Computers and Concrete, *Vol. 21, No. 1 (2018) 11-20*
DOI: https://doi.org/10.12989/cac.2018.21.1.011

Recommendation for the modelling of 3D non-linear analysis of RC beam tests

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(Received May 16, 2017, Revised August 22, 2017, Accepted September 11, 2017)

Abstract. The possibilities of non-linear analysis of reinforced-concrete structures are under development. In particular, current research areas include structural analysis with the application of advanced computational and material models. The submitted article aims to evaluate the possibilities of the determination of material properties, involving the tensile strength of concrete, fracture energy and the modulus of elasticity. To evaluate the recommendations for concrete, volume computational models are employed on a comprehensive series of tests. The article particularly deals with the issue of the specific properties of fracture-plastic material models. This information is often unavailable. The determination of material properties is based on the recommendations of Model Code 1990, Model Code 2010 and specialized literature. For numerical modelling, the experiments with the so called "classic" concrete beams executed by Bresler and Scordelis were selected. It is also based on the series of experiments executed by Vecchio. The experiments involve a large number of reinforcement, cross-section and span variants, which subsequently enabled a wider verification and discussion of the usability of the non-linear analysis and constitutive concrete model selected.

Keywords: reinforced concrete; classic beams; material properties; three-point bending test; computational mechanics

1. Introduction

Worldwide, great attention is paid to the area of concrete research and its use for engineering design. A fundamental part of the research is also devoted to experiments which are subsequently utilized to formulate theoretical relations and numerical modelling. The crucial comprehensive series of the published experiments involve the beams tested by Bresler and Scordelis (1963) in the 1960's. The series of 12 beams includes various variants of reinforcement, cross-section and span. Moreover, the series of experiments well documents the typical failure modes of reinforced-concrete beams. The published and well-documented experiments are often used for the verification of analytical and numerical models.

With respect to engineering and structural requirements, consideration of the non-linear behaviour of concrete in the constitutive material models is much more appreciated today. The experiments mentioned are particularly suitable for validating numerical computations because they have been described in detail and include load-deflection responses. In 2003, Vecchio from the University of Toronto continued with the experiments from the 1960's. The series of performed tests was based on the original information which was also supplemented with a more detailed testing of material properties within the new tests (Vecchio and Shim 2004). Such an updated experimental campaign is another vital source of data suitable for numerical models' verification. The original as well as repeated experiments have good agreement in peak load value, however there were observed differences of up to 40% in the case of peak load displacement.

In the case of modelling concrete structures, this particularly involves the application of the finite element method. To describe material properties and the application of non-linear models of concrete structures within the finite element method, procedures and recommendations can particularly be found in (ASCE 1982, Vecchio and Collins 1986, Willam and Tanabe 2001). The most important recommendations in the area of concrete structure design include Model Code 1990 (1993) and 2010 (2012).

Recommendations of *ceb-fip* Model Code 1990 (1993) are implemented in the current design code in Europe (Eurocode 2). However with respect to the progress in the concrete research and technology, new recommendation of *fib* Model Code 2010 (2012) was prepared. Even though those new recommendations are not codified in the design code yet.

Here one can find the essential principles of using the finite element models and non-linear analysis.

There is still a large space left here for research, the case studies and recommendations for the non-linear analysis of concrete structures. In particular, in terms of spatial models of concrete structures, there are a limited number of comprehensive studies involving various variants of reinforcement, cross-section and span. This is primarily the case of the selected type of the constitutive concrete model and computational model, the validation of which typically includes selected cases only. The Model Code

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ISSN: 1598-8198 (Print), 1598-818X (Online)

recommendations determine the basic framework and preconditions for non-linear analysis, which are, nevertheless, limited when it comes to broader and practical computations. In the case of concrete models for non-linear analysis, the recommendations particularly distinguish elastic-plastic models (Chen 1982, Yu *et al*. 2010, Sucharda, and Brožovský 2013), models based on fracture mechanics (Cervenka *et al*. 2007, Gamino *et al*. 2010) and/or combined models (Cervenka and Papanikolaou 2008). Above all, in the area of fracture mechanics, there is extensive research in the field of fracture parameters (see e.g., Lehky *et al.* 2014, Vesely *et al.* 2013, Strauss *et al*. 2014, Sucharda *et al*. 2017). The most significant material properties of the further selected fracture-plastic model of concrete are uni-axial compressive and tensile strength, and fracture energy. These material properties, however, are often unavailable in the laboratory or concrete plant. A detailed theoretical description of the mentioned issue of constitutive concrete models, modelling and size-effect can be found in (Bazant and Planas 1998, Planas *et al*. 1999, Shah 1990). Furthermore, the well-known concrete models and approaches include the Disturbed Stress Field Model for reinforced concrete (Vecchio 2000) and the Microplane Model (Bažant *et al*. 2000). Subsequently, the actual application of the constitutive concrete models enables the solution of demanding and complex concrete structures which would be solved otherwise only in a very complicated way. Typically, they include lightweight structures and beams. A certain limitation of the constitutive concrete models, however, often consists in the limited number of conducted complex analyses which would cover a broader area of the behavior of the numerical models in comparison with the experiments. This is also the case of the constitutive concrete model discussed in the article.

It is worth mentioning that the detailed analysis respecting more precisely real behavior of concrete structures requires 2D or even 3D models. Planar and spatial models allows for the better evaluation of the progress of fracture process as well as collapse compering to beams. The 3D model is suitable especially in more complicated cases of the structural member geometry and/or reinforcement. Such typical cases are light weighted hollow beams or members with spatial reinforcement. Also the capturing of crack opening progress is to be considered as a spatial problem.

