Computers and Concrete, *Vol. 21, No. 1 (2018) 67-74*
DOI: https://doi.org/10.12989/cac.2018.21.1.067

The effective properties of saturated concrete healed by EDM with the ITZs

Qing Chen[1,2], Zhengwu Jiang[*1], Hehua Zhu[3,4], J.W. Ju[5], Zhiguo Yan[3,4] and Haoxin Li[1]

[1]*Key Laboratory of Advanced Civil Engineering Materials of Ministry of Education, Tongji University, 4800 Cao'an Road, Shanghai 201804, China*
[2]*Shaanxi Provincial Major Laboratory for Highway Bridge and Tunnel, Chang'an University, Xi'an, Shaanxi 710064, China*
[3]*State Key Laboratory for Disaster Reduction in Civil Engineering, Tongji University, 1239 Siping Road, Shanghai 200092, China*
[4]*Key Laboratory of Geotechnical and Underground Engineering of the Ministry of Education, Tongji University, 1239 Siping Road, Shanghai 200092, China*
[5]*Department of Civil and Environmental Engineering, University of California, Los Angeles, CA 90095, USA*

(Received September 8, 2016, Revised November 9, 2017, Accepted November 14, 2017)

Abstract. A differential scheme based micromechanical framework is proposed to obtain the effective properties of the saturated concrete repaired by the electrochemical deposition method (EDM) considering the interfacial transition zone (ITZ) effects. The constituents of the repaired concrete are treated as different phases, consisting of (micro-)cracks, (micro-)voids and (micro-)pores (occupied by water), deposition products, intrinsic concrete made up by the three traditional solid phases (i.e., mortar, coarse aggregates and their interfaces) and the ITZs. By incorporating the composite sphere assemblage (CSA) model and the differential approach, a new multilevel homogenization scheme is utilized to quantitatively estimate the mechanical performance of the repaired concrete with the ITZs. The CSA model is modified to obtain the effective properties of the equivalent particle, which is a three-phase composite made up of the water, deposition products and the ITZs. The differential scheme is employed to reach the equivalent composite of the concrete repaired by EDM considering the ITZ effects. Moreover, modification procedures considering the ITZ effects are presented to attain the properties of the repaired concrete in the dry state. Results in this study are compared with those of the existing models and the experimental data. It is found that the predictions herein agree better with the experimental data than the previous models.

Keywords: electrochemical deposition; healing; saturated concrete; interfacial transition zone; effective properties; differential scheme; micromechanical framework

1. Introduction

The (micro-)cracks, (micro-)voids and (micro-)pores typically occur in concrete members and structures due to the influences of the surrounding environment and seriously cause damage to concrete's strength, moduli and durability (Ju and Lee 1991, Zhu *et al.* 2014, Jiang, *et al.* 2015). As a promising repairing approach for concrete in aqueous environment, the electrochemical deposition method (EDM) has been applied to marine structures and other situations in which traditional repairing methods are limited (Yokoda and Fukute 1992, Sasaki and Yokoda 1992, Jiang *et al.* 2015). Over the past 20 years, many experimental studies have been published on EDM (Ryu 2003a, Ryu and Otsuki 2005, Chang *et al.* 2009, Otsuki and Ryu 2001, Ryu 2003b, Chu and Jiang 2009, Otsuki *et al.* 1999, Ryu and Otsuki 2002, Jiang *et al*. 2008, Chen 2014). However, there are few theoretical models pertaining to mechanical properties of concrete during the EDM healing process. Therefore the authors present micromechanical frameworks to obtain the effective linear elastic properties of saturated concrete repaired by EDM (Zhu *et al.* 2014, Chen *et al.* 2015a, b). It should be mentioned that the interfaces between the deposition products and the concrete are assumed to be well bonded in their preliminary studies (Zhu *et al.* 2014, Chen *et al.* 2015a, b) and the effective properties of repaired concrete are predicted by Mori-Tanaka method, whose accuracy will decrease when the volume fraction of the inclusion phase increases (Yan *et al.* 2013, Zhu *et al.* 2014, Chen *et al.* 2015a).

As disclosed by many experiments, there are interfacial transition zones (ITZs) between the deposition product and the concrete when the EDM is applied to repair the deteriorated concrete. Meanwhile the ITZ properties are influenced by many factors, like the solution type and the current density (Otsuki and Ryu 2001, Ryu 2003b, Chu and Jiang 2009, Jiang *et al.* 2004, Jiang *et al.* 2008, Chen 2014). In this paper, an improved micromechanical approach is proposed to represent the microstructures of the saturated concrete repaired by the EDM with the ITZs. Meanwhile, differential scheme based multilevel homogenization procedures are employed to predict the effective properties of repaired concrete considering the ITZ effects. Specifically, the composite sphere assemblage (CSA) model is modified to obtain the effective properties of the equivalent particle, which is a three-phase composite made up of the water, the deposition products and the ITZs. The differential scheme is employed to reach the equivalent composite of the concrete repaired by EDM considering the ITZs. Moreover, modification procedures are presented to

∗Corresponding author, Professor
E-mail: m2ludear@126.com

http://www.techno-press.org/?journal=cac&subpage=8
ISSN: 1598-8198 (Print), 1598-818X (Online)

reach the properties of the repaired concrete with the ITZs in the dry state.

An outline of this paper is as follows. Section 2 illustrates the differential approach for two-phase composite. In section 3, a micromechanical framework is presented for the healed concrete with the ITZs. Specifically, an improved micromechanical model is proposed to represent the microstructures of the saturated concrete repaired by the EDM with the ITZs. New multilevel homogenization procedures are proposed to estimate the material's effective properties through incorporating the CSA model and differential scheme. Meanwhile, micromechanical procedures considering the ITZ effects are performed to modify the properties of repaired concrete in the dry state. Numerical examples including experimental validations and comparisons with existing micromechanical models are presented in section 4. And some conclusions are reached in the final section.

