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Fuzzy methodology application for modeling uncertainties in chloride ingress models of RC building structure

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Abstract. Chloride ingress is a common cause of deterioration of reinforced concrete located in coastal zone. Modeling the chloride ingress is an important basis for designing reinforced concrete structures and for assessing the reliability of an existing structure. The modeling is also needed for predicting the deterioration of a reinforced structure. The existing deterministic solution for prediction model of corrosion initiation cannot reflect uncertainties which input variables have. This paper presents an approach to the fuzzy arithmetic based modeling of the chloride-induced corrosion of reinforcement in concrete structures that takes into account the uncertainties in the physical models of chloride penetration into concrete and corrosion of steel reinforcement, as well as the uncertainties in the governing parameters, including concrete diffusivity, concrete cover depth, surface chloride concentration and critical chloride level for corrosion initiation. There are a lot of prediction model for predicting the time of reinforcement corrosion of structures exposed to chloride-induced corrosion environment. In this work, RILEM model formula and Crank's solution of Fick's second law of diffusion is used. The parameters of the models are regarded as fuzzy numbers with proper membership function adapted to statistical data of the governing parameters instead of random variables of probabilistic modeling of Monte Carlo Simulation and the fuzziness of the time to corrosion initiation is determined by the fuzzy arithmetic of interval arithmetic and extension principle. An analysis is implemented by comparing deterministic calculation with fuzzy arithmetic for above two prediction models.

Keywords: corrosion initiation time; service life; fuzzy; reinforced concrete; Monte Carlo simulation; probabilistic.

1. Introduction

The chloride induced corrosion of steel reinforcement embedded in concrete is the major cause to deterioration and reducing its related service life of RC structure.

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In normal environment case, concrete protects steel reinforcement from corrosion by forming a passive film around the steel due to the high alkalinity of the concrete pore solution. However when chloride ions from the origin such as deicing salts or seawater penetrate into the concrete and reach the steel surface, they disrupt the passive film and initiate corrosion. The corrosion of steel reinforcement will start immediately after the chloride content of concrete near the embedded steel reaches a critical level, which defines the resistance of steel to corrosion. Consequently, the onset of corrosion is governed by the surface chloride concentration, concrete diffusivity, concrete cover depth of the steel, corrosion critical level, as well as moisture level in terms of the pore solution, and the availability of oxygen. Herein, the prediction of the onset of corrosion is very important in order to reduce life cycle costs and enlarge the service life of RC structure as earlier stated. Thus, a reliable prediction model of chloride penetration into reinforced concrete structures is critical for predicting the time to onset of corrosion of steel reinforcement (AIJ 2004 and Lounis 2001).

Mathematical models of chloride ingress currently being developed are primarily based on chloride diffusion, which can be used as starting points in the development of service life prediction tools and performance-based specifications. Even if chloride ingress into concrete is complex, models are constructed around Fick's second law of diffusion and the error function solution by Crank. Fick's second law of diffusion concerns the rate of change of concentration with respect to time as follows:

$$\frac{\delta C}{\delta t} = D\frac{\partial^2 C}{\partial x^2} \tag{1}$$

with boundary condition of $C_x = 0$ at $t = 0$ and $0 < x < \infty$ or $C_x = C_0$ at $t = 0$ and $0 < x < \infty$.

It can be noted from Eq. (1) that the determination of time to corrosion initiation requires the values of D_{eff}, C_0, C_x and x. These variables of the deterministic prediction model assume the uncertainty associated with governing parameters such as exposure condition, type and quality of concrete, and quality of construction.

Where D_{eff} = apparent diffusion coefficient of concrete
C_o = surface chloride concentration
C_x = chloride concentration at the concrete cover depth from the concrete surface
x = concrete cover thickness

However, existing physical deterministic models for the time of corrosion initiation by chloride ingress provide point estimates (or fixed values) to determine a possible prediction time. This is generally not sufficient to identify predominant contributory prediction time that accounts for the uncertainties identified earlier. Therefore, the model needs to be further developed to include uncertainties in order that the 'probability' or 'possibility' of corrosion initiation time can be quantified. Possible approaches for doing this are Monte Carlo simulations, first order reliability methods and possibilistic analysis using fuzzy arithmetic (Do 2004).

In recent years, there has been much study about processing the uncertainty of the variables by using Monte Carlo (MC) simulation, by which the parameters of the models are modeled as random variables and the distribution of the time to corrosion initiation and probability of corrosion are determined. MC simulation proves to be well applicable but it is very difficult to be manually calculated because it generates the relevant random values of a number of variables (Abebe 2000 and Lounis 2001).

In this paper, a possibilistic approach based on the fuzzy method is pursued to include uncertainties in prediction models. The uncertainties of the deterministic model associated with various conditions are treated by fuzzy arithmetic which is a successful tool to solve engineering problems with uncertain parameters. The shape of variable derived from measured data is modeled as the standard form of triangular fuzzy numbers (TFN) which are just a rough approximation of the really existing uncertainty (Hanss 2000 and 2002). The prediction capability of fuzzy variable treated-prediction model is illustrated in a case study of a reinforced concrete building structure in coastal environment.

2. Uncertainty in prediction model and its process methodology

2.1. Classification of existing prediction models

There are a variety of mathematical models explicitly predicting corrosion initiation time in a quantitative way. These existing prediction models can be usually made a divide between physical and empirical models as described below.

1) Physical models: Physical models are based on theories on how transport of different substances takes place in a material.
2) Empirical models: Empirical models are based on observations of response from structures, exposed either in field or in laboratory. The observations are used to derive and quantify the parameters in the models.

There is a danger to use observations from already built structures, which is the case with empirical models, since they are influenced both by the materials, environments and the workmanship during construction. Thus, it is necessary to consider that the results from a prediction model never better quality than the input data (Ciampoli 1999).

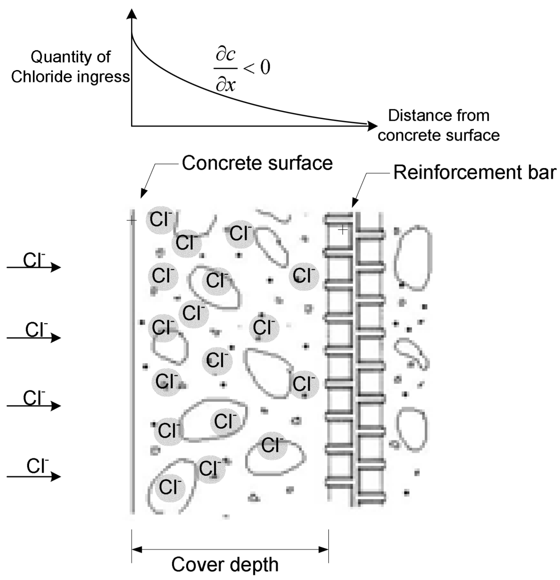


Fig. 1 Schematic representation for chloride ingress into concrete

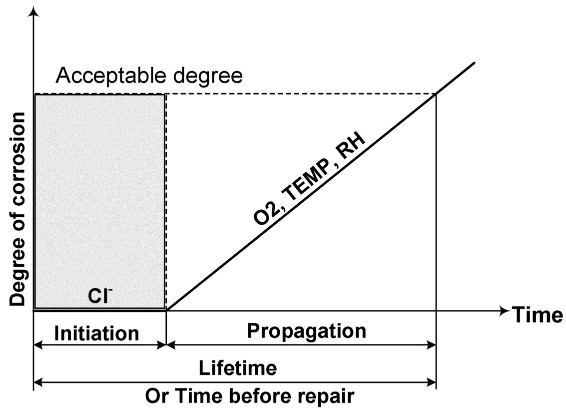


Fig. 2 Model for reinforcement corrosion, with a division into initiation and propagation periods (Tutti 1982)

