Computers and Concrete, *Vol. 20, No. 3 (2017) 329-338*
DOI: https://doi.org/10.12989/cac.2017.20.3.329

Interactive strut-and-tie-model for shear strength prediction of RC pile caps

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(Received March 5, 2016, Revised May 1, 2017, Accepted May 2, 2017)

Abstract. A new simple and practical strut-and-tie model (STM) for predicting the shear strength of RC pile caps is proposed in this paper. Two approaches are adopted to take into account the concrete softening effect. In the first approach, a concrete efficiency factor based on compression field theory is employed to determine the effective strength of a concrete strut, assumed to control the shear strength of the whole member. The second adopted Kupfer and Gerstle's biaxial failure criterion of concrete to derive the simple nominal shear strength of pile caps containing the interaction between strut and tie capacity. The validation of these two methods is investigated using 110 RC pile cap test results and other STMs available in the literature. It was found that the failure criterion approach appears to provide more accurate and consistent predictions, and hence is chosen to be the proposed STM. Finally, the predictions of the proposed STM are also compared with those obtained by using seven other STMs from codes of practice and the literature, and were found to give better accuracy and consistency.

Keywords: pile caps; discontinuity region; strut-and-tie model; shear strength

1. Introduction

The RC pile cap is one of the important structural components for transferring a load from one or more columns to a group of piles. Currently, there are two approaches for the design of pile caps which are available in international codes of practice (ACI 318-14, CSA 2004, BS 8110). The first, the so called sectional design method, assumes that a pile cap behaves as a reinforced concrete beam spanning between two or more piles. The sectional depth and the amount of tension reinforcement are determined using the conventional beam theory by assuming that the plane section remains plane. The design of pile caps as recommended in the ACI code, for example, applies the same sectional approach used for footings supported on soil and for two-way slabs directly supported on columns.

The second approach refers to a strut-and-tie model (STM), which can successfully apply to predict the behavior of various D-region members such as Hwang *et al.* (2000), Tang and Tan (2004), Wang and Meng (2008), Chetchotisak *et al*. (2014a, b), Hong *et al.* (2016a, b) and Yavuz (2016). According to this approach, the complex flow of stresses in a pile cap can be idealized by the 3D-STM as space truss-like members consisting of diagonal concrete struts and steel reinforcement ties connected at each node. In addition, previous researchers (Adebar *et al*. 1990, Adebar and Zhou 1996, Cavers and Fenton 2004) have confirmed that the design of pile caps using STM is more appropriate than the former approach.

Several researchers have proposed the STMs for analysis and design of pile caps (Adebar and Zhou 1996, Park *et al*. 2008, Souza *et al*. 2009, Guo 2015). Jensen and Hoang (2012) also proposed another approach, i.e., the upper bound plasticity method used to predict the critical failure mode and the load-carrying capacity of pile caps. However, uncertainty in strength predictions of pile caps failing in shear can be found in some approaches, as reported by Park *et al.* (2008). Moreover, some methods are considered too complex for practical design purposes. For this reason, more accurate and practical methods for computing the shear strengths of pile caps still require more development and this is considered as an active research field.

In this study, a new simple and practical STM is proposed for accurate prediction of shear strengths of RC pile caps. To allow for the softening effect in concrete struts controlling the shear strength of a whole member, the proposed STM is developed by considering two different approaches, namely, the efficiency factor approach and the failure criterion approach. These are investigated to determine the most appropriate one to be used in the proposed STM. A database of 110 pile caps test results assembled from a selection of published literature is employed for this analysis. All specimens are four–pile caps tested to fail in shear modes. Furthermore, the existing methods selected from the international codes of practice and available literature, are also compared with the proposed STM.

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http://www.techno-press.org/?journal=cac&subpage=8 ISSN: 1598-8198 (Print), 1598-818X (Online)

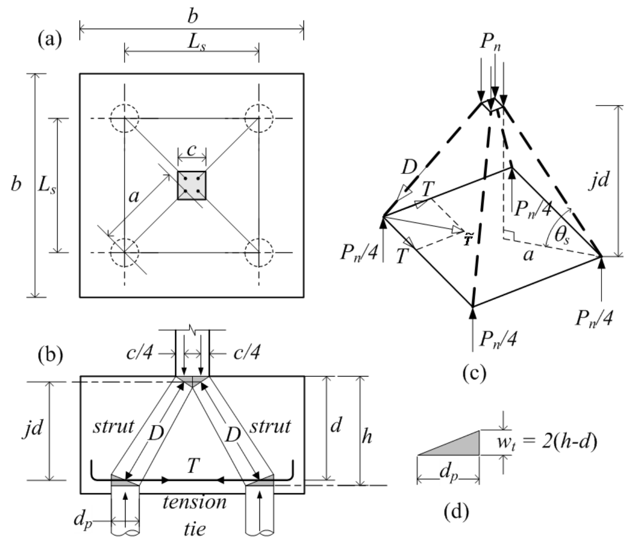

Fig. 1 a four-pile cap represented by STM: (a) plan; (b) section; (c) 3-D STM; (d) detail of LNZ

