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Statistical flexural toughness modeling of ultra high performance concrete using response surface method

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Abstract. This paper aims to model the effects of five different variables which includes: cement content (C), the steel fiber amount (F), the silica fume amount (SF), the superplasticizer (SP), the silica fume amount (SF), and the water to cementitious ratio (w/c) on 28 days flexural toughness of Ultra High Performance Concrete (UHPC) as well as, a study on the variable interactions and correlations by using analyze of variance (ANOVA) and response surface methodology (RSM). The variables were compared by fine aggregate mass. The model will be valid for the mixes with 0.18 to 0.32 w/c ratio, 4 to 8 percent steel fiber, 7 to 13 percent cement, 15 to 30 percent silica fume, and 4 to 8 percent superplasticizer by fine aggregate mass.

Keywords: ultra high performance concrete; response surface method; flexural toughness; central composite methodology

1. Introduction

Concrete is one of the most used materials in construction. However, it still has some inherent drawbacks like tensile strength or brittleness (Yoo *et al.* 2013a, b), therefore, attention to improve the properties of concrete for higher strength and ductility and tending to improve the durability resulted in innovation of several types of concrete (Zhang *et al.* 2014a, b). Ultra high performance concrete (UHPC) is one of the latest concrete that has unique properties (Wang 2014) such as high compressive strength, exhibiting the tensile and flexural strength with increase in energy absorption (toughness), improved high durability, improved resistance against freezing-thawing and various chemical attacks (Ma *et al.* 2004a, b).

Toughness is a quantity of energy absorption capacity, and it is used to describe the ability of UHPC to resist fracture when applied to static, dynamic and impact loads in different mix design. Energy absorption or toughness capability could be calculated from the area under the load-deflection curve in flexure, which will be the total energy absorbed prior to complete separation of the specimen (Marar *et al.* 2011a, b).

Effects of steel fiber content and shape on flexural toughness of ultra high performance concrete was done by Wu *et al.* (2016). The effect of just steel fiber orientation on flexural

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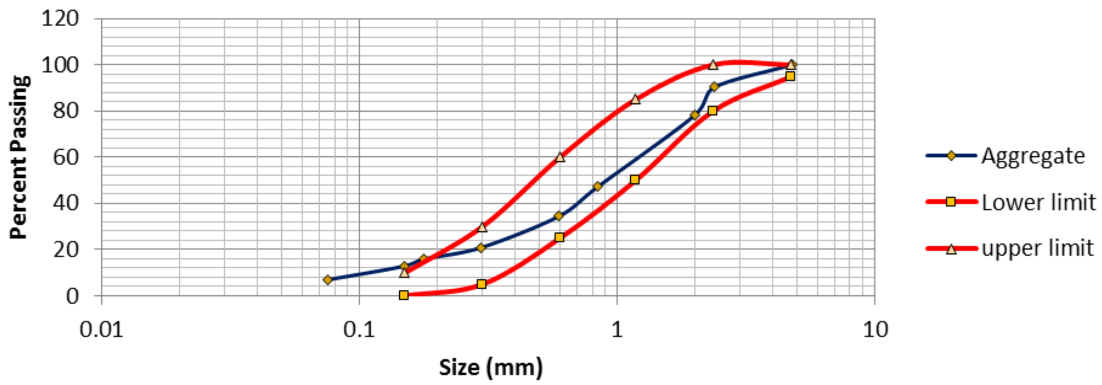

Fig. 1 Size distribution of sand

toughness were studied by Barnett *et al.* (2010). The effect of steel fiber and silica fume on flexural toughness were studied by Zhang *et al.* (2014). In most studies the single or some effects of concrete ingredients on flexural toughness were modeled and studied while in this study the effect of five independent variables together and the interactions between them on flexural toughness strength was modelled.

Statistical method with based of experimental design is used for this research work. Response surface method is a combination between statistical and mathematical techniques (Mohammed *et al.* 2014a, b), which can be used for modeling and analyzing with giving good interpretations by finding the relations between variables

A lot of studies were made on how to increase the toughness by different fiber properties (Tuan *et al.* 2014a, b), but they didn't focus on the effect of some other ingredients and interaction between them. This research tried to monitor the effect of concrete ingredients, separately or together on flexural toughness as well as offering the model for energy absorption prediction.