As mentioned earlier, reinforcement in concrete is normally in a passive state due to the properties of the surrounding cement paste (high alkalinity). When the reinforcement is in the passive state the rebars are protected with thin layer of iron oxide. This passive state can be changed in main ways of the chemical process of reinforcement corrosion. It is normally divided into two different periods, initiation and propagation. The initiation period is the time during which changes in the concrete takes place, in interaction with the exposure environment, until a limit is reached and damage starts to propagate. The propagation period begins when certain defined event occurs, e.g., initiation of corrosion, and it goes on until a specified limit state is reached. During the initiation period the reinforcement steel is depassivated due to ingress of chloride ions into the concrete. A simple, but useful, model to predict the lifetime of a concrete structure, exposed to chloride ingress, is proposed. The model is shown in Fig. 2 (Tutti 1982).

2.2. Uncertainties existing in prediction models

To achieve reliable results for the numerical solution of the deterministic prediction problem, exact values for the parameters for the problem equations should be available. In practice, however, exact values can not be provided. The model parameters exhibit variability, e.g., due to both irregularities in manufacturing when considering the physical properties of a material and uncertainties in measuring when considering the environmental condition.

Theses uncertainties can be grouped into aleatoric uncertainty and epistemic uncertainty. The aleatoric uncertainty arises from the physical or inherent uncertainty identified with the random nature of the basic parameters that govern the chloride penetration and corrosion mechanisms. This uncertainty is associated with variability of the concrete cover depth, uncertainty of the chloride concentration at the surface, and uncertainty of the chloride diffusion coefficient.

The epistemic uncertainty arises from the uncertainty in the models for chloride transport and corrosion initiation. The model uncertainty results from the use of a simplified physical model of the actual phenomenon, such as assumption of chloride transport mechanism governed by diffusion, use of simplified models of the diffusion coefficient and driving chloride concentration and use of simplified chloride critical level to define the corrosion resistance of steel reinforcement. The epistemic uncertainty also arises from statistical uncertainty due to estimating statistical representative value of an average from a limited sample size. Thus, it is clear that a deterministic prediction model can be quite improper in predicting the actual structural response to environmental condition (Lounis 2001 and Thoft-Christensen 2001).

To solve this limitation, the application of fuzzy set theory proves to be a practical approach. More specifically, the uncertainties in the model parameters can be taken into account by representing the effects of scatter by fuzzy numbers with their shape derived from statistical data (Hanss 2002).

The elementary mathematical operations like addition, multiplication, etc. must then be carried out using generalized versions of the operations that ensure the handling of fuzzy numbers. By this technique, one can demonstrate how initially assumed uncertainties are processed through the calculation procedure leading finally to fuzzy results that reflect the reliability of the problem solution. Additionally, the fuzzy results allow the computation of a crisp value as the most likely result for the problem which in general differs from the result achieved by an initially non-fuzzy approach using only crisp parameters.

3. Presentation of methodology to apply fuzzy numbers concept in this study

3.1. Definition of fuzzy number and fuzzy arithmetic

To qualify as a fuzzy number, a fuzzy set A on real numbers must be normal and convex. The fuzzy set must be normal, since the concept of a set of "real numbers close to a given real number *R*" is fully satisfied by *R* itself; hence the membership grade of *R* in any fuzzy set that attempts to capture a fuzzy number must be 1 (George 1995).

The bounded support of a fuzzy number and all its α-cuts for $\alpha \neq 0$ must be closed intervals to allow definition of arithmetic operations on fuzzy numbers in terms of standard arithmetic operations on closed intervals. Since α-cuts of any fuzzy number are required to be closed intervals for all $\alpha \in [0, 1]$, every fuzzy number is a convex fuzzy set. A fuzzy number is represented as an ordered set of confidence intervals, each of them providing the related numerical value at a given presumption level $\alpha \in [0, 1]$.

These confidence intervals should comply with the relation $\alpha_1 > \alpha_2 \Rightarrow {}^{\alpha_1}A \subset {}^{\alpha_2}A$. where $\alpha_1 > \alpha_2 \in [0, 1]$ and ${}^{\alpha_1}A$, ${}^{\alpha_2}A$ are the confidence intervals at presumption levels α_1 and α_2 respectively.

The four basic arithmetic operations on fuzzy numbers (addition, subtraction, multiplication, and division) can be described as sequences of operations among confidence intervals. In particular, let *A* and *B* be fuzzy numbers and let $\otimes$ be a generic arithmetic operator.

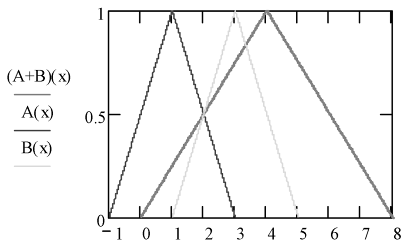
The fuzzy number $A \otimes B$ is obtained by computing the operation ${}^{\alpha}A \otimes {}^{\alpha}B$ for each $\alpha \in [0, 1]$, where ${}^{\alpha}A$ and ${}^{\alpha}B$ are the confidence interval of *A* and *B* at presumption level α. It was proved that this approach complies with the extension principle of Zadeh as

$$(A \otimes B)(z) = \operatorname*{Sup}_{z = x \otimes y} \min[A(x), B(y)] \tag{2}$$

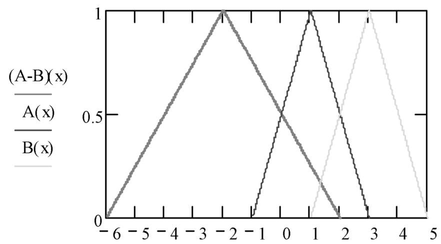
As an example of employing the above explanation, consider two triangular fuzzy numbers A and B defined as follows:

$$A(x) := \begin{cases} 0 & \text{if } (x \le -1) \wedge (x > 3) \\ \dfrac{(x+1)}{2} & \text{if } -1 < x \le 1 \\ \dfrac{(3-x)}{2} & \text{if } 1 < x \le 3 \end{cases} \qquad B(x) := \begin{cases} 0 & \text{if } x \le 1 \wedge x > 5 \\ \dfrac{(x-1)}{2} & \text{if } 1 < x \le 3 \\ \dfrac{(5-x)}{2} & \text{if } 3 < x \le 5 \end{cases}$$

