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# Analysis of flexural fatigue failure of concrete made with 100% coarse recycled and natural aggregates

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**Abstract.** In this study, the flexural fatigue performance of concrete beams made with 100% Coarse Recycled Concrete Aggregates (RCA) and 100% Coarse Natural Aggregates (NA) were statistically commanded. For this purpose, the experimental fatigue test results of earlier researcher were investigated using two parameter Weibull distribution. The shape and scale parameters of Weibull distribution function was evaluated using seven numerical methods namely, Graphical method (GM), Least-Squares (LS) regression of Y on X, Least-Squares (LS) regression of X on Y, Empherical Method of Lysen (EML), Mean Standard Deviation Method (MSDM), Energy Pattern Factor Method (EPFM) and Method of Moments (MOM). The average of Weibull parameters was used to incorporate survival probability into stress (S)-fatigue life (N) relationships. Based on the Weibull theory, as single and double logarithm fatigue equations for RCA and NA under different survival probability were provided. The results revealed that, by considering 0.9 level survival probability, the theoretical stress level corresponding to a fatigue failure number equal to one million cycle, decreases by 8.77% (calculated using single-logarithm fatigue equation) and 6.62% (calculated using double logarithm fatigue equation) in RCA when compared to NA concrete.

**Keywords:** fatigue; stress level; survival probability; Weibull parameters; regression; aggregates

## 1. Introduction

In the last few decades, there has been an upsurge in utilization of composite materials for various advanced applications like bridge deck and piers, airport pavement concrete, high rise building and other deliberate applications owing high demands on material performance placed by advanced technologies (Shokrieh *et al.* 2017). Most of these structures are typically exposed to millions of cycles of repetitive fatigue loads during their service life, making fatigue strength a crucial parameter in the design of these structures (Shi *et al*. 1993, Lee and Barr 2004). The effective and efficient use of composite materials in these structures thus necessitates the investigation of material performance under fatigue load. The fatigue behavior of concrete which was characterized by S-N curve, making the mean fatigue life of concrete under a given stress level to be approximated (Singh and Kaushik 2001, Singh *et al*. 2007).

Over the last decades, numerous researches have been conducted to predict the fatigue life and failure probability of concrete. The stress level (Hilsdorf and Kesler 1966, Oh 1991 a, b, Kim and Kim 1996, Lee and Barr 2004), stress ratio (Murdock and Kesler 1958, Tepfers and Kutti 1979) and loading frequency (Mohammadi and Kaushik 2005, Saucedo *et al*. 2013) are the major factors which influence the fatigue life of concrete. Due to the statistical nature of fatigue phenomenon, large scatter usually exists in the subsequent fatigue life results even under prudently controlled test procedures (Singh and Kaushik 2001, Ballinger 1972, Oh 1986, Singh *et al*. 2005, Goel *et al*. 2012). Several variables such as specimen manufacturing, preparation, handling, storage, test rig design and experimental methods play a vital role in scattered test results. This scatter was considered quite trivial in past because of the use of large safety factor, however the recent advanced techniques entail accurate characterization. Hence, this scatter in fatigue test results of composite materials is usually resolved by statistical approach (Singh and Kaushik 2003, Michael *et al*. 2004, Singh *et al*. 2005, Selmy *et al*. 2013, Ganesan *et al*. 2013, Feng *et al.* 2015, Nirosha and Adasooriya 2016).

In order to evaluate the reliability of composite materials and its properties, Weibull distribution function has proved to be a proficient technique (Dirikolu and Aktas 2002, Li *et al*. 2007, Chen *et al*. 2011), since the probability density function of the Weibull distribution has a wide variety of shapes. For example, when the shape parameter of the distribution is equal to 1.0, it becomes two-parameter exponential distribution; for shape parameter value nearing 2.0, the function is approximating to Rayleigh distribution (Abdallah*et al*. 1996) and it becomes normal distribution when the shape parameter value nears 3.0 (Bedi and Chandra 2009). Thus, the Weibull distribution has proven to be an appropriate means for describing composite material

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properties due to the availability of physically valid assumptions, comprehensive experimental verifications, relative ease in its use and better developed statistics.

