

Vibration analysis of silica nanoparticles-reinforced concrete beams considering agglomeration effects

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Abstract. In this paper, nonlinear vibration of embedded nanocomposite concrete is investigated based on Timoshenko beam model. The beam is reinforced by with agglomerated silicon dioxide (SiO₂) nanoparticles. Mori-Tanaka model is used for considering agglomeration effects and calculating the equivalent characteristics of the structure. The surrounding foundation is simulated with Pasternak medium. Energy method and Hamilton's principal are used for deriving the motion equations. Differential quadrature method (DQM) is applied in order to obtain the frequency of structure. The effects of different parameters such as volume percent of SiO₂ nanoparticles, nanoparticles agglomeration, elastic medium, boundary conditions and geometrical parameters of beam are shown on the frequency of system. Numerical results indicate that with increasing the SiO₂ nanoparticles, the frequency of structure increases. In addition, considering agglomeration effects leads to decrease in frequency of system.

Keywords: vibration of concrete beam; agglomerated SiO₂ nanoparticles; Pasternak medium; DQM; Mori-Tanaka model

1. Introduction

Reinforced concrete (RC) is a composite material with a reinforcement which can be steel bars, plates, fibers and nanoparticles. Recently, the usage of different type of nanoparticles in concrete structures has been an intense interest among researchers since the nanoparticles can improve the quality and material properties of concrete. With respect to this fact that the nanoparticles can be agglomerated in the concrete, however, in this paper, a mathematical model is introduced for concrete beam reinforced with nanoparticles for estimating the vibration behaviour of mentioned structures considering agglomeration effects.

With respect to the developed works in the field of RC structures, Kim and Aboutaha (2004) presented investigation of a three-dimensional (3-D) nonlinear finite element model analysis to examine the behavior of reinforced concrete beams strengthened with Carbon Fiber Reinforced Polymer (CFRP) composites to enhance the flexural capacity and ductility of the beams. Strength assessments and coefficient of water absorption of high performance self-compacting concrete containing different amounts of TiO₂ nanoparticles were presented by Nazari and Riahi (2010). Khalaj and Nazari (2012) studied split tensile strength of self-compacting concrete with SiO₂ nanoparticles and different amount of randomly oriented steel fibers. Compressive, flexural and split tensile strength

together with coefficient of water absorption of high performance self-compacting concrete containing different amount of Fe₂O₃ nanoparticles were investigated by Khoshakhlagh *et al.* (2012). Strength enhancement and durability-related characteristics along with rheological, thermal and microstructural properties of high strength self-compacting concrete (HSSCC) containing nano TiO₂ and industrial waste ash namely as fly ash (FA) were investigated by Jalala *et al.* (2013). The addition of steel fibers in concrete mixture was recognized by Ibraheem *et al.* (2014) as a non-conventional mass reinforcement scheme that improves the torsional, flexural, and shears behavior of structural members. Nominal moment-axial load interaction diagrams, moment-curvature relationships, and ductility of rectangular hybrid beam-column concrete sections were analyzed by El-Helou and Aboutaha (2015) using the modified Hognestad concrete model. The physico-mechanical properties of self-compacting lightweight aggregate concrete (SCLC) containing artificial lightweight aggregate (LWA) made from fly ash (FA) through cold-bonded process were investigated by Güneyisi *et al.* (2015). An experimental investigation was conducted by Ibraheem (2015) to examine the behavior and cracking of steel fiber reinforced concrete spandrel L-shaped beams subjected to combined torsion, bending, and shear. Le *et al.* (2016) presented the experiment results for three large-scale concrete composite beams with a newly puzzle shape of crestbond. The RC specimens were produced by Saribiyik and Caglar (2016) taking into consideration the RC beams with insufficient shear and tensile reinforcement having been manufactured with the use of concrete with low strength. Ding *et al.* (2016) investigated the flexural stiffness of simply supported steel-concrete composite I-beams under positive bending moment through combined