2. The differential approach for two-phase composite

2.1 The effective properties of the two-phase composite

By introducing the volume-average strain $\bar{\boldsymbol{\varepsilon}}$ and stress $\bar{\boldsymbol{\sigma}}$, the effective elastic stiffness tensor **D** of the two-phase composite can be defined through (Ju and Chen 1994a)

$$\bar{\boldsymbol{\sigma}} = \mathbf{D} : \bar{\boldsymbol{\varepsilon}} \tag{1}$$

With

$$\bar{\boldsymbol{\sigma}} \equiv \frac{1}{V}\int_V \boldsymbol{\sigma}(\mathbf{x})d\mathbf{x} = \frac{1}{V}\left[\int_{V_m} \boldsymbol{\sigma}(\mathbf{x})d\mathbf{x} + \int_{V_i} \boldsymbol{\sigma}(\mathbf{x})d\mathbf{x}\right] \tag{2}$$

$$\bar{\boldsymbol{\varepsilon}} \equiv \frac{1}{V}\int_V \boldsymbol{\varepsilon}(\mathbf{x})d\mathbf{x} = \frac{1}{V}\left[\int_{V_m} \boldsymbol{\varepsilon}(\mathbf{x})d\mathbf{x} + \int_{V_i} \boldsymbol{\varepsilon}(\mathbf{x})d\mathbf{x}\right] \tag{3}$$

where V is the volume of representative volume element (RVE), V_m is the volume of the matrix, V_i is the volume of the inhomogeneity.

2.2 The differential scheme

In terms of the inclusion-based micromechanical theory and the average stress method (Qu and Cherkaoui 2006, Mura 1987, Ju and Chen 1994a, b), the effective elastic stiffness tensor of the two-phase composite can be rephrased as Eq. (4)

$$\mathbf{D} = \mathbf{D}_0 + \phi(\mathbf{D}_I - \mathbf{D}_0)\mathbf{A} \tag{4}$$

where $\mathbf{D_0}$ is the elastic stiffness tensor of the matrix phase, $\mathbf{D}_I$ is the elastic stiffness tensor of the inhomogeneity, **A** is the strain concentration tensor for the inhomogeneity; V_m denotes the volume fraction of the inhomogeneity.

Let us define $\phi = \Omega_1/(\Omega_0+\Omega_1)$ and $\phi+\Delta\phi = (\Omega_1+\Delta\Omega)/(\Omega_0+\Omega_1+\Delta\Omega)$, where Ω_0 and Ω_1 represent the volume of the matrix phase and the inclusion phase in the current composite, respectively; $\Delta\Omega$ denotes the increment of inclusion volume. For the differential method, a composite with the volume fraction of inclusion equal to $\phi+\Delta\phi$, can be treated as the equivalent composite with the volume fraction of inclusion equal to $\Delta\Omega/(\Omega_0+\Omega_1+\Delta\Omega)$. It is noted that the matrix phase in the equivalent composite is the current composite, which includes the current matrix (Ω_0) and current inclusion (Ω_1). According to Eq. (4), the effective properties of the material can be obtained (McLaughlin 1977, Norris 1985, Mura 1987, Qu and Cherkaoui 2006)

$$\mathbf{D}(\phi+\Delta\phi) = \mathbf{D}(\phi) + \frac{\Delta\Omega}{(\Omega_0+\Omega_1+\Delta\Omega)}(\mathbf{D}_I - \mathbf{D}(\phi))\mathbf{A}(\mathbf{D}(\phi)) \tag{5}$$

Eq. (5) can be rephrased as below through the simple derivations

$$\frac{\mathbf{D}(\phi+\Delta\phi)-\mathbf{D}(\phi)}{\Delta\phi} = \frac{1}{1-\phi}(\mathbf{D}_I - \mathbf{D}(\phi))\mathbf{A}(\mathbf{D}(\phi)) \tag{6}$$

With

$$\Delta\phi = \frac{\Omega_1+\Delta\Omega}{\Omega_0+\Omega_1+\Delta\Omega} - \frac{\Omega_1}{\Omega_0+\Omega_1} = \frac{(1-\phi)\Delta\Omega}{(\Omega_0+\Omega_1+\Delta\Omega)} \tag{7}$$

When $\Delta\phi \to 0$, Eq. (6) can be expressed as

$$\frac{d\mathbf{D}(\phi)}{d\phi} = \frac{1}{1-\phi} \bullet (\mathbf{D}_I - \mathbf{D}(\phi)) : \mathbf{A}(\mathbf{D}(\phi)) \tag{8}$$

The composite effective properties with no inclusion effects should be the same as those of the matrix phase, which implies

$$\mathbf{D}(\phi)\big|_{\phi=0} = \mathbf{D}_0 \tag{9}$$

When the Eshelby method is considered, we write

$$\mathbf{A} = \left[\mathbf{I} + \mathbf{S}\mathbf{D}(\phi)^{-1}(\mathbf{D}_I - \mathbf{D}(\phi))\right]^{-1} \tag{10}$$

where **S** is Eshelby's tensor, which depends on $\mathbf{D}(\phi)$ and the shape of the inclusions; **I** defines the fourth-order isotropic identity tensor, whose components can be represented as $I_{ijkl} = \frac{1}{3}\delta_{ij}\delta_{kl} + \frac{1}{2}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl})$, where δ_{ij} is the Kronecker delta.

3. Micromechanical framework for saturated concrete repaired using EDM with the ITZs

3.1 Micromechanical model for the saturated concrete healed using EDM with the ITZs

According to the previous studies (Zhu *et al.* 2014, Chen *et al.* 2015a, b), the (micro-)cracks, (micro-)voids and (micro-)pores in the concrete can be assumed to be saturated for simplicity since the EDM is adopted in the aqueous environment. Meanwhile the three traditional solid phases (i.e., mortar, coarse aggregates and their interfaces) can be merged into one matrix phase, namely intrinsic concrete, in representative volume element (RVE) (Zhu *et al.* 2014). Different from the previous work (Zhu *et al.*

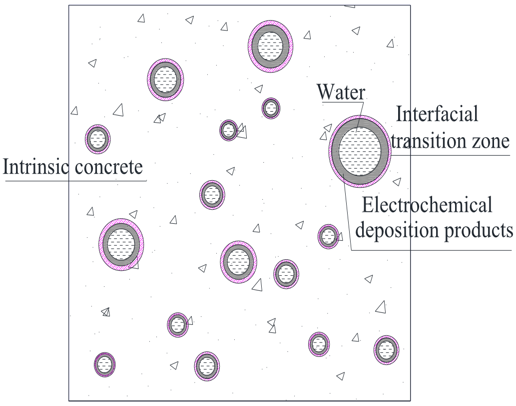


Fig. 1 Micromechanical model for saturated concrete healed using EDM with interfacial transition zones (ITZs)

2014, Chen *et al.* 2015a, b), the ITZs between the deposition products and concrete are incorporated in the present study. Therefore, in this extension the repaired concrete is described as a multiphase composite composed by water, deposition products, intrinsic concrete and ITZs, as shown in Fig. 1. The matrix phase is the intrinsic concrete and the inclusions are made of water, deposition products and the ITZs (Zhu *et al.* 2014, Chen *et al.* 2015a, b).

