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Modelling the rheological behaviour of fresh concrete: An elasto-viscoplastic finite element approach

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Abstract. Rheological behaviour of fresh concrete is an important factor in controlling concrete quality. It is recognized that the measurement of the slump is not a sufficient test method to adequately characterize the rheology of fresh concrete. To further understand the slump measurement and its relationship to the rheological properties, an elasto-viscoplastic, 2-D axisymmetric finite element (FE) model is developed. The FE model employs the Bingham material model to simulate the flow of a slump test. An experimental program is carried out using the Slump Rate Machine (SLRM_II) to evaluate the finite element simulation results. The simulated slump-versus-time curves are found to be in good agreement with the measured data. A sensitivity study is performed to evaluate the effects of yield stress, plastic viscosity and cone withdrawal rate on the measured flow curve using the FE model. The results demonstrate that the computed yield stress compares well with reported experimental data. The flow behaviour is shown to be influenced by the yield stress, plastic viscosity and the cone withdrawal rate. Further, it is found that the value of the apparent plastic viscosity is different from the true viscosity, with the difference depending on the cone withdrawal rate. It is also confirmed that the value of the final slump is most influenced by the yield stress.

Keywords: concrete; FE modelling; rheological properties; slump test; Bingham model.

1. Introduction

The quality of fresh concrete can be characterized by its homogeneity, and the ease, with which it can be mixed, transported, compacted and finished (Chidiac, *et al.* 2000). Rheology, which is defined as the science of the deformation and the flow of matter with time, describes the material property that can be employed to control quality. Concrete rheology has been qualitatively characterized through "workability", a measure of easiness to place and finish concrete; "consistency", the ability of concrete to flow; and "plasticity", which is the ease in the molding of concrete (Ferraris 1999, ACI 116R 1990).

The interplay of the foregoing parameters has been investigated (Ferraris and de Larrrard 1998, Chidiac, *et al.* 2000, Ferraris and Brower 2000). The results have shown that different concrete having the same consistency can exhibit different workability. The consistency of fresh concrete is

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reported to depend on water to cement ratio (*w/c*), aggregate to cement ratio, sand to aggregate ratio, and on the type, size and shape of the aggregate, on water content, and on type and amount of chemical and mineral admixtures (Wong, *et al*. 2001, Hobbs 1976, Struble, *et al*. 1998, Chingyi and Powell 1994, Topcu and Kocataskin 1995, Wallevik, *et al*. 1995, Faroug, *et al*. 1999, Ferraris and Brower 2000). This finding suggests that it is difficult to predict the flow properties of fresh concrete on the basis of the composition of the mixture or on the basis of workability measures such as slump test. According to Chidiac, *et al*. (2003), it is paramount to quantify the rheological properties for controlling the quality of fresh concrete.

Various simple rheological models exist for describing the flow of matter, namely Newtonian, Bingham, Herschel-Bulkley (H-B), shear thinning and shear thickening. These models are referred to as simple because they are suited to study the flow behaviour of materials only over a small shear range or where a simple flow behaviour is required. Complex models such as Moore, Cross, Sisko, Ellis and Casson, to name a few, have been developed to provide a more realistic prediction of the flow over a broader range of conditions (Papo 1988). The flow behaviour of fresh concrete has been shown to follow either the Bingham material model or the H-B material model (Tattersall and Banfill 1983, Tattersall and Baker 1988, Banfill 1993, Ferraris and de Larrard 1998). Materials that flow as Bingham fluid exhibit an infinite absolute viscosity until a sufficiently high stress is applied to initiate flow. Absolute viscosity is a constant of proportionality and is defined as the ratio of shear stress to shear strain rate. Above this stress, which is called the yield stress, the material shows simple linear flow where the gradient of shear stress is linearly proportional to shear strain rate. The proportionality constant is the material plastic viscosity, which is constant and independent of the shear strain rate. Plastic viscosity is the slope of the stress versus strain rate for a plastic material. H-B model incorporates elements of the Newtonian, Power law and Bingham Material model. The H-B model can be used to simulate shear thinning and shear thickening by modifying the power law index of the material. When the power law index equals to one, the H-B model becomes the Bingham's model. The added benefits of using the H-B model to simulate the flow of fresh concrete has yet to be demonstrated given the added difficulty of quantifying the power law index, which tends to be strain-rate dependent. This quantification is required in addition to the yield stress and the plastic viscosity. Furthermore, H-B does not give the plastic viscosity as defined by Bingham.