Table 1 Summary of the selected efficiency factor models

Researchers	Efficiency factor models
Vecchio and Collins (1986)	$\nu = \dfrac{1}{0.8+170\,\varepsilon_1} \le 0.85$
Zhang and Hsu (1998)	$\nu = \dfrac{5.8}{\sqrt{f'_c}} \dfrac{1}{\sqrt{1+400\,\varepsilon_1}} \le \dfrac{0.9}{\sqrt{1+400\,\varepsilon_1}}$
Kaufmann and Marti (1998)	$\nu = \dfrac{1}{(0.4+30\,\varepsilon_1) f_c'^{1/3}}$
Zwicky and Vogel (2006)	$\nu = (1.8-38\varepsilon_1)\cdot(f'_c)^{-1/3}$ and $0.85\cdot(f'_c)^{-1/3} \le \nu \le 1.6\cdot(f'_c)^{-1/3}$

2. Shape and geometry of the proposed STM

An RC pile cap can be generally idealized by a 3D-STM (Fig. 1) consisting of diagonal concrete struts and steel ties connected at each node. Prior to developing the mathematical models, the shape and geometry of STM should be identified. According to Fig. 1(c), the inclined angle between the diagonal concrete strut and the horizontal direction, θ_s, can be written in the form

$$\tan\theta_s = \frac{jd}{a} \tag{1}$$

where the term jd is the distance of the lever arm from the resultant compressive force to the centroid of the main tension steel, j=1−k/3, while k is derived from classical bending theory for a singly reinforced concrete section as

$$k = \sqrt{(n\rho)^2 + 2(n\rho)} - (n\rho) \tag{2}$$

where n is the modular ratio and ρ is the longitudinal reinforcement ratio. The term a is defined as the smallest length between the center of pile at the lower nodal zone (LNZ) and the column quarter point at the upper nodal zone (UNZ) of the pile cap as shown in Fig. 1(a). Next, by using the geometry of the LNZ (Fig. 1(d)), the effective area of the diagonal concrete strut A_{str} assumed to be the ellipsoid cross-section (Cavers and Fenton 2004, Park *et al.* 2008), can be estimated as

$$A_{str} = \frac{\pi}{4} d_p \left(w_t \cos\theta_s + d_p \sin\theta_s \right) \tag{3a}$$

or

$$A_{str} = \frac{\pi}{4} d_p \sqrt{w_t^2 + d_p^2} \tag{3b}$$

where d_p, c, and w_t are the pile diameter, the width of column, and the effective width of tie (w_t=2(h−d)), respectively. Eq. (3a) is developed based on ACI 318-14 and Park *et al*. (2008) whereas Eq. (3b) is modified from Hwang *et al*. (2000). These two equations will be investigated to obtain the most appropriate one for the proposed model.

3. Effective strength of concrete strut due to softening effect

Generally, the shear strength of the D-region members such as pile caps is controlled by the capacity of the concrete strut. In order to achieve this, the softening effect in concrete compressive strength due to transverse tensile strain needs to be considered. Currently, there have mainly been two approaches taking into account the effect: 1) a concrete efficiency factor (i.e., also called softening coefficient) generally found in the form of a constant value or a function developed in terms of the concrete compressive strength, the principal tensile strain (e.g., Vecchio and Collins 1986, Zhang and Hsu 1998, Kaufmann and Marti 1998, Zwicky and Vogel 2006), and the geometry of struts (Warwick and Foster 1993) and 2) a failure criterion such as the modified Mohr-Coulomb failure criterion and the Kupfer and Gerstle's bi-axial failure criterion (Kupfer and Gerstle 1973) employed to account for the effect of the bi-axial state of compressive and tensile stresses on the compressive strength of concrete. In general, the former approach is mostly based on compression field theory (CFT) and widely used in conjunction with STM for analysis and design of various D-region members (Hwang *et al*. 2000, Hwang and Lee 2002, Park *et al*. 2008, Chetchotisak *et al*. 2014a, b, etc.) while the latter is also successfully employed in several models (Tang and Tan 2004, Zhang and Tan 2007, Wang and Meng 2008).

3.1 The STM using efficiency factor approach

This approach is adapted from Hwang *et al*. (2000) and Park *et al*. (2008) based on equilibrium, compatibility, and the constitutive laws of cracked reinforced concrete. State-of-the-art efficiency factor models will also be evaluated to determine the most appropriate one to be used in conjunction with the proposed model. The details of these are listed in Table 1.

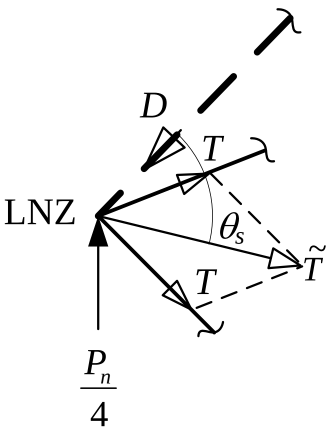

Fig. 2 The force equilibrium at LNZ

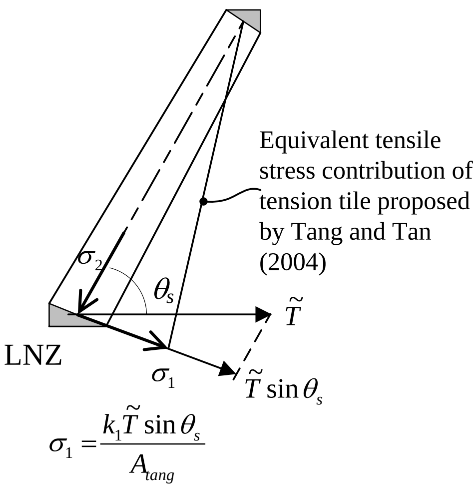

Fig. 3 Distribution of the tensile stress along strut

3.1.1 Equilibrium condition

Referring to Fig. 1, using the equilibrium condition, the load-carrying capacity of the pile cap failing in shear can be written in the terms of the capacity of concrete struts as

$$P_n = 4D\sin\theta_s \tag{4}$$

where D is the maximum compression force in the diagonal concrete strut.