## 2. Research significance

The Although there are numerous studies reporting the flexural fatigue performance of concrete using graphical method and method of moments of Weibull distribution (Bedi and Chandra 2009, Ganesan *et al*. 2013, Selmy *et al*. 2013, Zhou *et al.* 2016), only few studies are available for assessing the fatigue performance of recycled aggregate concrete beams made with 100% Coarse RCA (Thomas *et al*. 2014, Yan *et al*. 2010, 2011) and hence one is provoked to work in this direction. Also, the absence of comparative study on the fatigue performance of concrete using Weibull parameters obtained from seven methods viz., GM, LS-Y on X, LS-X on Y, EML, MSDM, EPFM and MOM has led to the development of more precise systematic statistical analysis and it is attained through Arora and Singh (2016) test results. The Weibull parameters were assessed using seven method such as GM, LS-Y on X, LS-X on Y, EML, MSDM, EPFM and MOM. Also, the mean and design fatigue lives for RCA and NA concrete was determined. According to the Weibull theory, a single and double logarithm fatigue equations for RCA and NA concrete corresponding to different reliabilities/survival probabilities were also proposed. The aim of this study is to provide a systematic support for the extensive statistical analysis to describe the fatigue strength of concrete beams made with 100 % RCA and NA, which becomes significant for the designer to choose appropriate fatigue strength value corresponding to different reliabilities/survival probabilities.

Table 1 Laboratory fatigue life data (number of cycles to failure N, in ascending order) of Arora and Singh (2016)

| Mix Id | Specimen No. | Stress level 'S' | | |
|---|---|---|---|---|
| | | 0.85 | 0.75 | 0.65 |
| RCA | 1 | 567 | - | 67225 |
| | 2 | 789 | 4353 | 68738 |
| | 3 | 1054 | 5615 | 88969 |
| | 4 | 1188 | 9382 | 90371 |
| | 5 | 1345 | 9792 | 120805 |
| | 6 | 1765 | 12829 | 189763 |
| | 7 | 1897 | 13702 | 249867 |
| | 8 | 2098 | 14045 | 261009 |
| | 9 | 2156 | 23020 | 319551 |
| | 10 | 2354 | 26079 | 409876 |
| NA | 1 | - | 10781 | 100801 |
| | 2 | 1137 | 13879 | 142054 |
| | 3 | 1367 | 18489 | 187623 |
| | 4 | 1678 | 21945 | 220075 |
| | 5 | 1945 | 25467 | 260685 |
| | 6 | 2271 | 31256 | 323068 |
| | 7 | 2605 | 36543 | 360845 |
| | 8 | 2647 | 42842 | 456944 |
| | 9 | 3096 | 46951 | 512089 |
| | 10 | 3987 | 51348 | 558973 |

## 3. Analysis of fatigue test data and discussion

The number of specimens tested at applied stress levels and corresponding fatigue results on RCA and NA concrete are shown in Table 1. The flexural failure results of Arora and Singh (2016) were used for the statistical analysis.

### *3.1 Weibull distribution theory*

The two parameter Weibull distribution function is the most popular and broadly utilized method in fatigue life investigations (Singh and Kaushik 2003, Singh *et al*. 2005, Selmy *et al*. 2013, Ganesan *et al*. 2013, Feng *et al.* 2015, Nirosha and Adasooriya 2016). The statistical analysis of fatigue-life results was performed using this method and itis characterized by a probability distribution function which is given by (Manwell *et al.* 2002, Mathew 2006)

$$(n) = \frac{\propto}{u}\left(\frac{n}{u}\right)^{\propto-1} exp\left[-\left(\frac{n}{u}\right)\right] \quad (1)$$

The cumulative function can be obtained by computing the integral of the probability distribution function. The cumulative distribution function is represented by (Manwell *et al.* 2002, Mathew 2006)

$$F(n) = 1 - exp\left[-\left(\frac{n}{u}\right)^{\propto}\right] \quad (2)$$

Where, $n$ = specific value of the random variable N; $\alpha$ = shape parameter or Weibull slope at certain stress level (S) and $u$ = scale parameter or characteristic life at certain stress level. Several numerical methods have been recommended in the literature to calculate the shape and scales parameters (Seguro and Lambert 2000, Celik 2003, Chang *et al.* 2003, Akpinar and Akpinar 2004, Lai and Lin 2006, Zhou *et al.* 2006, Akdag and Dinler 2009, Kwon 2010, Costa *et al.* 2012, Shu *et al.* 2015).

### *3.2 Analysis of fatigue-life data by the graphical method*

The probability of survival or survivorship function or reliability function, $L_N(n)$ of the two-parameter Weibull distribution, may be defined as $L_N(n)=1-F(n)$, and substituting this value of F(n) in Eq. (2) it is converted to (Gumble 1963, Wirsching and Yao 1970, Oh 1986, Oh 1991-b, Mohammadi and Kaushik 2005, Goel *et al*. 2012, Murali *et al*. 2017a, 2017b, 2018)

$$L_N(n) = \exp\left[\frac{n}{u}\right]^{\propto} \quad (3)$$

By taking logarithm twice for Eq. (3), the equation for the graphical method can be written as (Chang 2011, Levent *et al.* 2015)

$$\ln\left[\ln\left(\frac{1}{L_N}\right)\right] = \propto \ln n - \propto \ln u \quad (4)$$