Adopting the Bingham model for characterizing the rheological behaviour of fresh concrete requires being able to quantify two material properties, namely the yield stress and plastic viscosity, which are the fundamental rheological properties of fresh concrete. A two-point test is the first method developed to characterize the flow properties of fresh concrete (Tattersall and Bloomer 1970, Tattersall and Banfill 1983). At present, the rheological properties of fresh concrete can be estimated by rotation viscometer (Tattersall and Banfill 1983), BML rheometer (Wallevik 1990, Wallevik and Gjorv 1990), LaFarge rheometer (Tattersall 1990), ViscoCorder rheometer (Banfill 1990), Cemagref_IMG rheometer (Coussot 1993), rotation viscometer with inserted sphere (Teranish, *et al*. 1994), IBB rheometer (Tattersall 1976, Beaupre 1994) and BTHREOM (Hu, *et al*. 1995, Ferraris, *et al*. 1998). Ferraris and Brower (2000) carried out an experimental study to compare the measured rheological properties using five different concrete rheometers. Their results show that a high correlation exists between each pair of concrete rheometers even-though the test methods produce different values for the rheological properties. They also confirm earlier experimental findings, in which it is found that a good correlation exists only between slump and yield stress and not between slump and plastic viscosity.

The slump test remains the most common test for measuring concrete workability, due to being easy to perform, inexpensive and portable, and requiring very little training. As such, numerous empirical and analytical equations have been developed for estimating yield stress and plastic viscosity on the basis of slump test measurements; specifically the slump, slump flow, and time of slump (Tanigawa and Mori 1985, Murata and Kikukawa 1992, Hu, *et al.* 1992, Yoshiyuki, *et al.* 1994, Ferraris and de Larrard 1998, Chidiac, *et al.* 2000, Chidiac and Habibbeigi 2004).

Tanigawa, *et al.* (1985, 1986, 1989, 1992) developed a viscoplastic finite element model using axisymmetric triangular elements to simulate the flow of fresh concrete based on Bingham's constitutive model. The simulated results showed that the flow of fresh concrete during the slump test is greatly affected by the withdrawal rate of the cone. On the other hand, the final slump value is found not to be influenced by the cone withdrawal rate. Unfortunately, the simulated curves are not in the range of the withdrawal rates specified by the applicable standard ASTM C 143-90. It is also noted that friction between the concrete and the surface of the cone within the range of experimentally determined parameters has no significant effect on the slumping of the concrete (Mori and Tanigawa 1992). Of interest to this study is the finding that the slump value is influenced by yield stress and plastic viscosity (Tanigawa and Mori 1989, Mori and Tanigawa 1992). This conclusion is however not supported by published experimental data, which shows that the slump value is only correlated with the yield stress (Tattersal and Banfill 1983, Ferraris and de Larrard 1998, Ferraris and Brower 2000). The results also showed that the computed slump-versus-time curve using a viscoplastic finite element model is representative of the experimental data (Mori and Tanigawa 1992).

The objective of this study is to present a description of the elasto-viscoplastic finite element model that is based on the Bingham material model and incorporates different boundary conditions including a contact boundary. The FE model is employed to investigate the effects of the rheological properties on the slump and the effect of cone withdrawal rates on the apparent rheological properties of fresh concrete. An experimental component is carried out to validate the simulated flow behaviour of fresh concrete.

2. Numerical model

2.1. Stress formulation

According to the Bingham model, the shear stress, τ, of a viscoplastic material is described by

$$\tau = \tau_y + \eta \cdot \dot{\gamma} \tag{1}$$

in which, τ_y and η are the yield stress and plastic viscosity, respectively, and $\dot{\gamma}$ the strain rate. The generalized form of the Bingham model for an arbitrary state of stress is given by Hohenemser and Prager (Fung 1977)

$$\dot{\varepsilon}_{ij}^{vp} = \frac{1}{2\eta}\langle 1 - \frac{\tau_y}{\sqrt{J_2}}\rangle \sigma'_{ij} \tag{2}$$

where $\dot{\varepsilon}_{ij}^{vp}$, σ'_{ij} are viscoplastic strain rate and deviatoric stress tensors, respectively, and J_2 the second invariant of deviatoric stress. The brackets <> imply:

$$\langle \Phi(F) \rangle = \begin{cases} \Phi(F) : F > 0 \\ 0 : F \le 0 \end{cases} \quad (3)$$

With regard to the numerical implementation, the elasto-viscoplastic formulation follows the work of Zienkiewicz and Cormeau (1974) and Kanchi, *et al.* (1978). Accordingly, the element stiffness matrix and consistent load vector for every time step are computed using

$$[K] = \int_V [B]^T [D^{vp}][B] dV \quad (4)$$

$$\{F_{vp}\} = \int_V [B]^T [D^{vp}][\varepsilon_{vp}] \Delta t dV \quad (5)$$

in which

$$[D^{vp}] = ([D^e]^{-1} + \theta \Delta t [H])^{-1} \quad (6)$$

and

$$[H] = \frac{\partial \{\varepsilon_{vp}\}}{\partial \{\sigma\}} \quad (7)$$

2.2. Contact boundary

The slump cone is assumed rigid with the interface between the cone and the concrete being modelled via contact boundaries, as shown in Fig. 1. The springs have a compressive stiffness, with zero tensile stiffness, and the compressive stiffness is taken to be much greater than that of the adjoining element normal to the interface to prevent penetration from occurring (Sauve, *et al.* 2002). The revised equilibrium equations which include the contact boundary is given by

$$([K_s] + [K_e])\{\Delta x\} = \{F\} + [K_s](\{\Delta x\} - \{\Delta x_s\}) \quad (8)$$

where, $[K_s]$ is the stiffness matrix of the spring, $[K_e]$ the stiffness of the element, $\{F\}$ the nodal force vector, $\{\Delta x\}$ the nodal displacement during the time increment, and $\{\Delta x_s\}$ the compression in the spring during the time increment. The free span between the penetrating node and master boundary is given by the difference between $\{\Delta x\}$ and $\{\Delta x_s\}$.

