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Structural response relationship between scaled and prototype concrete load bearing systems using similarity requirements

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Abstract. This study is focused on the investigation for similitude the requirements between prototype and scaled models to determine the structural behavior of concrete load bearing systems. The scaling concept has been utilized in many engineering branches, has been assisted to engineers and scientists for obtain the behavior of the prototype by using scaled model. The scaling can be done for two purposes, either scaling up or scaling down depending upon the application. Because, scaled down models are the experimentation on scaled models is cheaper than huge structures. These models also provide facilities for experimental work. Similarity relationships between systems are created either by field equations of the system or by dimensional analysis. Within this study, similarity relationships were obtained by both methods. The similarity relations obtained are applied to different load bearing systems and it is determined that the similarity relation is a general expression. In this study, as an example, column, frame, cantilever beam and simple beam are chosen and 1/2, 1/5 and 1/10 scales are applied. The results are compared with the analytical results which are obtained by creating of the finite element models with SAP2000 software of different scaled load bearing systems. The analysis results of all systems are examined and it is determined that the scale factors are constant depending on the scale types for different load bearing systems.

Keywords: load bearing systems; dimensional analysis; scaled model; similitude requirement; structural behavior

1. Introduction

The greatness of the engineering constructions constitutes difficulties in experimental work. For this purpose, small scale models of the systems are created and experimental studies are carried out. There are many studies conducted within the scope of this subject in literature. The effects of gun voices to structures have been investigated in around the military area by Jiang and Shu (2005). For this purpose, using scaled models created in the laboratory environment with different acceleration records on the shake table and examined the changes in dynamic characteristics. Oliveira and Faria (2006) aimed to determine damage levels for arch dams in a study conducted using small scale models. Ramu *et al*. (2013) demonstrate the applicability of the similarity theory on designed models small scale. Three comparisons were made in the study, on the brink of the validity of analytical, numerical and experimental analysis. Lu *et al*. (2015) investigated the earthquake performance of a multi-story structure with steel reinforced columns and a concrete core tube. A scaled model was constructed to test the 54-story structure with 242 m height at the shaking table and the structural performance under different earthquakes was examined. In this study, a scaled model and prototype of the drilling rig located on the deck are produced to investigate the numerical estimation of the pressure distribution of large oil drilling tools and the elastic deformation under continuous loads by Shehadeh *et al*. (2015). Maximum stresses and displacements of prototype and model exposure to loads were calculated according to the parameters obtained from the Buckingham Π theorem. Scaled models studies carried out on dams, one of the important engineering structures (Zhou *et al*. 2000, Wang and Li 2006, Wang and He 2007, Wang and Li 2007, Sevim *et al*. 2011, Sevim *et al*. 2012, Sevim *et al*. 2013, Sevim *et al*. 2014)

Buckingham pi Theorem is the most common use in dimensional analysis used to establish similarity relationships between systems. There are many theoretical studies conducted using Buckingham Pi Theorem. Scale factors obtained by dimensional analysis is reflected the relation between prototype and model. In this study, Carpinteri and Corrado (2010) examined structural behaviors such as cracking or fracture of concrete and yielding or rupture of reinforced during the loading process in reinforced concrete beams subjected to bending effect by means of dimensional analysis. Ramu *et al*. (2010) aimed to develop the rules of scaling with respect to Buckingham Π Theorem and the similarity relationships. It is stated that scaled models similar to prototype can be created and use load. Balaguer and Claramonte (2011), in the work presented, investigated importance, advantages and the goals of dimensional analysis in detailed. It has correlated in between prototype and scaled models with the help of dimensional analysis that created with Buckingham Pi Theorem. In those studies, is stated that provide enable in experimental study fields of scaled models that used in many scientific area (Ghosh 2011). Datin and Prevatt

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(2013) aimed to improve the parts related to the codes in order to increase the wind resistance of the structures in the hurricane regions. To investigate the structural effects of the wind load, 1/3 scale wooden roofs were produced from lightweight wooden rods. The Buckingham theorem is used to establish the similarity between the prototype and the model roofs. Balawi *et al*. (2015) aimed at examining the scale law based on the theory of similarity in plates and beams under low velocity impact and static loading. In this study, in order to ensure compliance with prototypes, the scale factor is presented as the ratio of each change in the scaled models. Hafeez and Almaskari (2015) are stated that studies on the behavior of composite tubes under lateral loads have been continuing for many years, but studies investigating the effect of scaling using cylindrical specimens are not sufficient. Tubes that can be damaged under lateral load used Buckingham Theorem for the purpose of dimensional analysis.

Concrete load bearing system elements have been studied by taking the scaling principles into consideration. In the scope of the study, 1/2, 1/5 and 1/10 scaled models of column, frame, beam and cantilever prototypes were created. Dynamic characteristics and structural behavior such as displacements, stresses and strains between prototypes and models were investigated.

2. Scaling

2.1 Scope and usage area of the scaling

Scaling is a work that can be done in every area, by shrinking large elements or systems and by enlarge small elements or systems in a certain way to make them easy to work on. The main purpose of the scaling concept used in many areas is basically the same, and the main purpose is to simplify the work by making the hard and time-consuming systems smaller and simpler to test. The study areas of Civil Engineering are multi-story buildings, dams, airports etc. examining and testing such prototypes are a very expensive, time-consuming and difficult-to-control process. For this reason, it is very easy and convenient to do the desired work on the small models created by scaling the prototype. Due to the similarity between the prototype and the model, the results obtained in the small model will be interpreted so that the behavior of the prototype can be predicted. There are basically two types of scaling;

√ Scaling physical dimension of element or system

√ Scaling the internal behavior of the element or system

2.2 Similarity between systems

The aim of similarity analysis is to obtain design information of a large and expensive system by correlating the information obtained from experiments on a small and inexpensive model. The similarity conditions of the systems require that the relevant system parameters be the same, and these systems are controlled by the creation of special equations. Thus, equality or relativity of variables written for a system is valid for all systems. Each variable in a model is proportional with the matching variable in the prototype.

2.3 Dimensional analysis

The solution of the problems is obtained with respect to the results of the analytical and experimental studies. First of all, mathematical model is applied to obtain the solution of the problem. Experimental measurements are then made to check the analytical results. Experimental studies, which are an important step in comparing results and confirming correctness, require analysis of appropriate experimental data and careful examination. The main purpose of these experimental studies is having more knowledge with the least experiment. This purpose is being used in dimensional analysis as an important means of reaching. Dimensional analysis; is a method used to reduce the number and complexity of experimental variables affecting a physical phenomenon. If an event is based on n dimensional variables, this event can reduce to k ($k<n$) dimensionless variables with an appropriate dimensional analysis (URL-1).

Any physical state can be expressed together with size and unity. While the dimension describes the measurable physical size, the unit is important in terms of describing the intensity of this physical size. When doing dimensional analysis, character is much more important than the quantity of physical size. For this reason, only dimensions are taken into consideration in dimensional analysis. The size of each physical size is expressed by the size and symbol of the basic sizes (Table 1). Thus, all dimensions can be collected and processed in the same way.

In its broadest terms, there are two main ways to relate the model to the prototype. Similarity conditions are derived from the relevant field equations if the system has a mathematical model or by dimensional analysis if the mathematical model of the system is not valid. In dimensional analysis, all parameters and variables that affect the behavior of the system have to be known. The obtained equation is the dimensionless product of system parameters and variables. Thus, similarity conditions can be created on the basis of the generated equation.