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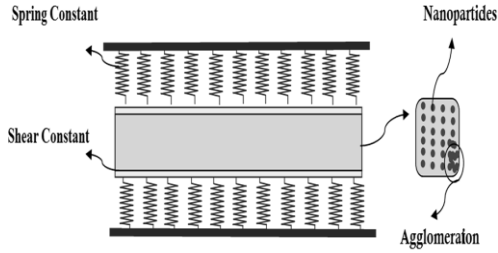


Fig. 1 Schematic of an embedded concrete column reinforced with agglomerated SiO₂ nanoparticles

experimental, numerical, and different standard methods. Hind *et al.* (2016) presented numerical analysis results carried out on a set of concrete beams reinforced with short fibers. To this purpose, a database of experimental results was collected from an available literature.

Mathematical modelling of concrete structure is a novel topic which is recently has been an intense interest among researchers. The nonlinear buckling of straight concrete columns armed with single-walled carbon nanotubes (SWCNTs) and SiO₂ nanoparticles resting on foundation was investigated by Jafarian *et al.* (2016) and Zamanian *et al.* (2016). The nonlinear buckling of straight concrete columns armed with single-walled carbon nanotubes (SWCNTs) resting on foundation was investigated by Safari Bilouei *et al.* (2016). Stress analysis of concrete pipes reinforced with AL₂O₃ nanoparticles was presented by Heidarzadeh *et al.* (2016) considering agglomeration effects.

To the best of author knowledge, no theoretical report has been found in the literature on vibration analysis of concrete beams reinforced with nanoparticles. Motivated by these considerations, we aim to present a mathematical model for vibration analysis of embedded concrete columns reinforced with SiO₂ nanoparticles considering agglomeration effects based on Mori-Tanaka approach. Based on Timoshenko beam model, the motion equations are derived using energy method and Hamilton's principal. Using DQM, the frequency of structure is calculated and the effects of different parameters such as volume percent of SiO₂ nanoparticles, SiO₂ agglomeration, geometrical parameters, elastic foundation and boundary conditions on the frequency of concrete beam are shown.

2. Formulation

Fig. 1 shows an embedded concrete beam reinforced with agglomerated SiO₂ nanoparticles. The surrounding foundation is described by Pasternak model containing the spring and shear constants.

2.1 Timoshenko and Mori-Tanaka theories

Based on the Timoshenko beam model, the displacement field of structure can be written as (Brush and Almorh 1975)

$$U_1(x, y, z, t) = u(x, t) + z\psi(x, t), \quad (1)$$

$$U_2(x, y, z, t) = 0, \quad (2)$$

$$U_3(x, y, z, t) = w(x, t), \quad (3)$$

Where U_1 , U_2 , and U_3 are the displacement of beam in x , y - and z -directions, respectively; $u(x, t)$ and $w(x, t)$ are the displacement components in the mid-plane; Ψ is the rotation of beam cross-section. However, the strain-displacement relation can be obtained as follows

$$\epsilon_{xx} = \frac{\partial u}{\partial x} + z \frac{\partial \psi}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2, \quad (4)$$

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \psi. \quad (5)$$

Based on Hook's law, the isotropic stress-strain relations can be written as

$$\sigma_{xx} = \frac{E}{1-\nu^2} \left[\frac{\partial u}{\partial x} + z \frac{\partial \psi}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right], \quad (6)$$

$$\sigma_{xz} = \frac{E}{2(1+\nu)} \left[\frac{\partial w}{\partial x} + \psi \right], \quad (7)$$