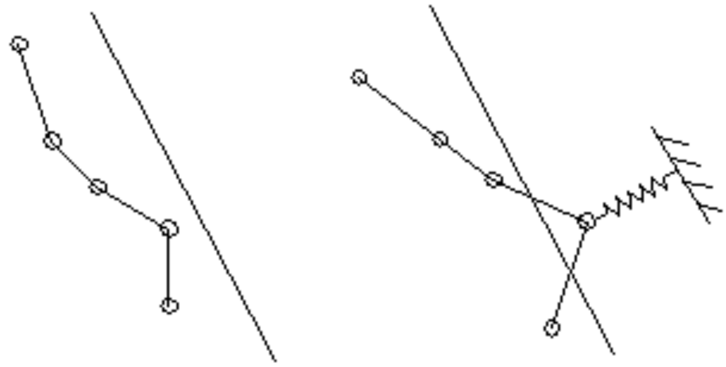

Fig. 1 Schematic view of the spring enforcing contact boundary condition

2.3. FE model

Since the flow response is assumed to be axisymmetrical, a 2-D, axisymmetric, bi-linear quadrilateral finite element is used to model the continuum. Mesh sensitivity is investigated by comparing the final slump value for different element sizes. Table 2 shows that the percent difference in the final slump decreases from 12% to less than 2%. On the basis of these results, a mesh size of 50 x 20 is selected for the study. The appropriateness of the finite element program is demonstrated by reproducing the simulated slumping of fresh concrete at different time of slumping as shown in Fig. 2 (Habibbeigi 2003).

3. Experimental program

An experimental program is carried out to collect the data required to verify the capability of the elasto-viscoplastic finite element model to simulate the flow of fresh concrete. To that effect, the Slump Rate Machine (SLRM), developed by Chidiac, *et al.* (2000, 2004), is used to measure the slump of the concrete, the withdrawal rate of the cone, and the flow of concrete in two perpendicular directions with time. Fig. 3 shows a view of SLRM_II. All readings are recorded and transferred to a computer via a data acquisition device.

Five concrete mixes were studied using the SLRM_II. The material composition of the five mixes including the measured properties are given in Table 1. The measurements of the slump cone

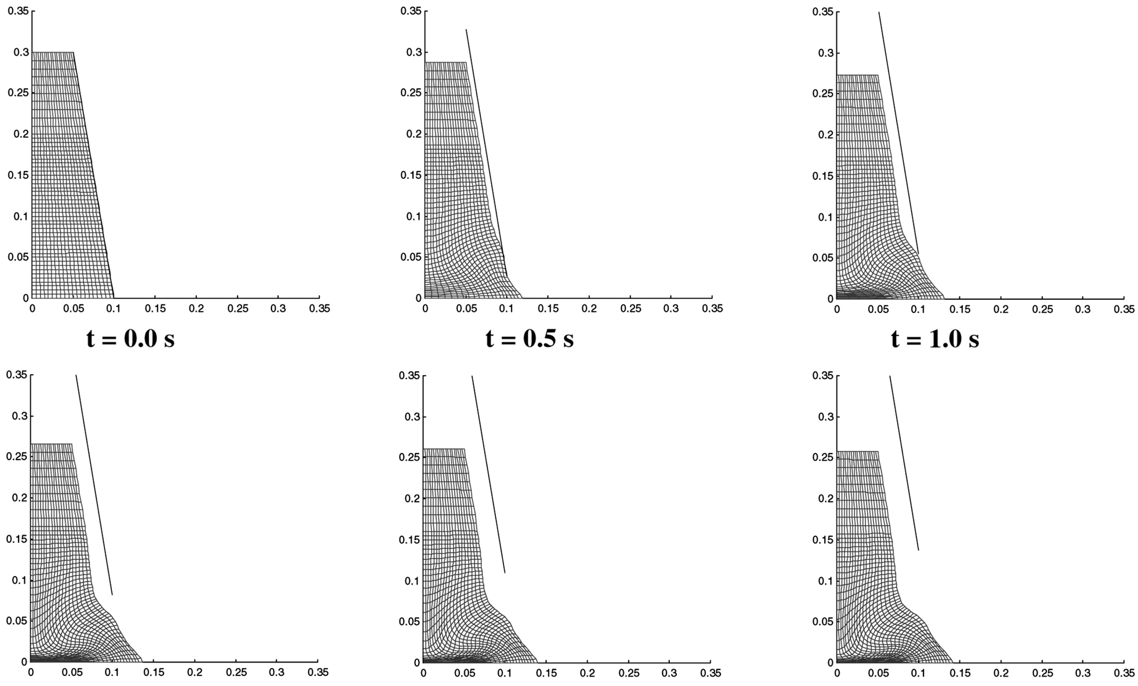

Fig. 2 FE simulation (ρ = 2000 kg/m^3, τ_y = 1800 Pa, η = 50 Pa.s and v_{cone} = 55 mm/s).

