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Camber calculation of prestressed concrete I-Girder considering geometric nonlinearity

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Abstract. Prestressed concrete I-girders are subject to different load types at their construction stages. At the time of strand release, i.e., detensioning, prestressed concrete girders are under the effect of dead and prestressing loads. At this stage, the camber, total net upward deflection, of prestressed girder is summation of the upward deflection due to the prestressing force and the downward deflection due to dead loads. For the calculation of the upward deflection, it is generally considered that prestressed concrete I-girder behaves linear-elastic. However, the field measurements on total net upward deflection of prestressed I-girder after detensioning show contradictory results. In this paper, camber calculations with the linear-elastic beam and elastic-stability theories are presented. One of a typical precast I-girder with 120 cm height and 31.5 m effective span length is selected as a case study. 3D finite element model (FEM) of the girder is developed by SAP2000 software, and the deflections of girder are obtained from linear and nonlinear-static analyses. Only geometric nonlinearity is taken into account. The material test and field measurement of this study are performed at prestressing girder plant. The results of the linear-elastic beam and elastic-stability theories are compared with FEM results and field measurements. It is seen that the camber predicted by elastic-stability theory gives acceptable results than the linear-elastic beam theory while strand releasing.

Keywords: camber; deflection; detensioning; finite element model; linear and non-linear static analyses; prestressed concrete

1. Introduction

Prestressed concrete has found extensive application in construction of medium and long span bridges because of its better stability, serviceability, economy, aesthetic appearance, structural efficiency, ease of fabrication, and low maintenance. Prestressed concrete girder is smaller in depth than an equivalent reinforced concrete member; therefore, the deflection of a prestressed concrete girder tends to be larger. For a prestressed concrete structure, deflections must be in certain limits. Prestressed concrete girder bridge construction consists of some stages. At these stages, girders are under the effect of different material properties, prestress losses, and load conditions. For all these reasons, deflection behavior of girder at any stage is different from each other. Generally the deflection of prestressed concrete is classified into two groups: short-term and long-term. Short-term deflection refers to the immediate deflection after detensioning, while the long-term deflection occurs over a long period of time largely due to losses of prestress, shrinkage and creep of the materials. In order to better predict long-term camber of girder, it is important to be able to better predict the camber at release French (2012). The uncertainty of the predicted camber in precast, prestressed girders can lead to problems during construction. Prediction of camber accurately is difficult because the camber depends on many random variables, some of which are interdependent and change over time. Some of the most important variables are the compressive strength and Young's modulus of concrete, amounts of creep and shrinkage, thermal gradients within the girder, and the time-dependent variations in prestressing force (Storm *et al*. 2013).

Numerous theoretical as well as experimental investigations on structural behavior and load-deflection behavior of prestressed concrete structures have been carried out over the last decades. Martin (1977) developed a set of multipliers for estimating camber at various time intervals that are still widely used by prestressed concrete designers today. (Kelly *et al*. 1987) found considerable variations between the predicted and actual cambers of eight identical American Association of State Highway and Transportation Officials (AASHTO) Type IV girders. Ghali and Azarnejad (1999) studied the effects of types of reinforcements and concrete strength on the deflections. Rodriguez-Gutierrez and Aristizabal-Ochoa (2007) presented a model for the calculation of both short and long-term deformations in reinforced, prestressed, and composite concrete beams with generalized end conditions subjected to bending about any transverse axis. (Rosa *et al.* 2007) conducted to develop improved methods of predicting camber in prestressed concrete girders. They used adjustment factor for Young's modulus to minimize the predicted error on the camber immediately after release. (Rizkalla *et al*. 2010) presented several factors related to girder production that have a significant impact on the prediction of camber and examined the accuracy of the