To obtain the effective properties of the repaired concrete, a new multilevel micromechanical homogenization scheme is proposed through incorporating the CSA model and the differential scheme based on the previous work (Li *et al.* 1999, Nguyen *et al.* 2011, Ju and Zhang 1998, Ju and Sun 1999, Ju and Sun 2001, Ju and Yanase 2010, Ju and Yanase 2011, Sun and Ju 2001, Sun and Ju 2004, Yanase and Ju 2012, Zhu *et al.* 2014, Zhu *et al.* 2015, Yan *et al.* 2013, Chen *et al.* 2015c, Yoo and Banthia 2015, Gal and Kryvoruk 2011, Pichler and Lackner 2008, Chen *et al.* 2016a, b, c, Mousavi Nezhad *et al.* 2016, Chen *et al.*2017). Specifically, by modifying the CSA model, the effective properties of the equivalent particle are calculated with the first and second level homogenizations to a three-phase composite composed by the water, the deposition products, and the ITZs. The equivalent homogeneous composite of the repaired concrete is attained with the third level homogenization by adopting the differential scheme.

3.2 The effective properties of the equivalent inclusion composed of the water and the deposition products

According to Hashin (1962), the bulk modulus of the equivalent inclusion composed of water and deposition product can be obtained with CSA model using following expressions

$$K_F = K_2 + \frac{\phi_{Fw}(K_1 - K_2)(3K_2 + 4\mu_2)}{3K_2 + 4\mu_2 + 3(1-\phi_{Fw})(K_1 - K_2)} \tag{11}$$

$$\phi_{Fw} = \frac{V_{wat}}{V_{wat} + V_{dep}} \tag{12}$$

Where K_1, K_2 and K_F denote the bulk modulus of the water and the deposition products and the equivalent inclusions, respectively. μ_2 is the shear modulus of the deposition product; ϕ_{Fw} is the volume fraction of the water phase in the two-phase composite composed by the water and the deposition products, V_{wat} represents the volume of the water and V_{dep} signifies the volume of the deposition products.

By replacing the inner and outer material as the water and deposition product, the effective shear modulus of the equivalent inclusion can be reached by modifying the results of Smith (1974, 1975) for the CSA model as follows

$$\alpha\left(\frac{\mu_F}{\mu_2} - 1\right)^2 + \beta\left(\frac{\mu_F}{\mu_2} - 1\right) + \gamma = 0 \tag{13}$$

Where

$$\begin{aligned}\alpha = &\left[4P(7-10\nu_2) - S\phi_{Fw}^{7/3}\right]\left[Q - (8-10\nu_2)(M-1)\phi_{Fw}\right] \\ &-126P(M-1)\phi_{Fw}\left(1-\phi_{Fw}^{2/3}\right)^2\end{aligned} \tag{14}$$

$$\begin{aligned}\beta = &35(1-\nu_2)P\left[Q - (8-10\nu_2)(M-1)\phi_{Fw}\right] \\ &-15(1-\nu_2)\left[4P(7-10\nu_2) - S\phi_{Fw}^{7/3}\right](M-1)\phi_{Fw}\end{aligned} \tag{15}$$

$$\gamma = -525P(1-\nu_2)^2(M-1)\phi_{Fw} \tag{16}$$

With

$$M = \mu_1 / \mu_2 \tag{17}$$

$$P = (7+5\nu_1)M + 4(7-10\nu_1) \tag{18}$$

$$Q = (8-10\nu_2)M + (7-5\nu_2) \tag{19}$$

$$S = 35(7+5\nu_1)M(1-\nu_2) - P(7+5\nu_2) \tag{20}$$

where μ_1 and μ_F respectively denote the shear modulus of water and equivalent inclusion, ν_1 and ν_2 are the Poison's ratio of water and deposition product, respectively.

3.3 The effective properties of the equivalent particle composed of the ITZ and the equivalent inclusion

By respectively replacing K_1, K_2, μ_2, ϕ_{Fw} and K_F in Eqs. (11)-(12) with K_F, K_{itz}, μ_{itz}, ϕ_{Se} and K_S, the bulk modulus of equivalent particle composed of the ITZ and the equivalent inclusion can be calculated. Here K_{itz} and μ_{itz} represent the bulk modulus and shear modulus of the ITZ; K_S signifies the bulk modulus of the equivalent particles made up of the ITZ and the equivalent inclusion; ϕ_{Se} is the volume fraction of the equivalent inclusion in the equivalent particle, which can be defined as below

$$\phi_{Se} = \frac{V_{wat} + V_{dep}}{V_{itz} + V_{dep} + V_{wat}} \tag{21}$$

where V_{itz} is the volume of the deposition product.

To obtain the effective shear modulus of the equivalent particle, μ_1, μ_2, ν_1, ν_2, ϕ_{Fw} and μ_F in Eqs.(13)-(20) should be replaced by μ_F, μ_{itz}, ν_F, ν_{itz}, ϕ_{Se} and μ_S, respectively. Here, μ_S

is the effective shear modulus of the equivalent particle; v_{itz} is the Poison's ratio of ITZ; v_F is the Poison's ratio of the equivalent inclusion, which can be obtained with the effective bulk modulus and shear modulus according to the theorem of elastic mechanics as below

$$\nu_F = 0.5\left(1-\frac{1}{\frac{1}{3}+\frac{K_F}{\mu_F}}\right) \tag{22}$$

3.4 The effective properties of the equivalent homogenous composite composed of the equivalent particle and the intrinsic concrete

For the two-phase composite composed by the equivalent particle and the intrinsic concrete as the matrix phase, the tensorial components of Eshelby tensor S can be expressed as

$$S_{ijkl} = \frac{K_3}{3K_3+4\mu_3}\delta_{ij}\delta_{kl} + \frac{3(K_3+2\mu_3)}{5(3K_3+4\mu_3)}(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}-\frac{2}{3}\delta_{ij}\delta_{kl}) \tag{23}$$

where K_3, μ_3 are respectively the bulk modulus and the shear modulus of the intrinsic concrete.