2.4 Buckingham Π theorem

Many methods are used to form equations by dimensional analysis. One of the most widely used method is the Buckingham Π Theorem. If the number of physical quantities (velocity, density, etc.) in a system is n and the number of basic dimensions (M, L, T) constituting these physical quantities is r, there is $k=n-r$ dimensionless number (Π group) that defining the system. In other words, the physical quantities that make up a system (Ramu 2010)

$$f = \left(A_1, A_2, A_3, \ldots, A_i, \ldots, A_n\right) = 0 \tag{1}$$

When represented by the Eq. (1), indicating the relationship between variables Π groups

$$F = \left(\Pi_1, \Pi_2, \Pi_3, \ldots, \Pi_i, \ldots, \Pi_{n-r}\right) = 0 \tag{2}$$

can be represented by the Eq. (2). Each Π group is determined to be a function of r repeating variables that

Table 1 Identification of according to M, L, T and F, L, T of the units of selected some sizes

	FLT System	MLT System
Gravity	LT^{-2}	LT^{-2}
Angle	$F^0L^0T^0$	$M^0L^0T^0$
Area	L^2	L^2
Density	$FL^{-4}T^2$	ML^{-3}
Force	F	MLT^{-2}
Frequency	T^{-1}	T^{-1}
Length	L	L
Mass	$FL^{-1}T^2$	M
Modulus of Elasticity	FL^{-2}	$ML^{-1}T^{-2}$
Moment of force	FL	ML^2T^{-2}
Moment of inertia	L^4	L^4
Pressure	FL^{-2}	$ML^{-1}T^{-2}$
Specify Weight	FL^{-3}	$ML^{-2}T^{-2}$
Strain	$F^0L^0T^0$	$M^0L^0T^0$
Stress	FL^{-2}	$ML^{-1}T^{-2}$
Surface Stress	FL^{-1}	MT^{-2}
Time	T	T
Velocity	LT^{-1}	LT^{-1}
Volume	L^3	L^3

directly affect the physical problem and at least one different variable. In accordance with the Buckingham Π theorem, the following considerations must be taken into consideration when choosing r variables repeating in all Π groups;

- Repeating variables must be dimensional
- The size of any of the repeating variables must not be a combination of the size of the other repeating variables. For example; if $r_1=L$, $r_2=T^1$, cannot be $r_3=LT^{-1}$
- Each of the base dimensions (M, L, T) must be present in at least one of the repeating variables. For example; cannot be $r_1=L^3T^{-1}$, $r_2=LT^{-1}$ and $r_3=L$. Because there is no M in the repeating parameters

According to Buckingham Π theorem;

√ Base quantity: Quantities in law of motion (length, time, force)

√ Basic dimensions: Dimensions of base quantity

In Buckingham's theorem, the size of the quantities is obtained either by the "F, L, T" dimensional system (force, length, time) or the "M, L, T" dimension system (mass, length, time). Table 1 show that some of the sizes can be compared with these systems. Among these basic dimensions, the use of "M, L, T" is more common and r=3 is used.

There are some situations that need to be taken into account choosing the parameters when forming the dimensionless Π groups. Selection unnecessary parameters that are not important for the physical problem can lead to the formation of an extra Π group. On the other hand, is not selecting parameters that are important for the physical problem can lead to deficiency an important Π group will be overlooked and the interpretation of the results may be lacking.

3. Obtaining similarity formulas of structural behaviour by dimensional analysis and mathematical formulas

3.1 Similarity formulas with dimensional analysis

Dimensional analysis is carried out taking into consideration the points described related to Buckingham Π Theorem. The purpose here is to show the relationship between the prototype and the model. In order to compare the results of the dimensional analysis, it is first to formulate the frequency change in the columns with the cross-sectional dimensions and height a, b, c, respectively, by the Buckingham Π theorem. In the calculation of the frequency value, Eq. (3) and different forms of this equation are used.

$$f_n = \frac{\omega_n}{2\pi}, \quad \omega_n = \sqrt{\frac{k}{m}}, \quad k = \frac{XEI}{h^3} \tag{3}$$

Parameters related to these equations; is state that M mass, k rigidity, π dimensionless parameter, E modulus of elasticity, I moment of inertia and X constant value. The steps of process applied according to the Buckingham Π theorem are presented below.

Step 1: Determining the number of parameters (n)

The parameters determined according to the frequency formula and the units of these parameters in terms of M, L, T are given in Table 2. There are total 8 parameters (n=8).

Step 2: Identification of the limiter number (r)

The rules expressed in the selection of r, which is the number of repeating parameters, are taken into account and is chosen as r=(E, h, m).

Step 3: Determining the number of non-repeating parameters (k) and Π-group number

There are total of 5 Π-groups for k=n−r. This expression, when represented by the Eq. (2)

$$f = (\pi_1, \pi_2, \pi_3, \pi_4, \pi_5) = 0 \tag{4}$$

Eq. (4) is obtained. In this way, each non-repeating parameter π_1=f, π_2=a, π_3=b. π_4=I and π_5=k, is determined.

Step 4: Calculation of non-repeating variables depending on repeating variables.

Each non-repeating parameter can be obtained as follows

$$\left.\begin{aligned} \pi_1 &= f.\left(E^a.h^b.m^c\right) \\ \pi_2 &= a.\left(E^a.h^b.m^c\right) \\ \pi_3 &= b.\left(E^a.h^b.m^c\right) \\ \pi_4 &= I.\left(E^a.h^b.m^c\right) \\ \pi_5 &= k.\left(E^a.h^b.m^c\right) \end{aligned}\right\} \tag{5}$$

In order to form groups (Eq. (5)) in this way to be dimensionless, the basic condition of $M^0L^0T^0$ must also be

Table 2 Identification of according to *M*, *L*, *T* of the units of selected parameters

a	*b*	*h*	*m*	*E*	*I*	*k*	*f*
L	L	L	M	$ML^{-1}T^{-2}$	L^4	MT^{-2}	T^{-1}

provided.
for π_1

$$\left.\begin{aligned} \pi_1 &= a.\left(E^a.h^b.m^c\right) \\ M^0L^0T^0 &= L.\left(ML^{-1}T^{-2}\right)^a.L^b.M^c \end{aligned}\right\} \quad (6)$$

expressions are obtained (Eq. (6)). The coefficients of *a*, *b*, *c* are obtained when the equation is solved considering the fact that the upper force of the same basic dimension is the same on both sides of the equation. When each parameter is calculated in all Π-groups, the final part is obtained as follows.

$$\left.\begin{aligned} \pi_{1(model)} &= \pi_{1(prototype)} \\ \pi_{2(model)} &= \pi_{2(prototype)} \\ \pi_{3(model)} &= \pi_{3(prototype)} \\ \pi_{4(model)} &= \pi_{4(prototype)} \\ \pi_{5(model)} &= \pi_{5(prototype)} \end{aligned}\right\} \quad (7)$$

With Eq. (7), the relation between prototype and model is established and solutions of Π-groups are made respectively.
for $\pi_1 = f.\ (E,h,m)$

$$\left.\begin{aligned} \pi_1 &= f.\left(E^a.h^b.m^c\right) \\ M^0L^0T^0 &= T^{-1}.\left(ML^{-1}T^{-2}\right)^a.L^b.M^c \\ M^0L^0T^0 &= M^{a+c}.L^{-a+b}.T^{-2a-1} \end{aligned}\right\} \quad (8)$$