$$Y = \alpha X - \beta \quad (5)$$

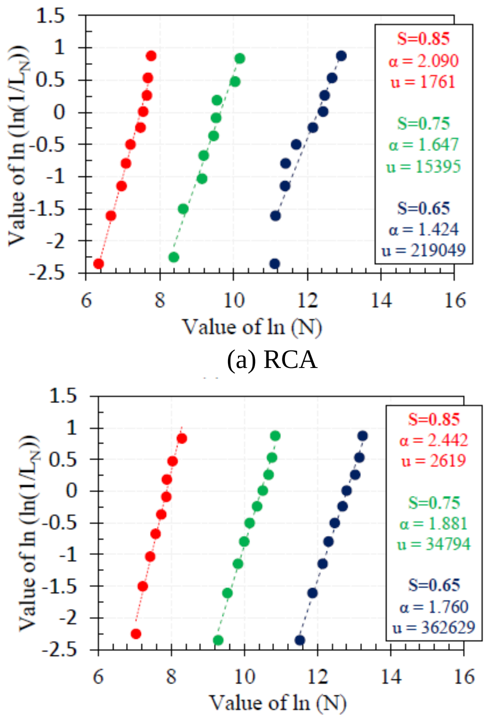


Fig. 1 Graphical analysis of fatigue life data at different stress levels (a) RCA and (b) NA

where, $Y$=ln[ln(1/$L_N$)], $X$=ln ($n$), and $\beta$=$\alpha$ ln $u$.

Eq. (4), represents that the relationship between ln[ln(1/$L_N$)] and ln($n$) is a linear one. The line slope represents the shape parameter ($\alpha$) and scale parameter ($u$) can be obtained from the second term of Eq. (4). In order to obtain a graph from Eq. (4), the number of failure cycles (N) corresponding to each stress level (S) are first arranged in ascending order; serial number is assigned for each value ($i$=1, 2, 3, . . . , $n$) and the reliability function $L_N$ for each N value at a given stress level S is calculated from the following expression (Gumble 1963, Wirsching and Yao 1970, Oh 1986, Oh 1991-b, Mohammadi and Kaushik 2005, Goel *et al*. 2012)

$$L_N = 1 - \frac{i}{k+1} \tag{6}$$

where $i$ is the failure order number and $k$ is number of fatigue data points at a given stress level S. The parameters of the distribution can be obtained directly from the plot between ln[ln(1/$L_N$)] and ln($N$) (Chang 2011, Levent *et al.* 2015). The analysis of fatigue life of RCA and NA concrete at all stress levels ($S$=0.85, S=0.75, $S$=0.65) and the obtained Weibull parameters from the graphical method of regression analysis are shown in Fig. 1. The equivalent values of the correlation coefficients are 0.984, 0.982 and 0.952 for RCA and 0.99, 0.994 and 0.996 for NA, at stress levels 0.85, 0.75 and 0.65 respectively. It can be seen that all the values of correlation coefficients are greater than 0.95, which shows a linear relationship between ln[ln(1/$L_N$)] and ln($N$) at all stress levels. The obtained Weibull parameters for all stress levels are summarised in Tables 3 and 4. At a particular stress level, if this relationship approximately follows a linear trend it indicates that the two parameter Weibull distribution is a reasonable assumption for the statistical description of fatigue test data at that stress level.

### *3.3 Analysis of fatigue-life data by the LS Y on X*

The LS method is usually used to fit a straight line instead of by eye. The straight line can be fit by minimizing the sum of squares of the vertical residuals, which is called LS Y on X method, or it can be fit by minimizing the sum of squares of the horizontal residuals, which is called LS X on Y method. When using least square method, the sum of the squares of the deviations S is defined in Eq. (7) (Zhang *et al*. 2007).

$$S = \sum_{i=1}^{n} [Y_i - (\alpha X_i - \alpha \ln u)]^2 \tag{7}$$

Differentiating the Eq. (7) with respect to $\alpha$ and $u$ respectively, and setting the partial derivatives equal to 0, the estimating equations of the LS Y on X method can be written as follows.

$$\alpha = \frac{\sum_{i=1}^{n}[(X_i - \bar{X})(Y_i - \bar{Y})]}{\sum_{i=1}^{n}(X_i - X)^2} \tag{8}$$

$$u = \exp[-(\bar{Y}/\alpha - \bar{X})] \tag{9}$$

$$\bar{X}_i = \sum_{I=1}^{n} X_i/n \qquad \bar{Y}_i = \sum_{I=1}^{n} Y_i/n \tag{10}$$

### *3.4 Analysis of fatigue-life data by the LS X on Y*

The estimating equation of LS X on Y can be obtained in a similar approach. The objective function is to minimize the value of $S = \sum_{i=1}^{n}[X_i - (\frac{1}{\alpha Y_i} + \ln u)]^2$ and the formula can be written as (Zhang *et al*. 2007).

$$\alpha = \frac{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}{\sum_{i=1}^{n}[(X_i - X)(Y_i - \bar{Y})]} \tag{11}$$

$$u = \exp[-(\bar{X} - \bar{Y}/\alpha)] \tag{12}$$

$$\bar{X}_i = \sum_{I=1}^{n} X_i/n \qquad \bar{Y}_i = \sum_{I=1}^{n} Y_i/n \tag{13}$$

The reliability function $L_N$is used in LS method is given below (Bernard and Bosi 1953).

$$L_N = 1 - \frac{i - 0.3}{k + 0.4} \tag{14}$$

The sample spread sheet of RCA at stress level of 0.85 for calculating Weibull parameters by LS method is shown in Table 2.