Where E and ν are Yong modulus and poisson's ratio of the SiO₂-reinforced concrete beam which can be calculated by the Mori-Tanaka model as

$$E = \frac{9KG}{3K+G}, \quad (8)$$

$$\nu = \frac{3K-2G}{6K+2G}. \quad (9)$$

Where the effective bulk modulus (K) and effective shear modulus (G) may be expressed as

$$K = K_{out} \left[1 + \frac{\xi \left(\frac{K_{in}}{K_{out}} - 1 \right)}{1 + \alpha(1-\xi) \left(\frac{K_{in}}{K_{out}} - 1 \right)} \right], \quad (10)$$

$$G = G_{out} \left[1 + \frac{\xi \left(\frac{G_{in}}{G_{out}} - 1 \right)}{1 + \beta(1-\xi) \left(\frac{G_{in}}{G_{out}} - 1 \right)} \right], \quad (11)$$

Where

$$K_{in} = K_m + \frac{(\delta_r - 3K_m\chi_r)C_r\xi}{3(\xi - C_r\xi + C_r\xi\chi_r)}, \quad (12)$$

$$K_{out} = K_m + \frac{C_r(\delta_r - 3K_m\chi_r)(1-\xi)}{3[1-\xi - C_r(1-\xi) + C_r\chi_r(1-\xi)]}, \quad (13)$$

$$G_{in} = G_m + \frac{(\eta_r - 3G_m\beta_r)C_r\zeta}{2(\xi - C_r\zeta + C_r\zeta\beta_r)}, \quad (14)$$

$$G_{out} = G_m + \frac{C_r(\eta_r - 3G_m\beta_r)(1-\zeta)}{2[1-\xi - C_r(1-\zeta) + C_r\beta_r(1-\zeta)]}, \quad (15)$$

Where two parameters ξ and ζ describe the agglomeration of nanoparticles and C_r is relates to the SiO_2 volume fraction. In addition, χ_r , β_r , δ_r , η_r may be calculated as

$$\chi_r = \frac{3(K_m + G_m) + k_r - l_r}{3(k_r + G_m)}, \quad (16)$$

$$\beta_r = \frac{1}{5} \left\{ \frac{4G_m + 2k_r + l_r}{3(k_r + G_m)} + \frac{4G_m}{(p_r + G_m)} + \frac{2[G_m(3K_m + G_m) + G_m(3K_m + 7G_m)]}{G_m(3K_m + G_m) + m_r(3K_m + 7G_m)} \right\}, \quad (17)$$

$$\delta_r = \frac{1}{3} \left[n_r + 2l_r + \frac{(2k_r - l_r)(3K_m + 2G_m - l_r)}{k_r + G_m} \right], \quad (18)$$

$$\eta_r = \frac{1}{5} \left[\frac{2}{3}(n_r - l_r) + \frac{4G_m p_r}{(p_r + G_m)} + \frac{8G_m m_r(3K_m + 4G_m)}{3K_m(m_r + G_m) + G_m(7m_r + G_m)} + \frac{2(k_r - l_r)(2G_m + l_r)}{3(k_r + G_m)} \right]. \quad (19)$$

Where k_r , l_r , n_r , p_r , m_r are the Hills elastic modulus for the nanoparticles (Mori and Tanaka 1973); K_m and G_m are the bulk and shear moduli of the matrix which can be written as

$$K_m = \frac{E_m}{3(1-2\nu_m)}, \quad (20)$$

$$G_m = \frac{E_m}{2(1+\nu_m)}. \quad (21)$$

Where E_m and ν_m are Young's modulus and the Poisson's ratio of concrete beam, respectively. Furthermore, β , α can be obtained from

$$\alpha = \frac{(1+\nu_{out})}{3(1-\nu_{out})}, \quad (22)$$

$$\beta = \frac{2(4-5\nu_{out})}{15(1-\nu_{out})}, \quad (23)$$

$$\nu_{out} = \frac{3K_{out} - 2G_{out}}{6K_{out} + 2G_{out}}. \quad (24)$$

2.2 Energy method and Hamilton's principal

The strain energy of the nanocomposite concrete beam can be expressed as

$$U = \frac{1}{2} \int_0^L \int_A (\sigma_{xx} \varepsilon_{xx} + \sigma_{xz} \gamma_{xz}) dV. \quad (25)$$