$a=-\frac{1}{2}$, $c=\frac{1}{2}$ and $b=-\frac{1}{2}$ are obtained with Eq. (8). When the relevant expressions are written to

$$\left.\begin{aligned} \pi_1 &= f.\left(E^{-\frac{1}{2}}.h^{-\frac{1}{2}}.m^{-\frac{1}{2}}\right)? \\ \pi_1 &= \frac{f\sqrt{m}}{\sqrt{E}\sqrt{h}} \end{aligned}\right\} \quad (9)$$

expressions are obtained (Eq. (9)).
$\pi_{1(model)} = \pi_{1(prototype)}$

When considering that m and p represent the model and the prototype, respectively, it's the expression of the equation

$$\left.\begin{aligned} \frac{f_m\sqrt{m_m}}{\sqrt{E_m}\sqrt{h_m}} &= \frac{f_p\sqrt{m_p}}{\sqrt{E_p}\sqrt{h_p}} \\ \frac{f_m}{f_p} &= \frac{\sqrt{m_p}}{\sqrt{m_m}}\frac{\sqrt{E_m}}{\sqrt{E_p}}\frac{\sqrt{h_m}}{\sqrt{h_p}} \end{aligned}\right\} \quad (10)$$

it occurs. Eq. (10) is shown as the scale factor of the frequency. Also

$$f_m = \frac{\sqrt{m_p}}{\sqrt{m_m}}\frac{\sqrt{E_m}}{\sqrt{E_p}}\frac{\sqrt{h_m}}{\sqrt{h_p}} f_p \quad (11)$$

Table 3 Scale factors

$\frac{f_m}{f_p}$	$\frac{\sqrt{m_p}}{\sqrt{m_m}}\frac{\sqrt{E_m}}{\sqrt{E_p}}\frac{\sqrt{h_m}}{\sqrt{h_p}}$
$\frac{a_m}{a_p}$	$\frac{1}{S}$
$\frac{b_m}{b_p}$	$\frac{1}{S}$
$\frac{I_m}{I_p}$	$\frac{1}{S^4}$
$\frac{k_m}{k_p}$	$\frac{1}{ES}$

The frequency of the model with the equation is obtained depend on the prototype frequency and the parameters (Eq. (11)). The same procedures for π_2, π_3, π_4 and π_5 are repeated and each of the scale factors presented in Table 3.

In Eq. (11)

$$\left.\begin{aligned} \frac{h_m}{h_p} &= \frac{1}{S} \quad ;\text{Geometry scale} \\ \frac{E_m}{E_p} &= \frac{1}{E} \quad ;\text{Material scale} \\ \frac{m_m}{m_p} &= \frac{1}{M} ;\text{Mass scale} \end{aligned}\right\} \quad (12)$$

If expressions are given in Eq. (12) are taken into consideration and written in their places

$$f_m = \sqrt{M\frac{1}{E}\frac{1}{S}} f_p \quad (13)$$

General relation is obtained.

For example;

• If the mass is assumed to be constant and only the geometry is scaled

$$\left.\begin{aligned} \frac{h_m}{h_p} &= \frac{1}{S} \\ \frac{E_m}{E_p} &= \frac{1}{E} = 1 \\ \frac{m_m}{m_p} &= \frac{1}{M} = 1 \end{aligned}\right\} \quad (14)$$

expressions are obtained, and when these expressions Eq. (14) are written in the Eq. (13)

$$f_m = \sqrt{\frac{1}{S}} f_p \quad (15)$$

connection is obtained (Eq. (15)).

• If only geometry scaling is done by considering mass change

$$\left.\begin{aligned} \frac{h_m}{h_p} &= \frac{1}{S} \\ \frac{E_m}{E_p} &= \frac{1}{E} = 1 \\ \frac{m_m}{m_p} &= \frac{1}{M} = \frac{1}{S^3} \end{aligned}\right\} \tag{16}$$

expressions are obtained, and when these expressions Eq. (16) are written in the Eq. (13)

$$\left.\begin{aligned} f_m &= \sqrt{S^3 \frac{1}{S}}\, f_p \\ f_m &= S.f_p \end{aligned}\right\} \tag{17}$$

connection is obtained (Eq. (17)).

3.2 Similarity formulas with mathematical formulas

According to different scaling states, the change of the general formula is obtained by Buckingham Π theorem. With the aid of the mathematical formula of frequency, dimensional analysis is carried out with different combinations according to geometric, mass and material scales and the results obtained are examined comparatively.

• The solution of the column prototype with the help of the mathematical formula is summarized as follows;

Column dimensions $= a \times b \times l$ m

Other data in the system $= \gamma, E_p, g$

System rigidity $= k = \dfrac{12E_p I}{L^3}$

The corresponding solution can be made as follows and

$$\left.\begin{aligned} I &= \frac{ab^3}{12} \\ k &= \frac{12E_p \frac{ab^3}{12}}{L^3} = \frac{12E_p ab^3}{12L^3} = \frac{E_p ab^3}{L^3} \\ m &= \frac{abL\gamma}{g} \\ \omega_n &= \sqrt{\frac{k}{m}} = \sqrt{\frac{E_p ab^3}{L^3}\frac{g}{abL\gamma}} = \sqrt{\frac{E_p b^2 g}{L^4\gamma}} \\ f_{n(prototype)} &= \frac{\omega_n}{2\pi} = \sqrt{\frac{E_p b^2 g}{L^4\gamma}}\,\frac{1}{2\pi} \end{aligned}\right\} \tag{18}$$

Eq. (18) is obtained. For the different scaled model situations of the column, the following steps are performed with the help of mathematical model.

• In the case of only 1/S geometry scaling by considering the mass change;

Column dimensions $= \dfrac{a}{S} \times \dfrac{b}{S} \times \dfrac{l}{S}$ m

Other data in the system $= \gamma, E_p, g$

as considering data, the corresponding solution can be made as follows and

$$\left.\begin{aligned} I &= \frac{\frac{a}{S}\left(\frac{b}{S}\right)^3}{12} = \frac{ab^3}{S^4 12} \\ k &= \frac{12E_p \frac{ab^3}{S^4 12}}{\left(\frac{L}{S}\right)^3} = \frac{12E_p ab^3 S^3}{12 S^4 L^3} = \frac{E_p ab^3}{L^3}\frac{1}{S} \\ m &= \frac{\frac{abL}{SSS}\gamma}{g} = \frac{abL\gamma}{S^3 g} = \frac{abL\gamma}{g}\frac{1}{S^3} \\ \omega_n &= \sqrt{\frac{k}{m}} = \sqrt{\frac{E_p ab^3}{L^3}\frac{1}{S}\frac{g}{abL\gamma}\frac{S^3}{1}} = \sqrt{\frac{E_p b^2 g}{L^4\gamma}}\, S \\ f_{n(Model)} &= \frac{\omega_n}{2\pi} = \sqrt{\frac{E_p b^2 g}{L^4\gamma}}\,\frac{1}{2\pi} S \end{aligned}\right\} \tag{19}$$

Eq. (19) is obtained. By establishing a relationship between the prototype and the scaled model of the column

$$\left.\begin{aligned} \frac{f_{n(Model)}}{f_{n(prototype)}} &= \frac{S}{1} \\ f_{n(m)} &= S\, f_{n(p)} \end{aligned}\right\} \tag{20}$$

equality is reached (Eq. (20)).

• If mass is assumed to be constant and only 1/S geometry scaling is considered;

Column dimensions $= \dfrac{a}{S} \times \dfrac{b}{S} \times \dfrac{l}{S}$ m

Other data in the system $= \gamma, E_p, g$

as considering data, the corresponding solution can be made as follows and

$$\left.\begin{aligned} I &= \frac{\frac{a}{S}\left(\frac{b}{S}\right)^3}{12} = \frac{ab^3}{S^4 12} \\ k &= \frac{12E_p \frac{ab^3}{S^4 12}}{\left(\frac{L}{S}\right)^3} = \frac{12E_p ab^3 S^3}{12 S^4 L^3} = \frac{E_p ab^3}{L^3}\frac{1}{S} \\ m &= \frac{abL\gamma}{g} \\ \omega_n &= \sqrt{\frac{k}{m}} = \sqrt{\frac{E_p ab^3}{L^3}\frac{1}{S}\frac{g}{abL\gamma}} = \sqrt{\frac{E_p b^2 g}{L^4\gamma}}\,\frac{1}{\sqrt{S}} \\ f_{n(Model)} &= \frac{\omega_n}{2\pi} = \sqrt{\frac{E_p b^2 g}{L^4\gamma}}\,\frac{1}{2\pi}\frac{1}{\sqrt{S}} \end{aligned}\right\} \tag{21}$$

Eq. (21) is obtained. By establishing a relationship between the prototype and the scaled model of the column

$$\left.\begin{aligned} \frac{f_{n(Model)}}{f_{n(prototype)}} &= \frac{\frac{1}{\sqrt{S}}}{1} \\ f_{n(m)} &= \frac{1}{\sqrt{S}} f_{n(p)} \end{aligned}\right\} \tag{22}$$

Equality is reached (Eq. (22)).