Table 1 Concrete mix design and the corresponding measured and predicted properties

Test	1	2	3	4	5
Strength (MPa)	32	32	30	25	25
Cement (kg/m^3)	488	514	213	208	186
Slag (kg/m^3)	0	0	71	39	59
Fly ash (kg/m^3)	0	0	0	29	0
Water (kg/m^3)	205	216	124	125	114
Water reducer (L/m^3)	0	0	0.57	0.56	0.50
Coarse aggregate (kg/m^3)	992	1080	1103	1076	1118
Sand (kg/m^3)	718	574	812	839	952
Slump (mm)	120	185	178	104	97
Slump flow (mm)	220	309	283	243	222
Simulated τ_y (Pa)	1525	900	950	1630	1725
Simulated η (Pa.s)	50	100	80	20	10

Table 2 Comparison of slump values for different mesh sizes

Number of elements in z direction	Number of elements in r direction	Total number of elements	τ_y = 1000Pa η = 100Pa.s	τ_y = 2000Pa η = 100Pa.s
75	30	2250	177.0	61.3
60	24	1440	175.0	60.3
50	20	1000	173.0	59.6
40	16	640	169.5	57.7
25	10	250	157.4	54.1

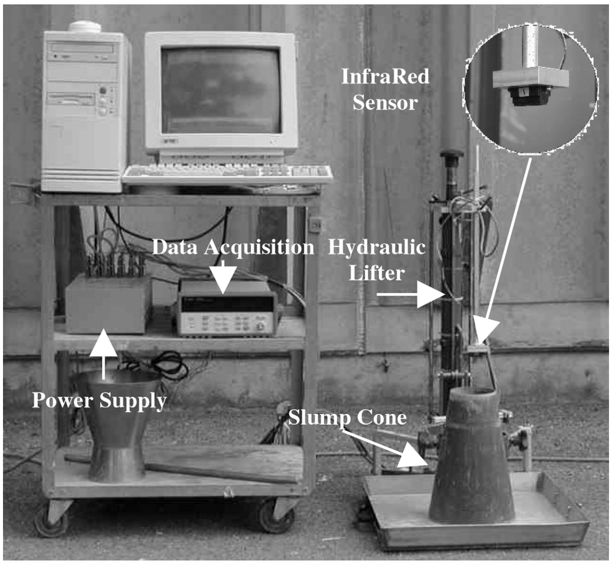

Fig. 3 View of the slump rate machine, SLRM_II

displacement for these experiments shows a withdrawal rate of 55 mm/s, which is in good agreement with the specified standard range of 43 mm/s to 100 mm/s (ASTM 1990) for slump testing. Figs. 4 through 8 show the measured slump-versus-time and corresponding simulated curves. A comparison of the simulated curves and experimental data confirms that the model is generally capable of describing the observed phenomenon, which implies that the rheological properties of fresh concrete can be estimated using the elasto-viscoplastic FE model. Accordingly, the rheological properties corresponding to the five mixes are given in Table 1.

A sensitivity study is carried out to investigate the effects of varying the values of the estimated rheological properties on the flow behaviour of the concrete and the final slump. The yield stress is varied by ±10% and the plastic viscosity by ±25%. The computed results are reproduced in Figs. 9 through 13. The results show that the final slump is most sensitive to the yield stress and that by varying the yield value by ±10% results in different changes in the final calculated slump for the five mixes. Furthermore, one observes that varying the value of plastic viscosity by ±25% affects mostly the flow behaviour. These results demonstrate that the FE model is capable of discriminating between mixes that have different rheological properties by measuring the slump as a function of time.

4. Analytical results

The motivation for developing the FE model is to provide a means to simulate and understand the slump test, with emphasis on examining the flow behaviour of fresh concrete. Specifically, investigation of the effects of varying the values of the yield stress, plastic viscosity, and withdrawal rate of the cone on the flow behaviour of fresh concrete is sought. The properties of the concrete considered for the analytical study are noted on the figures.