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current NCDOT method for predicting the prestress losses and camber for prestressed concrete girders as compared to field measurements. (Tadros *et al*. 2011) proposed a method for incorporating new AASHTO prediction formulas into a spreadsheet to predict initial and long-term camber, as well as an investigation of camber variability. Gocic and Sadovic (2012) presented an approach based on the Newton-Raphson method for obtaining the stresses and strains in the section at mid-span of prestressed girders according the equilibrium state. French (2012) carried out an experimental research to determine both short-term and long-term deflections of prestressed girders. (Storm *et al.* 2013) investigated factors related to prestressed concrete girder production that could affect the camber, and recommended the camber prediction methods. (Colajanni *et al*. 2014) presented the theoretical basis and the main results of a design procedure, which attempts to provide the optimal layout of ordinary reinforcement in prestressed concrete beams, subjected to bending moment and shear force. (Parrotta *et al*. 2014) carried out an analytical-experimental study for deformation behavior of RC beams. Hossain and Okeil (2014) presented three dimensional (3-D) finite element (FE) detailed joint model, which takes into account the gradual increase in prestressing force along the transfer length, the existence of cold joints between the cast in place concrete and the precast concrete, and the actual 180°-hook hairpin bar configuration. (Lou *et al.* 2015) presented the development of a finite element model for the geometric and material nonlinear analysis of bonded prestressed concrete continuous beams.

The main objective of this research is to investigate the accuracy of calculation of camber after detensioning the strand with linear-elastic beam and elastic-stability theories. For this purpose one of the typical precast I-girder with 120 cm height and 31.5 m effective span length is selected as an application. The 3D FEM of the girder is developed by SAP2000 software (2016). Nonlinear and linear-static analyses are performed under dead and prestressing loads to obtain deflection of the selected girder. Only geometric nonlinearity is taken into account. The material test and field measurement of this study is performed at a prestressing girder plant. The results of linear-elastic beam and elastic-stability theories are compared with those of FEM and field measurement.

2. Camber calculations

During the manufacturing process of prestressed concrete girders, prestressing force is transferred from strands to concrete by bond at their interface. Due to existence of the eccentrically located tendons, the girders tend to deflect upwards also called camber. Camber of prestressed girder at detensioning is a summation of the upward deflection due to the prestressing force and the downward deflection due to the dead loads.

The magnitude of camber depends on the self-weight of the girder, the amount of strand and prestressing force. In this study, deflection of girder is calculated with the linear-elastic beam and elastic-stability theories. According to the linear-elastic beam theory, the equilibrium equation is formulated for undeformed state of girder, in which secondary moment is neglected. However for the elastic-stability beam theory the equilibrium equation is formulated for deformed state of girder. According to the linear-elastic beam theory, mid-span downward deflection of prestressed concrete girder under the effect of self-weight is determined by

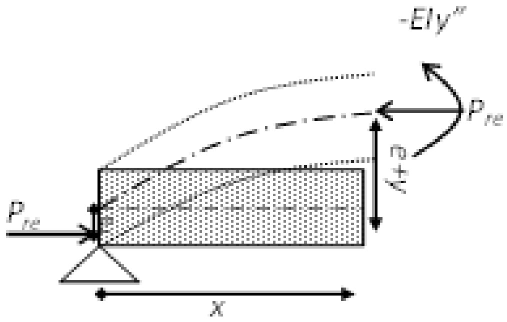

Fig. 1 Eccentrically loaded girder

$$\delta_{sw} = \left(\frac{5 \times q_{beam} \times L^4}{384 E_{ci} I_{girder}} \right) \tag{1}$$

Where L is the length of girder, q_{beam} is the linearly distributed dead load over the length of girder, E_{ci} is the Young's modulus of concrete and I_{girder} is the moment of inertia.

The upward deflection of prestressed concrete girders at mid-span due to prestressing force is determined by

$$\delta_{ps} = -\frac{P_{re} \times e \times L^2}{8 E_{ci} I_{girder}} \tag{2}$$

Where e and P_{re} are the eccentricity of strand and the total prestress force at release, respectively. Eq. (2) is applicable to beams with straight strands.

The downward deflection due to dead loads can be calculated with linear-elastic beam theory because the secondary moment does not occur with deformation of girder. However, under the effect of prestressing force the direction of force and displacement are perpendicular to each other, so the effects of secondary moment due to prestressing force must be taken into consideration. In this case, the elastic-stability theory formulated the equations of equilibrium in the deformed state and updated with the deformation should be used. According to the elastic-stability theory it is assumed that the girder is initially straight and materials obey Hooke's law (Chajes 1974).