### *3.5 Analysis of fatigue-life data by Empirical Method of Lysen (EML)*

In the empirical method suggested by Lysen 1983, $\alpha$ is calculated by Eq. (15). In the empirical method of Lysen, $u$ is obtained by Eq. (16)

$$\alpha = \left(\frac{\sigma}{\bar{N}}\right)^{-1.086} \tag{15}$$

Table 2 Spread sheet for calculating LS estimates

| $i$ | $N$ | $L_N$ | $Xi$=ln($N$) | $Y_i$=ln(-ln(1-$L_N$)) | $Xi$-$\bar{X}$ | $Yi$-$\bar{Y}$ | ($Xi$-$\bar{X}$)($Yi$-$\bar{Y}$) | ($Xi$-$\bar{X}$)$^2$ | ($Yi$-$\bar{Y}$)$^2$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 567 | 0.067 | 6.340 | -2.664 | -0.896 | -2.141 | 1.919 | 0.804 | 4.583 |
| 2 | 789 | 0.163 | 6.671 | -1.723 | -0.566 | -1.200 | 0.679 | 0.320 | 1.440 |
| 3 | 1054 | 0.260 | 6.960 | -1.202 | -0.276 | -0.679 | 0.188 | 0.076 | 0.461 |
| 4 | 1188 | 0.356 | 7.080 | -0.822 | -0.157 | -0.299 | 0.047 | 0.025 | 0.089 |
| 5 | 1345 | 0.452 | 7.204 | -0.509 | -0.033 | 0.015 | 0.000 | 0.001 | 0.000 |
| 6 | 1765 | 0.548 | 7.476 | -0.230 | 0.239 | 0.293 | 0.070 | 0.057 | 0.086 |
| 7 | 1897 | 0.644 | 7.548 | 0.033 | 0.311 | 0.556 | 0.173 | 0.097 | 0.309 |
| 8 | 2098 | 0.740 | 7.649 | 0.299 | 0.412 | 0.822 | 0.339 | 0.170 | 0.676 |
| 9 | 2156 | 0.837 | 7.676 | 0.594 | 0.439 | 1.117 | 0.491 | 0.193 | 1.248 |
| 10 | 2354 | 0.933 | 7.764 | 0.993 | 0.527 | 1.516 | 0.799 | 0.278 | 2.298 |
| Sum | - | - | 72.368 | -5.231 | - | - | 4.704 | 2.021 | 11.190 |
| Avg | - | - | 7.237 | -0.523 | - | - | - | - | - |

$$u = \bar{\mathrm{N}}\left(0.568 + \frac{0.433}{\alpha}\right)^{1/\alpha} \quad (16)$$

### 3.6 Analysis of fatigue-life data by Mean standard deviation method (MSDM)

MSDM is constructive where only the two parameters such as mean fatigue life and standard deviations are available. Therefore, the Weibull shape is assessed by Eq. (15) and scale parameter is estimated by Eq. (17) (Mohammadi *et al.* 2016).

$$u = \frac{\bar{\mathrm{N}}\alpha^{2.6674}}{0.184 + 0.816\,\alpha^{2.73855}} \quad (17)$$

### 3.7 Analysis of fatigue-life data by Energy pattern factor method (EPFM)

Energy pattern factor method is expressed as mean of the sum of cubes of all individual fatigue life considered in a sample, divided by the cube of mean fatigue life of sample. Once energy pattern factor is calculated the scale and shape parameter can be estimated from the following equations (Akdag and Dinler 2009, Saxena and Subba Rao 2015).

$$Epf = \frac{\overline{N^3}}{\bar{N}^3} \quad (18)$$

$$\alpha = 1 + \frac{3.69}{(Epf)^2} \quad (19)$$

where *Epf* is the energy pattern factor and the gamma function is defined by

$$\Gamma(\mathrm{x}) = \int_0^\infty t^{x-1}\exp(-t)\,dt \quad (20)$$

### 3.8 Analysis of fatigue-life data by Method of Moments (MOM)

It is based on the numerical iteration of mean fatigue life and standard deviation (σ). The dimensionless shape and