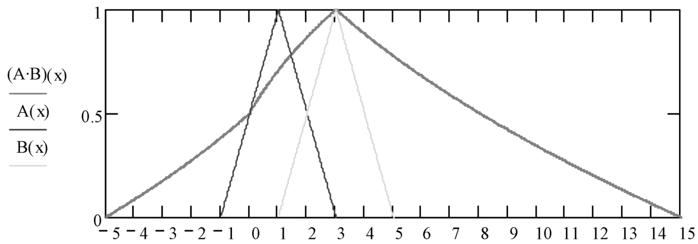
$$\mathrm{Add}(x) := \begin{cases} 0 & \text{if } x \le 0 \wedge x > 8 \\ \dfrac{x}{4} & \text{if } 0 < x \le 4 \\ \dfrac{(8-x)}{4} & \text{if } 4 < x \le 8 \end{cases}$$



$$\mathrm{Sub}(x) := \begin{cases} 0 & \text{if } x \le -6 \wedge x > 2 \\ \dfrac{x+6}{4} & \text{if } -6 < x \le -2 \\ \dfrac{(2-x)}{4} & \text{if } -2 < x \le 2 \end{cases}$$



$$\mathrm{Mul}(x) := \begin{cases} 0 & \text{if } x \le -5 \wedge x > 15 \\ \dfrac{3-\sqrt{4-x}}{2} & \text{if } -5 \le x < 0 \\ \dfrac{\sqrt{1+x}}{2} & \text{if } 0 \le x < 3 \\ \dfrac{4-\sqrt{1+x}}{2} & \text{if } 3 < x \le 15 \end{cases}$$



$$\mathrm{Div}(x) := \begin{cases} 0 & \text{if } x < -1 \wedge x \ge 3 \\ \dfrac{x+1}{2-2x} & \text{if } -1 \le x < 0 \\ \dfrac{(5x+1)}{2x+2} & \text{if } 0 \le x < \dfrac{1}{3} \\ \dfrac{(3-x)}{2x+2} & \text{if } \dfrac{1}{3} \le x < 3 \end{cases}$$

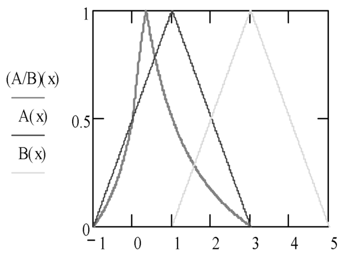


Fig. 3 Illustration of arithmetic operations on fuzzy number

Their α-cuts are:

$$^{\alpha}A = [2\alpha - 1, 3 - 2\alpha]$$
$$^{\alpha}B = [2\alpha + 1, 5 - 2\alpha]$$

Using fuzzy interval arithmetic, we can obtain

$$^{\alpha}(A + B) = [4\alpha, 8\text{-}4\alpha] \qquad \text{for } \alpha \in (0.1]$$
$$^{\alpha}(A - B) = [4\alpha - 6, 2 - 4\alpha] \qquad \text{for } \alpha \in (0.1]$$
$$^{\alpha}(A \cdot B) = \begin{cases} [-4\alpha^2 + 12\alpha - 5, 4\alpha^2 - 16\alpha^2 + 15] & \text{for } \alpha \in (0, 0.5] \\ [4\alpha^2 - (1,4)\alpha^2 - 16\alpha + 15] & \text{for } \alpha \in (0, 5.1] \end{cases}$$
$$^{\alpha}(A / B) = \begin{cases} [(2\alpha - 1)/(2\alpha + 1), (3 - 2\alpha)/(2\alpha + 1)] & \text{for } \alpha \in (0, 0.5] \\ [(2\alpha - 1)/(5 - 2\alpha), (3 - 2\alpha)/(2\alpha + 1)] & \text{for } \alpha \in (0, 5.1] \end{cases}$$

The resulting fuzzy numbers are then Fig. 3.

3.2. Formulation with Triangular Fuzzy Number (TFN)

To include uncertainties into the solution procedures of deterministic prediction model, the fuzzy numbers that are used to represent the uncertain model parameters was implemented in a standard form of TFN due to those simplicity in both calculation and just three components (Hanss 2000).

Considering a definite uncertain parameter A, measured data for the parameter are assumed to be available from which a normalized distribution function $N_A(x)$ can be derived that expresses the frequency of occurrence of a certain measured value x for the parameter A within the interval Δx. In most cases, these data approximately show Gaussian distribution, i.e., normal distribution. The uncertainty in the parameter A can then be approximately modeled by a fuzzy number $\tilde{A}$ with the membership function $A(x)$ of Eq. (3), which has the support of $2 \times 2\sigma_a$ set up for around 95% confidence interval of a normalized distribution function $N_a(x)$.

$$A(x) = \begin{vmatrix} \frac{x-a}{m-a}, & \text{if } a < x \le m \\ \frac{b-x}{b-m}, & \text{if } m < x \le a \\ 0, & \textit{otherwise} \end{vmatrix} \tag{3}$$

where m are the mean value of the normal distribution in Fig. 4 and a and b is lower bound and upper bound that obtained from lower and upper bound of 5% of the normal distribution in Fig. 4.

Considering an uncertain parameter B showing lognormal distribution, similarly to an uncertain parameter A of normal distribution, a triangular form of membership function is identified as shown in Fig. 5.

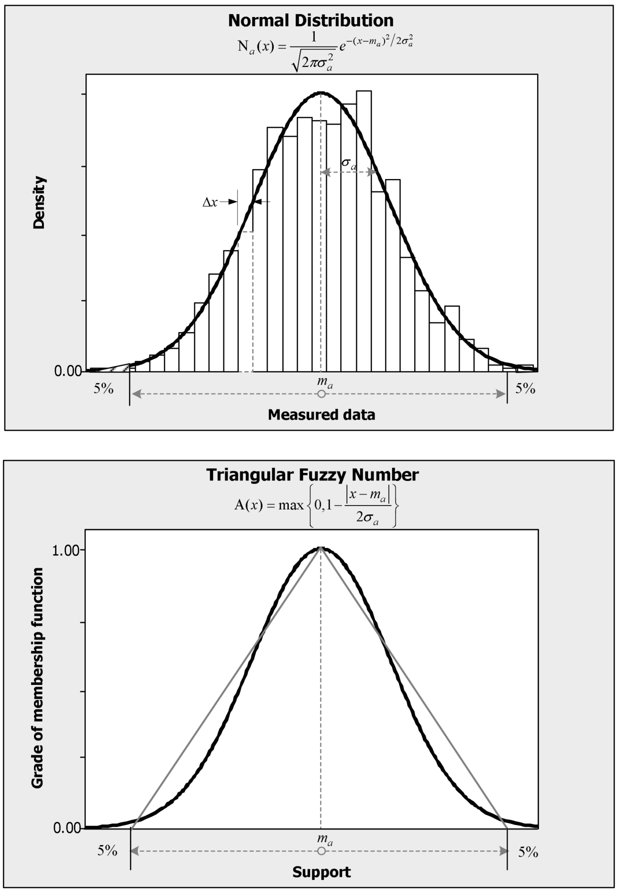


Fig. 4 Normal distribution of any parameter and its adaptation to membership function of triangular fuzzy number