2. Experimental activities

2.1 Materials

2.1.1 Fine aggregate

In this study normal mining sand with maximum particle size of 5mm were used. Particle size distribution based on ASTM C136 (1995) and controlled by ASTM C33-03 (2004) as shown in Fig. 1.

2.1.2 Mixing water

The tap water was used for mixing and curing.

2.1.3 Superplasticizer

The superplasticizer was a polycarboxylic ether based with high range water reducing and new generation superplasticizer admixture developed for using in UHPC which is called GLENIUM 27 and manufactured by BASF. The superplasticizer is consistent with EN 934-2 (2009).

Table 1 The variables with their levels

Variables	Code	Levels of Variables		
		-1	0	+1
Silica fume	A	15%	25%	30%
Superplasticizer	B	4%	6%	8%
Fiber	C	10%	15%	20%
Cement	D	70%	100%	130%
w/c	E	0.18	0.225	0.32

*Percentages are based on fine aggregate mass used

2.1.4 Steel fiber

The dimension of steel fiber used was 0.55 mm in diameter and 13 mm in length with the tensile strength of 1345 MPa and young's modulus of 210000 MPa. Steel fiber was manufactured by Dramix, and confirmed by ASTM A820 (2001).

2.1.5 Silica fume

A white undensified silica fume with more than 95% purity of silicon dioxide and particle sizes between 0.1-1 μm as pozzolanic material was used.

2.1.6 Cement

The property of the cement used was, type 2, 42.5N Portland sulfate resistance slag cement and is controlled by European standard EN 197-1 (2012) cement composition. Amount of slag and clinker used for its manufacture were between 21-35% and 65-79%.

2.2. Experimental design

With an objective of developing relations between the variables and response (empirical model for flexural toughness), a statistical method using design of experiment (DOE) based on response surface methodology (RSM) was adopted. For providing equivalent precision of estimates in all directions, central composite design with equal precision was selected.

2.3. Methodology

In this research, the flexural toughness of UHPC with local materials at different levels as well as mix proportion for each responses was considered and the interactions of variables were monitored by using response surface methodology with central composition design of α=1 (face centered) quadratic model for response. The correlation between the variables and the effect of response was analyzed by ANOVA. The statistical software "Design- Expert version 9.0.3", Stat-Ease, Inc., was used for experimental design

In this study, the flexural toughness test which was monitored through ASTM C1609 was defined as the response and 5 variables as SF (A), superplasticizer content (B), steel fiber content (C), cement content (D), w/c ratio (E) is defined to explain the modeling. Based on previous studies as reported by Yu *et al.* (2014), Máca *et al.* (2014), Wille *et al.* (2012), the range of variables were selected as follows: SF amount is from 15 to 30 percent of fine aggregate mass, the