Let $\mathbf{D}_S$, $\mathbf{D}_3$ and $\mathbf{D}_T$ represent the stiffness tensor of the equivalent particle, the intrinsic concrete and the equivalent composite of the saturated concrete repaired by EDM with the ITZs. The components for these tensors can be read as below

$$D_{Sijkl} = K_S\delta_{ij}\delta_{kl} + \mu_S(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}-\frac{2}{3}\delta_{ij}\delta_{kl}) \tag{24}$$

$$D_{3ijkl} = K_3\delta_{ij}\delta_{kl} + \mu_3(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}-\frac{2}{3}\delta_{ij}\delta_{kl}) \tag{25}$$

$$D_{Tijkl} = K_T\delta_{ij}\delta_{kl} + \mu_T(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}-\frac{2}{3}\delta_{ij}\delta_{kl}) \tag{26}$$

where $\mathbf{K}_T$, μ_T are the bulk modulus and the shear modulus of the equivalent composite of the concrete repaired by EDM with the ITZs, correspondingly.

By substituting Eqs. (23)-(26) into Eqs. (8)-(10), the effective bulk modulus and shear modulus of the concrete repaired by EDM with the ITZs can be calculated by solving the following nonlinear ordinary differential equations after some derivations

$$\frac{dK_T}{d\phi_T} + \frac{(K_T-K_S)(3K_T+4\mu_T)}{(1-\phi_T)(3K_S+4\mu_T)} = 0 \tag{27}$$

$$\frac{d\mu_T}{d\phi_T} + \frac{5\mu_T(\mu_T-\mu_S)(3K_T+4\mu_T)}{(1-\phi_T)\left[3K_T(3\mu_T+2\mu_S)+4\mu_T(2\mu_T+3\mu_S)\right]} = 0 \tag{28}$$

$$\phi_T = \frac{V_{wat}+V_{dep}+V_{itz}}{V_{wat}+V_{dep}+V_{itz}+V_{mat}} = \frac{V_{wat}+V_{dep}+V_{itz}}{V_{tot}} \tag{29}$$

with the initial conditions as below

$$K_T(0) = K_3 \tag{30}$$

$$\mu_T(0) = \mu_3 \tag{31}$$

where V_{mat} and V_{tot} means the volume of the intrinsic concrete matrix and the total volume of the composite. With regard to the effect of water viscosity in pores of the saturated concrete (Wang and Li 2007, Jiang *et al.* 2005, 2006, 2013), μ_T should be multiply the modification function as below (Wang and Li 2007)

$$F = 1 + f_1\phi_T^{\,2} + f_2\phi_T \tag{32}$$

where f_1 and f_2 are parameters investigated by the experiment (Wang and Li 2007).

Furthermore, the Young's modulus of repaired concrete can be obtained based on the theorem of elastic mechanics, provided that the bulk modulus and shear modulus are known

$$E_T = \frac{9K_T\mu_T}{3K_T+\mu_T} \tag{33}$$

where E_T is the Young's modulus of the equivalent homogenous composite (i.e., the healed concrete).

3.5 Modifications to estimations of effective properties in dry conditions

When the properties of the concrete repaired using EDM in the dry state is considered, three modification coefficients, χ_K, χ_μ and χ_E are introduced to modify the predicting results of the proposed model, apart from replacing the properties of water with those of air according to our previous work (Zhu *et al.* 2014). Since the pores should not be assumed to be spherical in the dry state, χ_K, χ_μ and χ_E are employed to reflect the influence of the crack (pore) shape in the following

$$\chi_K = \frac{K_\alpha^*}{K_{\alpha=1}^*} \tag{34}$$

$$\chi_\mu = \frac{\mu_\alpha^*}{\mu_{\alpha=1}^*} \tag{35}$$

$$\chi_E = \frac{E_\alpha^*}{E_{\alpha=1}^*} \tag{36}$$

$$\alpha = \frac{1}{N}\sum_{i=1}^{N}\frac{a_i}{b_i} \tag{37}$$

where $K_{\alpha=1}^*$, $\mu_{\alpha=1}^*$ and $E_{\alpha=1}^*$ are the predicted effective bulk modulus, effective shear modulus and Young's modulus, respectively, when the crack (pore) shape is spherical (α=1). Moreover, K_α^*, μ_α^* and E_α^* are the predicted effective bulk modulus, effective shear modulus and Young's modulus, respectively, when the pore shape is not spherical (α<1) according to Berryman (1980); α is the equivalent aspect ratio of the pores; a_i and b_i are the lengths of the pores' minor and major axes, respectively; and N is the number of different pores in the concrete. We refer to Zhu *et al.* (2014) for details. However, due to the ITZ effects, K_2

Table 1 Three types of ITZs between the deposition products and the concrete, with ϕ_{itz} representing the volume fraction of ITZ in the equivalent particle