4.1. Yield stress

Fig. 14 shows the slump-versus-time curves for a range of yield stress. The results clearly confirm

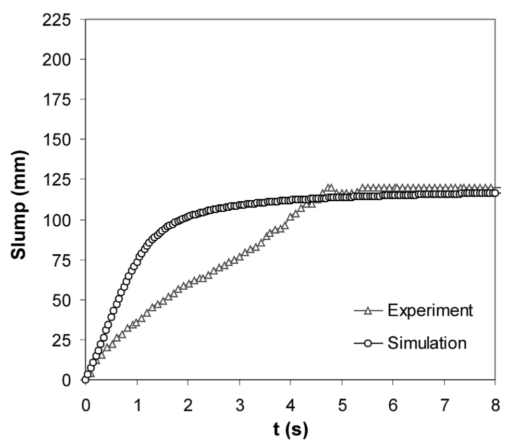

Fig. 4 Recorded and simulated slump-versus-time curve for mix #1

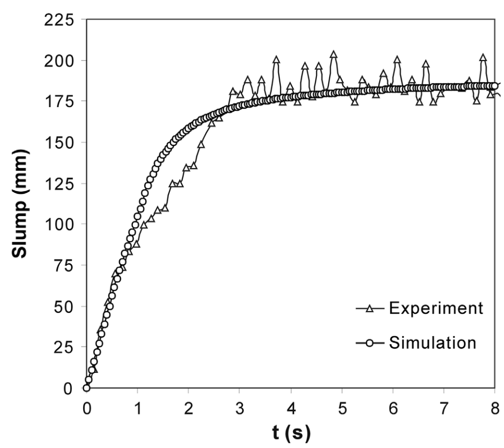

Fig. 5 Recorded and simulated slump-versus-time curve for mix #2

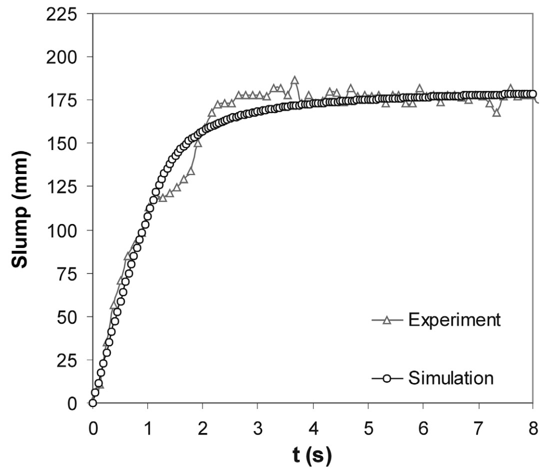

Fig. 6 Recorded and simulated slump-versus-time curve for mix #3

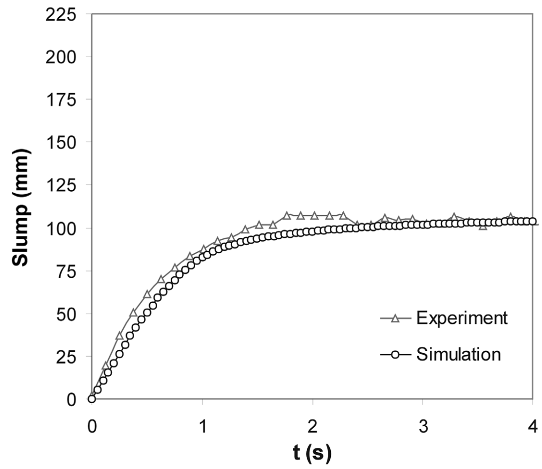

Fig. 7 Recorded and simulated slump-versus-time curve for mix #4

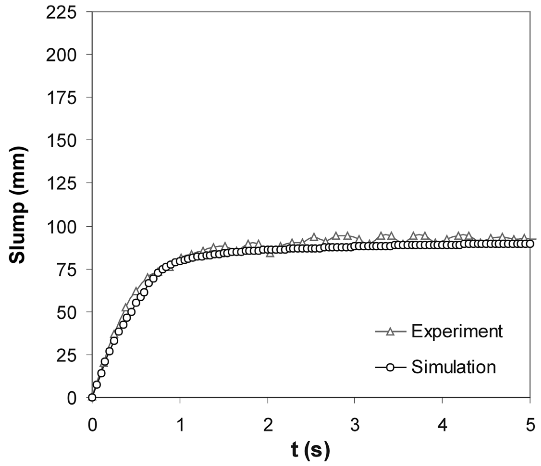

Fig. 8 Recorded and simulated slump-versus-time curve for mix #5

that the final slump value is mostly affected by the yield stress. Examining the slump-versus-time curves, it can be observed that the flow is also influenced by the value of the yield stress, but to a lesser degree. These findings are found to be in agreement with the literature.

Additional finite element simulations corresponding to light weight, normal weight and heavy weigh concrete are carried out, with the results, in the form of slump versus yield stress, shown in Figs. 15 to 17, respectively. These results show the correlation between the values of the slump and yield stress. They indicate that plastic viscosity has a negligible effect on the slump value. Using the FE results together with a linear regression analysis reveals that the simulated yield stress depends solely on the density of the concrete and the final slump, with

$$\tau_y = 0.3635 \rho g (H - S1) \tag{9}$$

in which R^2 is 0.99.