• If only material scaling is considered;

Column dimensions $= a \times b \times l \text{ m}$

Other data in the system $= \gamma, E_m, g$

as considering data, the corresponding solution can be made as follows and

$$\left.\begin{aligned}
I &= \frac{ab^3}{12} \\
k &= \frac{12E_m \frac{ab^3}{12}}{L^3} = \frac{12E_m ab^3}{12L^3} = \frac{E_m ab^3}{L^3} \\
m &= \frac{abL\gamma}{g} \\
\omega_n &= \sqrt{\frac{k}{m}} = \sqrt{\frac{E_m ab^3}{L^3}\frac{g}{abL\gamma}} = \sqrt{\frac{E_m b^2 g}{L^4\gamma}} \\
f_{n(\text{Model})} &= \frac{\omega_n}{2\pi} = \sqrt{\frac{E_m b^2 g}{L^4\gamma}}\frac{1}{2\pi}
\end{aligned}\right\} \tag{23}$$

Eq. (23) is obtained. By establishing a relationship between the prototype and the scaled model of the column

$$\left.\begin{aligned}
\frac{f_{n(\text{Model})}}{f_{n(\text{prototype})}} &= \frac{\sqrt{E_m}}{\sqrt{E_p}} \\
f_{n(m)} &= \sqrt{\frac{E_m}{E_p}} f_{n(p)}
\end{aligned}\right\} \tag{24}$$

Equality is reached (Eq. (24)).

• If mass is assumed to be constant and 1/S geometry and material scaling are taken into account;

Column dimensions $= \frac{a}{S} \times \frac{b}{S} \times \frac{l}{S} \text{ m}$

Other data in the system $= \gamma, E_m, g$

as considering data, the corresponding solution can be made as follows and

$$\left.\begin{aligned}
I &= \frac{\frac{a}{S}\left(\frac{b}{S}\right)^3}{12} = \frac{ab^3}{S^4 12} \\
k &= \frac{12E_m \frac{ab^3}{S^4 12}}{\left(\frac{L}{S}\right)^3} = \frac{12E_m ab^3 S^3}{12S^4 L^3} = \frac{E_m ab^3}{L^3}\frac{1}{S} \\
m &= \frac{abL\gamma}{g} \\
\omega_n &= \sqrt{\frac{k}{m}} = \sqrt{\frac{E_m ab^3}{L^3}\frac{1}{S}\frac{g}{abL\gamma}} = \sqrt{\frac{E_m b^2 g}{L^4\gamma}}\frac{1}{\sqrt{S}} \\
f_{n(\text{Model})} &= \frac{\omega_n}{2\pi} = \sqrt{\frac{E_m b^2 g}{L^4\gamma}}\frac{1}{2\pi}\frac{1}{\sqrt{S}}
\end{aligned}\right. \tag{25}$$

Eq. (25) is obtained. By establishing a relationship between the prototype and the scaled model of the column

$$\left.\begin{aligned}
\frac{f_{n(\text{Model})}}{f_{n(\text{prototype})}} &= \frac{\sqrt{E_m}\frac{1}{\sqrt{S}}}{\sqrt{E_p}} \\
f_{n(m)} &= \frac{1}{\sqrt{S}}\sqrt{\frac{E_m}{E_p}} f_{n(p)}
\end{aligned}\right\} \tag{26}$$

Equality is reached (Eq. (26)).

• By considering the mass change, in the case of 1/S geometry and material scaling;

Column dimensions $= \frac{a}{S} \times \frac{b}{S} \times \frac{l}{S} \text{ m}$

Other data in the system $= \gamma, E_m, g$

as considering data, the corresponding solution can be made as follows and

$$\begin{aligned}
I &= \frac{\frac{a}{S}\left(\frac{b}{S}\right)^3}{12} = \frac{ab^3}{S^4 12} \\
k &= \frac{12E_m \frac{ab^3}{S^4 12}}{\left(\frac{L}{S}\right)^3} = \frac{12E_m ab^3 S^3}{12S^4 L^3} = \frac{E_m ab^3}{L^3}\frac{1}{S} \\
m &= \frac{\frac{abL}{SSS}\gamma}{g} = \frac{abL\gamma}{S^3 g} = \frac{abL\gamma}{g}\frac{1}{S^3} \\
\omega_n &= \sqrt{\frac{k}{m}} = \sqrt{\frac{E_m ab^3}{L^3}\frac{1}{S}\frac{g}{abL\gamma}\frac{S^3}{1}} = \sqrt{\frac{E_m b^2 g}{L^4\gamma}}S \\
f_{n(\text{Model})} &= \frac{\omega_n}{2\pi} = \sqrt{\frac{E_m b^2 g}{L^4\gamma}}\frac{1}{2\pi}S
\end{aligned} \tag{27}$$

Eq. (27) is obtained. By establishing a relationship between the prototype and the scaled model of the column

$$\frac{f_{n(\text{Model})}}{f_{n(\text{prototype})}} = \frac{\sqrt{E_m}S}{\sqrt{E_p}} \quad \Rightarrow \quad f_{n(m)} = S\sqrt{\frac{E_m}{E_p}} f_{n(p)} \tag{28}$$

Equality is reached (Eq. (28)). The frequency formulas obtained according to different scale types using the equations and expressions tried to be explained in detail above are summarized in Table 4.

Table 4 Frequency formulas according to scale types

Scale Types	Formulas
Geometry	$f_{n(m)} = \frac{1}{\sqrt{S}} f_{n(p)}$
Geometry (constant mass)	$f_{n(m)} = S f_{n(p)}$
Material	$f_{n(m)} = \sqrt{\frac{E_m}{E_p}} f_{n(p)}$
Geometry and material (constant mass)	$f_{n(m)} = \frac{1}{\sqrt{S}}\sqrt{\frac{E_m}{E_p}} f_{n(p)}$
Geometry and material	$f_{n(m)} = S\sqrt{\frac{E_m}{E_p}} f_{n(p)}$

When made work and the obtained equations are evaluated, it is seen that the frequency relations obtained from Buckingham Π theorem and field equations are the same. In the most general way, when is written prototype and model data in the Eq. (10), the frequency value of the scaled model is obtained. It is also possible to convert the data into different formats. When conversion is done for mass

$$\left.\begin{aligned} m &= V.\gamma \\ m_p &= V_p.\gamma_p \\ m_m &= V_m.\gamma_m \end{aligned}\right\} \tag{29}$$

If the expressions (Eq. (29)) are written in Eq. (10)

$$f_m = \frac{\sqrt{V_p\,\gamma_p}}{\sqrt{V_m.\gamma_m}} \frac{\sqrt{E_m}}{\sqrt{E_p}} \frac{\sqrt{h_m}}{\sqrt{h_p}} f_p \tag{30}$$