Table 3 Values of the Weibull parameters for fatigue-life of RCA

| Method | S-0.85 | | S-0.75 | | S-0.65 | |
|---|---|---|---|---|---|---|
| | $\alpha$ | $u$ | $\alpha$ | $u$ | $\alpha$ | $u$ |
| GM | 2.09 | 1761.00 | 1.65 | 15394.61 | 1.42 | 219048.96 |
| LS-Y on X | 2.33 | 1739.81 | 1.84 | 15142.66 | 1.57 | 214140.73 |
| LS-X on Y | 2.38 | 1731.45 | 1.92 | 1731.45 | 1.77 | 206435.00 |
| EML | 2.66 | 1352.10 | 1.91 | 11706.27 | 1.62 | 166959.54 |
| MSDM | 2.66 | 1712.48 | 1.91 | 14879.34 | 1.62 | 208482.07 |
| EPFM | 2.79 | 1708.75 | 2.02 | 14900.55 | 1.74 | 209353.15 |
| MOM | 2.65 | 1711.83 | 1.89 | 14877.04 | 1.61 | 208278.35 |
| Average | 2.51 | 1673.92 | 1.88 | 12661.70 | 1.62 | 204671.11 |

Table 4 Values of the Weibull parameters for fatigue-life of NA

| Method | S-0.85 | | S-0.75 | | S-0.65 | |
|---|---|---|---|---|---|---|
| | $\alpha$ | $u$ | $\alpha$ | $u$ | $\alpha$ | $u$ |
| GM | 2.44 | 2619.13 | 1.88 | 34794.12 | 1.76 | 362629.29 |
| LS-Y on X | 2.73 | 2599.70 | 2.09 | 34261.91 | 1.96 | 357477.24 |
| LS-X on Y | 2.79 | 2588.70 | 2.13 | 34119.37 | 1.98 | 356339.14 |
| EML | 2.79 | 2050.95 | 2.26 | 26518.97 | 2.10 | 276482.73 |
| MSDM | 2.79 | 2589.31 | 2.26 | 33817.35 | 2.10 | 352630.74 |
| EPFM | 2.81 | 2586.94 | 2.41 | 33784.66 | 2.25 | 352620.2 |
| MOM | 2.78 | 2587.81 | 2.25 | 33815.17 | 2.08 | 352620.19 |
| Average | 2.73 | 2517.50 | 2.18 | 33015.94 | 2.03 | 344399.93 |

scale parameters can be expressed as follows (Kumar and Gaddada 2012)

$$\alpha = \left(\frac{0.9874}{\frac{\sigma}{\bar{N}}}\right)^{-1.086} \quad (21)$$

$$\bar{N} = u\Gamma\,(1 + 1/\gamma) \quad (22)$$

The fatigue life in terms of survival probability (R) i.e., reliability is given by the Eq. (23) (Fredy and Artur 2015).