3.1.2 Stress-strain relationship

According to Vecchio and Collins (1986) and Zhang and Hsu (1998), the softened stress-strain relationship of the cracked concrete can be written as

$$\sigma_2 = \nu f_c'\left(2\left(\frac{\varepsilon_2}{\nu\varepsilon_0}\right)-\left(\frac{\varepsilon_2}{\nu\varepsilon_0}\right)^2\right), \text{ for } \frac{\varepsilon_2}{\nu\varepsilon_0}\le 1 \tag{5a}$$

$$\sigma_2 = \nu f_c'\left(1-\left(\frac{\varepsilon_2/\nu\varepsilon_0 - 1}{2/\nu - 1}\right)^2\right), \text{ for } \frac{\varepsilon_2}{\nu\varepsilon_0} > 1 \tag{5b}$$

where σ_2 and ε_2 are the average principal stress and strain of concrete in the direction of the strut, respectively, v can be taken from one of the selected efficiency factors listed in Table 1, and ε_0 is the concrete cylinder strain corresponding to the cylinder strength f_c', which can be approximately defined as (Foster and Gilbert 1996)

$$\varepsilon_0 = 0.002 + 0.001\left(\frac{f_c' - 20}{80}\right), \text{ for } 20 \le f_c' \le 100 \text{ MPa} \tag{6}$$

According to Eq. (5), a pile cap is assumed to fail in shear whenever the compressive stress and strain of the concrete strut reach the following conditions

$$\sigma_2 = \nu f_c' \tag{7}$$

$$\varepsilon_2 = \nu\varepsilon_0 \tag{8}$$

3.1.3 Compatibility

To estimate the principal tensile strain ε_1 directly used to calculate the efficiency factor v, the compatibility condition of Mohr's circle is employed as

$$\varepsilon_1 + \varepsilon_2 = \varepsilon_h + \varepsilon_v \tag{9}$$

where ε_1 is the average principal tensile strains of concrete in the direction perpendicular to the concrete strut, ε_h and ε_v are the average normal strains of concrete in the horizontal and vertical directions, respectively. Both ε_h and ε_v are difficult to exactly determine, however, they can be approximately taken as 0.002. This assumption can provide reasonable prediction of shear strengths of various D-region members (Hwang *et al*. 2000, Hwang and Lee 2002, Park *et al*. 2008).

3.1.4 Procedure for calculating the shear strength of pile caps

The solution procedure of this problem is achieved by using the same algorithm as proposed by Hwang *et al*. (2000). This can be summarized as follows.

1) For a trial value of P_n, and using the equilibrium condition, the value of D can be computed by Eq. (4), and then, $\sigma_{2\max}=P_n/(4A_{str}\sin\theta_s)$.

2) Using the assumption that the shear strength of pile caps is reached when the compression stress in a concrete strut is equal to the maximum strength, an initial value of v is obtained by Eq. (7), while the strain of the concrete strut ε_2 is computed using Eq. (8).

3) By using ε_1 computed from Eq. (9), an updated value of v can be obtained. If the value of v from Step 2 is sufficiently close to the updated one, then P_n selected in Step 1 is the shear strength of the pile cap, otherwise update the value of P_n and repeat Steps 1 through 3.

3.2 The STM using Kupfer- Gerstle's biaxial failure criterion

Referring to Fig. 2, the LNZ is subjected to tri-axial stresses. However, because of the symmetry of the pile cap in each of two orthogonal directions, the simplification of using the bi-axial tension-compression stress state is made and is shown in Fig. 3.

Although the compressive strength of concrete is reduced due to the softening effect of the transverse tensile strain, when the Kupfer and Gerstle's biaxial failure criterion at the LNZ is adopted, the stress based failure formulation is

$$\frac{\sigma_1}{\sigma_{tn}} + 0.8\frac{\sigma_2}{f_c'} = 1 \tag{10}$$

where σ_1 denotes the principal tensile stress and σ_{tn} symbolizes the combined tensile strength of both reinforcement and concrete in the σ_1 direction. Detailed derivation information for each term will be given in the following section. According to Tang and Tan (2004) and considering the LNZ shown in Fig. 3, the principal tensile stress σ_1 perpendicular to the diagonal concrete strut can be determined as

$$\sigma_1 = \frac{k_1\tilde{T}\sin\theta_s}{A_{tang}} \tag{11}$$

where $\frac{\tilde{T}\sin\theta_s}{A_{tang}}$ is the average tensile stress across the diagonal strut due to the component of $\tilde{T}$ in the principal tensile direction of the LNZ. In addition, by using the condition of symmetry as described above, the force $\tilde{T}$ is a resultant of the tension tie forces T at the LNZ, which can be calculated as

$$\tilde{T} = \sqrt{2}T = \frac{P_n}{4\tan\,\theta_s} \tag{12}$$

The term A_{tang} is the area of the tangential-section of the strut. In addition, the term constant k_1 indicates a factor taking account of the non-uniformity of the stress distribution of $\tilde{T}\sin\theta_s$, because it is difficult to determine the exact magnitude and distribution of the principal tensile stress. By substituting Eq. (12) into Eq. (11), the principal tensile stress σ_1 can be rewritten as