Eq. (30) is obtained. When converting for volume

$$\left.\begin{aligned} V &= a.b.h \ (br^3) \\ V_p &= a_p.b_p.h_p \\ V_m &= a_m.b_m.h_m \end{aligned}\right\} \tag{31}$$

If the expressions (Eq. (31)) are written in Eq. (10)

$$\left.\begin{aligned} f_m &= \frac{\sqrt{a_p.b_p.h_p.\gamma_p}}{\sqrt{a_m.b_m.h_m.\gamma_m}} \frac{\sqrt{E_m}}{\sqrt{E_p}} \frac{\sqrt{h_m}}{\sqrt{h_p}} f_p \\ f_m &= \frac{\sqrt{a_p.b_p.h_p}}{\sqrt{a_m.b_m.h_m}} \frac{\sqrt{\gamma_p}}{\sqrt{\gamma_m}} \frac{\sqrt{E_m}}{\sqrt{E_p}} \frac{\sqrt{h_m}}{\sqrt{h_p}} f_p \end{aligned}\right\} \tag{32}$$

Volume scale factor $\frac{h_m}{h_p} = \frac{1}{S}$ is written in formula obtained with Eq. (32)

$$\left.\begin{aligned} f_m &= \frac{\sqrt{S^3}}{1} \frac{\sqrt{\gamma_p}}{\sqrt{\gamma_m}} \frac{\sqrt{E_m}}{\sqrt{E_p}} \frac{1}{\sqrt{S}} f_p \\ f_m &= S \frac{\sqrt{\gamma_p}}{\sqrt{\gamma_m}} \frac{\sqrt{E_m}}{\sqrt{E_p}} f_p \end{aligned}\right\} \tag{33}$$

Eq. (33) is obtained. Conclusions can be obtained with different density values *γ* in formula obtained.

The weight of the build varies with the scale. In the scale study, the constant mass is formed by adding to the structure as extra load of the difference the mass formed by volume reduction. Different equations can be obtained by keeping the desired parameter scaled or fixed.

The equations obtained by using the column are generalized by simplifying the system specific coefficients and obtaining the basic parameters, and these equations can be used in every system.

In order to obtain the displacement relations, similarity conditions were formed according to different scale types using mathematical models. Using the general displacement equation, the relationship between the prototype and the scaled model for the column

$$\begin{aligned} F &= K.U \\ F &= V.\gamma \\ k &= \frac{12EI}{L^3} \end{aligned} \tag{34}$$

using the expressions (Eq. (34)), for prototype

$$V_p.\gamma_p = \frac{12E_pI_p}{{L_p}^3} U_p \tag{35}$$

Eq. (35) is obtained. For model

$$V_m.\gamma_m = \frac{12E_mI_m}{{L_m}^3} U_m \tag{36}$$

Eq. (36) is obtained. When $\frac{U_p}{U_m}$ similarity condition is applied

$$\frac{U_p}{U_m} = \frac{E_mI_m{L_p}^3V_p\gamma_p}{E_pI_p{L_m}^3V_m\gamma_m} \tag{37}$$

Eq. (37) is obtained. When the scale factor for the moment of inertia is taken as S^4

$$\left.\begin{aligned} \frac{U_p}{U_m} &= \frac{E_m}{E_p}\frac{1}{S^4}S^3\frac{V_p}{V_m}\frac{\gamma_p}{\gamma_m} \\ \frac{U_p}{U_m} &= \frac{E_m}{E_p}\frac{1}{S}\frac{V_p}{V_m}\frac{\gamma_p}{\gamma_m} \end{aligned}\right\} \tag{38}$$

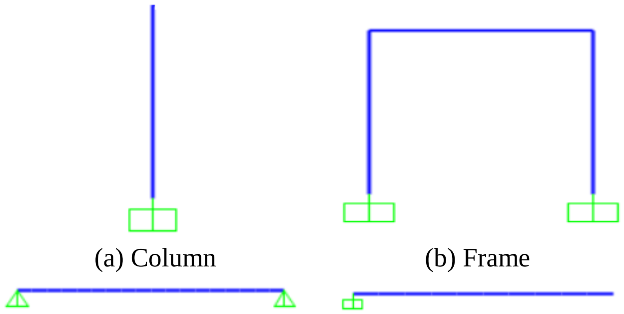
A general similarity formula is obtained for the displacement (Eq. (38)). In the equation, when the scale factor for volume S^3 is written

$$\left.\begin{aligned} \frac{U_p}{U_m} &= \frac{E_m}{E_p}\frac{1}{S}S^3\frac{\gamma_p}{\gamma_m} \\ U_m &= \frac{E_p}{E_m}\frac{1}{S^2}\frac{\gamma_m}{\gamma_p}U_p \end{aligned}\right\} \tag{39}$$

equation is obtained. The displacement value of the scaled model is obtained by writing the prototype displacement values in Eq. (39). To some different types of scaling, the corresponding formulas are summarized in Table 5.

Table 5 Displacement formulas according to scale types

Scale Types	Formulas
Geometry	$\delta_m = S\,\delta_p$
Geometry (constant mass)	$\delta_m = \frac{1}{S^2}\delta_p$
Material	$\delta_m = \frac{E_p}{E_m}\delta_p$
Geometry and material (constant mass)	$\delta_m = \frac{E_p}{E_m}S\delta_p$
Geometry and material	$\delta_m = \frac{E_p}{E_m}\frac{1}{S^2}\delta_p$



(a) Column (b) Frame

(c) Simple beam (d) Cantilever beam

Fig. 1 Finite element models of load bearing systems

Table 6 Size properties of load bearing systems

Load Bearing Systems	System Dimensions (m)			
	Column Section	Height	Beam Section	Span
Column	0.5´ 0.5	3	----	----
Frame	0.5´ 0.5	3	0.3´ 0.5	3
Cantilever Beam	----	----	0.5´ 0.5	3
Simple Beam	----	----	0.5´ 0.5	3

4. Calculation of dynamic characteristics and structural properties of load bearing systems

4.1 Finite element analysis

Finite element models of selected columns, frames, simple beams and cantilever beam models are created with SAP2000 (2016) program in order to obtain the frequency and displacement of the structural system elements by using the finite element method. In order to evaluate the results obtained under different scale values, the corresponding processes were carried out with consideration of the 1/2, 1/5 and 1/10 scale situations. In this way, the results of the finite element analysis obtained for the five different scaling states of the related systems are compared with the theoretical solutions calculated by using mathematical models. The finite element models generated by Sap2000 program are shown in Fig. 1.

The acceptance of bulk mass, which is taken into account during the mathematical modelling, is also taken into account during the finite element analysis.

In the study, it was observed that the results obtained on the simple beam and cantilever beam elements are close to each other. In order to further improve the obtained results, singular forces representing their own weight were applied to the center of the simple beam span and to the cantilever beam end. Table 6 gives the dimension properties of the selected systems. The acceptance of bulk mass, which is taken into account during the mathematical modelling, is also taken into account during the analysis of the finite elements.

In the study, it was observed that the results obtained on the simple beam and cantilever beam elements are close to each other. In order to further improve the obtained results, singular force representing weight of structure is applied to the simple beam and the cantilever beam. Table 6 show that the dimension properties of the selected systems.