Submitting Eqs. (4) and (5) into Eq. (25) yields

$$U = \frac{1}{2} \int_0^L \int_A \left[\sigma_{xx} \left(\frac{\partial u}{\partial x} + z \frac{\partial \psi}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right) + \sigma_{xz} \left(\frac{\partial w}{\partial x} + \psi \right) \right] dV. \quad (26)$$

The kinetic energy of the structure can be written as

$$K = \frac{\rho}{2} \int_0^L \int_A [(\dot{U}_1)^2 + (\dot{U}_2)^2 + (\dot{U}_3)^2] dV, \quad (27)$$

Where ρ is the density of the nanocomposite concrete beam. Submitting Eqs. (1) to (3) into Eq. (27) yields

$$K = \frac{\rho}{2} \int_0^L \int_A \left[\left(\frac{\partial u}{\partial t} + z \frac{\partial \psi}{\partial t} \right)^2 + \left(\frac{\partial w}{\partial t} \right)^2 \right] dV. \quad (28)$$

The external work due to the surrounding foundation can be expressed as (Kolahchi *et al.* 2016a)

$$W = \int_0^L (-k_w w + k_g \nabla^2 w) w dx, \quad (29)$$

Where k_w and k_g are spring and shear constants of foundation, respectively. Using Hamilton's principle as follows

$$\int_0^t (\delta U - \delta K - \delta W) dt = 0. \quad (30)$$

The motion equations of the structure can be derived as follows

$$\frac{Eh}{1-\nu^2} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x^2} \frac{\partial w}{\partial x} \right) = \rho h \frac{\partial^2 u}{\partial t^2}, \quad (31)$$

$$\frac{K_s Eh}{2(1+\nu)} \left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial \psi}{\partial x} \right] - k_w w + k_g \frac{\partial^2 w}{\partial x^2} = \rho h \frac{\partial^2 w}{\partial t^2}, \quad (32)$$

$$\frac{Eh^3}{12(1-\nu^2)} \frac{\partial^2 \psi}{\partial x^2} - \frac{K_s Eh}{2(1+\nu)} \left[\frac{\partial w}{\partial x} + \psi \right] = \frac{\rho h^3}{12} \frac{\partial^2 \psi}{\partial t^2}. \quad (33)$$

Where K_s is the shear correction factor. The associated boundary conditions can be expressed as

- Clamped-clamped boundary condition (C-C)

$$\begin{aligned} w = u = \psi = \frac{\partial w}{\partial x} &= 0, & @ \quad x = 0 \\ w = u = \psi = \frac{\partial w}{\partial x} &= 0, & @ \quad x = L \end{aligned} \quad (34)$$

- Clamped-simply boundary condition (C-S)

$$\begin{aligned} w = u = \psi = \frac{\partial w}{\partial x} &= 0, & @ \quad x = 0 \\ w = u = \frac{\partial \psi}{\partial x} = \frac{\partial^2 w}{\partial x^2} &= 0, & @ \quad x = L \end{aligned} \quad (35)$$

- Simply-Simply boundary condition (S-S)

$$\begin{aligned} w = u = \frac{\partial \psi}{\partial x} = \frac{\partial^2 w}{\partial x^2} &= 0, & @ \quad x = 0 \\ w = u = \frac{\partial \psi}{\partial x} = \frac{\partial^2 w}{\partial x^2} &= 0, & @ \quad x = L \end{aligned} \quad (36)$$

3. DQM

The main idea of the DQM is that the derivative of a function at a sample point can be approximated as a weighted linear summation of the function value at all of the sample points in the domain. The functions f and their k^{th} derivatives with respect to x can be approximated as (Kolahchi *et al.* 2015, 2016b)

$$\frac{d^n f(x_i)}{dx^n} = \sum_{j=1}^N C_{ij}^{(n)} f(x_j) \quad n=1, \dots, N-1, \quad (37)$$

Where N is the total number of nodes distributed along the x -axis which can be calculated as

$$x_i = \frac{L}{2} \left[1 - \cos\left(\frac{\pi i}{N_x}\right) \right], \quad (38)$$

