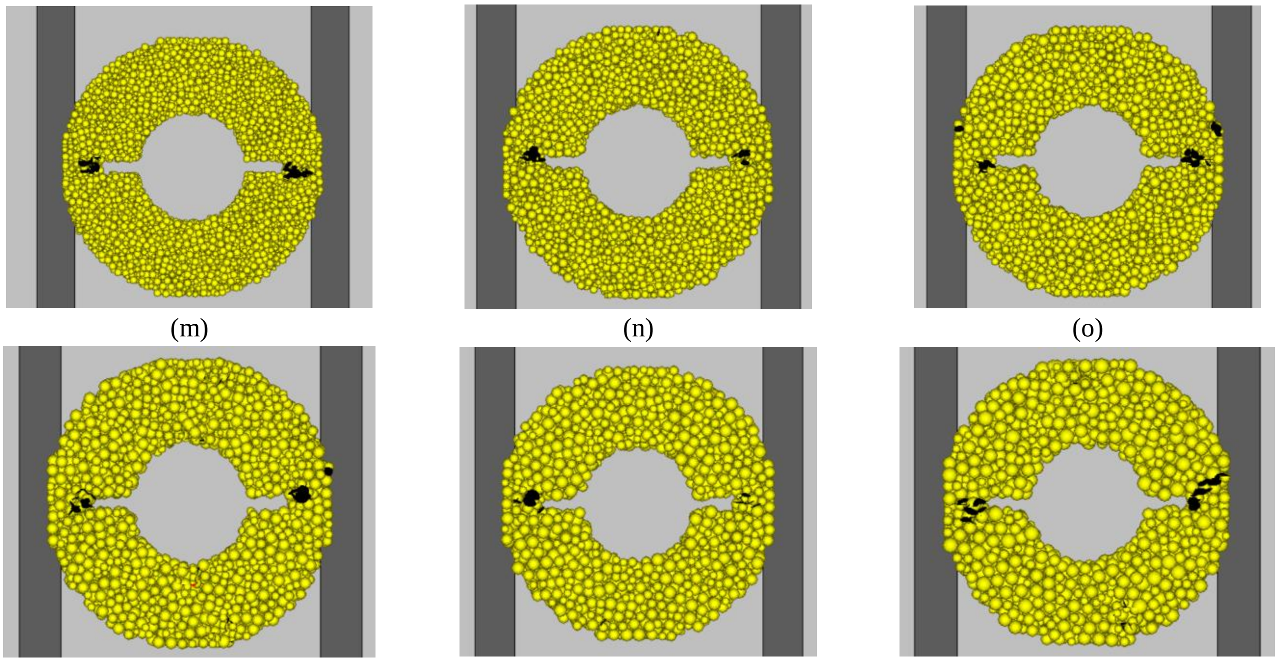


Fig. 4 The HCCD specimen with different ball radius of, (g) 1.45 mm, (h) 1.55 mm, (i) 1.65 mm, (j) 1.75 mm, (k) 1.85 mm, (l) 1.95 mm