Table 2 Design of experiments

Mix no	SF (A)	SP (B)	Fiber (C)	Cement (D)	w/c (E)	Sand	SF (A)	SP (B)	Fiber (C)	Cement (D)	w/c (E)
1	-1	1	-1	-1	1	1	0.15	0.08	0.10	0.7	0.320
2	1	-1	-1	1	1	1	0.30	0.04	0.10	1.3	0.320
3	1	-1	1	-1	-1	1	0.30	0.04	0.20	0.7	0.180
4	0	0	1	0	0	1	0.20	0.06	0.20	1.0	0.225
5	1	-1	-1	-1	-1	1	0.30	0.04	0.10	0.7	0.180
6	0	0	0	0	-1	1	0.20	0.06	0.15	1.0	0.180
7	-1	1	-1	-1	-1	1	0.15	0.08	0.10	0.7	0.180
8	1	-1	1	1	-1	1	0.30	0.04	0.20	1.3	0.180
9	-1	0	0	0	0	1	0.15	0.06	0.15	1.0	0.225
10	-1	-1	1	-1	1	1	0.15	0.04	0.20	0.7	0.320
11	0	-1	0	0	0	1	0.20	0.04	0.15	1.0	0.225
12	1	1	-1	1	1	1	0.30	0.08	0.10	1.3	0.320
13	0	0	0	1	0	1	0.20	0.06	0.15	1.3	0.225
14	1	-1	1	-1	1	1	0.30	0.04	0.20	0.7	0.320
15	1	-1	1	1	1	1	0.30	0.04	0.20	1.3	0.320
16	-1	-1	-1	1	1	1	0.15	0.04	0.10	1.3	0.320
17	0	0	0	0	1	1	0.20	0.06	0.15	1.0	0.320
18	-1	-1	1	1	1	1	0.15	0.04	0.20	1.3	0.320
19	-1	-1	-1	1	-1	1	0.15	0.04	0.10	1.3	0.180
20	0	0	0	0	0	1	0.20	0.06	0.15	1.0	0.225
21	1	-1	-1	-1	1	1	0.30	0.04	0.10	0.7	0.320
22	0	0	0	0	0	1	0.20	0.06	0.15	1.0	0.225
23	-1	1	-1	1	1	1	0.15	0.08	0.10	1.3	0.320
24	1	1	1	1	-1	1	0.30	0.08	0.20	1.3	0.180
25	1	1	1	1	1	1	0.30	0.08	0.20	1.3	0.320
26	-1	1	1	-1	-1	1	0.15	0.08	0.20	0.7	0.180
27	-1	-1	1	1	-1	1	0.15	0.04	0.20	1.3	0.180
28	-1	-1	1	-1	-1	1	0.15	0.04	0.20	0.7	0.180
29	1	1	-1	-1	-1	1	0.30	0.08	0.10	0.7	0.180
30	1	1	1	-1	1	1	0.30	0.08	0.20	0.7	0.320
31	-1	-1	-1	-1	-1	1	0.15	0.04	0.10	0.7	0.180
32	1	1	1	-1	-1	1	0.30	0.08	0.20	0.7	0.180
33	1	-1	-1	1	-1	1	0.30	0.04	0.10	1.3	0.180
34	-1	-1	-1	-1	1	1	0.15	0.04	0.10	0.7	0.320
35	1	1	-1	1	-1	1	0.30	0.08	0.10	1.3	0.180
36	-1	1	1	1	1	1	0.15	0.08	0.20	1.3	0.320
37	-1	1	1	-1	1	1	0.15	0.08	0.20	0.7	0.320
38	-1	1	-1	1	-1	1	0.15	0.08	0.10	1.3	0.180
39	1	1	-1	-1	1	1	0.30	0.08	0.10	0.7	0.320
40	0	1	0	0	0	1	0.20	0.08	0.15	1.0	0.225
41	1	0	0	0	0	1	0.30	0.06	0.15	1.0	0.225
42	0	0	0	-1	0	1	0.20	0.06	0.15	0.7	0.225
43	0	0	-1	0	0	1	0.20	0.06	0.10	1.0	0.225
44	0	0	0	0	0	1	0.20	0.06	0.15	1.0	0.225
45	-1	1	1	1	-1	1	0.15	0.08	0.20	1.3	0.180

Fig. 2 Beam under loading to do flexural toughness test

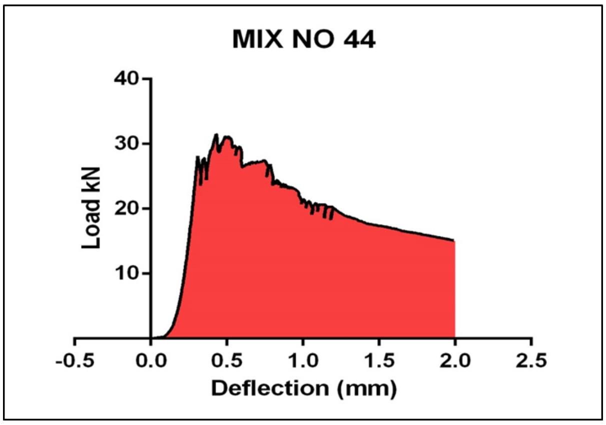

Fig. 3 Load- deflection of mix No 44

superplasticizer content is from 4 to 8 percent, the steel fiber content is from 10 to 20 percent, the OPC amount is from 70 to 130 percent of fine aggregate mass, and w/c ratio from 0.18 to 0.32. The variables with their various levels limitation are given in Table 1.

2.4 Specimen preparation and test specimen

In this research, 45 batches were prepared (Table 2) which were mixed in a drum rotating mixer. Firstly, premix which is included dry materials (cement, SF, sand) except steel fiber were blended in a well determined proportion for 5 minutes, then proportional amount of superplasticizer was added to suitable water as well as steel fiber, thereafter, mixed of water, superplasticizer, and steel fiber was added to premixed mixture and continuation mixing as the ultra high performance concrete changes from a dry mixture to a thick fresh concrete. Three 10×10×50 mm beams were used for 28 days for the flexural toughness test. After casting, all samples were compacted by vibration table and kept in the moist curing room for 24 hours. They were then molded out and transferred to the curing water tank at 23±2℃ until testing.