Table 7 Modulus of elasticity values of concrete class

Concrete Classes	E (N/mm^2)	F_{ck} (N/mm^2)	Calculated E (N/mm^2)
C14/16	26000	14	26160.38651
C16/20	27000	16	27000.00000
C18/22.5	27500	18	27788.58223
C20/25	28000	20	28534.44185
C25/30	30000	25	30250.00000
C30/37	32000	30	31800.98312
C35/45	33000	35	33227.25930
C40/50	34000	40	34554.80479
C45/55	36000	45	35801.66278
C50/60	37000	50	36980.97039

C20/25 concrete is used in all the load bearing systems given in Fig. 1 and the unit volume weight of concrete is taken as 24,0345 kN/m^3. For the Modulus of Elasticity value, the numerical values calculated using Eq. (40) specified in TERDC (2007) are taken into account. For different concrete classes, the Modulus of Elasticity values calculated using Eq. (40) are given in Table 7.

$$E = 3250\sqrt{f_{ck}} + 14000 \,(\text{Unity N/mm}^2) \qquad (40)$$

The scaling of the Modulus of Elasticity has been taken into account with scaling of the material property. For example, for C20/25 concrete class, the strength corresponding to a 1/10 scale concrete is 2.0 N/mm^2. The corresponding Modulus of Elasticity is obtained at E_m=18596194.08 kN/m^2 using Eq. (40).

It has been observed that there are certain differences between the dynamic characteristic values obtained from the modal analysis results of the systems modelled in SAP2000 program and the values obtained as the result of mathematical formula. In order to reduce the specified differences to a minimum level, the model of the finite element mesh is modified and the most suitable model to improve the results is obtained. In the scope of the study, frequency, period, mode shape and displacement values are obtained. The obtained values are presented in next chapter.

4.2 Analytical solution

The frequency and displacement values of these systems are obtained analytically using mathematical models of columns, frames, simple beams and cantilever beams. The results obtained are explained in detail below on the cantilever beam example. The steps of calculation of the frequency value obtained as a result of the calculations performed considering the prototype and the 1/2 scaled model of the cantilever beam are presented as follows:

The prototype data of the cantilever beam

Dimensions $0.5\times0.5\times3\text{m}$

Other data in the system $\gamma = 24.0345\,\text{kN/m}^3$

$E_p = 2853441\,\text{kN/m}^2$

$$g = 9.81\,\mathrm{m/s^2}$$

System rigidity $\quad k = \dfrac{3E_p I}{L^3}$

The solution steps taken considering the above mentioned data are as follows. With the help of frequency equations

$$\left.\begin{aligned}
I &= \frac{0.5\times0.5^3}{12} = 5.20833\,E-3\mathrm{m}^4\\
k &= \frac{3\times 28534441\ \times 5.20833\,E-3}{3^3} = 16512.976\ \mathrm{kN/m}\\
m &= \frac{0.5\times0.5\times3\times24.0345}{9.81} = 1.8375\ \mathrm{kNs^2/m}\\
\omega_n &= \sqrt{\frac{k}{m}} = \sqrt{\frac{16512.97612}{1.8375}} = 94.797962\ \mathrm{rad/s}\\
f_{n(p)} &= \frac{\omega_n}{2\pi} = \frac{94.797962}{2\pi} = 15.087564\ \mathrm{Hz}
\end{aligned}\right\} \tag{41}$$

frequency value is obtained (Eq. (41)).

<u>*1/2 geometry and mass scaling model data of the cantilever beam*</u>

Dimensions $\quad 0.25\times0.25\times1.5\,\mathrm{m}$

Other data in the system $\quad \gamma = 24.0345\,\mathrm{kN/m^3}$

$$E_p = 2853441\,\mathrm{kN/m^2}$$

$$g = 9.81\,\mathrm{m/s^2}$$

System rigidity $\quad k = \dfrac{3E_m I}{L^3}$

The solution steps taken considering the above mentioned data are as follows. With the help of equation

$$\begin{aligned}
&= \frac{0.25\times0.25^3}{12} = 3.255208333\ E-3\mathrm{m}^4\\
k &= \frac{3\times 28534441\ \times 3.255208333\ E-3}{1.5^3} = 8256.493\ \mathrm{kN/m}\\
m &= \frac{0.25\times0.25\times1.5\times24.0345}{9.81} = 0.2296875\ \mathrm{kNs^2/m}\\
\omega_n &= \sqrt{\frac{k}{m}} = \sqrt{\frac{8256.493344}{0.2296875}} = 189.5959854\ \mathrm{rad/s}\\
f_{n(m)} &= \frac{\omega_n}{2\pi} = \frac{189.5959854}{2\pi} = 30.17513827\ \mathrm{Hz}
\end{aligned} \tag{42}$$

frequency value is obtained (Eq. (42)). Using the obtained geometric and mass scale similarity formula, the frequency value of the model obtained from the frequency value of the prototype

$$\frac{f_{n(m)}}{f_{n(p)}} = \frac{30.17513827}{15.087564} = 2.0000 \tag{43}$$

is obtained (Eq. (43)). The frequency values of the other load bearing systems are presented in next chapter.

The steps in the displacement calculation for the prototype and the 1/2 scaled model of the cantilever beam are as follows;

<u>*The prototype data of the cantilever beam.*</u>

Dimensions $\quad 0.5\times0.5\times3\,\mathrm{m}$

Other data in the system $\quad \gamma = 24.0345\,\mathrm{kN/m^3}$

$$E_p = 2853441\,\mathrm{kN/m^2}$$

System displacement $\quad \delta = \dfrac{PL^3}{3EI}$

The solution steps taken considering the above mentioned data are as follows. With the help of displacement equation

$$\left.\begin{aligned}
I &= \frac{0.5\times0.5^3}{12} = 5.20833\,E-3\mathrm{m}^4\\
P &= W = 0.5\times0.5\times3\times24.0345 = 18.025875\ \mathrm{kN}\\
\delta_p &= \frac{18.025875\ \times 3^3}{3\times 28534441\ \times 5.208333\ E-3} = 0.0010916\ \mathrm{m}\\
\delta_p &= 1.09\ \mathrm{mm}
\end{aligned}\right\} \tag{44}$$

displacement value is obtained (Eq. (44)). P load according to bulk mass acceptance is applied as system weight.

<u>*1/2 geometry and mass scaling model data of cantilever beam*</u>

Dimensions $\quad 0.25\times0.25\times1.5\,\mathrm{m}$

Other data in the system $\quad \gamma = 24.0345\,\mathrm{kN/m^3}$

$$E_p = 2853441\,\mathrm{kN/m^2}$$

System displacement $\quad \delta = \dfrac{PL^3}{3EI}$

The solution steps taken considering the above mentioned data are as follows. With the help of equation

$$\begin{aligned}
I &= \frac{0.25\times0.25^3}{12} = 3.255208333\ E-3\mathrm{m}^4\\
P &= W = 0.25\times0.25\times1.5\times24.0345 = 2.253234375\ \mathrm{kN}\\
\delta_p &= \frac{2.253234375\ \times 3-1.5^3}{3\times 28534441\ \times 3.255208333\ E-4} = 0.00218324\ \mathrm{m}\\
\delta_p &= 2.18\ \mathrm{mm}
\end{aligned} \tag{45}$$

Displacement value is obtained (Eq. (45)). P load according to bulk mass acceptance is applied as system weight. Using the obtained geometric and mass scale similarity formula, the displacement value of the model obtained from the displacement value of the prototype

$$\frac{\delta_m}{\delta_p} = \frac{2.18}{1.09} = 2.0000 \tag{46}$$

is obtained (Eq. (46)). The displacement values of the other load bearing systems are presented in next chapter.

4.3 Comparison of results

The frequency values of the all load bearing systems are presented in detail in Tables 8-10 for different scaling types of the results of finite element analysis and mathematical formula.