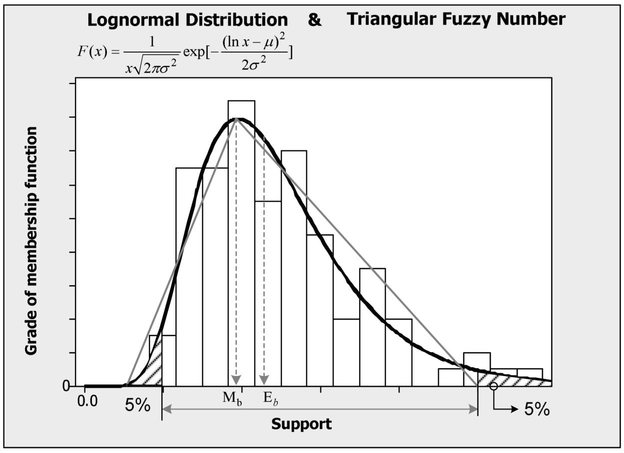


Fig. 5 Lognormal distribution of any parameter and its adaptation to membership function of triangular fuzzy number

4. Case study (calculating the prediction model by fuzzy arithmetic method detailed previously)

4.1. Preparation of input parameters and related uncertainties

Fig. 6 illustrates the calculation procedure of fuzzy arithmetic for prediction model of RILEM model and Crank's solution. In implementing fuzzy arithmetic for RILEM model and Crank's solution, the method treating input variables with fuzzy numbers is same. Just a difference stems from selecting deterministic formula.

The mean value and standard deviations of all parameters which are used to apply fuzzy

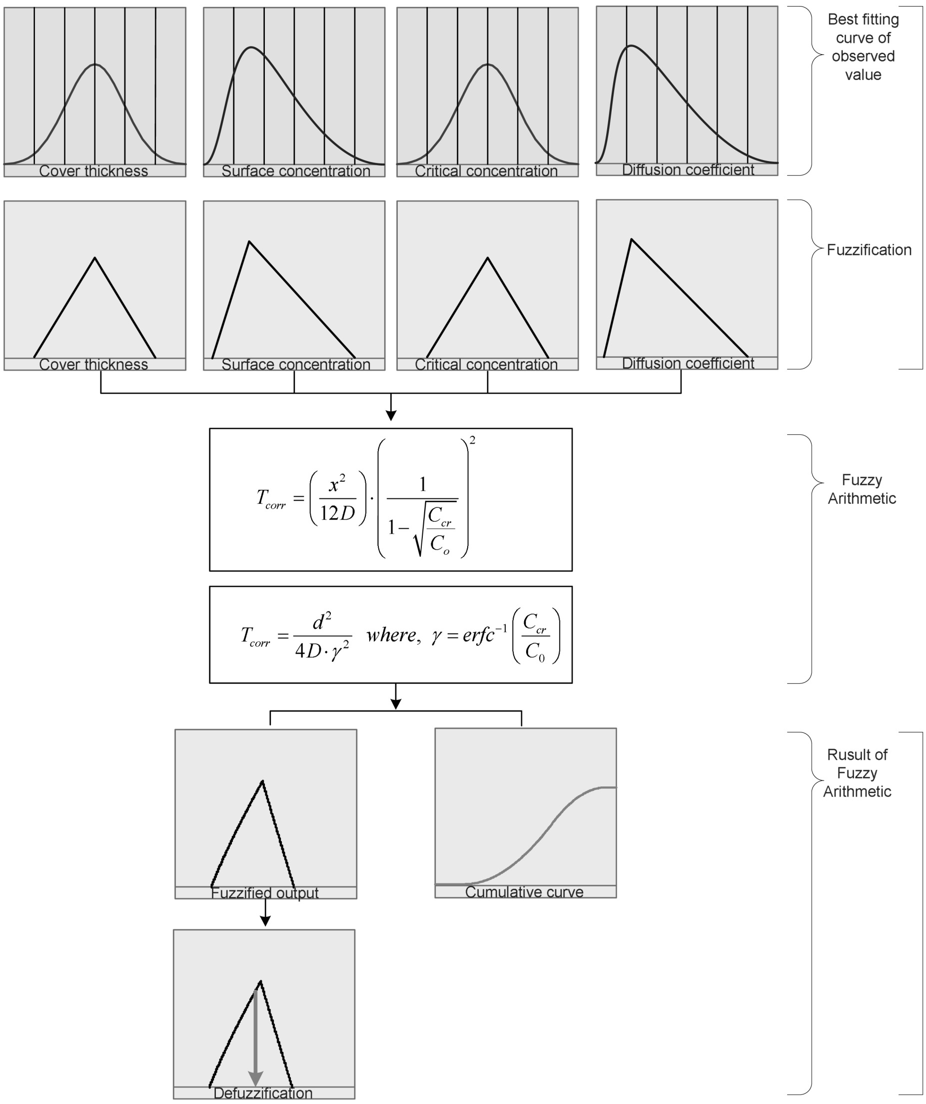
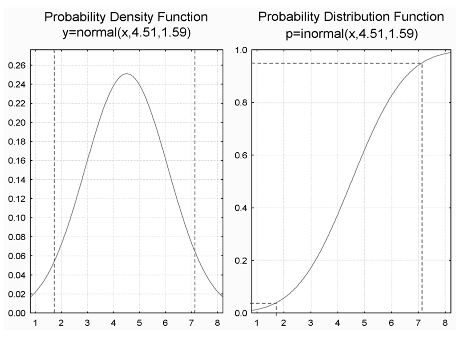
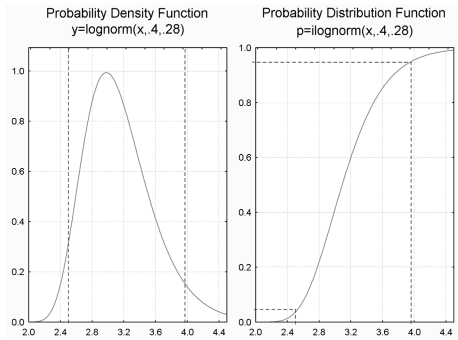


Fig. 6 Calculation procedure of fuzzy arithmetic for prediction model of RILEM model and Crank' solution