The important aspects of non-linear analyses of concrete structures include not only a correct selection of the concrete material model, but also the selection recommended for the additional computation of the material properties which are not available in the laboratory. Regarding the suitability of the use of the detailed description of the concrete in non-linear analysis, typically, a question arises about the model fidelity of the relations used for the additional computation of specific properties of the concrete, particularly including the concrete tensile strength, fracture energy and the modulus of elasticity Model Code 1990 (1993), Model Code 2010 (2012). It also results from the fact that specialized articles and papers describing many experiments (Barzegar 1988, Lu *et al*. 2015) only indicate the basic material properties of the concrete. This is insufficient for non-linear computations of concrete structures; therefore, it is the recommended relations that must be used for additional computation of the missing parameters. With regard to the variable concrete composition, however, it is important to pay attention to the range of validity of the relations applied. In some cases, however, it is possible to find the values recommended for specific materials, including their dispersion, in standards and recommendations (ISO 2394, 1998) and (JCSS, 2016).

2. Research significance

The article deals with the verification of the applicability of spatial computational models and the fracture-plastic model of concrete (Cervenka *et al*. 2007) on a coherent series of tests of reinforced-concrete beams. The simulated experiments involve a great number of reinforcement, cross-section and span variants. The computations aimed to determine which of the input parameters of concrete are the most suitable for representation of the non-linear behavior. Within the numerical computations, load diagrams, total loading capacity and deformation are evaluated at the place of the maximum loading capacity. This also includes the way in which a beam collapses. The input parameters evaluated are specific properties of the concrete, which include fracture energy, concrete tensile strength and the modulus of elasticity. For numerical modelling, the experiments executed by Bresler and Scordelis and later by Vecchio were selected. Determination of material properties is based on the recommendations of Model Code 1990 (1993), Model Code 2010 (2012) and specialized literature (Vos, E. 1983).

Model Code recommendations are baseline for conducted numerical computations. First set of computations, marked as MC1990, is based on the Model Code 1990 (1993). The second set of computations marked as MC2010 is based on the newer Model Code 2010 (2012) that is not reflected in design codes yet.

The subsequently executed computations enable a broader verification of the applicability of the models, as well as a discussion on the selected non-linear analysis constitutive concrete model. Thus enabling the comparison of the numerical models based on the concrete strength and Model Code recommendations or concrete strength and more comprehensive experimental data.

3. Material properties

Modelling and non-linear analysis of concrete structures involve use of constitutive concrete models. These models aim to take the real concrete behavior into consideration. Thus, the concrete fails under compression by crushing. This is modelled by means of plasticity. Under tension, cracks form and the theory of fracture mechanics is applied. However, the number of necessary input parameters of the concrete is always increasing by more detailed description. This is typical of, for instance, the selected computational models and the constitutive concrete model combining plasticity for compression and fracture mechanics for

Table 1 Variants of additional computation of input parameters of concrete

	Experiment			
	MC2010		MC1990	
	I	II	I	II
Poisson number	υ=0.2		υ=0.2	
Modulus of elasticity	Exp.	$E_c = 21500\left(\frac{f_c+8}{10}\right)^{\frac{1}{3}}$	Exp.	E_c=(6000-15.5 f_{cu}) $(f_{cu})^{(1/2)}$
Tensile strength	Exp.	f_t=0.21 $(f_c)^{(2/3)}$	Exp.	f_t=0.24 $(f_{cu})^{(2/3)}$
Fracture energy	G_F=73 $f_c^{0.18}$		G_F=25 f_t	

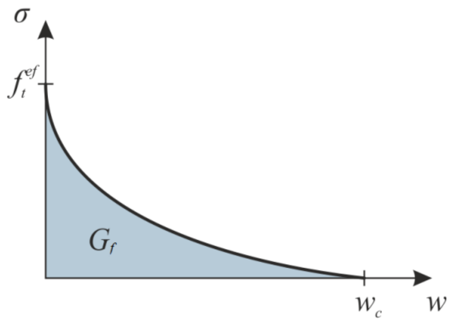

Fig. 1 Exponential crack opening law

tension. Its basic description is indicated in (Yu *et al*. 2010). The selected constitutive concrete model is based on the application of the Menetrey-Willam law in (Menétrey and Willam 1995), smeared crack model and fracture energy for the computation of softening (Cervenka and Papanikolaou 2008). The most significant material parameters include uni-axial compressive and tensile strength and/or fracture energy. Furthermore, the Crack Opening Law is often used to describe concrete behavior. The Exponential Crack Opening Law can be expressed as (Cervenka *et al*. 2007)

$$\frac{\sigma}{f_t^{ef}} = \left\{1 + \left(c_1 \frac{w}{w_c}\right)^3\right\} \exp\left(-c_2 \frac{w}{w_c}\right) - \frac{w}{w_c}(1 + c_1^3)\exp(-c_2) \quad (1)$$

Numerical analyses are prepared in several alternatives with respect to above mentioned possibilities of modeling of concrete and the available amount of input parameters from laboratory experiments (concrete strength, tensile strength, modulus of elasticity-set marked as I) or available during typical construction (usually concrete strength only-set marked II). Bot set of results have also subsets based on the Model Code selection (1990 or 2010). Thus four set of analyses have been conducted as given in Table 1. The goal is verification of the resulting non-linear analyses with available experimental data from literature.

Graphically, the Crack Opening Law can be represented as in Fig. 1.