	Bulk modulus (GPa)	Shear modulus (GPa)	ϕ_{itz}
Type 1	18.61	12.3	0.1
Type 2	18.61*0.4	12.3*0.4	0.1
Type 3	18.61*0.4	12.3*0.4	0.3

and μ_2 in Eqs.(37)-(43) of Zhu *et al.*(2014) should be replaced by K_{ave} and μ_{ave}, which are obtained by the following expressions

$$K_{ave}=0.5\left(\phi_{Md}K_2+\phi_{Mi}K_{itz}+\left(1-\phi_{Md}-\phi_{Mi}\right)K_3\right)+0.5\left(1/\left(\frac{\phi_{Md}}{K_2}+\frac{\phi_{Mi}}{K_{itz}}+\frac{1-\phi_{Md}-\phi_{Mi}}{K_3}\right)\right) \tag{38}$$

$$\mu_{ave}=0.5\left(\phi_{Md}\mu_2+\phi_{Mi}\mu_{itz}+\left(1-\phi_{Md}-\phi_{Mi}\right)\mu_3\right)+0.5\left(1/\left(\frac{\phi_{Md}}{\mu_2}+\frac{\phi_{Mi}}{\mu_{itz}}+\frac{1-\phi_{Md}-\phi_{Mi}}{\mu_3}\right)\right) \tag{39}$$

$$\phi_{Md}=\frac{V_{dep}}{V_{dep}+V_{itz}+V_{mat}} \tag{40}$$

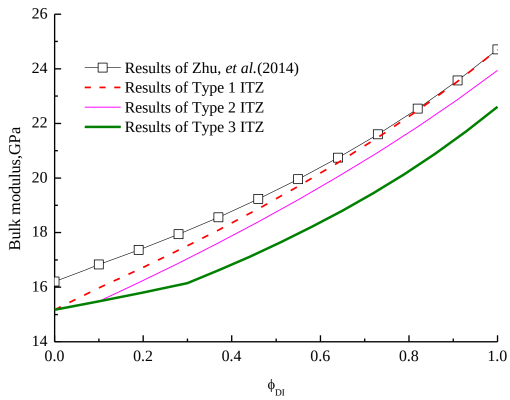
$$\phi_{Mi}=\frac{V_{itz}}{V_{dep}+V_{itz}+V_{mat}} \tag{41}$$

4. Verification and discussion

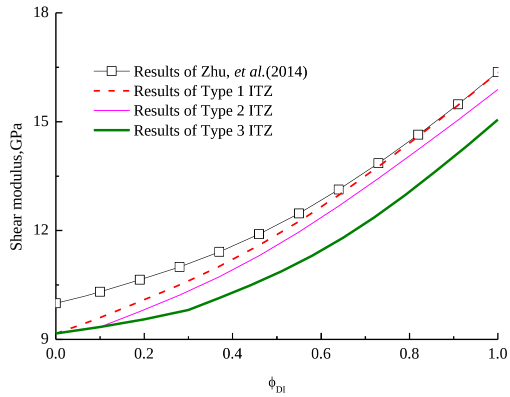
4.1 Comparison with the existing model and experiments during the healing process

The exact values for the ITZ thickness and properties are not the main interest of this study. Instead we focus on the quantitative effects of a given ITZ on the healing effectiveness of EDM. Therefore, three different types of ITZs, which are listed in Table 1, are utilized as examples to perform the simulations. The thicknesses or volume fractions are the same and the properties are different when the first and second types of ITZs are considered. As to the second and third types of ITZs, they have the same properties but different thicknesses. The properties of intrinsic concrete and deposition products are from Zhu *et al.* (2014). Fig. 2(a) presents the comparisons of the shear modulus among predictions of the equivalent particle with different types of ITZs. It can be found that the predictions of Zhu *et al.* (2014) can be reached using the proposed framework in this study with the perfect bonding (i.e., the properties of ITZ are equal to those of the deposition products). When the properties of ITZ decrease, the equivalent particle demonstrates lower shear modulus. With the increase of volume fractions of the ITZ, the values decrease for the predicted shear modulus of the equivalent particle. Similar conclusion can be reached when the effective bulk modulus of the equivalent particle is considered, which is shown in Fig. 2(b).

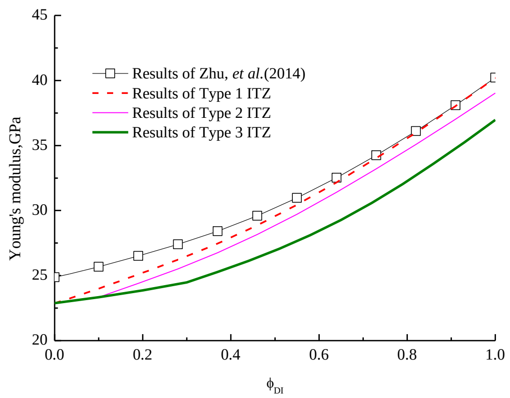
Fig. 3(a) presents the variations of the predicted bulk modulus of the repaired concrete during the healing process with different types of ITZ. It can be found that the



(a) The bulk modulus



(b)The shear modulus



(c)The Young's modulus

Fig. 3 Comparisons among predictions for the effective properties using different models

estimations of Zhu *et al.* (2014) are very near to the results in this study when the interfacial bonds are perfect. The maximum relative difference is less than 7% between the results of Zhu *et al.* (2014) and those in this study. With the increase of the ITZ properties, the repaired concrete demonstrates greater bulk modulus. However, the effective properties of the repaired concrete decrease when the volume fraction of the ITZ turns higher. Meanwhile, the properties of the repaired concrete all increase during the healing process with different type of ITZs. As to the effective shear modulus and Young's modulus, the similar conclusions can be reached, as shown in Fig. 3(b) and Fig. 3(c).

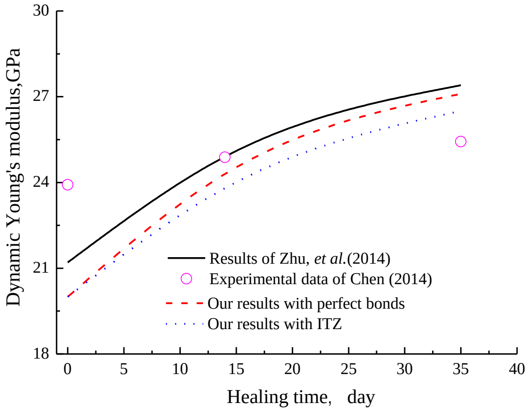


Fig. 4 Comparison between the results obtained with different models and those obtained experimentally (Chen, *et al.* 2014) for the dynamic Young's modulus in the dry state