Table 1 Statistic properties of all parameters in corrosion prediction model

<table>
<tr><td rowspan="2">Parameter</td><td rowspan="2">Cover thickness:</td><td>Mean value</td><td>4.51</td><td rowspan="2">Surface concentration</td><td>Mean value</td><td>3.09</td></tr>
<tr><td>Standard variation</td><td>1.59</td><td>Standard variation</td><td>0.44</td></tr>
<tr><td>Probability density function
and
Probability distribution function</td><td colspan="3">
Lower bound:1.89, Mode: 4.51, Upper bound:7.13</td><td colspan="3">
Lower bound:2.54, Mode: 2.98, Upper bound:3.96</td></tr>
<tr><td rowspan="2">Parameter</td><td rowspan="2">Critical concentration</td><td>Mean value</td><td>1.25</td><td rowspan="2">Diffusion coefficient</td><td>Mean value</td><td>1.26</td></tr>
<tr><td>Standard variation</td><td>0.23</td><td>Standard variation</td><td>0.37</td></tr>
<tr><td>Probability density function
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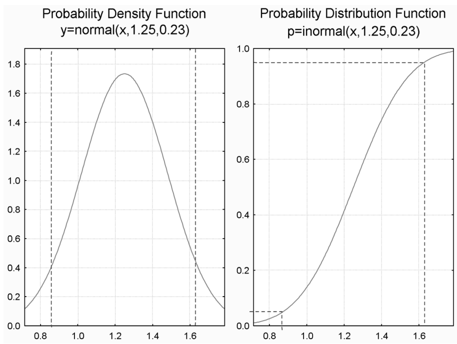
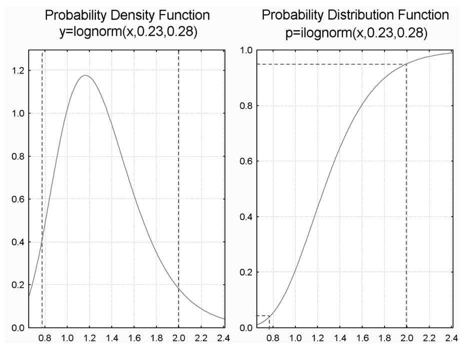
Probability distribution function</td><td colspan="3">
Lower bound:0.87, Mode: 1.25, Upper bound:1.63</td><td colspan="3">
Lower bound:0.79, Mode: 1.16, Upper bound:1.99</td></tr>
</table>

arithmetic to solving corrosion prediction problems with uncertain parameters are listed in Table 1, where cover thickness, diffusion coefficient and surface chloride concentration among input parameters was prepared with the measured data from the actual reinforced concrete building structure located at coastal environment or in reference to literature survey.

1) Surface chloride concentration: Between about 10 and 20 mm, the chloride concentration reaches a maximum value and is be relatively constant, so called, considered to quasi-constant after an initial time (Martin-Perez 1999). This depth varies depending on concrete quality and exposure condition, with a value usually taken as 12.7 mm (Weyers, *et al*. 1993). In this study surface chloride concentration is characterized by the measured data located 12.7 mm below the surface of specimen extracted from the object. The surface chloride concentration is identified by being best described by lognormal distribution having a mean value of 3.09 and a standard deviation of 0.44.
2) Effective cover depth: As explained above, the effective cover depth is taken as the concrete cover depth less the depth of maximum chloride concentration. The chloride ingress is therefore modeled as a Fick's second law over the effective cover depth, which is thought to be best described by normal distribution having a mean value of 4.51 and a standard deviation of

1.59.

3) Apparent diffusion coefficient: The diffusion coefficient of concrete is known to depend on the pore structure, which is related to the water/cement ratio, kind of cement, mix proportion and quality. The two variables recognized to have the most influence on the coefficient of diffusion were the water-cement ratio and the presence of mineral admixtures (Weyers, *et al.* 1994). Supposed water-cement ratio of concrete used is 0.5, the apparent chloride diffusion coefficient was estimated by using Eq. (4) (JSCE 2002).

$$\log D_{eff} = -3.9(w/c)^2 + 7.2(w/c) - 2.5 \tag{4}$$

The coefficient of variance (COV) and distribution function was regarded as 30% and lognormal function in reference to literature survey (Weyers, *et al.* 1994), respectively.

Critical chloride concentration: The critical chloride concentration indicates the chloride threshold level which is the concentration of chlorides necessary to break down the protective passive film on the reinforcing steel surface and initiate corrosion. As for the parameters discussed earlier, chloride thresholds proposed in the literature cover a wide range of values. Some investigations (Miyagawa, *et al.* 1985 and Otsuki, *et al.* 1985) show that the chloride threshold level to initiate corrosion of steel in concrete is ordinarily considered to be 1.2 to 2.5 kg/m^3. In this study, the mean value and COV of critical chloride concentration was regarded as 1.2 and 18%, respectively and was supposed to be normally distributed.

All characteristics of input parameters described above are listed in Table 1.

4.2. Calculation of T_{corr} based on RILEM model

As stated earlier, the prediction model by Fick's 2nd law like Eq. (1) has been widely used due to its simplicity.

To calculate the deterministic prediction model by using fuzzy arithmetic in this study, the approximation model of Crank's solution of Fick's second law presented by RILEM like Eq. (5) is used (Rilem report 14 1996).

$$C_{cr} = (C_0 - C_{init}) \cdot \left\{ 1 - \frac{x}{\sqrt{3D \cdot T_{corr}}} \right\}^2 + C_{init} \tag{5}$$

Eq. (6) is rewritten as follows:

$$T_{corr} = \frac{x^2}{12D} \cdot \left(\frac{1}{1 - \sqrt{\frac{C_{cr}}{C_o}}} \right)^2 \tag{6}$$

When Eq. (6) is calculated by fuzzy arithmetic, it needs to stop it in specific point since a linear approximation of simple TFN become function of higher degree and too difficult. Thus fuzzy arithmetic is applied to calculate the time T_{corr} to initiation of reinforcement corrosion separated into two parts like Eq. (7). Normally, in stochastic model by Monte Carlo Simulation (MC simulation), all parameters of Eq. (6) are taken into consideration by modeling them as random variables which