$$N = u(-\ln(R)^{\frac{1}{\alpha}}) \quad (23)$$

### 3.9 Weibull parameters

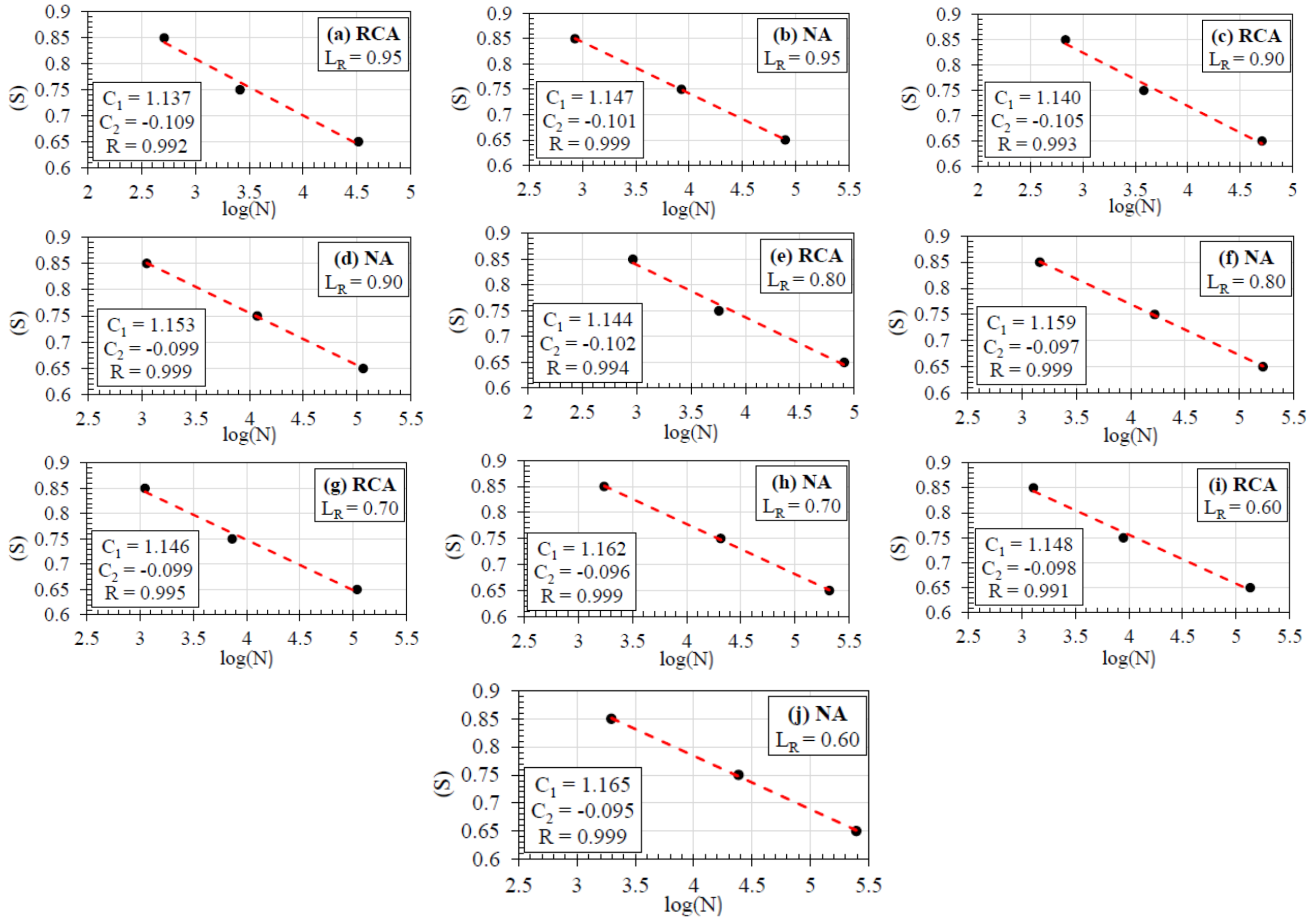


Fig. 2 Single logarithmic fatigue equation curves for determination of coefficients of the fatigue equation

Table 5 Calculated results of equivalent fatigue life corresponding to different reliabilities

| $L_R$ | RCA | | | NA | | |
|---|---|---|---|---|---|---|
| | $S$=0.85 | $S$=0.75 | $S$=0.65 | $S$=0.85 | $S$=0.75 | $S$=0.65 |
| 0.95 | 512 | 2599 | 32807 | 849 | 8465 | 79822 |
| 0.9 | 682 | 3815 | 51128 | 1105 | 11773 | 113763 |
| 0.85 | 921 | 5692 | 81197 | 1454 | 16604 | 164597 |
| 0.8 | 1110 | 7308 | 108415 | 1726 | 20585 | 207339 |
| 0.75 | 1281 | 8851 | 135283 | 1969 | 24269 | 247439 |
| 0.7 | 1446 | 10415 | 163284 | 2202 | 27912 | 287550 |

The values of the Weibull parameters attained from seven different methods for RCA are listed in Table 3 and for NA concrete are listed in Table 4. It can be seen from Table 3 that for concrete made with RCA, the average of Weibull parameters obtained from seven methods are $\alpha$=2.51, $u$=1673.92; $\alpha$=1.88, $u$=12661.70 and α=1.62, $u$=204671.11 corresponding to the stress level 0.85, 0.75 and 0.65 respectively. Likewise, the Weibull parameters for NA concrete are $\alpha$=2.73, $u$=2517.50; $\alpha$=2.18, $u$=33015.94 and α=2.03, $u$=344399.93 corresponding to the stress level 0.85, 0.75 and 0.65 respectively and it is listed in Table 4. It can be seen from the Tables 3 and 4 that, lower values of shape parameter for fatigue life were obtained for RCA with reference to NA concrete, thus representing higher variability in the distribution of flexural fatigue life of RCA. Higher shape parameters are observed at stress levels $S$=0.85, indicating less scatter in the fatigue- life data representing a more uniform damage mechanism. The value of the shape and scale parameters attained from seven methods slightly different from each other. Hence, in this case, the average values of the parameters obtained from seven methods were used for further analysis.

### *3.10 Failure probability and S-N relationships*

Using the values of Weibull parameters, the fatigue life at any stress level is listed in Tables 5. From Eq. (23) the fatigue lives corresponding to survival probabilities at different stress level has been calculated. These computed values of fatigue lives of the tested samples (RCA and NA) in forms of survival probabilities of 0.95, 0.9, 0.8, 0.7 and 0.6 are summarised in Table 5. To determine the one-million cycle fatigue strength of RCA and NA concrete, a linear regression analysis of each set of data has been carried out and plotted in the form of S-N curves as shown in Figs. 2 (a) to (i) and Figs. 3 (a) to (i).