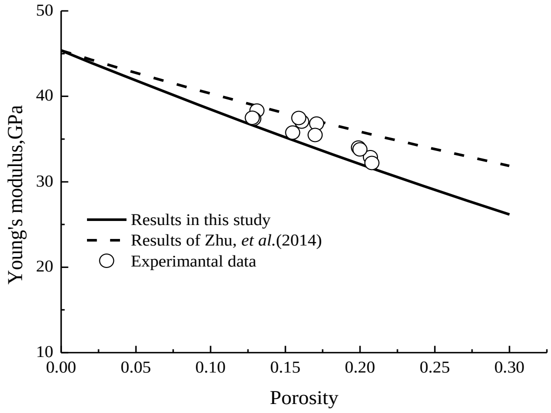


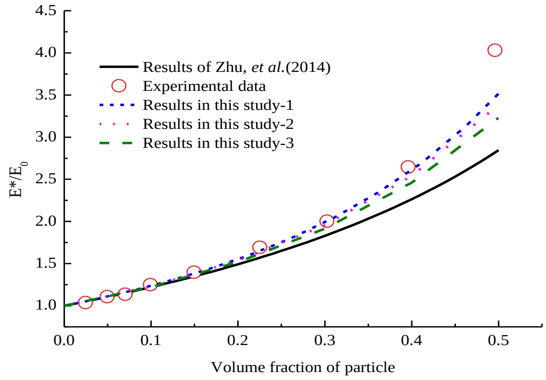
Fig. 5 The comparison among results in this study, those in Zhu *et al.* (2014) and those obtained experimentally (Yaman, *et al.* 2002) for the Young's modulus of saturated concrete

Fig. 4 displays the comparisons among results obtained with different models and experimental data. Here, the dynamic Young's moduli in the dry state of the specimen before and after healing in Chen's experiment (Chen 2014) are adopted to validate the proposed micromechanical model. The average initial porosity of the specimen is 0.299. The average pulse-velocity of its intrinsic concrete is 5134.5 m/s. The density is 2537.9 kg/m^3 and the Poisson's ratio is 0.229. It can be observed that predictions with different models meet well with the experimental data. Still the predictions herein are a little less than those of Zhu *et al.* (2014) when the perfect bonding assumption is adopted. Meanwhile, the ITZ influence can be quantitatively estimated with the proposed model.

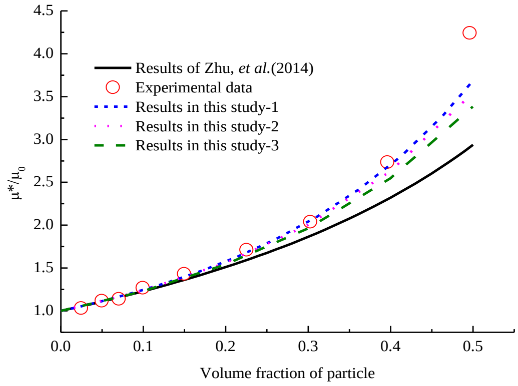
4.2 Comparison with the existing model and experiments at extreme states

The first extreme state is that there is absolutely no healing process in the concrete. The repaired concrete turns to the saturated concrete (i.e., the damaged zone is occupied by water). Fig. 5 exhibits the comparisons among results reached by different models and the experimental data of Yaman *et al.* (2002). It can be found that the predictions of Zhu *et al.* (2014) are close to those in this study. The estimations with different models respond the experimental data reasonably. Moreover, the results herein agree better with the experimental data with the increase of porosity.

The experimental data of Smith (1976) are employed to verify the second extreme state, which is that the concrete has been completely healed by the EDM. The properties of the matrix and the particles are E_0=3.0 GPa, v_0=0.4 and E_1=76 GPa, v_1=0.23, respectively (Smith 1976). Three different ITZ properties (E_{itz}/E_1=1, 0.4, 0.2 and $v_{itz}=v_1$) are adopted to perform the simulations herein. Fig. 6(a) presents the estimations of Young's modulus with different models and the experimental data. The predicted results are close to each other and agree well with the experimental data when the particle volume fraction is low for different micromechanical models. With the increase of particle volume fraction, the predicting results in this study meet the



(a)The Young's modulus



(b)The shear modulus

Fig. 6 The comparison among the results obtained with the different micromechanical models and those obtained experimentally (Smith 1976) for the effective Young's modulus

experimental data better than those of Zhu *et al.* (2014). Furthermore, the interfacial effects can be calculated quantitatively with the proposed model in this study. With the decrease of ITZ properties, the composite demonstrate lower effective properties. According to Fig. 6(b), the predicting results for the effective shear modulus herein are still near to those of Zhu *et al.* (2014) and respond better to the experimental data than those of Zhu *et al.* (2014) when the particle volume fraction is higher. Similarly, the quantitative effects of ITZs on the effective properties can be reached with our proposed micromechanical framework.

5. Conclusions

Since the interfacial bonds are not perfect when the EDM is utilized to repair the deteriorated concrete, a multiphase micromechanical framework is proposed for saturated concrete repaired by EDM considering the ITZ effects in this study. With different ITZ thicknesses and properties, the imperfect bonding effects are characterized quantitatively. By incorporating the CSA model and the differential approach, a new multilevel homogenization scheme is utilized to estimate the mechanical performance of the repaired concrete. Specifically, the CSA model is modified to obtain the effective properties of the equivalent particle, which is a three-phase composite made up of the water, the deposition product and the ITZs. The differential scheme is employed to reach the equivalent composite of the concrete repaired by EDM considering the imperfect interfacial bonds. Moreover, modification procedures considering the ITZ effects are presented to attain the properties of repaired concrete in the dry state. Results in this study are compared with those of the existing models and the experimental data. It is found that the proposed micromechanical framework is both feasible and capable of describing the mechanical performance of the saturated concrete during the EDM healing process with the imperfect bonds. Both of the properties and volume fractions of the ITZs play important roles in determining the effective properties of the equivalent particle and repaired concrete. For special cases, the proposed micromechanical framework can predict the properties of saturated concrete and particle reinforce composite with imperfect bonds. When the volume fraction of the inclusion is higher, the results in this study correspond better with the experimental data than those of previous models.