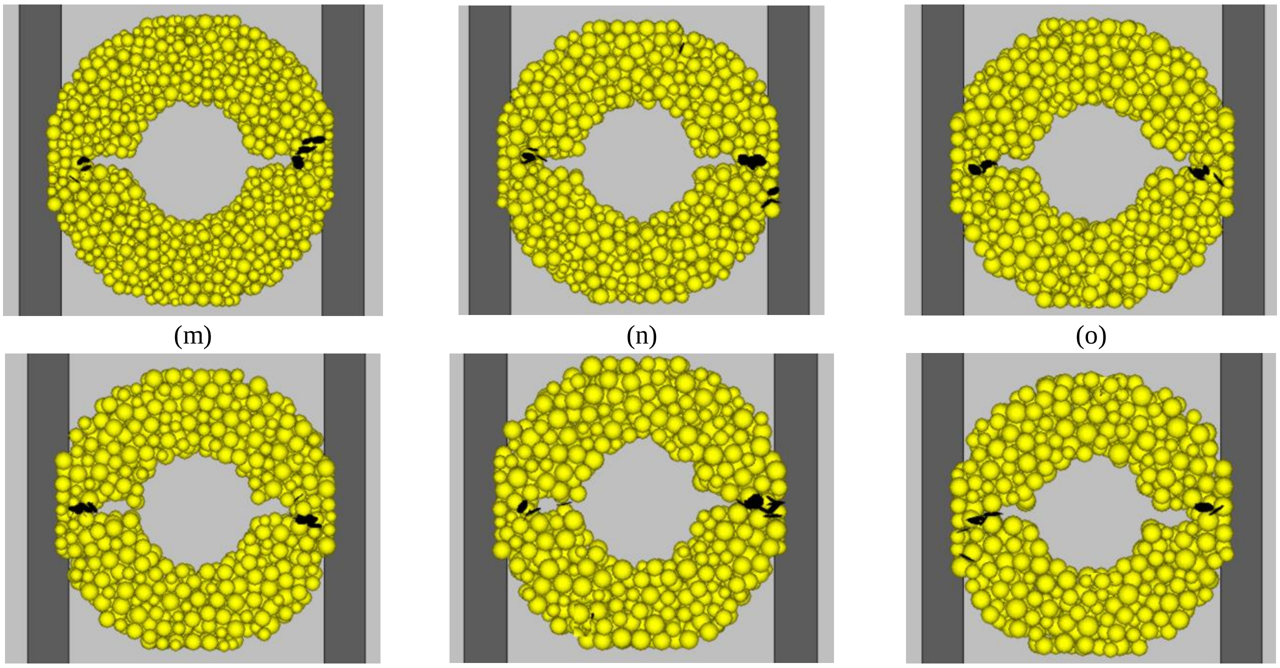
2521 balls, 2224 balls and 1973 balls, respectively. The black and red lines shown in this Figure represent the tensile and shear cracks, respectively. It is concluded that the failure process of the specimen is constant as the diameter of the ball increases. The tensile cracks are the dominant failure mode of these specimens because these cracks initiate from the notch tips during the loading process and then propagate parallel to the loading axis (direction) and finally interact with the upper boundary of the model.



(m) (n) (o)

(p) (kq) (r)

Fig. 5 Failure pattern in the HCCD specimen with different ball radius of, (a) 0.85 mm, (b) 0.95 mm, (c) 1.05 mm, (d) 1.15 mm, (e) 1.25 mm, (f) 1.35 mm



(m) (n) (o)

(p) (kq) (r)

Fig. 5 Failure pattern in the HCCD specimen with different ball radius of, (g) 1.45 mm, (h) 1.55 mm, (i) 1.65 mm, (j) 1.75 mm, (k) 1.85 mm, (l) 1.95 mm

3.2 The effect of particle size on the tensile fracture toughness

Tensile fracture toughness was measured by using equation 1 and 2. Fig 6 shows the effect of particle size on the tensile fracture toughness. The result shows that tensile fracture toughness was decreased with increasing the particle size.

4. Conclusions

In this work the effect of particle size on the failure pattern and tensile fracture toughness in HCCD specimen has been investigated using PFC3D. Firstly calibration of PFC2D was performed using Brazilian tensile strength. Secondly HCCD test models consisting different particle size was simulated numerically. The results show that:

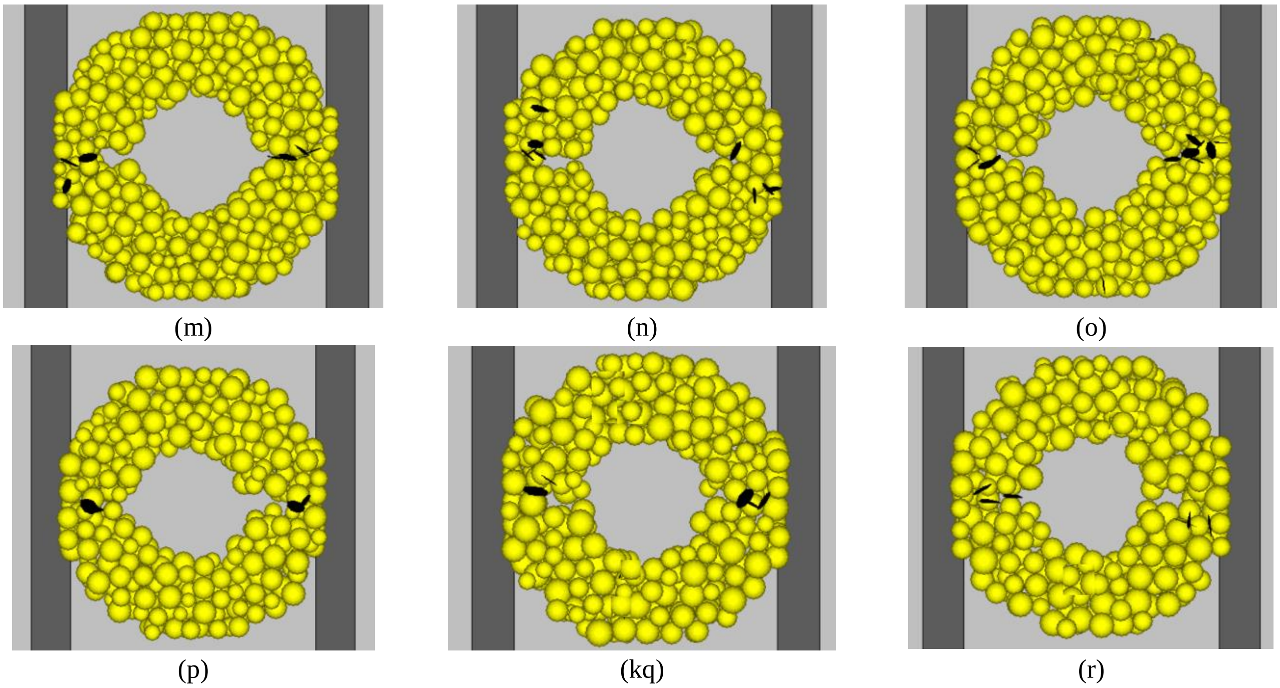


Fig. 5 Failure pattern in the HCCD specimen with different ball radius of, (m) 2.05 mm, (n) 2.15 mm, (o) 2.25 mm, (p) 2.35 mm, (q) 2.45 mm, (r) 2.55 mm

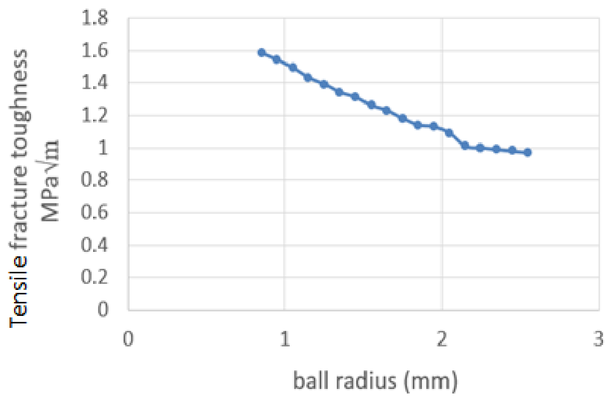


Fig. 6 the effect of particle size on the tensile fracture toughness

- The failure process and fracturing pattern is constant by increasing the diameter of the balls.
- The dominant mode of failure is that of tensile one
- The tensile cracks initiate from the notch tips
- These cracks propagate till they interact with the upper boundary of the specimen.
- As the ball diameter decreases the number of tensile cracks increases.
- As the particle size increases the tensile fracture toughness is decreased.

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