### *3.11 Fatigue equation*

Based on numerous studies, other researcher (Sun *et al.* 2011) has proposed the fatigue equations shown below

$$S = C_1 - C_2 \lg N \tag{24}$$

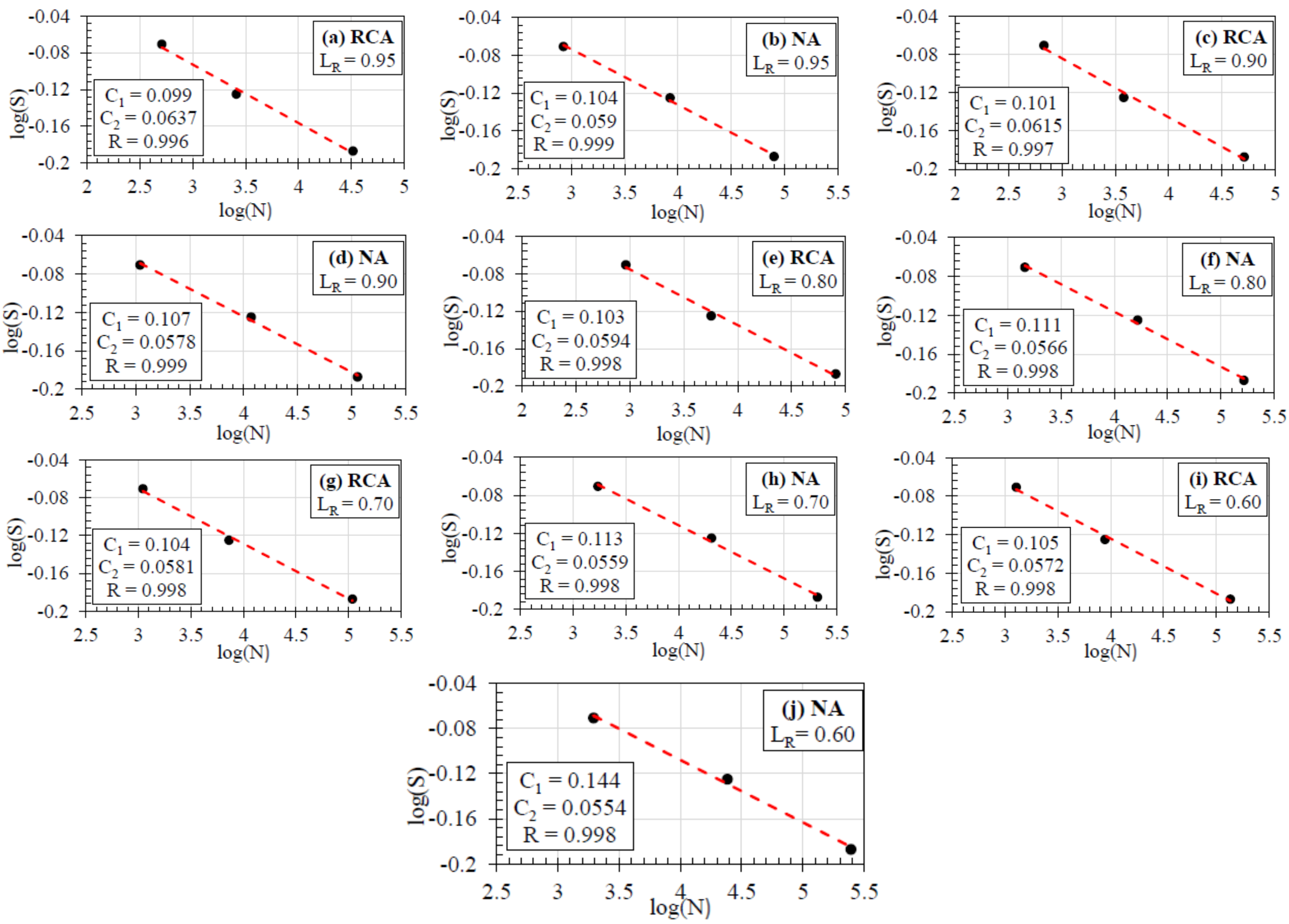


Fig. 3 Double logarithmic fatigue equation curves for determination of coefficients of the fatigue equation