Acknowledgements

This work is supported by the National Key Basic Research and Development Program (973 Program, No. 2011CB013800) and National Natural Science Foundation of China (51508404, 51478348, 51278360, 51308407; U1534207). This work is also supported by Key Laboratory of Advanced Civil Engineering Materials (Tongji University), State Key Laboratory of High Performance Civil Engineering Materials (No. 2015CEM008), Program of Shanghai Science and Technology Commission (15DZ1205003), the 1000 Talents Plan Short-Term Program by the Organization Department of the Central Committee of the CPC, Research Program of State Key Laboratory for Disaster Reduction in Civil Engineering, the Scientific Platform Open Funds of Fundamental Research Plan for the Central Universities, Chang'an University (310821151113) and Natural Science Foundation of Shandong Province (ZR2013EEL019). The authors declare that they have no conflict of interest.

References

Berryman, J.G. (1980), "Long-wave propagation in composite elastic media II. Ellipsoidal inclusion", *J. Acoust. Soc. Am.*, **68**(6), 1820-1831.

Chang, J.J., Yeih, W.C., Hsu, H.M. and Huang, N.M. (2009), "Performance evaluation of using electrochemical deposition as a repair method for reinforced concrete beams", *Struct. Longev.*, **1**(2), 75-93.

Chen, Q. (2014), "The stochastic micromechanical models of the multiphase materials and their applications for the concrete repaired by electrochemical deposition method", Ph.D. Dissertation Tongji Univ., Shanghai.

Chen, Q., Jiang, Z.W., Yang, Z.H., Zhu, H.H., Ju, J.W., Yan, Z.G. and Wang, Y.Q. (2016a), "Differential-scheme based micromechanical framework for saturated concrete repaired by the electrochemical deposition method", *Mater. Struct.*, **49**(12), 5183-5193.

Chen, Q., Mousavi Nezhad, M., Fisher, Q. and Zhu, H.H. (2016b), "Multi-scale approach for modeling the transversely isotropic elastic properties of shale considering multi-inclusions and interfacial transition zone", *Int. J. Rock Mech. Min.*, **84**, 95-104.

Chen, Q., Zhu, H.H., Ju, J.W., Guo, F., Wang, L.B., Yan, Z.G., Deng, T. and Zhou, S. (2015c), "A stochastic micromechanical model for multiphase composite containing spherical inhomogeneities", *Acta Mech.*, **226**(6), 1861-1880.

Chen, Q., Zhu, H.H., Ju, J.W., Jiang, Z.W., Yan, Z.G. and Li, H.X. (2017), "Stochastic micromechanical predictions for the effective properties of concrete considering the interfacial transition zone effects", *Int. J. Damage Mech.*, DOI: 10.1177/1056789517728501.

Chen, Q., Zhu, H.H., Yan, Z.G., Deng, T. and Zhou, S. (2015a), "Micro-scale description of the saturated concrete repaired by electrochemical deposition method based on Mori-Tanaka method", *J. Build Struct.*, **36**(1), 98-103.

Chen, Q., Zhu, H.H., Yan, Z.G., Ju, J.W., Deng, T. and Zhou, S. (2015b), "Micro-scale description of the saturated concrete repaired by electrochemical deposition method based on self-consistent method", *Chin. J. Theor. Appl. Mech.*, **47**(2), 367-371.

Chen, Q., Zhu, H.H., Yan, Z.G., Ju, J.W., Jiang, Z.W. and Wang, Y.Q. (2016c), "A multiphase micromechanical model for hybrid fiber reinforced concrete considering the aggregate and ITZ effects", *Constr. Build. Mater.*, **114**, 839-850.

Chu, H.Q. and Jiang, L.H. (2009), "Correlation analysis between concrete parameters and electrodeposition effect based on grey theory", *J. Wuhan Univ. Technol.*, **31**(7), 22-26.

Gal, E. and Kryvoruk, R. (2011), "Fiber reinforced concrete properties-a multiscale approach", *Comput. Concrete*, **8**(5), 525-539.

Hashin, Z. (1962), "The elastic moduli of heterogeneous materials", *J. Appl. Mech.*, **29**, 143-150.

Jiang, Z.W, Sun, Z.P. and Wang, P.M. (2005), "Autogenous relative humidity change and autogenous shrinkage of high-performance cement pastes", *Cement Concrete Res.*, **35**(8), 1539-1545

Jiang, Z.W, Sun, Z.P. and Wang, P.M. (2006), "Internal relative humidity distribution in cement paste due to moisture diffusion and self-desiccation", *Cement Concrete Res.*, **36**(2), 320-325.

Jiang, Z.W., Li, W.T. and Yuan, Z.C. (2015), "Influence of mineral additives and environmental conditions on the self-healing capabilities of cementitious materials", *Cement Concrete Compos.*, **57**, 116-127.

Jiang, Z.W., Li, W.T., Deng, Z.L. and Yan, Z.G. (2013), "Experimental investigation of the factors affecting accuracy and resolution of the pore structure of cement-based materials by thermoporometry", *J. Zhejiang Univ-Sci.*, **14**(10), 720-730.

Jiang, Z.W., Xing, F., Sun, Z.P. and Wang, P.M. (2008), "Healing effectiveness of cracks rehabilitation in reinforced concrete using electrodeposition method", *J. Wuhan Univ. Technol.*, **23**(6), 917-922.

Ju, J.W. and Chen, T.M. (1994a), "Micromechanics and effective moduli of elastic composites containing randomly dispersed ellipsoidal inhomogeneities", *Acta Mech.*, **103**, 103-121.

Ju, J.W. and Chen, T.M. (1994b), "Effective elastic moduli of two-phase composites containing randomly dispersed spherical inhomogeneities", *Acta Mech.*, **103**, 123-144.

Ju, J.W. and Lee, X. (1991), "Micromechanical damage models for brittle solids. Part I: tensile loadings", *J. Eng. Mech.*, **117**, 1495-1514.

Ju, J.W. and Sun, L.Z. (1999), "A novel formulation for the exterior-point Eshelby's tensor of an ellipsoidal inclusion", *J. Appl. Mech-T.*, ASME, **66**(2), 570-574.

Ju, J.W. and Sun, L.Z. (2001), "Effective elastoplastic behavior of metal matrix composites containing randomly located aligned spheroidal inhomogeneities. Part I: micromechanics-based formulation", *Int. J. Solid. Struct.*, **38**(2), 183-201.

Ju, J.W. and Yanase, K. (2010), "Micromechanics and effective elastic moduli of particle-reinforced composites with near-field particle interactions", *Acta Mech.*, **215**(1), 135-153.