2.5 Flexural toughness strength

Table 3 Mix design amounts and flexural Toughness of UHPC

Mix no	Sand (kg)	Silica Fume (kg) A	Super-plasticizer (kg) B	Steel Fiber (kg) C	Cement (kg) D	Flexural Toughness (kN.mm) Y
1	50	7.5	4.0	5.0	35	39.24
2	50	15.0	2.0	5.0	65	17.56
3	50	15.0	2.0	10.0	35	43.28
4	50	10.0	3.0	10.0	50	38.00
5	50	15.0	2.0	5.0	35	30.36
6	50	10.0	3.0	7.5	50	39.00
7	50	7.5	4.0	5.0	35	51.11
8	50	15.0	2.0	10.0	65	35.51
9	50	7.5	3.0	7.5	50	40.00
10	50	7.5	2.0	10.0	35	57.00
11	50	10.0	2.0	7.5	50	32.48
12	50	15.0	4.0	5.0	65	15.69
13	50	10.0	3.0	7.5	65	38.20
14	50	15.0	2.0	10.0	35	36.85
15	50	15.0	2.0	10.0	65	21.18
16	50	7.5	2.0	5.0	65	15.00
17	50	10.0	3.0	7.5	50	40.59
18	50	7.5	2.0	10.0	65	44.64
19	50	7.5	2.0	5.0	65	31.85
20	50	10.0	3.0	7.5	50	32.00
21	50	15.0	2.0	5.0	35	11.93
22	50	10.0	3.0	7.5	50	40.00
23	50	7.5	4.0	5.0	65	13.00
24	50	15.0	4.0	10.0	65	48.00
25	50	15.0	4.0	10.0	65	25.00
26	50	7.5	4.0	10.0	35	61.00
27	50	7.5	2.0	10.0	65	56.25
28	50	7.5	2.0	10.0	35	82.00
29	50	15.0	4.0	5.0	35	34.00
30	50	15.0	4.0	10.0	35	27.39
31	50	7.5	2.0	5.0	35	58.74
32	50	15.0	4.0	10.0	35	33.88
33	50	15.0	2.0	5.0	65	34.45
34	50	7.5	2.0	5.0	35	29.00
35	50	15.0	4.0	5.0	65	46.89
36	50	7.5	4.0	10.0	65	24.00
37	50	7.5	4.0	10.0	35	39.62
38	50	7.5	4.0	5.0	65	47.30
39	50	15.0	4.0	5.0	35	40.62
40	50	10.0	4.0	7.5	50	28.00
41	50	15.0	3.0	7.5	50	30.06
42	50	10.0	3.0	7.5	35	46.78
43	50	10.0	3.0	5.0	50	36.45
44	50	10.0	3.0	7.5	50	27.78
45	50	7.5	4.0	10.0	65	40.00

Table 4 Analysis result of regression models

Response	R^2	Adj-R^2	Pre-R^2	F-Value	Lack of fit	Model P-value
Flexural toughness	0.85	0.78	0.63	11.97	0.59	<0.0001