Each of the twelve reinforced-concrete beams has been computed in four variants. The first approach to the computations is based on the recommendations of Model Code 2010 (2012), i.e. MC2010. The second approach combines the recommendations of Model Code 1990 (1993), i.e., MC1990 and (Vos 1983). The basic computation variants marked as I are based on experimental test data; only the fracture energy G_F is computed additionally according to the model MC2010 or MC1990. In the most frequent case, only the compressive strength of the concrete is known, thus the computations marked as II are analysed as well. With these computational variants, the tensile strength f_t of concrete, fracture energy G_F and modulus of elasticity E_c are computed according to code specifications additionally. Once again, the computations are divided according to the model MC2010 or MC1990. Approaches to the computation in the four variants are indicated in detail in Table 1. The initial input value is the cube f_{cu} or cylindrical f_c compressive strength of concrete. To convert the concrete compressive strength, the well-known relation can be used

$$f_c = 0.85 f_{cu} \quad (2)$$

4. Experimental program

The experimental program includes three-point bending tests of 12 concrete beams. The tests were conducted by Bresler and Scordelis in 1963. Subsequently, the tests were repeated by Vecchio and Shim (2004). The initial testing parameters are very similar to each other. The results from the laboratory experiments are similar with respect to overall carrying capacity and pre-peak part of the load-deflection responses. However more significant differences are observed at the peak load level. The experimental programs involve four series of reinforced-concrete beams of a rectangular cross-section. Their designation and cross-sections are represented in Fig. 2.

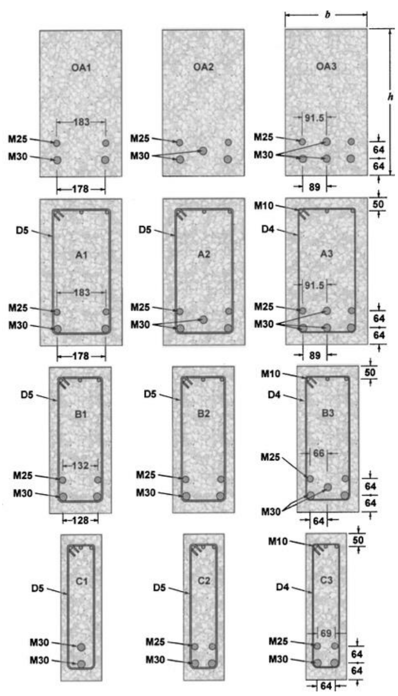

Fig. 2 Cross-sections and reinforcement of the individual beams (Vecchio and Shim 2004)

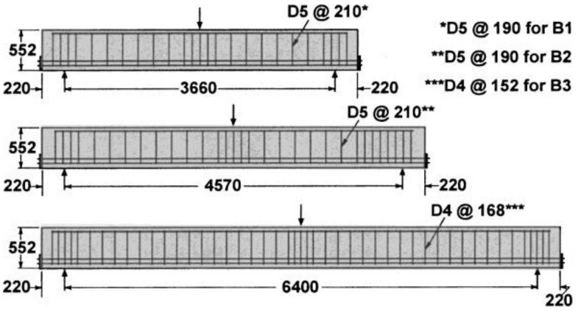

Fig. 3 Support and reinforcement diagram of the individual beams-three different lengths (Vecchio and Shim 2004)

Table 2 Characteristics of beam concrete (Vecchio and Shim 2004)

Designation	f_c [MPa]	f_t [MPa]	E_c [GPa]
OA1	22.6	1.68	36.50
OA2	25.9	1.84	32.90
OA3	43.5	2.60	34.30
A1	22.6	1.68	36.50
A2	25.9	1.84	32.90
A3	43.5	2.60	34.30
B1	22.6	1.68	36.50
B2	25.9	1.84	32.90
B3	43.5	2.60	34.30
C1	22.6	1.68	36.50
C2	25.9	1.84	32.90
C3	43.5	2.60	34.30

Table 3 Characteristics of reinforcing bars (Vecchio and Shim 2004)

Designation	d_s [mm]	A_s [mm^2]	f_y [MPa]	f_u [MPa]	E_s [GPa]
M10	11.3	100	315	460	200
M25/A[1]	25.2	500	440	615	210
M25/B[2]	25.2	500	445	680	220
M30	29.9	700	436	700	200
D4	3.7	25.7	600	651	200
D5	6.4	32.2	600	649	200

[1] For beams from the 2nd series (OA2, A2, B2 and C2).
[2] For beams from the 1st and 3rd series (OA1, A1, B1 and C1; OA3, A3, B3 and C3).

Table 4 Supplementary input parameters of the concrete model based on the MC2010 computation variants

Designation	MC2010 I	MC2010 II		
	G_F [N/m]	f_t [MPa]	E_c [GPa]	G_F [N/m]
OA1	128.0	1.79	28.2	128.0
OA2	131.1	2.05	32.3	131.1
OA3	144.0	3.24	37.1	144.0
A1	128.0	1.79	31.2	128.0
A2	131.1	2.05	32.3	131.1
A3	144.0	3.24	37.1	144.0
B1	128.0	1.79	31.2	128.0
B2	131.1	2.05	32.3	131.1
B3	144.0	3.24	37.1	144.0
C1	128.0	1.79	31.2	128.0
C2	131.1	2.05	32.3	131.1
C3	144.0	3.24	37.1	144.0

The individual series differ in the amount and type of reinforcement. Every series is comprised of three different beam lengths (3.66 m; 4.57 m; 6.40 m) represented in Fig. 3. The beam parameters are indicated in detail in (Vecchio and Shim 2004). All the beams had the same height of 552 mm. The beams of the OA series have the same geometry as those of the A series and the width of 305 mm. However, the OA series beams are not provided with shear reinforcement. The beams of the B and C series primarily differ in the height-width ratio of the cross-section. The B series of beams had a width of 229 mm; the C series that of 152 mm.