A simple supported girder exposed eccentricity load cut at a distance x from the pinned support is shown in Fig. 1. This partial section of the column must be equilibrium. Thus we have

$$EIy'' + P_{re}(e + y) = 0 \tag{3}$$

Where EIy'' is the internal resisting moment. By solving Eq. (3), the upward deflection due to prestressing force at the mid-span according to the elastic-stability theory can be obtained as

$$\delta_{ps} = e\left(\frac{1}{\cos\left(\frac{L}{2}\sqrt{\frac{P_{re}}{E_{ci}I_{girder}}}\right)} - 1\right) \quad (4)$$

3. Prestressed concrete girder models

In this paper, simply supported prestressed I-girder with 120 cm height and 31.5 m effective span length is selected as an application. A typical appearance and the dimensions of cross section are shown in Fig. 2. The low-relaxation Grade 270 prestressing strand (characteristic tensile strength f_u of 1860 MPa) with 15 mm (0.6 in.) diameter is used as a strand type. Strands layout along the girder length is assumed as linear, and the distance between the strands is 6 cm. The camber calculations are based on cross section and material properties and include the elastic shortening losses. Young's modulus and unit weight of strand determined by the producing company are used. In many reinforced concrete structures applications, to estimate the material properties such as compressive strength and Young's modulus is very important to meet design requirements. In this study, material tests are conducted to evaluate the compressive strength and unit weight of concrete. To determine the compressive strength, concrete cubic samples obtained from the plants are taken directly from a mix being used to make girders. The 15×15×15 cm plastic cubic molds are filled with concrete, and prepared according to the TS 3114 standard. The cubic molds are made adjacent to the girder bed line using concrete from the same batch, and are subsequently placed on the outside of the girder side-forms and under the tarps to subject to similar curing conditions as the girders. However, determination of Young's modulus is time-consuming and expensive so the common practice is to estimate it using empirical relationships, based on various codes of practice (Maia and Aslani 2016).

To determine the Young's modulus of concrete, there are many models and expressions. These models and expressions predict the Young's modulus of concrete based on the concrete strength and, possibly, other parameters (Carrasquillo *et al*. 1981, CEB-FIP 1990, Gardner and Lockman 2001, etc.). French (2012) determined that the Pauw (ACI 318-08, AASHTO LRFD 2010) equation most closely predicted the Young's modulus in comparison with the other models at detensioning. The expression for the Young's modulus of concrete according to Pauw equation is given as

$$E_{ci} = 0.043 K_1 \gamma_c^{1.5} \sqrt{f'_c} \quad (5)$$

Where γ_c is the unit weight of concrete at time of test, in kg/m^3, f_c is the concrete compressive strength at time of test, in MPa and K_1 is the correction factor for source of aggregate to be taken as 1.0 unless determined by physical test. Material properties considered in the numerical analysis are given in Table 1.

Efficient design of prestress concrete bridges demands

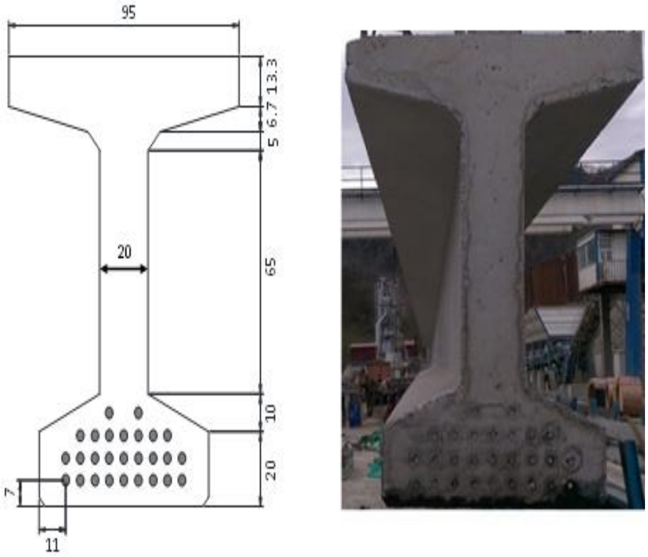

Fig. 2 Cross-section of the investigated girder

Table 1 Material properties

Material	Young's Modulus (MPa)	Poisson's Ratio	Density (kg/m^3)
Concrete	31095	0.2	2440
Strand	201605	0.3	7850

an accurate prediction of prestress losses. The prestress losses are defined as the loss of tensile stress in the prestress steel due elastic shortening, creep, shrinkage and relaxation. During the transfer operation of prestress, the girder tends to reduce in length, which is called elastic shortening. The prestress loss due to elastic shortening is