$$\sigma_1 = \frac{k_1P_n\sin\,\theta_s}{4A_{tang}\,\tan\,\theta_s} \tag{13}$$

According to Park *et al*. (2008) and Souza *et al*. (2009), half the tensile strength contributed by the flexural reinforcement and concrete are used as the tension tie strengths in each of two orthogonal directions, combined into the tensile strength in the direction of $\tilde{T}$ at the LNZ (Fig. 2). This concept can well predict the flexural strengths of pile caps. For this reason, and using the same manner as Eq. (13), the tensile capacity, σ_{tn} at the LNZ can be written as

$$\sigma_{tn} = \frac{k_2\sqrt{2}\left(\frac{A_{sd}}{2}f_y + F_{ct}\right)\sin\theta_s}{A_{tang}} \tag{14}$$

where k_2 is a factor representing the non-uniformity of the tensile capacity. f_y and A_{sd} are the yield strength and the area of reinforcement in the considered direction, respectively. F_{ct} is the tensile capacity of concrete given by

$$F_{ct} = f_{ct}A_{ct} \tag{15}$$

where f_{ct} and A_{ct} are the tensile strength and the effective area of the concrete tie, respectively, and can be expressed as

$$f_{ct} = 0.5\sqrt{f_c'} \tag{16}$$

and

$$A_{ct} = w_t\frac{b}{2} \tag{17}$$

In addition, the principal compressive stress σ_2 in the direction of the diagonal concrete strut at the LNZ can be computed by

$$\sigma_2 = \frac{D}{A_{str}} = \frac{P_n}{4A_{str}\sin\,\theta_s} \tag{18}$$

From Eqs. (10), (13), (14) and (18), the following expression can be derived for the nominal shear strength of pile caps P_n,

$$P_n = \left(\frac{1}{P_t} + \frac{0.8}{P_s}\right)^{-1} \tag{19}$$

where

$$P_s = 4\,f_c'A_{str}\sin\theta_s \tag{20}$$

$$P_t = 4\sqrt{2}\alpha\left(\frac{A_{sd}}{2}f_y + F_{ct}\right)\tan\theta_s \tag{21}$$

The term $\alpha = k_2/k_1$ can be determined by minimizing the COV of the ratio of the experimental shear strength to the shear strength computed using Eq. (19). Through a nonlinear optimization technique such as the conjugate gradient method, the optimal value for α is found to be 1.2. In addition, it should be noted that this equation contains the interaction between the two terms of load-carrying capacity of pile caps, namely, strut capacity P_s, and tension tie capacity P_t. The combination of these terms is similar to the equivalent stiffness of the two springs linked in series. Accordingly, this STM is named "the interactive STM".

4. Verification of the proposed STM

4.1 Database of pile cap tests for verification of the STMs

An experimental database of 110 reinforced concrete pile caps used in this study was assembled from the published literature (Blévot and Frémy 1967, Clarke 1973, Sabnis and Gogate 1984, Suzuki *et al*. 1998, Suzuki *et al*.

Table 2 Database of pile cap experiments used for this study

Reference	No. of tested samples	Concrete strength (MPa)	Shear strength (kN)
Blévot and Frémy (1967)	30	13-49	250-9,000
Clarke (1973)	10	21-31	1,110-2,080
Sabnis and Gogate (1984)	8	18-41	173-280
Suzuki *et al*. (1998)	17	19-31	480-1,039
Suzuki *et al*. (1999)	3	27-28	1,245-1,303
Suzuki *et al*. (2000)	15	25-29	549-1,117
Suzuki and Otsuki (2002)	18	20-38	735-1,103
Chan and Poh (2000)	3	33-40	870-1,250
Ahmad *et al*. (2009)	6	31-27	480-604

1999, Suzuki *et al*. 2000, Suzuki and Otsuki 2002, Chan and Poh 2000, Ahmad *et al*. 2009) that provide the capacity of the specimens failing in shear. This can be summarized in Table 2.

4.2 The most appropriate geometry and approach to account for softening effect

Before evaluating the developed STMs's performance in computation, two statistical parameters are introduced. The first is the average value (AVG) of the ratio of experimental to predicted strength, and can be used as a rough indicator of conservative or unconservative bias of the methods on the safety. The second is the coefficient of variation (COV), which may be used to reveal the level of consistency of models.

Overall performance in the prediction of the developed STMs is shown in Table 3. Initially, it can be found that all STMs used in conjunction with A_{str} from Eq. (3b) give more consistency than the others, as indicated by lower values of COV. Of these efficiency factor models, their accuracy in computation does not differ significantly. However, it seems that Zwicky and Vogel's model (2006) gives the most accurate predictions. This is in agreement with the earlier findings by Chetchotisak *et al*. (2014a, b) who developed the STM for deep beams. Nevertheless, it can be pointed out that the developed STM using the Kupfer-Gerstle's biaxial failure criterion gives the closest agreement with the experimental results in terms of both precision and reliability (AVG =1.01 , COV =16.4%).

The reason why the failure criterion approach gives the higher level of accuracy than the efficiency factor one is discussed below. The former was developed from tests of unreinforced specimens satisfying the conditions of concrete struts in a common pile cap containing no transverse reinforcement, but only small percentages of main steel. The latter was based on the CFT calibrated from the experimental program of RC panels constructed with a moderate amount of web reinforcement and tested using a 2D plane stress configuration. Therefore, the former is recommended to be the proposed STM for predicting the shear strengths of pile caps.