In addition, C_{ij} is the weighting coefficients which can be obtained by

$$A_{ij}^{(1)} = \frac{M(x_i)}{(x_i - x_j)M(x_j)}, \quad (39)$$

Where M is Lagrangian operator which is

$$M(x_i) = \prod_{j=1}^{N_x} (x_i - x_j), \quad i \neq j. \quad (40)$$

Using DQM and $d = d_0 e^{i\omega t}$, the motion equations can be expressed in matrix form as

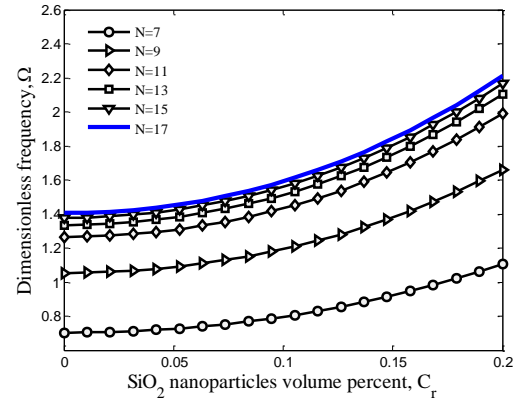


Fig. 2 Accuracy and convergence of DQM

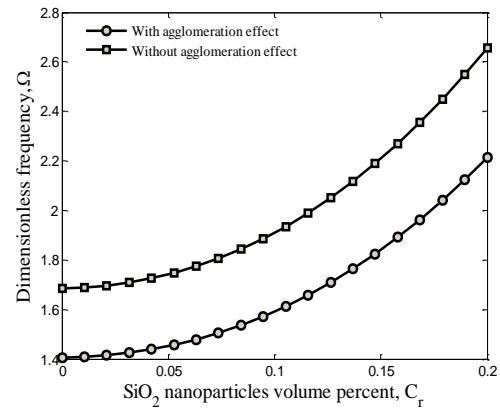


Fig. 3 Agglomeration effect on the frequency of structure

$$\{[K_G] - \omega^2[M]\}[d] = [0], \quad (41)$$

Where $(d) = (u \ w \ \psi)^T$; (K) is stiffness matrix and (M) is the mass matrix; ω is the frequency of structure. Finally, based on an iterative method and eigenvalue problem, the frequency of structure may be obtained.

4. Numerical results

Herein, a concrete column with length of $L=3$ m, thickness of $h=30$ cm, Yong modulus of $E_m=20$ Gpa and Poison's ratio of $\nu_m=0.3$ is considered which is reinforced with agglomerated SiO_2 nanoparticles with Yong modulus of $E_r=75$ Gpa and Poison's ratio of $\nu_r=0.3$.

The effect of the grid point number in DQM on the frequency of the concrete column is shown in Fig. 2. As can be seen, fast rate of convergence of the method are quite evident and it is found that 17 DQM grid points can yield accurate results. It can be found that with increasing the volume percent of SiO_2 nanoparticles, the nonlinear frequency increases. It is due to the fact that with increasing volume percent of SiO_2 nanoparticles, the stiffness of structure increases.

The effects of agglomeration (ξ) on the frequency of structure versus the SiO_2 nanoparticles volume percent are demonstrated in Fig. 3. It can be found that considering