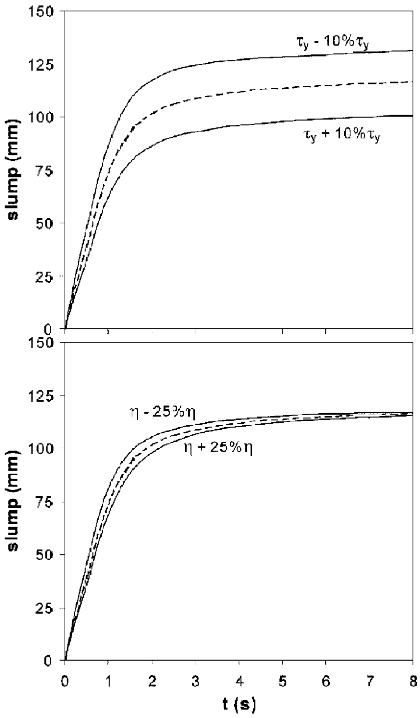

Fig. 9 Simulated slump-versus-time with varying yield stress and plastic viscosity for mix #1

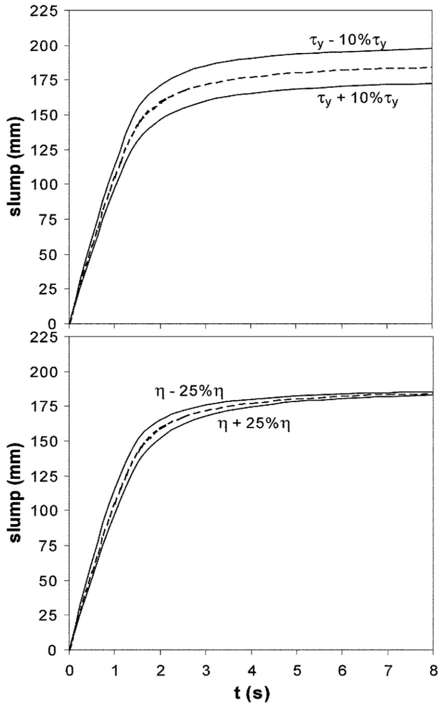

Fig. 10 Simulated slump-versus-time with varying yield stress and plastic viscosity for mix #2

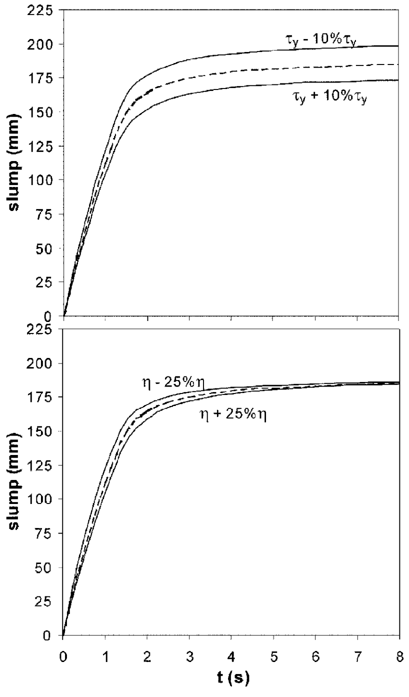

Fig. 11 Simulated slump-versus-time with varying yield stress and plastic viscosity for mix #3

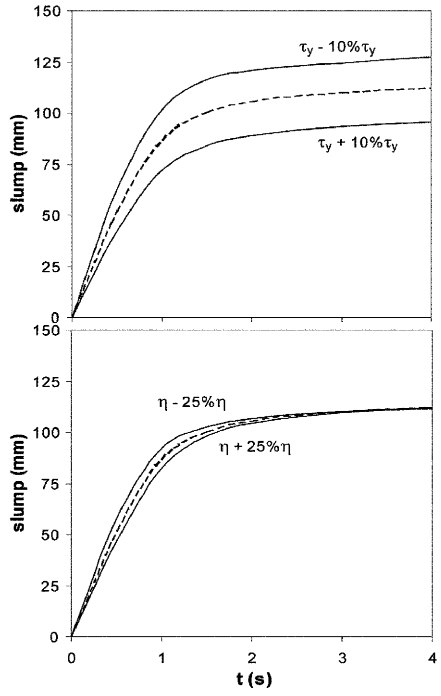

Fig. 12 Simulated slump-versus-time with varying yield stress and plastic viscosity for mix #4