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp} \quad (6)$$

Where, E_p is the modulus of elasticity of prestressing steel, f_{cqp} is the sum of concrete stresses at the center of gravity of prestressing tendons due to the prestressing force at transfer.

Strand relaxation that occurs between the time of strand pull and release is another effect that leads to prestress lose. However, this effect is rather small (i.e., prestress losses on the order of approximately 1%) and is neglected by MnDOT and by the precasters (French 2012).

4. Finite element modeling

Nonlinear-static and linear-static analyses of the girder are performed with SAP2000 (2016) in order to obtain the deflections of the girder. In nonlinear-static analysis, only geometric nonlinearity is taken into account. Three-dimensional finite element model (FEM) of girder is given in Fig. 3. The girder model consists of 20 frame elements and 27 tendons. The girder and strands are represented by frame and tendon, respectively. As a boundary condition, the left and right hand supports are selected as pinned and roller, respectively. Adjacent nodes between the frame

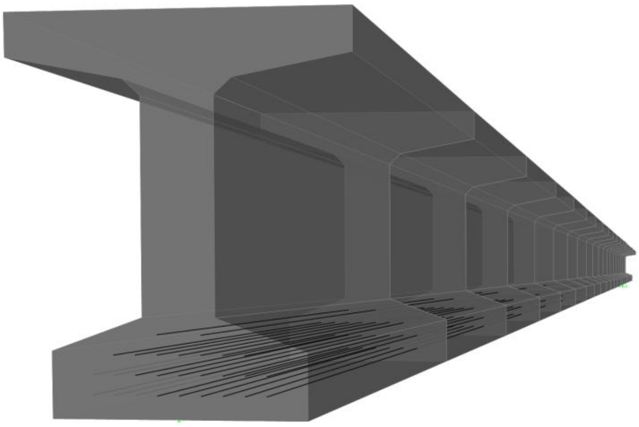

Fig. 3 Finite element model of the girder

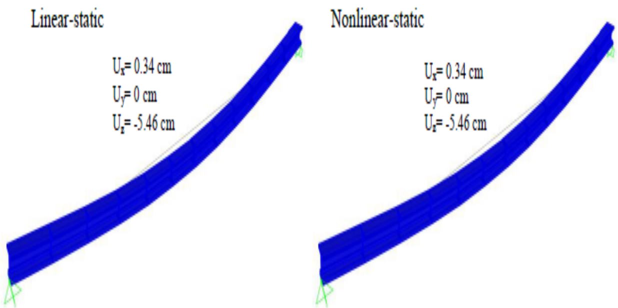

Fig. 4 Downward deflection due to the self-weight of beam

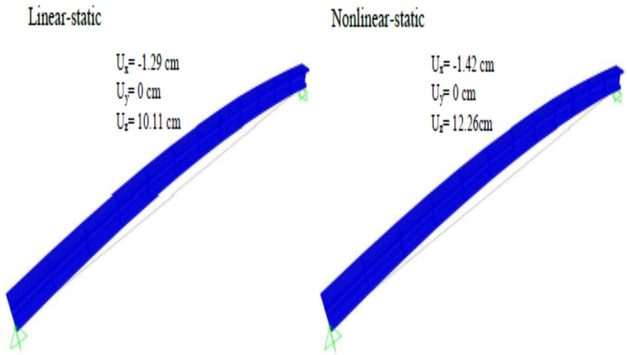

Fig. 5 Upward deflection due to the prestress force

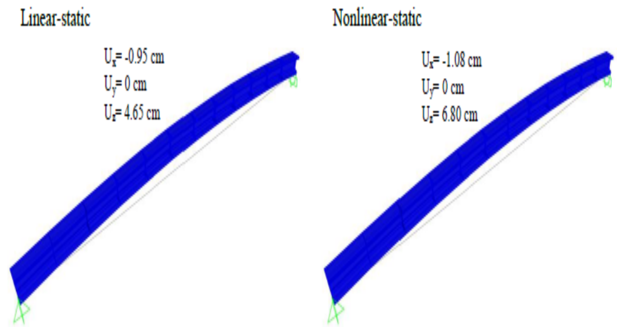

Fig. 6 Total net upward deflection at detensioning

elements and strands are connected to each other to represent the perfect bond assumption. Transfer length is not taken into consideration.

Self-weight of girder is calculated from the finite element software directly. Prestressing force is calculated as 181 kN when the prestressing losses are taken into account. This force is simultaneously applied to all strands at both sides. During the strand release time-dependent factors such as creep and shrinkage are not taken into consideration.

5. Numerical results

The deflections occur due to the self-weight, prestressing force and total of these forces obtained from the linear-static and nonlinear-static analyses are given separately. The distribution of the downward deflection along the girder under self-weight is given in Fig. 4. It is seen that, the downward deflection has an increasing trend towards the middle of the girder. The maximum deflection of the girder at the mid-span is obtained -5.46 cm from both analyses. It is clearly seen that calculation of downward deflection due to the self-weight of girder is not affected from the geometric nonlinearity. The distribution of upward deflection along the girder due to the prestressing force obtained from the linear and nonlinear analyses is given in Fig. 5. It is seen that, the upward deflection has an increasing trend towards the middle of the girder.