The beam experiments were accompanied by standardized tests of concrete and reinforcement according to Tables 2 and 3. Moreover, the production and testing procedure of the indicated beams have been described in detail. Load diagrams of the testing procedure have been recorded.

5. Computational models

For the modelling of concrete beams, volume computational elements were used. The mesh of finite elements has a regular shape and is created from cube elements. The cube edge length is approximately 75 mm. The regular mesh of finite elements is created by a generator. The mesh of finite elements is shown on graphic output (see e.g. Fig. 6). The size of the finite elements was selected with regard to the beam size and concrete aggregate. The maximum aggregate size was 20 mm. The reinforcement was modelled by means of bar elements. Steel was considered in the computation by the elastic-plastic model.

Steel reinforcement bars modelled as discrete reinforcement with special truss elements. Discrete reinforcement is in form of reinforcing bars and is modeled by special truss elements (Cervenka *et al.* 2007). The selection of elements is adopted with respect to the evaluation of input parameters for numerical computations of pre-peak part load-deflection responses. Selection of the elements was in line with the aim to evaluate the compare suitability of Model Code 1990 (1993) and 2010 (2012) with respect to recommendations for numerical modeling of input parameters for concrete. Selected finite elements well describe pre-peak part of a loading process where the bond between concrete and steel reinforcement is captured sufficiently based on the adopted model. In case of post-peak part of the load-deflection responses, different approaches would provide better match between model results and experiments. One of such approaches is e.g., bond-slip model that is more challenging with respect to input parameters that are not in available data (Vecchio and Shim 2004) sufficiently covered. The research dealing with concrete steel bond modelling is discussed in (Kwak and

Table 5 Supplementary input parameters of the concrete model based on the MC1990 computation variants

Designation	MC1990 I	MC1990 II		
	G_F [N/m]	f_t [MPa]	E_c [GPa]	G_F [N/m]
OA1	59.3	2.14	28.8	53.5
OA2	84.3	2.34	30.5	58.5
OA3	78.3	3.31	37.2	82.8
A1	59.3	2.14	28.8	53.5
A2	84.3	2.34	30.5	58.5
A3	78.3	3.31	37.2	82.8
B1	59.3	2.14	28.8	53.5
B2	84.3	2.34	30.5	58.5
B3	78.3	3.31	37.2	82.8
C1	59.3	2.14	28.8	53.5
C2	84.3	2.34	30.5	58.5
C3	78.3	3.31	37.2	82.8

Filippou 1990, Kwak 1990, Filippou 1986, Kwak and Filippou 1995). Summary of available approaches in case of steel reinforcement modeling is also available in (Sucharda and Brožovský 2011). It is worth mentioning that there are also other parameters influencing the computations. Among these belong loading character (force or displacement, monotonic or cyclic) and type of the non-linear problem solver.

Concrete was modelled by the above-mentioned fracture-plastic model. Specifically, the 3D Non Linear Cementitious 2 variant (Cervenka and Papanikolaou 2008) was selected from the library. The models of concrete beams also considered the boundary conditions. The support plates and loads were modelled by means of the linear-elastic material E=200 GPa. The beam and the plates were in rigid contact. Supports were axially placed on both of the support plates in the vertical direction; moreover, a support in the horizontal direction of the beam was located on one of the plates. To solve the non-linear system of equations, the Newton-Raphson method was selected. The load was induced by deformation. The highest number of iterations within one step was 50.

6. Numerical modelling and parametric study

This section contains the supplementary input parameters computed according to Model Code followed by the results from experiments and numerical models. The numerical modelling includes the parametric study for various input parameters of concrete indicated in Table 1 for non-linear computation. Specifically, four computations are conducted for each of the experiment variants. The additionally computed input parameters of concrete, according to MC2010, are indicated in Table 4.

In the computation marked I, only the fracture energy G_F was computed based on codes suggestions. With the variant marked II, it was the tensile strength of concrete f_t, fracture energy G_F and the modulus of elasticity of concrete E_c. The supplementary computed input data of concrete, according to MC1990, are indicated in Table 5.

The comparison of experiments and numerical simulation is presented next. Tables 6 and 7 compare values of the maximum loading capacity according to applied experimental or computed input data and code MC2010 or