Table 3 Overall performances in prediction of the approaches to represent softening effect

Approach to consider softening effect for	Performance measures of model using Eq. (3a)		Performance measures of model using Eq. (3b)	
	AVG	% COV	AVG	% COV
Vecchio & Collins	0.92	26.5	0.86	24.6
Zhang & Hsu	1.06	26.5	0.99	24.6
Kaufmann & Marti	0.90	22.4	0.84	21.5
Zwicky & Vogel	1.01	22.3	0.95	21.4
Kupfer-Gerstle's failure criterion	1.04	17.3	1.01	16.4

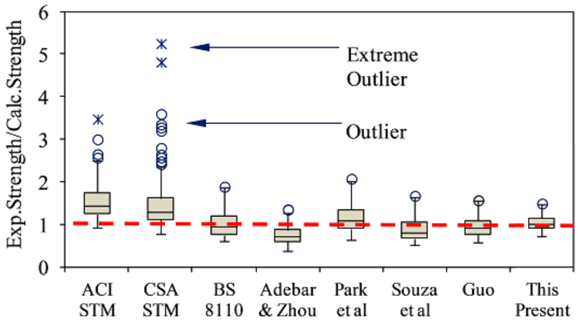

Fig. 4 Distribution of ratio of experimental strength to calculated strength for the different pile cap models

4.3 Comparison with other STMs

The format of the formula of the proposed STM is firstly compared with those of the methods available in codes of practice and state-of-the-art approaches, such as ACI-STM, CSA-STM, BS 8110, Adebar and Zhou (1996), Park *et al*. (2008), Souza *et al*. (2009) and Guo (2015). The details of these methods are shown in Table A1 (Appendix). As seen in this table, ACI-STM, CSA-STM and Park *et al*. (2008) have relatively similar formats, but different efficiency factor models. The constant value of efficiency factor was applied in ACI's approach, while Vecchio and Collins (1986) and Zhang and Hsu (1998) were employed in CSA-STM and Park *et al*. (2008)'s STM, respectively. According to BS 8110, the truss model was used for calculating the amount of main reinforcement, while the empirical formulas were applied for shear design. In addition, Adebar and Zhou (1996)'s model has been proposed in the simple form of nodal zone capacity, developed from an experimental and analytical study of compression struts confined by plain concrete. Among these selected models, the Souza *et al*. (2009) model adapted from Siao (1993) and the CEB-FIP Model Code seems to be the simplest. Finally, Guo (2015) used nonlinear finite element analysis in conjunction with the least-squares approach to develop his model. This has also been proposed in the uncomplicated form of the strut capacity expressed as functions of the concrete strength and the punching shear span-to-depth ratio. By comparing these selected formulations with the proposed STM Eq. (19), it is found that the proposed STM is the only STM containing both

Table 4 Overall performance in prediction of the different STMs

Pile cap shear strength models	Performance measures of the different STMs	
	AVG	% COV
ACI-STM	1.54	29.4
CSA-STM	1.54	48.0
BS 8110	1.00	29.1
Adebar and Zhou (1996)	0.76	28.4
Park *et al*. (2008)	1.12	25.8
Souza *et al*. (2009)	0.89	33.8
Guo (2015)	0.94	22.7
The proposed STM	1.01	16.4

strut and tie capacities combined into a unique and simple formula.

Secondly, the accuracy in predicting the shear strength of the proposed method is compared with the aforementioned seven approaches. Since the comparison is considered only the nominal capacity of the pile caps, the strength reduction factors for all codes of practice are taken equal to unity and any exceptions in the codes are neglected. The overall performances in prediction of the different methods are summarized in Fig. 4 as boxplots and Table 4.

The boxplots present the distributions of the ratios of test strength to the predicted strength for the eight different STMs. The boxplots provide the first quartile (Q1), second quartile (Q2), third quartile (Q3), as well as outliers. The whiskers extend only as far as the furthest points within 1.5 interquartile ranges (IQR) below Q1 or above Q3. The circle markers are used to represent the outliers within 3 IQR below Q1 or above Q3, whereas the asterisks illustrate the extreme outliers beyond 3 IQR of Q1 and Q3.

Although the selected four STMs, i.e., ACI-STM, CSA-STM, Adebar and Zhou (1996), and Park *et al*. (2008) have had their validity evaluated by Park *et al*. (2008) using 33 pile cap test results, in this paper, the authors employing 110 test specimens to prove the validity and confirm the findings of Park *et al*. (2008) that ACI-STM and CSA-STM still provide considerably conservative results with very large scatters. This can be indicated by the large size of boxplots, as well as the large values of AVG and COV. The reason for this conservatism can be described here. By observing the calculations of shear strengths of pile caps following both North American codes in this study, it is found that the effective areas of concrete struts directly used for calculating the shear strengths, are relatively small. This is also consistent with the evaluation of deep beams shear strengths using the same codes by Park and Kuchma (2007). Adebar and Zhou (1996)'s approach is also found to overestimate the shear strength. In addition, Park *et al*. (2008) shows relatively more accuracy and consistency than the aforementioned STMs. Compared to the proposed STM, the Park *et al*. (2008) STM adopting the minimum of the effective area of concrete strut between LNZ and UNZ as shown in Table A1, and the efficiency factor by Zhang and Hsu (1998) seems to be less accurate (AVG = 1.12, COV= 25.8%) than the proposed STM utilizing Eq. (3b) in conjunction with Kupfer-Gerstle biaxial tension-compression criterion (AVG=1.01 , COV=16.4%). This is because of using the more appropriate geometry for modeling the STM of pile caps and the approach accounting for the softening effect as described previously. For the three remaining STMs evaluated by the authors, i.e., BS 8110, Souza *et al*. (2009), and Guo (2015), the last one gives reasonably good prediction compared with others, but is still less accurate than the proposed STM. In brief, the proposed STM can be considered as a simple and practical approach while providing more precise and consistent predictions of the capacity of pile caps than the selected STMs.