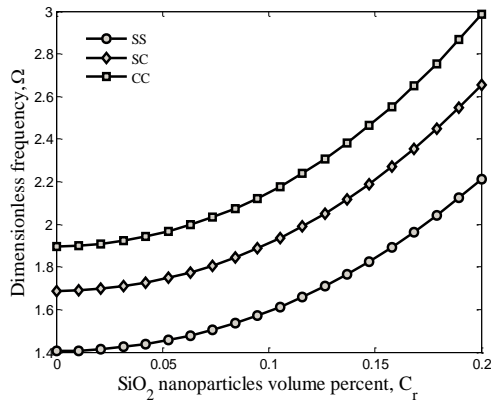


Fig. 4 Boundary condition effects on the frequency of structure

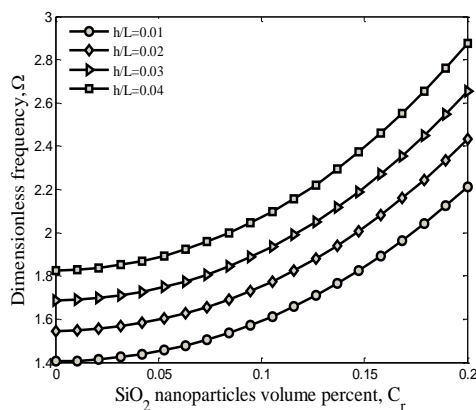


Fig. 5 Thickness to length ratio effects on the frequency of structure

agglomeration effects leads to lower frequency. It is due to the fact that considering agglomeration effect leads to lower stiffness in structure. However, the agglomeration effect has a major effect on the vibration behaviour of structure. In addition, with increasing C_r , the frequency is increased for both models.

Fig. 4 illustrates the influence of boundary conditions on the frequency along the volume percent of SiO_2 nanoparticles. It can be concluded that the frequency is higher for CC boundary conditions with respect to other types of considered ones. It is because in the CC boundary condition, the structure becomes stiffer.

The effect of thickness to length ratio of the concrete beam on the frequency versus the volume percent of SiO_2 nanoparticles is depicted in Fig. 5. As can be seen, with increasing the thickness to length ratio of the concrete beam, the frequency is increased since the stiffness of structure enhances.

Fig. 6 presents the influence of elastic medium on the frequency along the volume percent of SiO_2 nanoparticles. Obviously, the foundation has a significant effect on frequency of the beam, since the frequency of the system in the case of without foundation is lower than other cases. It can be concluded that the frequency for Pasternak model (spring and shear constants) is higher than Winkler (spring constant) one. The above results are reasonable, since the Pasternak medium considers not only the normal stresses

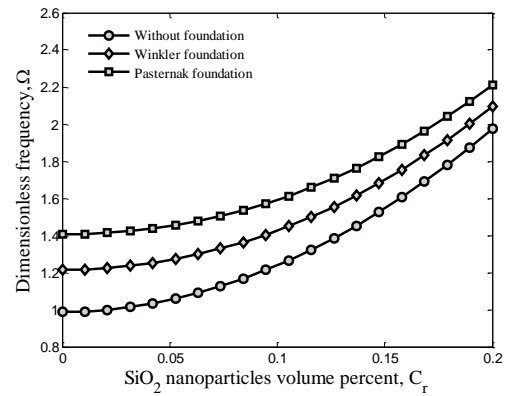


Fig. 6 Elastic medium effects on the frequency of structure

(i.e., Winkler foundation) but also the transverse shear deformation and continuity among the spring elements.

5. Conclusions

Due to lack of profound studies on the mathematical modelling of concrete beams for vibration analysis, in this paper vibration characteristics of the concrete Timoshenko beams reinforced with agglomerated SiO_2 nanoparticles was investigated. Using Mori-Tanaka mode, the effective material properties of structure were calculated considering agglomeration effects. Based on Hamilton's principle, and in accordance with aforementioned theory and beam model, motion equations were derived. Finally, obtained differential equations were solved numerically using DQM for obtaining the frequency of structure. The effects of volume percent of SiO_2 nanoparticles, agglomeration, boundary conditions, elastic medium and geometrical parameters of beam are shown on the frequency of system. Results indicate that with increasing the volume percent of SiO_2 nanoparticles, the nonlinear frequency increases. In addition, the frequency of concrete beam decreases with considering agglomeration effects. Obviously, considering elastic medium, the frequency was increased. Finally, it is hoped that the results presented in this paper would be helpful for mathematical modeling of concrete structures and using nanotechnology for production of them.

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