According to the linear-static and nonlinear-static analyses, the maximum upward deflection is obtained 10.11 cm and 12.26 cm, respectively. The upward deflection of girder obtained from the nonlinear-static analysis, in which P-Delta displacement is considered is greater than that of the linear-static analysis. Total net upward deflection of prestressed girder is a summation of the upward deflection due to the prestressing force and the downward deflection due to the self-weight of girder. The distribution of total net upward deflection (camber) along the girder after detensioning obtained from both analyses is given in Fig. 6. According to the linear-static and nonlinear-static analyses, the maximum camber is obtained 4.65 cm and 6.80 cm, espectively.

A simple stretch-wire system is used for measurement of total net deflection shortly after completion of strand release. The material used for the stretch-wire system is important because the line has minimal self-weight and elongation under tension, and be easy to handle. In this study, fishing line is used for measurement because it is proved to be strong, durable and easy to work (French 2012). To determine the initial camber after detensioning, a measurement is taken just after side-forms are removed at precasting bed (Fig. 7). The measurement taken from selected application of this study is seen in Fig. 10.

The results obtained by analytical methods, FEM and field measurements are shown in Table 2. The downward deflection obtained from FEM and beam theories are equal, but the upward deflection due to the prestressing force is different in all cases. The maximum downward deflection is obtained 5.46 cm. The upward deflection is obtained 10.11 cm and 12.26 cm from the linear-static and nonlinear-static analyses of FEM and 10.21 cm and 12.39 cm from the linear-elastic beam and elastic-stability theories,

Fig. 7 Precasting bed

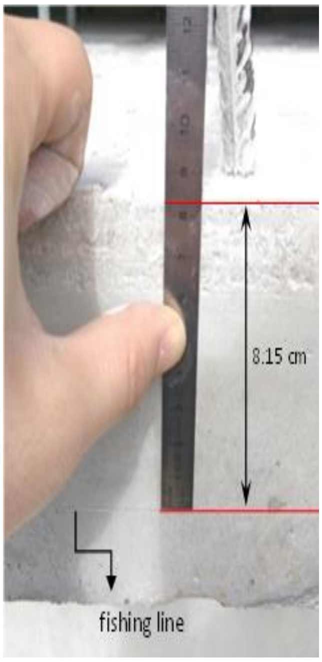

Fig. 8 Total camber of girder

respectively. As expected, the upward deflection obtained from the nonlinear-static analysis of FEM and the elastic-stability theory is greater than the linear-elastic beam theory and linear-static analysis of FEM. The total net camber is obtained 4.67 cm and 6.80 cm from linear-static and nonlinear-static analysis of FEM of girder and calculated 4.75 cm and 6.93 cm with linear-elastic beam and elastic-stability theories, respectively.