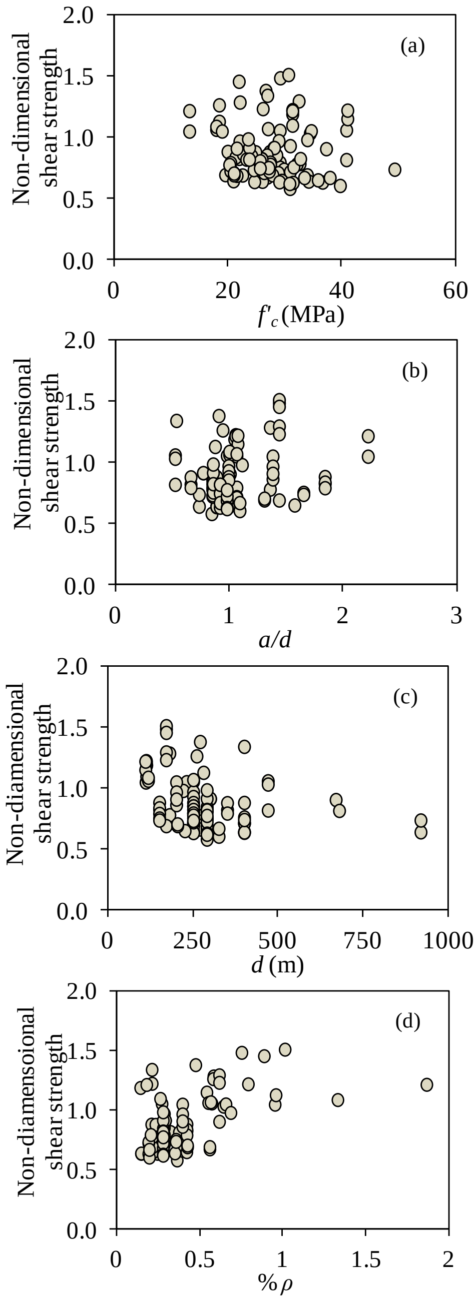

Fig. 5 Variation of the shear strength of pile caps with each of the four influencing parameters: (a) concrete strength; (b) shear span-to-depth ratio; (c) effective depth and (d) amount of main reinforcement