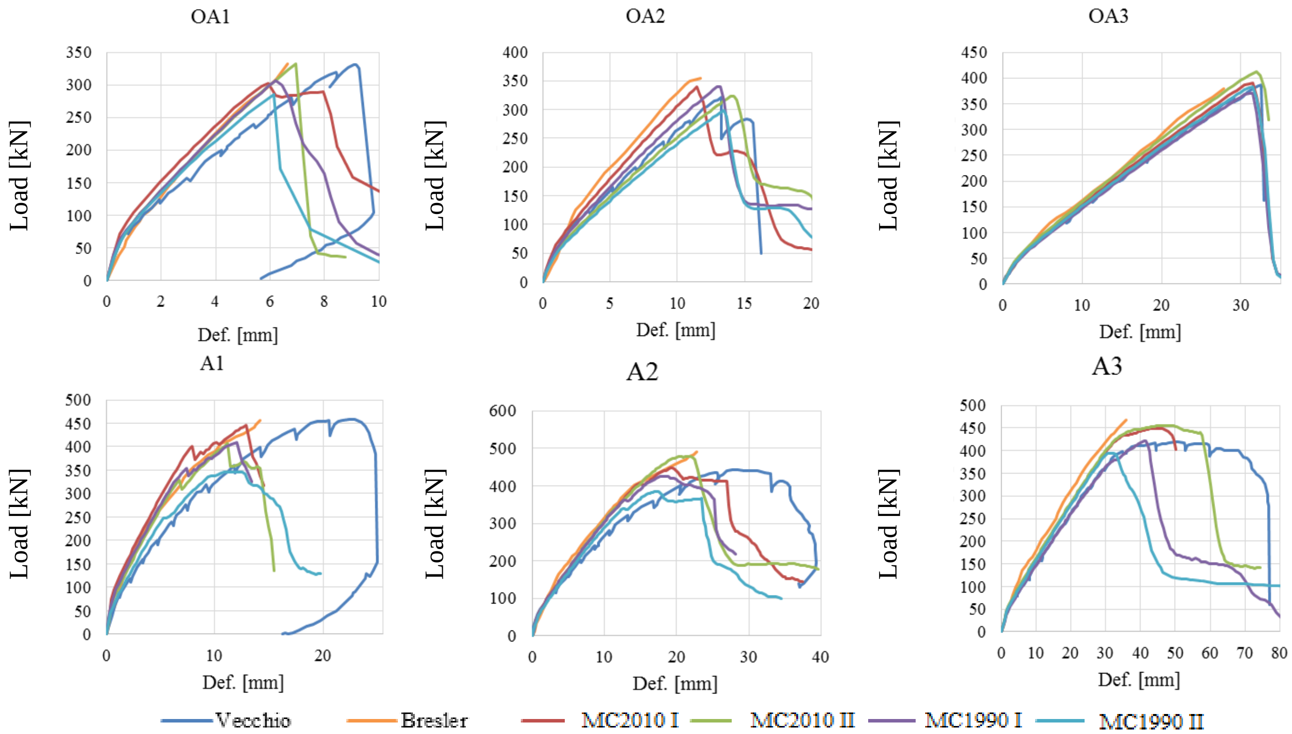

Fig. 4 Load-deflection responses for the OA and A series
(on the vertical axis: load [kN]; on the horizontal axis: vertical deflection in the middle of the beam [mm])

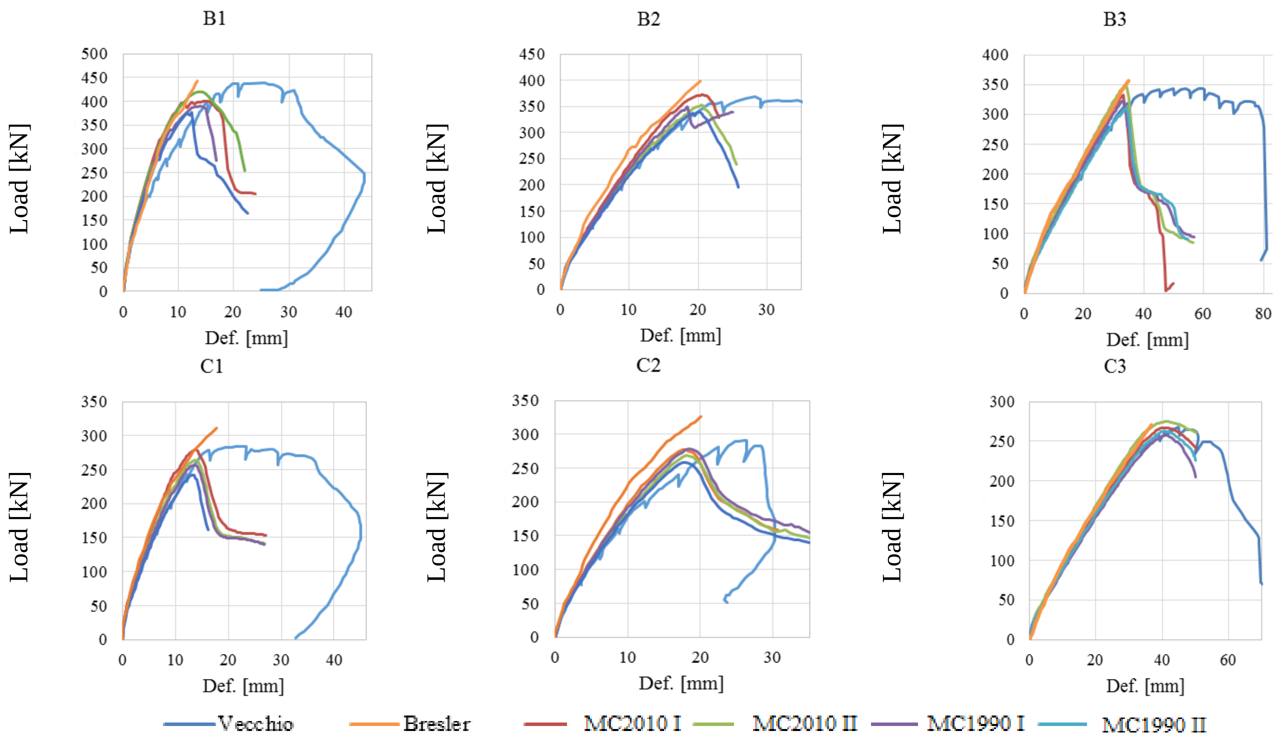

Fig. 5 Load-deflection responses for the B and C series
(on the vertical axis: load [kN]; on the horizontal axis: vertical deflection in the middle of the beam [mm])

Table 6 Evaluation of total loading capacity for the MC2010 computation variants

Designation	Experiment		MC2010 I		MC2010 II	
	Vecchio	Bresler	Vecchio	Bresler	Vecchio	Bresler
	[kN]	[kN]	$P_{u,exp}/P_{u,calc}$	$P_{u,exp}/P_{u,calc}$	$P_{u,exp}/P_{u,calc}$	$P_{u,exp}/P_{u,calc}$
OA1	331	334	1.096	1.106	0.998	1.007
OA2	320	356	0.997	1.109	0.990	1.102
OA3	385	378	0.996	0.978	0.936	0.919
A1	459	468	1.000	1.020	1.129	1.151
A2	439	490	0.989	1.104	0.918	1.024
A3	420	468	1.002	1.117	0.921	1.026
B1	434	446	0.991	1.019	1.034	1.062
B2	365	400	0.990	1.085	1.037	1.136
B3	342	356	0.995	1.036	0.981	1.021
C1	282	312	0.991	1.097	1.069	1.183
C2	290	324	0.998	1.115	1.080	1.207
C3	265	270	0.989	1.007	0.965	0.983
Average			1.003	1.066	1.005	1.068