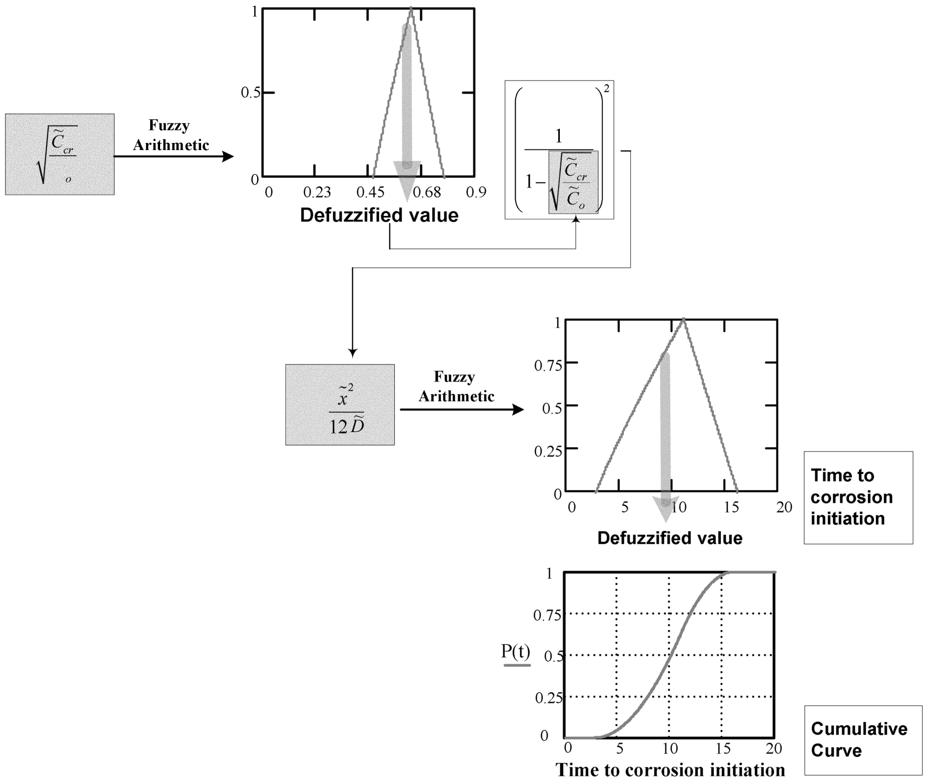


Fig. 7 Calculation procedure of time to corrosion initiation based on RILEM model

have probabilistic density functions (PDF) that are obtained from field measurements or from the survey analysis but in this study, they are treated as fuzzy variables with proper core and support by conforming to the procedure illustrated in Figs. 4 and 5.

Overall procedure of calculating the time T_{corr} to initiation of reinforcement corrosion by Eq.(6) based on fuzzy arithmetic is represented in Fig. 7. Crisp value of time to corrosion initiation is acquired by multiplying the defuzzified value of membership function of $\tilde{x}^2/12\tilde{D}$ by the crisp value of $[1/(1-\sqrt{\tilde{C}_{cr}/\tilde{C}_o})]^2$ calculated by inserting the defuzzified value of $\sqrt{\tilde{C}_{cr}/\tilde{C}_o}$, where each membership function is defuzzified by fuzzy centroid method, i.e., Center of Area(CoA), by way of the following Eq. (7).

$$\bar{y} = \frac{\int A(y) \cdot y dy}{\int A(y) dy} \tag{7}$$

In this study, arithmetic operation of addition, subtraction, division, and multiplication on fuzzy numbers is carried out by using fuzzy interval arithmetic. As mapping fuzzy numbers via functions, the extension principle is applied to those transformations.

Fig. 8 represents the shape of membership function of fuzzy number $\tilde{C}_{cr}$ and $\tilde{C}_o$ as well as the fuzzy arithmetic procedure of $\sqrt{\tilde{C}_{cr}/\tilde{C}_o}$ based on fuzzy interval arithmetic and extension principle.

Fig. 9 represents the membership function of fuzzy number $\tilde{x}$ and $\tilde{D}$, and also those transformations

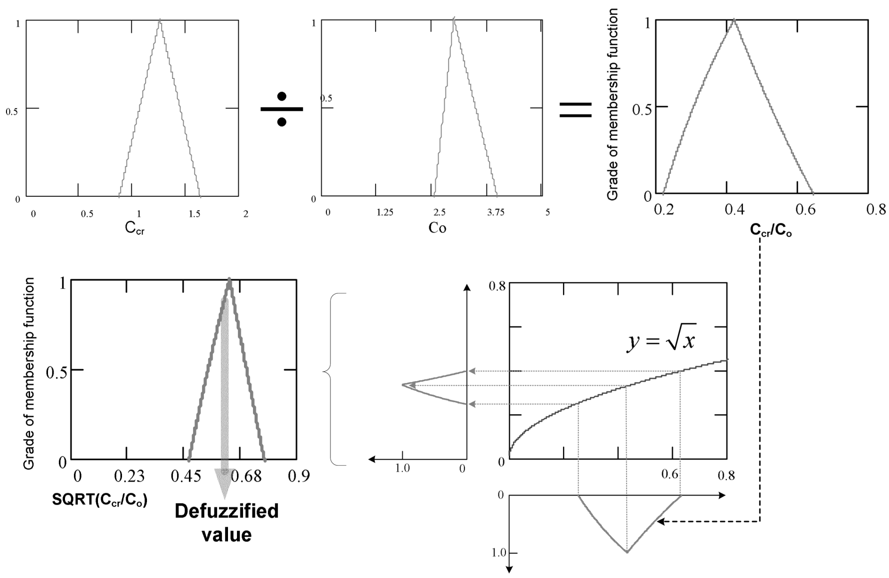


Fig. 8 Membership function of fuzzy number critical chloride concentration (C_{cr}) divided by fuzzy number surface chloride concentration (C_o) and its square root (RILEM model)

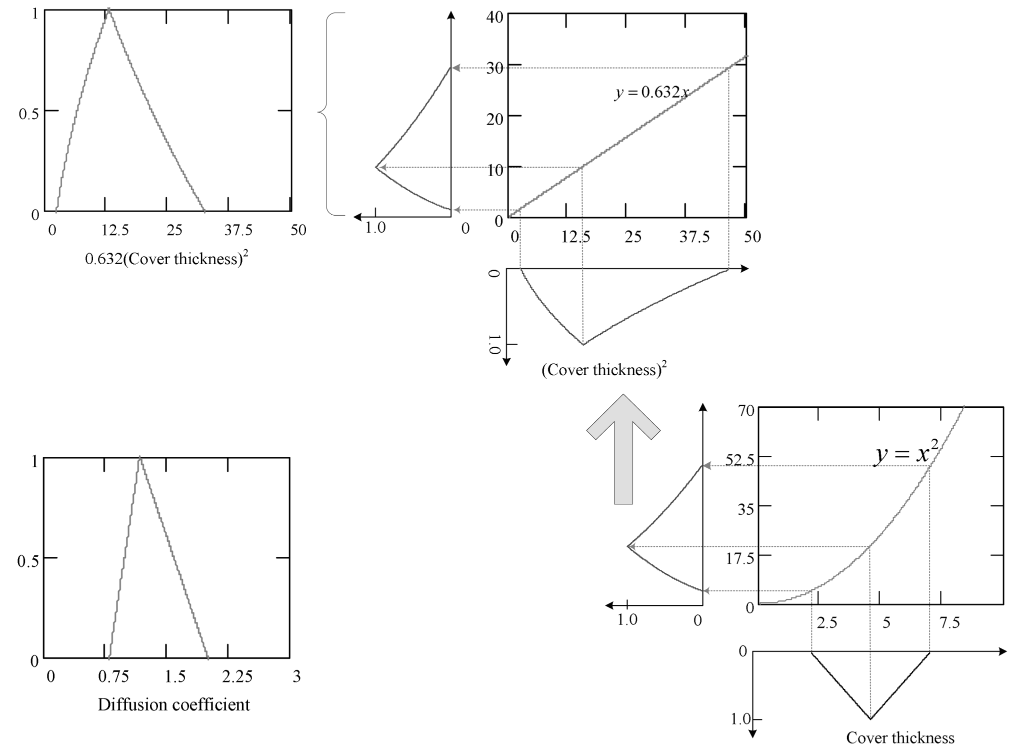


Fig. 9 Membership function of cover thickness (x) and diffusion coefficient (C), and transformation to 0.632(cover thickness)^2 based on extension principle (RILEM model)