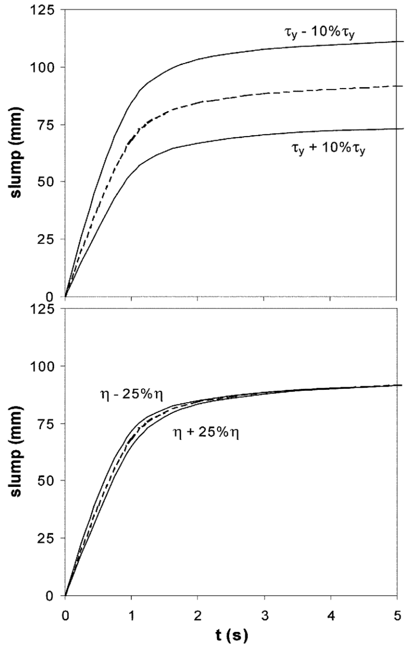

Fig. 13 Simulated slump-versus-time with varying yield stress and plastic viscosity for mix #5

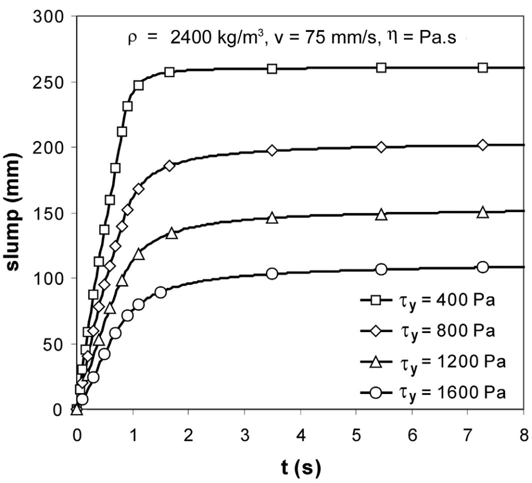

Fig. 14 The effect of yield stress variation on the slump-versus-time spectrum

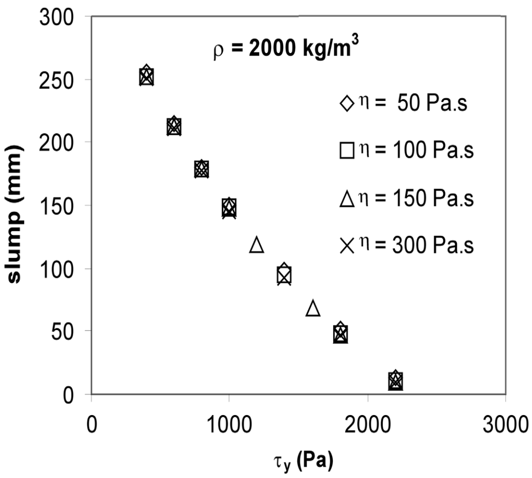

Fig. 15 Yield stress-versus-slump for lightweight concrete

Eq. (9) is further evaluated using published experimental data (Ferraris, *et al.* 1998, Hu, *et al.* 1992). Fig. 18 provides a comparison between the computed finite element values and those measured using the BTRHEOM (Hu, *et al.* 1995). Based on these results, one observes a one to one correlation between the model and the experimental data (R^2 is 0.71).

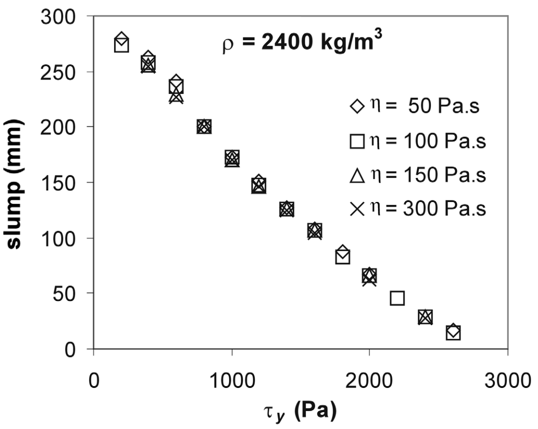

Fig. 16 Yield stress-versus-slump for normal-weight concrete

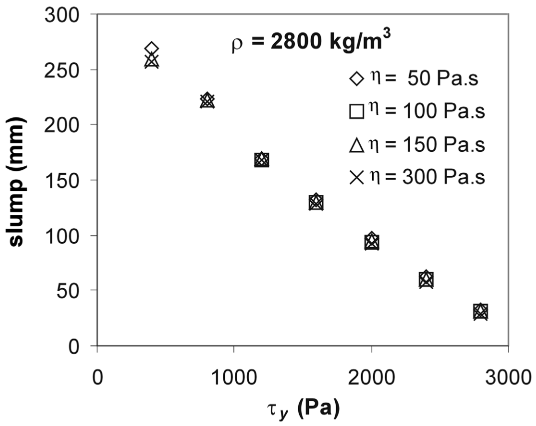

Fig. 17 Yield stress-versus-slump for heavyweight concrete

4.2. Plastic viscosity

Fig. 19 predicts the flow behaviour of fresh concrete when the plastic viscosity is varied from 50 to 300 Pa.s. The results confirm earlier observations that plastic viscosity has minimal effects on the final slump value. Furthermore, one observes that it greatly affects the flow behaviour as represented by the slump-versus-time curve. Based on this finding one can conclude that the slump test provides sufficient sensitivity to discriminate between mixes on the basis of their rheological properties.