Table 7 Evaluation of total loading capacity for the MC1990 computation variants

Designation	Experiment		MC1990 I		MC1990 II	
	Vecchio	Bresler	Vecchio	Bresler	Vecchio	Bresler
	[kN]	[kN]	$P_{u,exp}/P_{u,calc}$	$P_{u,exp}/P_{u,calc}$	$P_{u,exp}/P_{u,calc}$	$P_{u,exp}/P_{u,calc}$
OA1	331	334	1.081	1.091	1.160	1.170
OA2	320	356	0.942	1.048	1.074	1.195
OA3	385	378	1.039	1.020	1.005	0.987
A1	459	468	1.119	1.141	1.325	1.351
A2	439	490	1.030	1.150	1.138	1.270
A3	420	468	0.997	1.111	1.064	1.186
B1	434	446	1.113	1.144	1.151	1.183
B2	365	400	1.046	1.146	1.079	1.183
B3	342	356	1.059	1.103	1.097	1.142
C1	282	312	1.100	1.217	1.168	1.292
C2	290	324	1.041	1.163	1.121	1.252
C3	265	270	1.031	1.051	1.008	1.027
Average			1.050	1.115	1.116	1.186

MC1990 respectively. Evaluation of the experiments executed by Bresler-Scordelis and Vecchio are indicated separately here. In all, eight comparison variants have thus been achieved. The load diagrams from the experiments and computations are shown in Fig. 4 and 5. Table 8 to 9 contains deformation corresponding to the maximum loading capacity.

In view of the scope of the computations executed and results achieved, only the graphic outputs for the variant of the MC2010 I input parameters are presented next. The OA series of beams have the same geometry as the A series. However, the OA series of beams are not reinforced with shear reinforcement as mentioned before. The selected computation variants and experiments of the OA and A series shows that the beams reinforced with shear reinforcement typically feature a more distinctive loading capacity by tens of percent. Fig. 7 clearly shows the formation of shear failure of the compression strut of the OA beam. At the beginning of loading, cracks appeared in the middle of the beam span. Later, however, they started spreading to the supports, while shear cracks began to appear. The shear cracks caused the structure to collapse.

Table 8 Evaluation of the maximum deformation for the MC2010 computation variants

Designation	Experiment		MC2010 I		MC2010 II	
	Vecchio	Bresler	Vecchio	Bresler	Vecchio	Bresler
	[mm]	[mm]	$u_{u,exp}/u_{u,calc}$	$u_{u,exp}/u_{u,calc}$	$u_{u,exp}/u_{u,calc}$	$u_{u,exp}/u_{u,calc}$
OA1	9.1	6.6	1.531	1.111	1.313	0.952
OA2	13.2	11.7	1.155	1.024	0.930	0.824
OA3	32.4	27.9	1.030	0.887	1.014	0.873
A1	18.8	14.2	1.456	1.100	1.684	1.272
A2	29.1	22.9	1.497	1.178	1.423	1.120
A3	51.0	35.8	1.146	0.805	1.074	0.754
B1	22.0	13.7	1.520	0.947	1.635	1.018
B2	31.6	20.8	1.521	1.001	1.539	1.013
B3	59.6	35.3	1.806	1.069	1.753	1.038
C1	21.0	17.8	1.528	1.295	1.527	1.294
C2	25.7	20.1	1.427	1.116	1.427	1.116
C3	44.3	36.8	1.079	0.896	1.053	0.875
Average			1.391	1.036	1.364	1.012

Table 9 Evaluation of the maximum deformation for the MC1990 computation variants

Designation	Experiment		MC1990 I		MC1990 II	
	Vecchio	Bresler	Vecchio	Bresler	Vecchio	Bresler
	[mm]	[mm]	$u_{u,exp}/u_{u,calc}$	$u_{u,exp}/u_{u,calc}$	$u_{u,exp}/u_{u,calc}$	$u_{u,exp}/u_{u,calc}$
OA1	9.1	6.6	1.471	1.067	1.487	1.078
OA2	13.2	11.7	1.000	0.886	0.982	0.870
OA3	32.4	27.9	1.046	0.901	1.030	0.887
A1	18.8	14.2	1.566	1.183	1.623	1.226
A2	29.1	22.9	1.578	1.242	1.678	1.320
A3	51.0	35.8	1.230	0.863	1.596	1.120
B1	22.0	13.7	1.574	0.980	1.763	1.098
B2	31.6	20.8	1.706	1.123	1.557	1.025
B3	59.6	35.3	1.806	1.069	1.727	1.023
C1	21.0	17.8	1.527	1.294	1.584	1.343
C2	25.7	20.1	1.388	1.086	1.427	1.116
C3	44.3	36.8	1.079	0.896	1.092	0.907
Average			1.414	1.049	1.462	1.084

The most distinct shear cracks emerge on the longest beam of the OA series. This was similar during the real experiment as well.