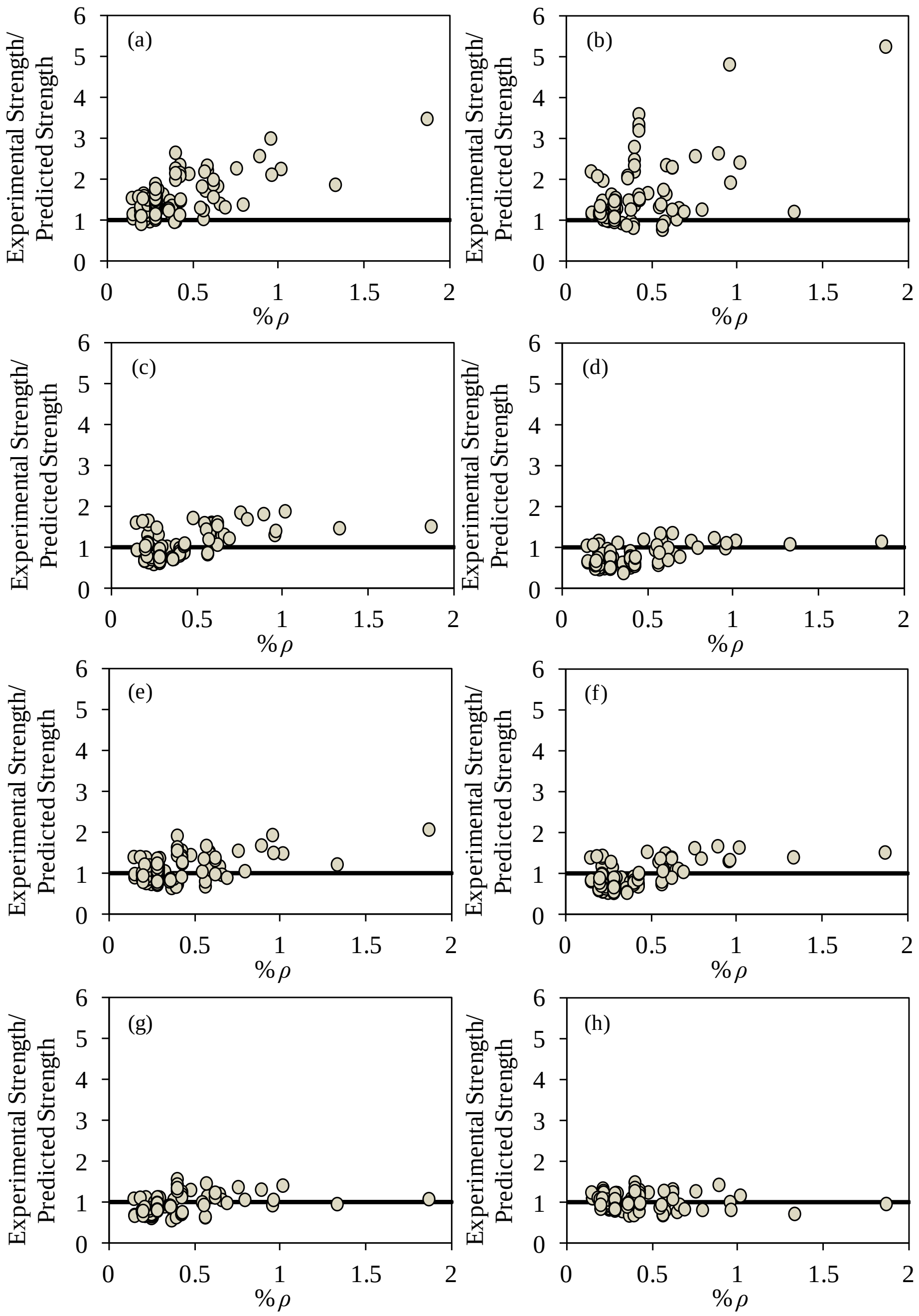

Fig. 6 Effect of the amount of main reinforcement on shear strength predictions using different approaches: (a) ACI-STM; (b) CSA-STM; (c) BS 8110; (d) Adebar and Zhou (1996) (e) Park *et al*. (2008); (f) Souza *et al*. (2009); (g) Guo (2015) and (h) The proposed STM

4.4 Investigation of the uniformity of the STMs

In general, the shear strength of an RC member is affected by important parameters such as concrete strength, shear span-to-depth ratio, effective depth, and the amount of main reinforcement. Therefore, the effects of these parameters on the non-dimensional shear strength, i.e., $P_n / bd\sqrt{f_c'}$, are illustrated in Fig. 5. It is demonstrated that the pile cap shear strength does not depend on concrete strength (Fig. 5(a)) and shear span-to-depth ratio (Fig. 5(b)) while one is relatively influenced by the effective depth (Fig. 5(c)) and particularly by the amount of main reinforcement (Fig. 5(d)). As a result, the amount of main steel is considered to be the key parameter to investigate the consistency of the STMs.

Fig. 6 shows the plots of the strength ratio for the different STMs also made for the same ranges of the aforementioned parameter. As illustrated in this figure, most STMs tend to underestimate the shear strengths for heavily reinforced footings, particularly for ACI-STM (Fig. 6(a)) and CSA-STM (Fig. 6(b)). This may indicate that the effect of the amount of main steel is not explicitly included in these STMs. For example, Guo (2015) assumed that the reinforcement ratio has little influence on the shear strength and thus a constant reinforcement ratio of 0.6% was adopted in his investigation. Therefore, this STM has a tendency to over-predict the shear strength for members with low reinforcement (Fig. 6(g)). On the other hand, it clearly confirms form Fig. 6(h) that the proposed STM in Eq. (19) containing the terms of strength of struts contributed by concrete and the strength of tension ties contributed by the main tension steel, has more robust accuracy over the considered ranges of this key parameter.

5. Conclusions

A new STM has been developed to theoretically analyze the shear strength of RC pile caps. Two approaches considering the concrete softening effect have been evaluated to determine the most appropriate formulation for use in the proposed STM. A database of 110 test specimens available in the literature was used to validate the STMs. Seven different models selected from codes of practice and published literature were also considered in the comparison. The following conclusions can be drawn from the present study:

- The approach to capture the concrete softening effect utilized the failure criterion approach, i.e., the Kupfer-Gerstle biaxial tension-compression criterion is found to provide a higher level of accuracy than the efficiency factor approach.
- The proposed STM adopted from the Kupfer-Gerstle biaxial tension-compression criterion contains an interaction between strut and tie capacity, therefore this is named as “the interactive STM”.
- The proposed STM can predict the shear strength of RC pile caps more accurately and robustly than the methods of ACI-STM, CSA-STM BS 8110, Adebar and Zhou (1996), Park *et al*. (2008), Souza *et al*. (2009), and Guo (2015). The average and coefficient of variation of the ratio between tested shear strengths and predicted shear strength using the proposed STM are 1.01 and 16.4%, respectively.
- Owing to its accuracy, uniformity and simplicity, the proposed STM may be considered to be useful for practical designs.

Acknowledgments

The authors would like to thank Rajamangala University of Technology Isan, Thailand for providing financial support for this research program. Any opinions, findings, and conclusions expressed in this paper are those of the authors, and do not necessarily reflect the views of the sponsors.

Notations

a	the smallest length between the center of pile at the lower nodal zone (LNZ) and the column quarter point at the upper nodal zone (UNZ) of the pile cap
A_{ct}	effective area of the concrete tie
A_{sd}	area of reinforcement in the considered direction
A_{str}	effective area of the diagonal concrete strut
A_{tang}	area of the tangential-section of the strut
b	width of pile cap
c	width of column
D	maximum compression force in diagonal strut
d	effective depth of pile cap
d_p	pile diameter
f_c'	concrete cylinder strength
F_{ct}	tensile capacity of concrete tie
f_{ct}	tensile strength of the concrete tie
f_{cu}	concrete cube strength
f_y	yield strength reinforcement
h	distance from top of pile cap to top of pile
h_c	depth of compression zone at section
L_s	pile spacing
n	modular ratio
P_n	predicted nominal shear strength of pile cap
P_s	capacity of pile cap contributed from the concrete strut
P_t	capacity of pile cap contributed from the tension tie
T	tension tie forces
$\tilde{T}$	resultant of the tension tie forces T at the LNZ
w_t	effective width of tension tie
ε_h	average normal strains in the horizontal directions
ε_v	average normal strains in the vertical directions
ε_0	strain at peak stress of standard cylinder
ε_1	average principal tensile strains in the direction perpendicular to the concrete strut
ε_2	average principal strain of concrete in the direction of the strut
ν	one of the selected efficiency factors listed in Table 1
θ_s	inclined angle between the diagonal concrete strut and the horizontal direction
ρ	longitudinal reinforcement ratio
σ_1	principal tensile stress of concrete at the LNZ
σ_2	principal compressive stress of concrete in the direction of the diagonal concrete strut at the LNZ
σ_{tn}	tensile capacity at the LNZ