Table 6 Fatigue equations of RCA and NA at different survival probability

| Mixture type | $L_R$ | Single-logarithm fatigue equation | | Double-logarithm fatigue equation | |
|---|---|---|---|---|---|
| | | $S=C_1-C_2$ lg N | Correlation coefficient (R) | lg $S$=lg $C_1-C_2$ lg N | Correlation coefficient (R) |
| RCA | 0.95 | $S$=1.137-0.109 lg N | 0.992 | lg $S$=0.099-0.064 lg N | 0.996 |
| | 0.90 | $S$=1.140-0.105 lg N | 0.993 | lg $S$=0.101-0.062lg N | 0.997 |
| | 0.80 | $S$=1.144-0.102 lg N | 0.994 | lg $S$=0.103-0.059 lg N | 0.998 |
| | 0.70 | $S$=1.146-0.099 lg N | 0.995 | lg $S$=0.104-0.058 lg N | 0.998 |
| | 0.60 | $S$=1.148-0.098 lg N | 0.991 | lg $S$=0.105-0.057 lg N | 0.998 |
| NA | 0.95 | $S$=1.147-0.101 lg N | 0.999 | lg $S$=0.104-0.059 lg N | 0.999 |
| | 0.90 | $S$=1.153-0.099 lg N | 0.999 | lg $S$=0.107-0.058 lg N | 0.999 |
| | 0.80 | $S$=1.159-0.097 lg N | 0.999 | lg $S$=0.111-0.057 lg N | 0.998 |
| | 0.70 | $S$=1.162-0.096 lg N | 0.999 | lg $S$=0.113-0.056 lg N | 0.998 |
| | 0.60 | $S$=1.165-0.095 lg N | 0.999 | lg $S$=0.114-0.055 lg N | 0.998 |

$$\lg S=\lg C_1-C_2 \lg N \quad (25)$$

The Eq. (24) assumes a linear relationship between $S$ and lg $N$, referred to as a single logarithm S-N line as shown in Fig. 2 (a) to (i). Similarly, Eq. (25) assumes a linear relationship between lg $S$ and lg $N$, referred to as a double logarithm S-N line as shown in Figs. 3 (a) to (i). A single and double logarithmic linear regression was performed on RCA and NA concrete to obtain the regression coefficients $C_1$ and $C_2$ under different survival probabilities. Table 6 shows that, under each stress level and for different survival probabilities, the absolute values of all correlation coefficients were above 0.97, which meant that the single and double logarithms equations show good fitness.

Based on the data and single and double logarithmic equations provided in Table 6, the calculated fatigue limit stress values corresponding to a fatigue life with one million cycles are shown in Table 7. The S-N regression equations provided in Table 6 can be used by the design engineers to estimate the flexural fatigue life of RCA and NA concrete for the desired level of survival probability.

Table 7 Theoretic stress level of RCA and NA concrete when the fatigue failure number N is equal to one million cycle

| Mixture type | Survival probability $L_N$ | Theoretical stress level | |
|---|---|---|---|
| | | Single-logarithm fatigue equation | Double-logarithm fatigue equation |
| RCA | 0.95 | 0.483 | 0.518 |
| | 0.90 | 0510 | 0.536 |
| | 0.80 | 0.532 | 0.561 |
| | 0.70 | 0.552 | 0.570 |
| | 0.60 | 0.560 | 0.579 |
| NA | 0.95 | 0.541 | 0.562 |
| | 0.90 | 0.559 | 0.574 |
| | 0.80 | 0.577 | 0.587 |
| | 0.70 | 0.586 | 0.598 |
| | 0.60 | 0.595 | 0.608 |

Table 7 shows the theoretical stress level of RCA and NA concrete calculated using single and double logarithm fatigue equations. Considering any level of survival probability, the theoretical stress level of RCA is decreased with reference to NA concrete when the fatigue failure number N is equal to one million cycle (Feng *et al.* 2015, Li *et al*. 2007). At 0.9 survival probability, the theoretical stress level of RCA decreases by 8.77% (calculated by single-logarithm fatigue equation) and 6.62% (calculated by double logarithm fatigue equation) when compared to NA concrete. This further validates the superior fatigue performance of NA concrete.

## 4. Conclusions

Based on the analytical results and discussions presented in this paper, the following conclusions can be drawn.

The probabilistic distribution of fatigue-life of RCA and NA concrete at a given stress level, approximately follows the two-parameter Weibull distribution with statistical correlation coefficient values exceeding 0.95. The shape parameter of the Weibull distribution for fatigue-life of RCA and NA concrete were different for different levels of the applied fatigue stress. The Seven methods of estimation of shape and scale parameters yielded slight different results from each other, therefore average value of seven methods were used for S-N relationship. Lower values of the shape parameter for fatigue life were obtained for RCA with reference to NA concrete, thus indicating higher variability in the distribution of flexural fatigue life of RCA. Further, higher shape parameters are obtained at stress levels $S$=0.85, which demonstrates a less scatter in the fatigue life data in turn indicating a more uniform damage mechanism. Using two-parameter Weibull distribution, the fatigue life for RCA and NA concrete at different survival probability was determined thereafter incorporating them in single and double logarithm equations for evaluating the coefficients of the fatigue equations. These fatigue equations become handy for the design engineers to find the fatigue strength of RCA and NA concrete at required level of failure probability. As a whole, the fatigue performance of NA concrete is better than RCA.

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