4.3. Withdrawal rate

Being a boundary value problem, it is important to examine the influence of the contact boundary

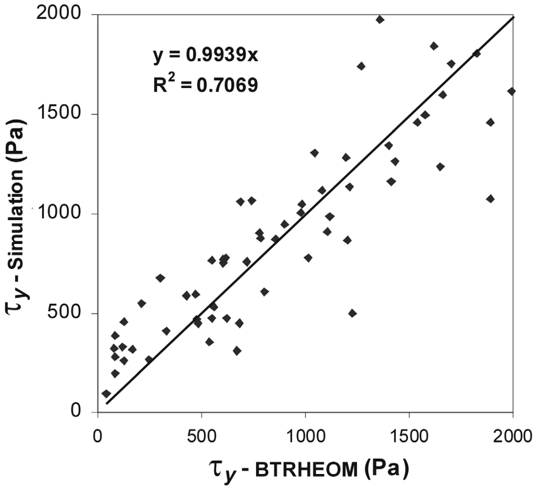

Fig. 18 Comparison between the simulated yield stress and the experimental data

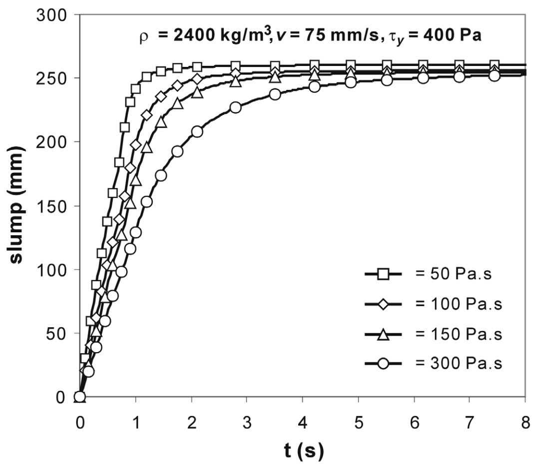

Fig. 19 The effect of plastic viscosity variation on the slump-versus-time spectrum

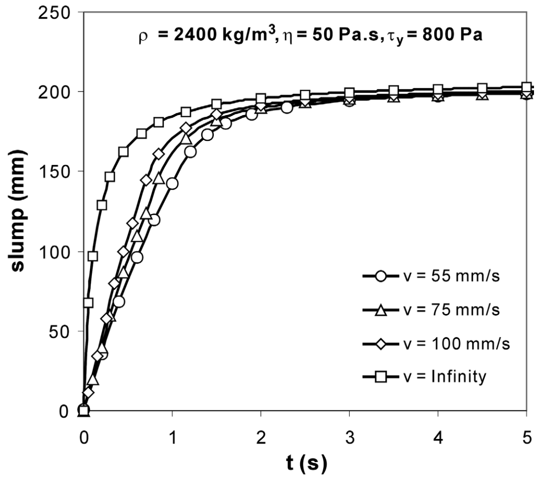

Fig. 20 The effect of withdrawal rate variation on the slump-versus-time spectrum

on the flow behaviour of fresh concrete. The computed slump-versus-time curves, for a withdrawal rate ranging from 55 mm/s to infinity, are shown in Fig. 20. The results confirm that the final slump is not affected by the withdrawal rates, however, the withdrawal rate significantly influences the flow behaviour. Although the slump-versus-time curves shown correspond to the same concrete mix, it is evident that the value of the apparent plastic viscosity changes, which implies that the estimation of the plastic viscosity without considering the withdrawal rate of the cone results in the apparent value rather than the true one. Viscosity is called apparent viscosity because it depends on the withdrawal rate. As the withdrawal rate decreases, the interactions between the fresh concrete and the cone increases, leading to a different slump versus time curve.

5. Conclusions

The computed FE slump-versus-time curves are found to be in good agreement with the measured data obtained using SLRM_II. The results confirm that the FE model can discriminate between concrete mixes with different rheological properties by measuring the slump-versus-time curve. Furthermore, the yield stress values derived using the FE model agree with the experimentally measured values reported in the literature.

The evaluation of the effects of yield stress, plastic viscosity and withdrawal rate on the flow behaviour and final slump value has led to the following conclusions:

- The yield stress value is linearly proportional to the slump and the density of the material. This relation is in agreement with the expressions reported in the literature.
- The final slump value is affected mainly by the yield stress.
- The yield stress has some effects on the flow behaviour of fresh concrete.
- The flow behaviour is most influenced by the plastic viscosity and the cone withdrawal rate.
- The measured flow behaviour provides a value for the apparent plastic viscosity, which is different from the material Bingham plastic viscosity. The latter can only be evaluated by considering the withdrawal rate of the cone. In that regard, the elasto-viscoplastic finite element

model can be used to determine the Bingham plastic viscosity of the fresh concrete.

- The proposed model in conjunction with the slump rate machine can therefore be used to evaluate the Bingham rheological properties of fresh concrete.

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