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Appendix

Table A1 Details of shear strength models for reinforced concrete pile caps for comparison

Reference	Shear Strength Model
ACI-STM	$P_n = 4\, f_{ce} A_{str} \sin\theta_s$, where $f_{ce} = 0.85\beta_s\, f_c'$, $\beta_s = 0.60$, $A_{str} = \min\begin{pmatrix} \pi/4 d_p \left(w_t \cos\theta_s + d_p \sin\theta_s\right), \\ c/\sqrt{2}\left(h_c \cos\theta_s + c/\sqrt{2}\, \sin\theta_s\right)\end{pmatrix}$
CSA-STM	$P_n = 4\, f_{ce} A_{str} \sin\theta_s$ where $f_{ce} = \frac{f_c'}{0.8+170\varepsilon_1} \le 0.85 f_c'$, $\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002)/\tan^2\theta_s$ $A_{str} = \min\begin{pmatrix} \pi/4 d_p \left(w_t \cos\theta_s + d_p \sin\theta_s\right), \\ c/\sqrt{2}\left(h_c \cos\theta_s + c/\sqrt{2}\, \sin\theta_s\right)\end{pmatrix}$

BS 8110	$P_n = \min\left(2v_{c1}b_v d,\, v_{c2}b_o d\right)$,
	 For $f_{cu} < 25$ MPa.
	 $v_{c1} = 0.79\left(100 A_s/b_v d\right)^{1/3}\left(400/d\right)^{1/4} 2d/a_v$
	 $\le 0.8\sqrt{f_{cu}}$ or 5 MPa.
	 For $f_{cu} > 25$ MPa.
	 $v_{c1} = 0.79\left(\frac{100\, A_s}{b_v\, d}\right)^{1/3}\left(\frac{400}{d}\right)^{1/4}\frac{2d}{a_v}\left(\frac{f_{cu}}{25}\right)^{1/3}$
	 $\le 0.8\sqrt{f_{cu}}$ or 5 MPa.
	 $f_{cu} \le 40$ MPa., $100 A_s/b_v d \le 3$, $\left(\frac{400}{d}\right)^{1/4} \le 0.67$,
	 If $L_s \le 3d_p$ then $b_v = b$, If $L_s > 3d_p$ then
	 $b_v = 3d_p$, $v_{c2} = 0.8\sqrt{f_{cu}} \le 5$ MPa.
	 If $L_s \le 3d_p$ then $b_o = 4c$, If $L_s > 3d_p$ then
	 $b_o = 4\left(L_s - 3d_p/5\right)$

Adebar and Zhou (1996)	$P_n = \min\left(\left(f_b\right)_{UNZ} \cdot A_{col},\ 4\left(f_b\right)_{LNZ} \cdot A_{pile}\right)$, A_{col} = sectional area of column, A_p = sectional area of pile $f_b \le 0.6 f_c' + \alpha\beta 6\sqrt{f_c'}$, $\alpha = 1/3\left(\sqrt{A_2/A_1} - 1\right) \le 1.0$, $\beta = 1/3\left(h_s/b_s - 1\right) \le 1.0$ The ratio A_2/A_1 is identical to that used in ACI code for calculating the bearing strength. $\left(h_s/b_s\right)_{UNZ} \approx 2d/c$, $\left(h_s/b_s\right)_{LNZ} \approx d/d_p$

Table A1 Continued

Reference	Shear Strength Model
Park *et al*. (2008)	$P_n = 4\nu\, f_c' A_{str} \sin\theta_s$, ν = Zhang and Hsu's efficiency factor (1998), $A_{str} = \min\begin{pmatrix} \pi/4 d_p \left(w_t \cos\theta_s + d_p \sin\theta_s\right), \\ c/\sqrt{2}\left(kd \cos\theta_s + c/\sqrt{2}\, \sin\theta_s\right)\end{pmatrix}$
Souza *et al*. (2009)	$P_n = 2.08bd\left(f_c'\right)^{2/3}$
Guo (2015)	$P_n = 4F\sin\gamma$, $F = 1.885R^2 \cdot \alpha\left(f_c'\right)\cdot\beta(\lambda)\cdot f_c'$, $\gamma = \tan^{-1}\left(0.9\sqrt{2}d/L_s\right)$, $\alpha\left(f_c'\right) = 2.05 - 0.22\sqrt{f_c'}$ for $6.7\,\text{MPa} \le f_c' \le 35\,\text{MPa}$ $\alpha\left(f_c'\right) = 0.75$ for $35\,\text{MPa} \le f_c' \le 50\,\text{MPa}$, $\beta(\lambda) = 2.1125 - 0.75\lambda$ for $0.15 \le \lambda \le 0.95$, $\beta(\lambda) = 1.4$ for $\lambda \ge 0.95$ $\lambda = w/d$, w = punching shear span,= the radius R of the pile