Ju, J.W. and Yanase, K. (2011), "Micromechanical effective elastic moduli of continuous fiber-reinforced composites with near-field fiber interactions", *Acta Mech.*, **216**(1), 87-103.

Ju, J.W. and Zhang, X.D. (1998), "Micromechanics and effective transverse elastic moduli of composites with randomly located aligned circular fibers", *Int. J. Solid. Struct.*, **35**(9-10), 941-960.

Li, G.Q., Zhao, Y. and Pang, S.S. (1999), "Four-phase sphere modeling of effective bulk modulus of concrete", *Cement Concrete Res.*, **29**, 839-845.

McLaughlin, R. (1977), "A study of the differential scheme for composite materials", *Int. J. Eng. Sci.*, **15**, 237-244.

Mousavi Nezhad, M., Zhu, H.H., Ju, J.W. and Chen, Q. (2016), "A simplified multiscale damage model for the transversely isotropic shale rocks under tensile loading", *Int. J. Damage Mech.*, **25**(5), 705-726.

Mura, T. (1987), *Micromechanics of Defects in Solids*, Martinus Nijhoff Publishers, the Netherlands.

Nguyen, N.B., Giraud, A. and Grgic, D. (2011), "A composite sphere assemblage model for porous oolitic rocks", *Int. J. Rock Mech. Min.*, **48**, 909-921.

Norris, A.N. (1985), "A differential scheme for the effective modulus of composites", *Mech. Mater.*, **4**, 1-16.

Otsuki, N. and Ryu, J.S. (2001), "Use of electrodeposition for repair of concrete with shrinkage cracks", *J. Mater. Civil Eng.*, **13**(2), 136-142.

Otsuki, N., Hisada, M., Ryu, J.S. and Banshoya, E.J. (1999), "Rehabilition of concrete cracks by electrodeposition", *Concrete Int.*, **21**(3), 58-62.

Pichler, C. and Lackner, R. (2008), "A multiscale creep model as basis for simulation of early-age concrete behavior", *Comput. Concrete*, **5**(4), 295-328.

Qu, J.M. and Cherkaoui, M. (2006), *Fundamentals of Micromechanics of Solids*, John Wiley & Sons, Inc., Hoboken, New Jersey.

Ryu, J.S. (2003a), "New waterproofing technique for leaking concrete", *J. Mater. Sci. Lett.*, **22**, 1023-1025.

Ryu, J.S. (2003b), "Influence of crack width，cover depth，water cement ratio and temperature on the formation of electrodeposition on the concrete surface", *Mag. Concrete Res.*, **55**(1), 35-40.

Ryu, J.S. and Otsuki, N. (2002), "Crack closure of reinforced concrete by electro deposition technique", *Cement Concrete Res.*, **32**(1), 159-264.

Ryu, J.S. and Otsuki, N. (2005), "Experimental study on repair of concrete structural members by electrochemical method", *Scripta Materialia*, **52**, 1123-1127.

Sasaki, H. and Yokoda, M. (1992), "Repair method of marine reinforced concrete by electro deposition technique", *Proceedings of the Annual Conference of Japanese Concrete Institute*, 849-854.

Smith, J.C. (1974), "Correction and extension of Van der Poel's method for calculating the shear modulus of a particulate composite", *J. Res. Nat. Bureau Stand. A. Phys. Chem.*, **78**(3), 355-361.

Smith, J.C. (1975), "Simplification of Van der Poel's formula for the shear modulus of a particulate composite", *J. Res. Nat. Bureau Stand. A. Phys. Chem.*, **79A**(2), 419-423.

Smith, J.C. (1976), "Experimental values for the elastic constants of a particulate-filled glassy polymer", *J. Res. NBS*, **80A**, 45-49.

Sun, L.Z. and Ju, J.W. (2001), "Effective elastoplastic behavior of metal matrix composites containing randomly located aligned spheroidal inhomogeneities. Part II: applications", *Int. J. Solid. Struct.*, **38**(2), 203-225.

Sun, L.Z. and Ju, J.W. (2004), "Elastoplastic modeling of metal matrix composites containing randomly located and oriented spheroidal particles", *J. Appl. Mech-T.*, ASME, **71**, 774-785.

Wang, H.L. and Li, Q.B. (2007), "Prediction of elastic modulus and Poisson's ratio for unsaturated concrete", *Int. J. Solid. Struct.*, **44**, 1370-1379.

Yaman, I.O., Hearn, N. and Aktan, H.M. (2002), "Active and non-active porosity in concrete part I: experimental evidence", *Mater. Struct.*, **35**(3), 102-109.

Yan, Z.G., Chen, Q., Zhu, H.H., Ju, J.W., Zhou, S. and Jiang, Z.W. (2013), "A multiphase micromechanical model for unsaturated concrete repaired by electrochemical deposition method", *Int. J. Solid. Struct.*, **50**(24), 3875-3885.

Yanase, K. and Ju, J.W. (2012), "Effective elastic moduli of spherical particle reinforced composites containing imperfect interfaces", *Int. J. Damage Mech.*, **21**(1), 97-127.

Yokoda, M. and Fukute, T. (1992), "Rehabilitation and protection of marine concrete structure using electrodeposition method", *Proceedings of the International RILEM/CSIRO/ACRA Conference on Rehabilitation of Concrete Structures, RILEM*, Melbourne.

Yoo, D.Y. and Banthia, N. (2015), "Numerical simulation on structural behavior of UHPFRC beams with steel and GFRP bars", *Comput. Concrete*, **16**(5), 759-774.

Zhu, H.H., Chen, Q., Ju, J.W., Yan, Z.G., Guo, F., Wang, Y.Q., Jiang, Z.W., Zhou, S. and Wu, B. (2015), "Maximum entropy based stochastic micromechanical model for two-phase composite considering the inter-particle interaction effect", *Acta Mech.*, **226**(9), 3069-3084.

Zhu, H.H., Chen, Q., Yan, Z.G., Ju, J.W. and Zhou, S. (2014), "Micromechanical model for saturated concrete repaired by electrochemical deposition method", *Mater. Struct.*, **47**, 1067-1082.

CC