As can be seen from the results, for the frequency values, the finite element model and the analytical solution results give approximate results with the results of the similarity formulas generated. The scale factors (f_m/f_p) between different systems with the same scale are constant. The scale factors of the different scaling types are be in the

Table 8 The frequency values obtained from a result of analytical solutions and finite element analysis of concrete load bearing systems for 1/2 scaled ratio

Scale Types	Load Bearing System Elements	Finite Element Analysis Results (Hz)		Formula Results	Analytical Results (Hz)		Formula Results
		Prototype	1/2 Scaled Model	1/2 Scaled Model	Prototype	1/2 Scaled Model	1/2 Scaled Model
Geometry	Column	30.1573	20.1049	21.3244	30.1750	21.3370	21.3370
	Frame	26.4612	18.0261	18.7109	26.4653	18.7138	18.7138
	Simple Beam	58.0622	41.0562	41.0562	60.3503	42.6741	42.6741
	Cantilever Beam	14.9363	10.5616	10.5616	15.0875	10.6685	10.6685
Geometry and Mass	Column	30.1573	60.3146	60.3146	30.1750	60.3503	60.3500
	Frame	26.4612	54.0359	52.9224	26.4653	52.9307	52.9306
	Simple Beam	58.0622	116.1243	116.1244	60.3503	120.7004	120.7006
	Cantilever Beam	14.9363	29.8727	29.8726	15.0875	30.1751	30.1750
Material	Column	30.1573	27.8169	28.9635	30.1750	27.8333	28.9805
	Frame	26.4612	24.4076	25.4137	26.4653	24.4115	25.4176
	Simple Beam	58.0622	53.5562	55.7637	60.3503	55.6667	57.9612
	Cantilever Beam	14.9363	13.7772	14.3450	15.0875	13.9167	14.4902
Geometry and Material	Column	30.1573	18.5446	20.4803	30.1750	19.6812	20.4923
	Frame	26.4612	16.6272	17.9702	26.4653	17.2616	17.9730
	Simple Beam	58.0622	37.8700	39.4309	60.3503	39.3623	40.9847
	Cantilever Beam	14.9363	9.7420	10.1435	15.0875	9.8406	10.2461
Geometry, Material and Mass	Column	30.1573	55.6339	57.9269	30.1750	55.6667	57.9609
	Frame	26.4612	49.8424	50.8274	26.4653	48.8229	50.8353
	Simple Beam	58.0622	107.1124	111.5274	60.3502	111.3334	115.9223
	Cantilever Beam	14.9363	27.5544	28.6900	15.0875	27.8334	28.9805

Table 9 The frequency values obtained from a result of analytical solutions and finite element analysis of concrete load bearing systems for 1/5 scaled ratio

Scale Types	Load Bearing System Elements	Finite Element Analysis Results (Hz)		Formula Results	Analytical Results (Hz)		Formula Results
		Prototype	1/5 Scaled Model	1/5 Scaled Model	Prototype	1/5 Scaled Model	1/5 Scaled Model
Geometry	Column	30.1573	13.4329	13.4868	30.1750	13.4947	13.4947
	Frame	26.4612	11.8849	11.8338	26.4653	11.8357	11.8356
	Simple Beam	58.0621	25.9662	25.9661	60.3503	26.9895	26.9895
	Cantilever Beam	14.9363	6.6798	6.6797	15.0875	6.7474	6.7473
Geometry and Mass	Column	30.1573	150.7844	150.7865	30.1750	150.8757	150.8750
	Frame	26.4612	133.4035	132.3060	26.4653	132.3267	132.3265
	Simple Beam	58.0622	290.3108	290.3110	60.3503	301.7513	301.7515
	Cantilever Beam	14.9363	74.6818	74.6815	15.0875	75.4378	75.4375
Material	Column	30.1573	25.5614	25.5614	30.1750	25.5765	25.5764
	Frame	26.4612	22.4286	22.4286	26.4653	22.4321	22.4320
	Simple Beam	58.0622	49.2136	49.2137	60.3503	51.1530	51.1531
	Cantilever Beam	14.9363	12.6601	12.6600	15.0875	12.7882	12.7882
Geometry and Material	Column	30.1573	11.3858	11.4314	30.1750	11.4382	11.4381
	Frame	26.4612	10.0736	10.0304	26.4653	10.0319	10.0319
	Simple Beam	58.0622	22.0090	22.0090	60.3502	22.8763	22.8763
	Cantilever Beam	14.9363	5.6618	5.6617	15.0875	5.7190	5.7191
Geometry, Material and Mass	Column	30.1573	127.8053	127.8070	30.1750	127.8825	127.8820
	Frame	26.4612	113.0732	112.1429	26.4653	112.1605	112.1603
	Simple Beam	58.0622	246.0682	246.0683	60.3503	255.7651	255.7653
	Cantilever Beam	14.9363	63.3005	63.3002	15.0875	63.8412	63.9410

position of multiplication. For example, when the scale factor of geometric scaling is multiplied by the scale factor of material scaling, the scaling factor of the system that is scaled as geometric and material is obtained.

The displacement values of the other load bearing systems are presented in detail in Tables 11-12-13 for different scaling types of the results of finite element analysis and mathematical formula.

As can be seen from the results, for the frequency values, the finite element model and the analytical solution results give approximate results with the results of the similarity formulas generated. The scale factors (δ_m/ δ_p)

Table 10 The frequency values obtained from a result of analytical solutions and finite element analysis of concrete load bearing systems for 1/10 scaled ratio

Scale Types	Load Bearing System Elements	Finite Element Analysis Results (Hz)		Formula Results	Analytical Results (Hz)		Formula Results
		Prototype	1/10 Scaled Model	1/10 Scaled Model	Prototype	1/10 Scaled Model	1/10 Scaled Model
Geometry	Column	30.1573	9.5318	9.5366	30.1750	9.5421	9.5421
	Frame	26.4617	8.3219	8.3679	26.4653	8.3691	8.3690
	Simple Beam	58.0621	18.3608	18.3608	60.3503	19.0844	19.0844
	Cantilever Beam	14.9363	4.7233	4.7233	15.0875	4.7711	4.7711
Geometry and Mass	Column	30.1573	301.5732	301.5730	30.1750	301.751	301.7500
	Frame	26.4617	263.2899	264.6170	26.4653	264.6535	264.6530
	Simple Beam	58.0622	580.6217	580.6220	60.3503	603.5027	603.5030
	Cantilever Beam	14.9363	149.3637	149.3630	15.0875	150.8757	150.8750
Material	Column	30.1573	24.3455	24.3456	30.1750	24.3599	24.3599
	Frame	26.4617	21.3617	21.3622	26.4653	21.3651	21.3651
	Simple Beam	58.0622	46.8728	46.8728	60.3503	48.7199	48.7199
	Cantilever Beam	14.9363	12.0579	12.0579	15.0875	12.1799	12.1799
Geometry and Material	Column	30.1573	7.6949	7.6987	30.1750	7.7032	7.7032
	Frame	26.4617	6.7182	6.7553	26.4653	6.7562	6.7562
	Simple Beam	58.0622	14.8224	14.8224	60.3503	15.4066	15.4066
	Cantilever Beam	14.9363	3.8130	3.8130	15.0875	3.8516	3.8516
Geometry, Material and Mass	Column	30.1573	243.4558	243.4560	30.1750	243.5995	243.5990
	Frame	26.4617	212.5503	213.6220	26.4653	213.6509	213.6510
	Simple Beam	58.0622	468.7277	468.7280	60.3503	487.1992	487.1990
	Cantilever Beam	14.9363	120.5791	120.5790	15.0875	121.7997	121.7990

Table 11 The displacement values obtained from a result of analytical solutions and finite element analysis of concrete load bearing systems for 1/2 scaled ratio