The total net camber measured at prestressing plant after detensioning is 8.15 cm. Prediction of total net camber after strand release with the linear-elastic beam theory and the linear-static analysis of FEM is far away from the field measurement than nonlinear-static analysis of FEM and elastic-stability beam theory.

6. Conclusions

The aim of this paper is to investigate the accuracy of the short-term deflection of prestressed concrete I-girder after detensioning with the linear-elastic beam and elastic-stability theories. According to the linear-elastic beam theory, the girder videlicet secondary moment is neglected, but in elastic-stability theory, the equilibrium equation is formulated for deformed state of girder. Simply supported prestressed I-girder with 1.20 m height and 31.5 m effective span length is selected as an application. FEM of girder are developed by SAP2000 software, and the nonlinear and linear-static analyses are performed under dead and prestressing load. In nonlinear static analysis, only geometric nonlinearity is taken into account. Material testing and deflection measurement of the selected girder are performed on construction plant. The prediction of girders' short-term deflection with linear-elastic beam and elastic-stability theories is compared with field measurement, and the result of FEM. The main conclusions drawn from this analytical study are:

Table 2 Results of FEM, beam theories and field measurement

Method	Deflection			
	Self-weight of girder (cm)	Prestressing force (cm)	Total net camber (cm)	Field measurement (cm)
Linear-elastic beam theory	-5.46	10.21	4.75	8.15
Elastic-stability theory	-5.46	12.39	6.93	
SAP2000 (Linear-static)	-5.46	10.11	4.67	
SAP2000 (Nonlinear-static)	-5.46	12.26	6.80	

• The downward deflection due to the self-weight of girder has an increasing trend towards middle of the girder. The downward deflection of the girder obtained from the linear and nonlinear analyses of FEM and beam theories are equal to each other. It is clearly seen that the calculation of downward deflection due to dead load is not affected from geometric nonlinearity because the deflection of girder is in the same direction with the load, thus secondary moment does not occur.

• The downward deflection due to the self-weight of irder can be accurately calculated with the linear-elastic beam theory.

• The upward deflection due to the prestressing force has an increasing trend towards the middle of the girder. The upward deflection of girder obtained from the nonlinear-static analysis, in which P-Delta displacement criterion is taken into account is greater than the linear-static analysis of FEM. Also, the result of elastic-stability theory is greater than the linear-elastic beam theory. The difference between linear-static and nonlinear-static analyses of FEM is 21%. Also, difference between the linear-elastic beam and elastic-stability theories is 21%, too. It is seen that, as the strands are released and the girder begins to camber up, the secondary moment occurs due to prestressing force.

• The maximum upward deflection obtained from the linear-elastic beam theory and linear-static analysis of FEM is pretty close to each other. Also, the nonlinear-static analysis of FEM has almost same results with the elastic-stability theory.

• The total net upward deflection (camber) of girder after detensioning has an increasing trend towards the middle of the girder. The prediction of camber with linear-static analysis of FEM and linear-elastic beam theory is far away from the value of field measurement. However, the results of nonlinear analysis of FEM and elastic-stability theory approach the value of the field measurement. The difference between the field measurement and the linear-elastic beam theory is 42%, but difference between the field measurement and elastic-stability theory is 15%.

The conclusion of this study is that the prediction of the downward deflection due to self-weight with the linear-elastic beam and elastic-stability theories is same, but the prediction of upward deflection due to prestressing force is different from each other. Under the effect of prestressing force, direction of force is perpendicular to the direction of the deflection, thus secondary moment occurs. Secondary moment is not taken into account the linear-elastic beam theory; therefore, the prediction of upward deflection with linear-elastic beam theory can be given misleading result. Using the elastic-stability theory taken into account P-Delta displacement criterion for prediction of short-term deflection of prestressed concrete I-girder gives more accurate result rather than the linear-elastic beam theory.

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