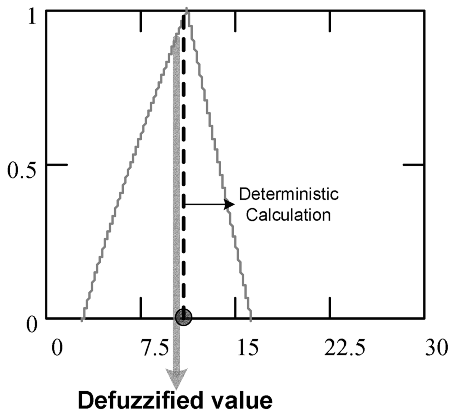


Fig. 10 The membership function of $0.632 \cdot \tilde{x}^2/\tilde{D}$

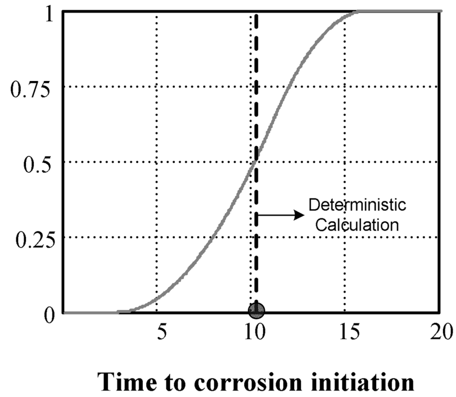


Fig. 11 The cumulative curve of $0.632 \cdot \tilde{x}^2/\tilde{D}$

via each function. They show that mathematical operation of division increases the variability that each parameter possesses.

As shown in Fig. 8, the defuzzification of $\sqrt{\tilde{C}_{cr}/\tilde{C}_o}$ with the Center of Area (CoA) leads to the crisp value

$$\sqrt{\tilde{C}_{cr}/\tilde{C}_o} = 0.637$$

which can be considered as the representative value for $\sqrt{\tilde{C}_{cr}/\tilde{C}_o}$.

By inserting the above expected value of $\sqrt{\tilde{C}_{cr}/\tilde{C}_o}$ into $1/(1-\sqrt{\tilde{C}_{cr}/\tilde{C}_o})$, the crisp value is as follow:

$$[1/(1-\sqrt{\tilde{C}_{cr}/\tilde{C}_o})]^2 = 7.589$$

Fig. 10 represents the membership function of $7.589 \times (\tilde{x}^2/12\tilde{D})$ and its cumulative distribution function, and $7.589 \times (\tilde{x}^2/12\tilde{D})$ defuzzified by CoA is as follow:

$$7.589 \times \frac{\tilde{x}^2}{12\tilde{D}} \rightarrow \frac{0.632 \cdot \tilde{x}^2}{\tilde{D}} = 9.912$$

Finally, the defuzzification of the time T_{corr} to initiation of reinforcement corrosion determined is as follows:

$$T_{corr} = \left(\frac{\tilde{x}^2}{12D^2}\right) \times \left(\frac{1}{1-\sqrt{\frac{\tilde{C}_{cr}}{\tilde{C}_o}}}\right)^2 = 9.912$$

Herein from the result of fuzzy arithmetic for RILEM formula, it can be understood the time to corrosion initiation ranges from 2.9 years to 16 years and the representative value of that distribution is about 9.9 years.

It means that reinforcement corrosion will be initiated after 2.9 to 16 years under given boundary condition and cover thickness in case of be calculated by using RILEM model.

And also the cumulative curve of the time to corrosion initiation is generated in Fig. 11. This cumulative curve is generated by representing the ratio of cumulative area of function to total area in given limitation of 2.9 to 16.

The crisp value of 9.9 years is interpreted to have the membership degree of about 0.9 in function shown in Fig. 10 is about 0.9. In other words, in view of cumulative curve of Fig. 11, it means that the possibility that reinforcement corrosion is initiated after 9.9 years can be estimated about to 50%. At about 16 years the occurrence of corrosion initiation is estimated to be 1.0, which it can be understood that corrosion of the embedded steel bar occur at 16 years after construction in case of the exposed environment.

4.3. Calculation for T_{corr} based on Crank's solution of Fick's second law

To calculate the prediction model by using fuzzy arithmetic, here the second model is Crank's solution of Fick's second law as follows;

$$C(x,t) = C_o\left\{1-erf\left[\frac{x}{2\sqrt{D\cdot t}}\right]\right\} \tag{8}$$

$$= C_o\left\{erfc\left[\frac{x}{2\sqrt{D\cdot t}}\right]\right\}$$

where $C(x, t)$: chloride ion concentration at a time t and a depth x in kg/m^3, C_o: boundary chloride concentration in kg/m^3, x : concrete cover depth in cm and erf : statistical error function as follows;

$$erf(z) = \frac{2}{\sqrt{\pi}}\int_z^\infty \exp(-t^2)dt$$

$$erfc = 1-erf$$

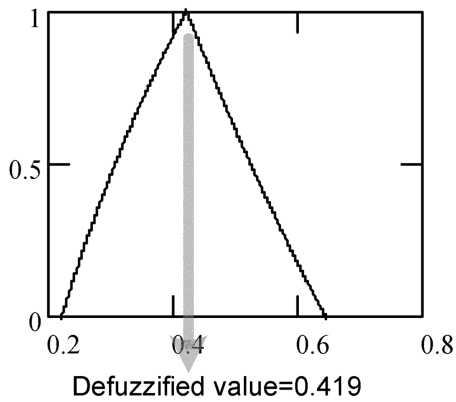


Fig. 12 Membership function of $\tilde{C}_{cr}/\tilde{C}_o$

The corrosion of steel reinforcement must be activated as the chloride content of concrete near the embedded steel reach a chloride threshold, i.e., critical corrosion level.

Thus to acquire the time to corrosion initiation Eq. (8) is rewritten as follows:

$$T_{corr} = \frac{x^2}{4D \cdot \gamma^2} \quad \text{with} \quad \gamma = erfc^{-1}\left(\frac{C_{cr}}{C_0}\right) \tag{9}$$

The prediction model used in this chapter is slightly different with RILEM formula in that RILEM formula is empirical solution while Crank's solution of Eq. (8) is based on diffusion.

The calculation procedure implementing fuzzy arithmetic for Crank's solution of Fick's second law of diffusion shown in Eq. (9) is illustrated in Fig. 6.