For the A series beam, cracks at the maximum load are plotted in Fig. 6. The A series beams also collapsed as a result of the shear failure. Before failure of the compression strut, the yield point is exceeded in the shear reinforcement. Subsequently, the concrete is being crushed along the compression strut and the beam collapses. Exceeding of the yield stress is illustrated in Fig. 8, in which the reinforcement stress just before the structure collapse is plotted. Apparently, the stress level in the shear reinforcement is almost at the ultimate strength of steel (f_u=615 MPa). A similar failure mechanism also occurred on the other two beams of the A series.

A failure mechanism similar to the A series beams repeats with the following B and C series. Selected graphic results for the B beam series are in Fig. 9. The crack inclination, position and width are significantly influenced

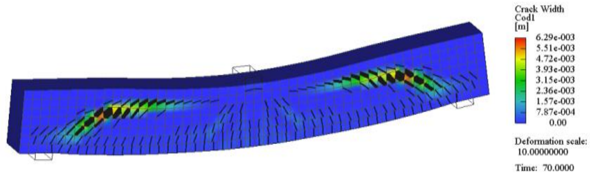

Fig. 6 Cracking including crack width contour plot at the peak load for the A1 beam series

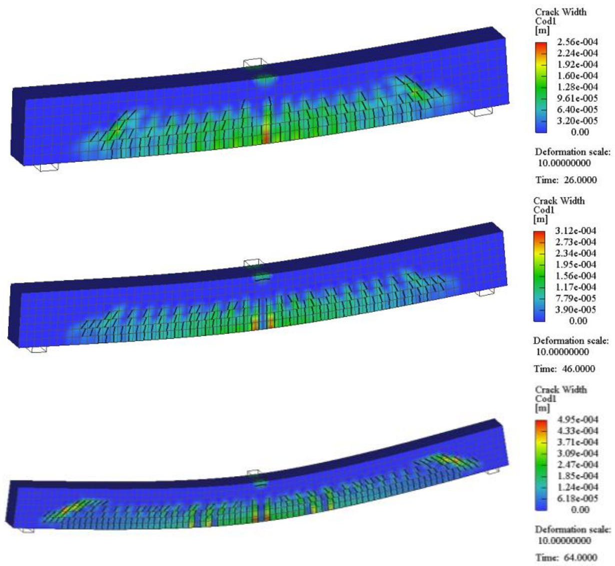

Fig. 7 Cracking including crack width contour plot at the peak load for the OA beam series

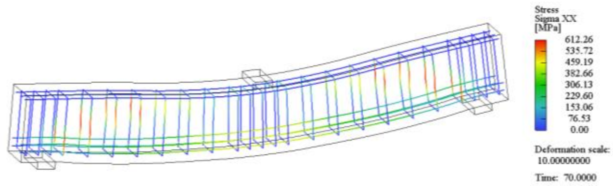

Fig. 8 Reinforcement stress immediately before the A1 beam collapse

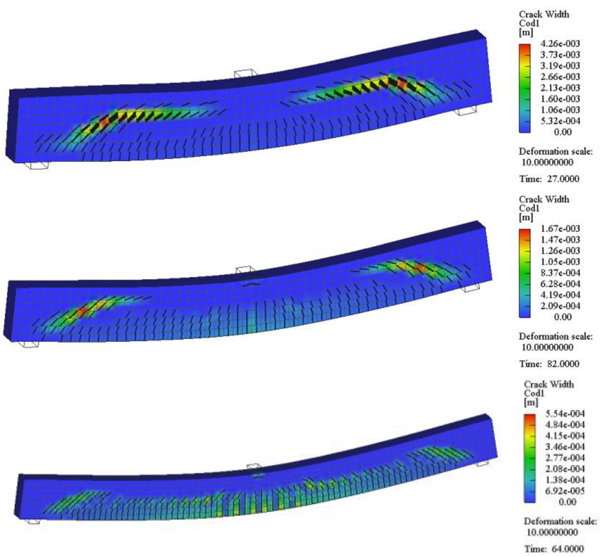

Fig. 9 Cracking including crack width contour plot at the peak load for the B beam series

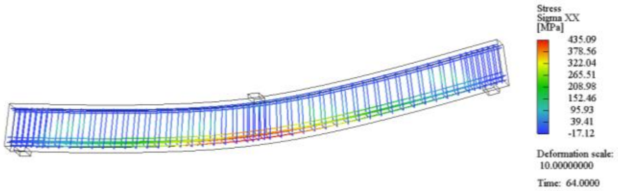

Fig. 10 Plotting of reinforcement stress immediately before the B3 beam collapse

Table 10 Summary results for computation of the peak loading capacity of beams

Model Code	Variant	I		II	
		Vecchio	Bresler	Vecchio	Bresler
		$P_{u,exp}/P_{u,calc}$	$P_{u,exp}/P_{u,calc}$	$P_{u,exp}/P_{u,calc}$	$P_{u,exp}/P_{u,calc}$
MC2010	Average	1.003	1.066	1.005	1.068
	Stand.	0.028	0.048	0.064	0.084
MC1990	Average	1.050	1.115	1.116	1.186
	Stand.	0.048	0.054	0.082	0.099

by the span. This behavior has also been very well captured in the computational models.

The difference, however, is primarily with the longest beam variants. Here the collapse occurred due to the main reinforcement failure. This state is illustrated in Fig. 10, which shows the stress in the reinforcement of the B3 beam at the pre-collapse computation step.

7. Discussion and results

By comparison of the numerical analyses executed and experiments selected, it is possible to evaluate that the computations very well reflect the actual behaviour of reinforced-concrete beams during loading. The summary results are then indicated in Table 10 for the peak loading capacity and in Table 11 for deformation. In most cases of the numerical analyses, typical loading phases can be distinguished. These involve the area of linear loading, crack development, reinforcement plasticization, formation of shear failure, etc. The numerical models also reflected the real collapse mode of the beams.