Scale Types	Load Bearing System Elements	Finite Element Analysis Results (mm)		Formula Results	Analytical Results (mm)		Formula Results
		Prototype	1/2 Scaled Model	1/2 Scaled Model	Prototype	1/2 Scaled Model	1/2 Scaled Model
Geometry	Column	0.0038	0.0076	0.0076	0.00379	0.00758	0.00758
	Frame	0.0061	0.01213	0.0122	0.0049	0.00985	0.00980
	Simple Beam	0.0737	0.1000	0.1468	0.0682	0.1364	0.1364
	Cantilever Beam	1.1000	2.2000	2.2000	1.0916	2.1832	2.1836
Geometry and Mass	Column	0.0038	0.0009	0.0009	0.00379	0.000948	0.000975
	Frame	0.0061	0.001516	0.0015	0.0049	0.000559	0.001225
	Simple Beam	0.0737	0.0184	0.0184	0.0682	0.0171	0.0171
	Cantilever Beam	1.1000	0.3000	0.2750	1.0916	0.2729	0.2729
Material	Column	0.0038	0.0045	0.0045	0.00379	0.00445	0.00445
	Frame	0.0061	0.007128	0.007169	0.0049	0.005791	0.005759
	Simple Beam	0.0737	0.0860	0.0866	0.0682	0.0802	0.0802
	Cantilever Beam	1.1000	1.3000	1.29288	1.0916	1.2830	1.2830
Geometry and Material	Column	0.0038	0.0089	0.00893	0.00379	0.008909	0.008909
	Frame	0.0061	0.01426	0.01434	0.0049	0.011583	0.011518
	Simple Beam	0.0737	0.2000	0.1732	0.0682	0.1604	0.1603
	Cantilever Beam	1.1000	2.6000	2.5858	1.0916	2.5660	2.5660
Geometry, Material and Mass	Column	0.0038	0.0011	0.0011	0.00379	0.001114	0.0011
	Frame	0.0061	0.001782	0.001792	0.0049	0.000657	0.00143
	Simple Beam	0.0737	0.0217	0.0217	0.0682	0.0200	0.0200
	Cantilever Beam	1.1000	0.3000	0.3232	1.0916	0.3208	0.3208

between different systems with the same scale are constant. The scale factors of the different scaling types are be in the position of multiplication. For example, when the scale factor of geometric scaling is multiplied by the scale factor of material scaling, the scaling factor of the system that is scaled as geometric and material is obtained.

5. Conclusions

In this study, it is aimed to determine the characteristics properties and structural behaviors of concrete load bearing systems by establishing similarity relation between prototype and scaled models. Finite element models of

Table 12 The displacement values obtained from a result of analytical solutions and finite element analysis of concrete load bearing systems for 1/5 scaled ratio

Scale Types	Load Bearing System Elements	Finite Element Analysis Results (mm)		Formula Results	Analytical Results (mm)		Formula Results
		Prototype	1/5 Scaled Model	1/5 Scaled Model	Prototype	1/5 Scaled Model	1/5 Scaled Model
Geometry	Column	0.0038	0.0190	0.0190	0.00379	0.01895	0.01895
	Frame	0.0061	0.03032	0.0305	0.0049	0.02464	0.02450
	Simple Beam	0.0737	0.4000	0.3685	0.0682	0.3411	0.3410
	Cantilever Beam	1.1000	5.6000	5.5000	1.0916	5.4581	5.4580
Geometry and Mass	Column	0.0038	0.0002	0.0002	0.00379	0.000152	0.000152
	Frame	0.0061	0.000243	0.000244	0.0049	0.000121	0.000196
	Simple Beam	0.0737	0.0029	0.00295	0.0682	0.0027	0.00273
	Cantilever Beam	1.1000	0.0445	0.0440	1.0916	0.0436	0.04366
Material	Column	0.0038	0.0053	0.0053	0.00379	0.00528	0.00528
	Frame	0.0061	0.008441	0.00849	0.0049	0.006859	0.00682
	Simple Beam	0.0737	0.1000	0.10258	0.0682	0.0950	0.09493
	Cantilever Beam	1.1000	1.5000	1.53111	1.0916	1.5194	1.5194
Geometry and Material	Column	0.0038	0.0264	0.0265	0.00379	0.026379	0.0264
	Frame	0.0061	0.04221	0.04245	0.0049	0.034293	0.0341
	Simple Beam	0.0737	0.5000	0.5129	0.0682	0.4749	0.4747
	Cantilever Beam	1.1000	7.7000	7.6555	1.0916	7.5972	5.4580
Geometry, Material and Mass	Column	0.0038	0.0002	0.0002	0.00379	0.000211	0.00211
	Frame	0.0061	0.0003377	0.00034	0.0049	0.000169	0.00027
	Simple Beam	0.0737	0.0041	0.0041	0.0682	0.0038	0.0038
	Cantilever Beam	1.1000	0.0620	0.0612	1.0916	0.0608	0.0608

Table 13 The displacement values obtained from a result of analytical solutions and finite element analysis of concrete load bearing systems for 1/10 scaled ratio

Scale Types	Load Bearing System Elements	Finite Element Analysis Results (mm)		Formula Results	Analytical Results (mm)		Formula Results
		Prototype	1/10 Scaled Model	1/10 Scaled Model	Prototype	1/10 Scaled Model	1/10 Scaled Model
Geometry	Column	0.0038	0.0379	0.03800	0.00379	0.0379	0.0379
	Frame	0.0061	0.0606	0.0610	0.0049	0.04927	0.0490
	Simple Beam	0.0737	0.7000	0.7370	0.0682	0.6823	0.6820
	Cantilever Beam	1.1000	11.1000	11.000	1.0916	10.9162	10.9160
Geometry and Mass	Column	0.0038	0.000038	0.00004	0.00379	0.000038	0.00004
	Frame	0.0061	0.00006065	0.00006	0.0049	0.000030	0.00004
	Simple Beam	0.0737	0.000737	0.00074	0.0682	0.00068	0.00068
	Cantilever Beam	1.1000	0.0111	0.0110	1.0916	0.01092	0.01092
Material	Column	0.0038	0.0058	0.00583	0.00379	0.00582	0.00582
	Frame	0.0061	0.009306	0.00936	0.0049	0.007561	0.00752
	Simple Beam	0.0737	0.1000	0.11309	0.0682	0.10469	0.10465
	Cantilever Beam	1.1000	1.7000	1.6879	1.0916	1.67500	1.67498
Geometry and Material	Column	0.0038	0.0582	0.05830	0.00379	0.058159	0.05820
	Frame	0.0061	0.09306	0.09360	0.0049	0.07561	0.07520
	Simple Beam	0.0737	1.1	1.13090	0.0682	1.04688	1.04650
	Cantilever Beam	1.1000	17.1	16.8790	1.0916	16.7500	16.7498
Geometry, Material and Mass	Column	0.0038	0.00005816	0.00006	0.00379	0.000058	0.00006
	Frame	0.0061	0.00009306	0.00009	0.0049	0.000047	0.00007
	Simple Beam	0.0737	0.001131	0.00113	0.0682	0.00105	0.00105
	Cantilever Beam	1.1000	0.01709	0.01688	1.0916	0.01675	0.01675

selected columns, frames, simple beams and cantilever beam systems are carried out with consideration of the 1/2, 1/5 and 1/10 scale situations with SAP2000 (2016) program in order to obtain the dynamic characteristics, displacement, stress and strain of the structural system elements by using the finite element method. Also, the results of the finite element analysis are compared with the theoretical solutions calculated by using mathematical models. In order to reduce the specified differences to a minimum level, the model of the finite element mesh is modified and the most suitable model to improve the results is obtained. As can be seen from the results, the scale factors (f_m/f_p and δ_m/δ_p) between different systems with the same scale are constant. The scale factors of the different scaling types are be in the position of multiplication. For the characteristic properties and structural behaviors, the finite element model and the analytical solution results give approximate results with the results of the similarity formulas generated. It is seen in this study that the characteristic properties and structural behavior of the systems can be obtained establishing a similarity relation, by establishing small scale models.

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