The input variable is firstly prepared in order to implement fuzzy arithmetic for Crank's solution, which obtained from the result of best curve fitting to the collected data from literature survey and field measurements. Secondly, the input variable is treated with triangular fuzzy number with proper core and support as a method represented in Figs. 4 and 5.

The ready input data of fuzzy number is substituted for each variable in Eq. (9) and fuzzy arithmetic is implemented using fuzzy interval and extension principle.

Definitely, fuzzy arithmetic is implemented as parted with two. First is to find the mapping of the inverse error function to the defuzzified value of $\tilde{C}_{cr}/\tilde{C}_o$ and second is to implement the fuzzy arithmetic to $\tilde{x}^2/1.348\tilde{D}$ in Fig. 13 in order to get the final output, where fuzzy arithmetic is implemented through fuzzy interval and extension principle and each membership function is defuzzified by fuzzy centroid method, i.e., Center of Area(CoA), by way of the following Eq.(8).

Fig. 14 illustrates the fuzzy number $\tilde{C}_{cr}$ divided by the fuzzy number $\tilde{C}_o$ and the defuzzification of $\tilde{C}_{cr}/\tilde{C}_o$ leads to the crisp value

$$\tilde{C}_{cr}/\tilde{C}_o = 0.419$$

and the inverse of complementary error function to the defuzzification value of $\tilde{C}_{cr}/\tilde{C}_o$ returns 0.5802.

Finally the membership function of $\tilde{x}^2/1.348\tilde{D}$ is plotted in Fig. 14 and its defuzzification value is about 11.6 years.

From the result of fuzzy arithmetic for Crank's solution of Fick's law of diffusion, it can be

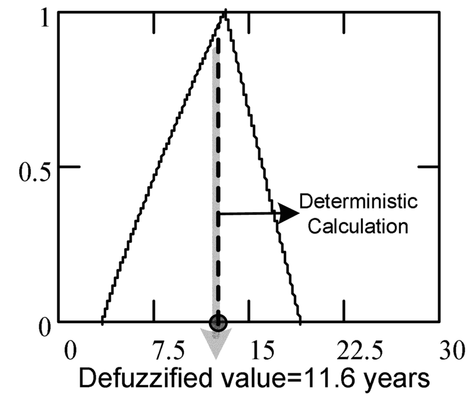


Fig. 13 Membership function of $\tilde{x}^2/1.348\tilde{D}$

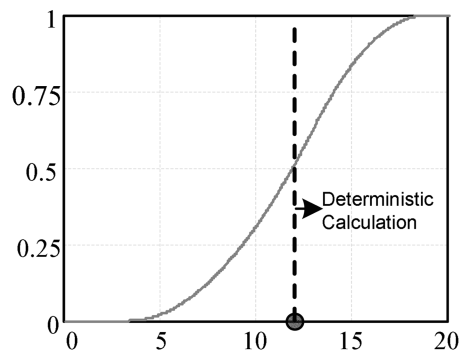


Fig. 14 Cumulative curve of $\tilde{x}^2/1.348\tilde{D}$

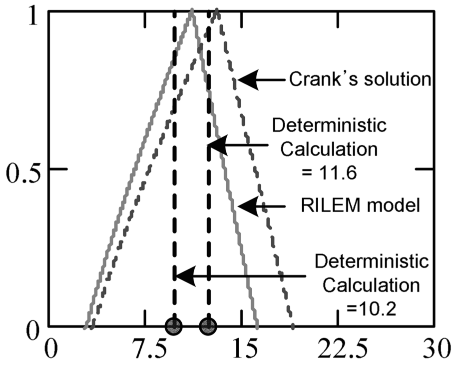


Fig. 15 Comparison of RILEM model and Crank's solution (Membership function and deterministic solution)

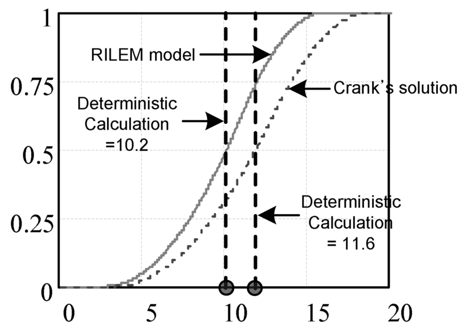


Fig. 16 Comparison of RILEM model and Crank's solution (Cumulative curve and deterministic solution)

understood the time to corrosion initiation ranges from 3 years to 19 years and the representative value of that distribution is about 11.6 years.

It means that reinforcement corrosion will be initiated after 3 to 19 years under given boundary condition and cover thickness in case of be calculated by using Crank's solution. And also the cumulative curve of the time to corrosion initiation is generated and is shown in Fig. 15, from the cumulative distribution function, membership degree of 11.6 to membership function shown in Fig. 15 is about 0.5. In other words, in view of cumulative curve it means that the possibility that reinforcement corrosion is initiated after 11.6 years can be estimated about to 50%. From this, it is understood that corrosion of the embedded steel bar occur at 11.6 years after construction in case of the exposed environment, because the occurrence of corrosion at 11.6 years is calculated to be 1.0.

4.4. Comparison of deterministic solution and fuzzy arithmetic solution

Fig. 15 is to plot each membership function of the time to corrosion initiation (T_{corr}) calculated by fuzzy arithmetic for RILEM model and Crank's solution and the result of its deterministic solution. And also Fig. 16 illustrates the cumulative curve of the fuzzified output of T_{corr} obtained from fuzzy arithmetic for RILEM model and Crank's solution, respectively.

The defuzzification value of Crank's solution is higher than that of RILEM model and also the defuzzification value of each model formula is well coincident with the solution value of deterministic calculation using a mean as the representative value.

The result of fuzzy arithmetic for RILEM model shows the corrosion initiation time prior to that of Crank's solution, which can interpret RILEM model as more conservative expression than Crank's solution.

5. Conclusions

This paper presented the application of a fuzzy arithmetic approach for the modeling and prediction of reinforcement corrosion in building structures that are subjected to coastal environment. The approach will take into account the uncertainties in the physical modeling, and variability of the material and structural parameters affecting the corrosion process, in addition to the statistical and decision uncertainties.

Of the prediction model for corrosion initiation used in this work, RILEM model is more conservative than Crank's solution and both models show good coincidence with deterministic solution. The fuzzy arithmetic-based prediction model can output the stochastic result and it make it possible for users to make more rational decision about the time to corrosion initiation and predicting service life. Thus, the proposed fuzzy arithmetic-based prediction model will overcome the shortcomings of existing deterministic prediction models. The implementation of this tool will provide more extensive predictions and will enable decision-makers to select cost-effective repair strategies.

6. Future study

This paper is concerned with presenting a fuzzy theory-based methodology for modeling the uncertainties in prediction model of time to corrosion initiation of RC structure. Even if it proved to be more extensive than a deterministic solution in that the occurrence of corrosion is numerically calculated, more case studies should be carried out to validate this method

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