As can be seen in the result summary given in Tables 10 and 11, the most suitable input data for the concrete model for numerical simulations are those directly from the experiment, when only the fracture energy had to be additionally computed. These were the computation variants designated as MC2010-I and MC1990-I. The more suitable additional computation of the fracture energy is that from the Model Code 2010 recommendations. In the case of the total loading capacity, the differences between the computed and actual results were only units of per cent. In the case of the variant I computation with experimental data on the concrete properties, there was an average compliance with the Vecchio 1.003 beams and Bresler 1.066 beams. In sum, for both series of experiments, the average deviation was 7.6%. In the case of the results of deformation at the maximum loading capacity, there was a more distinct difference between computations and experiments,

Table 11 Summary results for computation of deformation at the peak loading capacity of beams

Model Code	Variant	I		II	
		Vecchio	Bresler	Vecchio	Bresler
		$u_{u,exp}/u_{u,calc}$	$u_{u,exp}/u_{u,calc}$	$u_{u,exp}/u_{u,calc}$	$u_{u,exp}/u_{u,calc}$
MC2010	Average	1.391	1.036	1.364	1.012
	Stand.	0.224	0.132	0.272	0.162
MC1990	Average	1.414	1.049	1.462	1.084
	Stand.	0.256	0.140	0.263	0.150

particularly in the case of the experiments by Vecchio.

Experiments conducted by Vecchio had higher deformations in most of the studied cases. It is important for the comparison that the descending branch is not indicated for the experiments executed by Bresler and Scordelis. On the experiments executed by Vecchio, there is also a distinct loading cycle and the descending branch in the load diagrams. The numerical results showed better agreement in case of peak load with beams tested by Bresler-Scordelis. The difference were up to 8.4%. Typically, the resulting deformations differences between the experiments conducted by Vecchio and numerical solution were between 30-40%. Relative difference between maximum loading capacity obtained with the numerical solution and experiment were also dependent on the particular evaluated type of beams. It can be seen in case of beam type B.

The important finding was that even though there were difference in displacement is observed in peak load region of load-deflection responses, the numerical results lie between the results from experimental data set by Bressler and Vechio with closer match to Vecchio. The overall representing of load-deflection respons behavior by applied numerical models was however good. It is worth mentioning that the deformation of concrete structure itself is influenced among others by creep and shrinkage (typically in case of construction works). This can then have a significant influence for total displacement. The bond-slip effect may also significantly influence the peak load displacements. One needs into considerations in case of the bond-slip also shape of the reinforcement, and the aggregate type and size. However the displacements corresponding to the peak load are though usually important to analyze the reliability of the numerical results.

The numerical results achieved can be elaborated further:

- In the cases of advanced analysis of reinforced-concrete structures, it is suitable to use volume computational models and the fracture-plastic model of concrete. This has been validated on a coherent series of beams, involving a large number of reinforcement, cross-section and span variants. Very good compliance can be achieved in the case of determining the total loading capacity of the structure.
- The difference between the numerical results and the experiments based on the available input data (variant I) with concrete strength, modulus of elasticity, tensile strength and variant II with concrete strength only was up to 2.7% in case of MC 2010 and up to 7.1% in case

of MC 1990.

• In the cases of analyzing older concrete structures or experiments, it is suitable to consider application of the relations from Model Code 1990 (1993).

• The application of numerical models introduces uncertainties into the computation. Therefore, it is beneficial to respect the recommendations for the creation of computational models and parameters for non-linear computations.

8. Conclusions

A study has been conducted in order to compare the effect of application of experimental parameters to parameters computed according to the Model Code. The influence of tension strength and modulus of elasticity that are commonly not available experimentally was analyzed. The selected input parameters were applied in the numerical simulation both from experiments as well as according to recommendation of the Model Code.

In the case of non-linear analysis of new or present-day concrete structures, it is suitable to apply the recommendations of Model Code 2010 (2012) for additional computation of specific material properties of common concretes up to the C50/57 class.

The most suitable, however, is the use of input data for the specific concrete recipe and processing technology. Nevertheless, the differences in the results between the variant I with concrete strength, modulus of elasticity, tensile strength and variant II with the concrete strength only are minor especially in case of Model Code 2010 (2012) and peak load analysis.

The differences between the measured displacement and experiments are higher comparing to ultimate load. Thus the precision of simulated results prepared based on measured compressive strength only (variant II) seems to be satisfactory.

For real designs, however, it is more important to consider the inherent random character of the input data properly in view of the required level of safety and serviceability.

It was found that the concrete input parameters for the material model are better simulated according to recommendations of Model Code 2010 (2012). This is particularly apparent in the experiments executed by Vecchio and the peak loading capacity computation. It is assumed that it is influenced by the fact that the newer experiments and recommendations of MC2010 are based on the current method of concrete processing and technologies. Particularly, this also has an influence on the deformational properties of concrete. This takes effect, for instance, in the fracture energy. In the case of the experiments by Bresler and Scordelis, it is advantageous to use the relations indicated for the MC1990 for computation representing the LD diagram and total loading capacity. For the numerical analyses executed, it is beneficial to use 3D computational models which enable a detailed analysis of critical places of the beams.

Processed numerical analyzes conducted on top of the experimental data from Bresler and Scordelis beams, and the duplicate series conducted by Veccio represent important precedent for non-linear analysis of other equally complex and important concrete mechanisms and confirmed the suitability of numerical model for modeling shear-critical behavior.

Acknowledgments

The article preparation was supported from sources for conceptual development of research development and innovations at VŠB - Technical University of Ostrava for the year 2017, which were granted by the Ministry of Education, Youth and Sports of the Czech Republic.

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