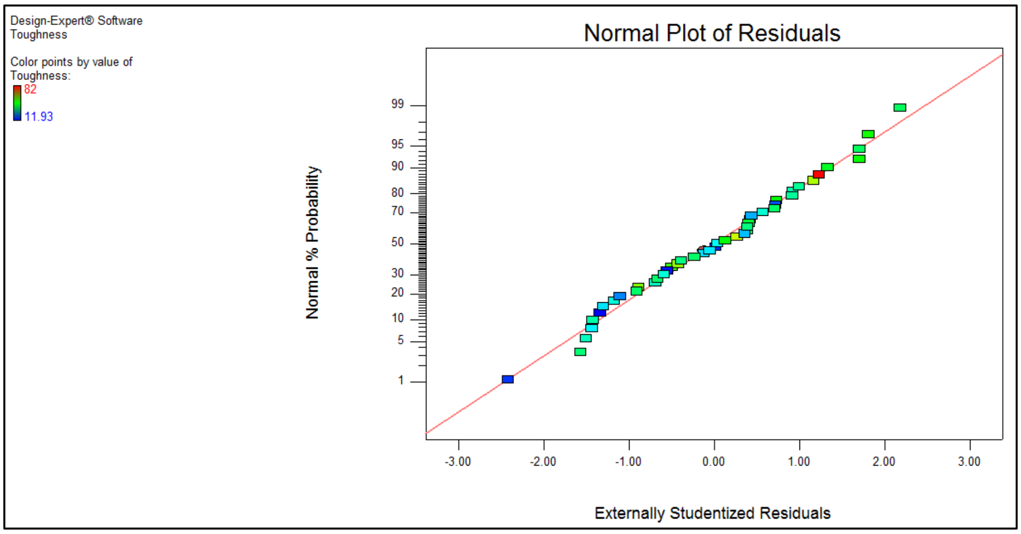

Fig. 4 Normal plot of residual value of flexural toughness

The ASTM C1609 (2006) was used in doing this test. This test involves four point flexural loading. The beam size which was 100×100×500 mm with the span of 300 mm and load distance of 100 mm. One sample with third-points within deflection measurement under universal machine loading with two LVDT at the middle of the span and two sides of beams is shown in Fig. 2. The flexural toughness is the area under the load versus net deflection curve 0 to 2 mm (l/150).

3. Results

The effects of five variables (silica fume content, superplasticizer content, steel fiber content, OPC content, and w/c) on flexural toughness of UHPC have been analyzed by using response surface method. For producing the model 45 points were selected such as 32 points for model making, 3 points for replication, and 8 points to consider the lack of fit.

Flexural toughness is obtained from the area under the loads versus net deflection curve 0 to 2 mm which is given as an example for mix no 44 in Fig. 3. In order to calculate the area under the load-deflection curve, Prism GraphPad Software Version 6.0 was performed. Table 3 shows the results of using five different variables in the mechanical properties of UHPC. Each result was derived by the average of 3 specimens. This experiment was followed through ASTM C1609 (2006).

The interaction and correlation between the variables and the floral toughness was calculated by ANOVA (analysis of variance). For the modeling, linear model, two-factor interaction, and quadratic models were considered to find best predictive model. In each model, the significant parameters were detected and then, by the backward elimination technique the insignificant terms

Table 5 Parameter estimated for model

	Constant	A	B	C	D	E	AB	AC	AD	AE	BC	DE	B^2	D^2	E^2
MODEL ESTIMATE	35.64	-5.80	-0.69	4.72	-4.95	-8.10	3.11	-2.49	4.11	1.77	-4.27	-1.61	-6.70	5.55	2.85
Prob > F	-----	0.00	0.54	0.00	0.00	0.00	0.01	0.04	0.00	0.13	0.00	0.17	0.09	0.16	0.46

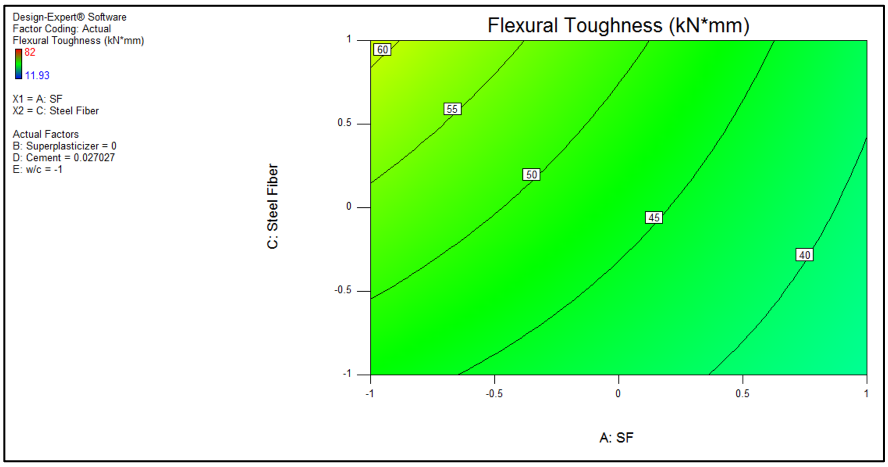

Fig. 5 Contour plot of flextural strength changes, X1=SF amount and X2=Steel fiber

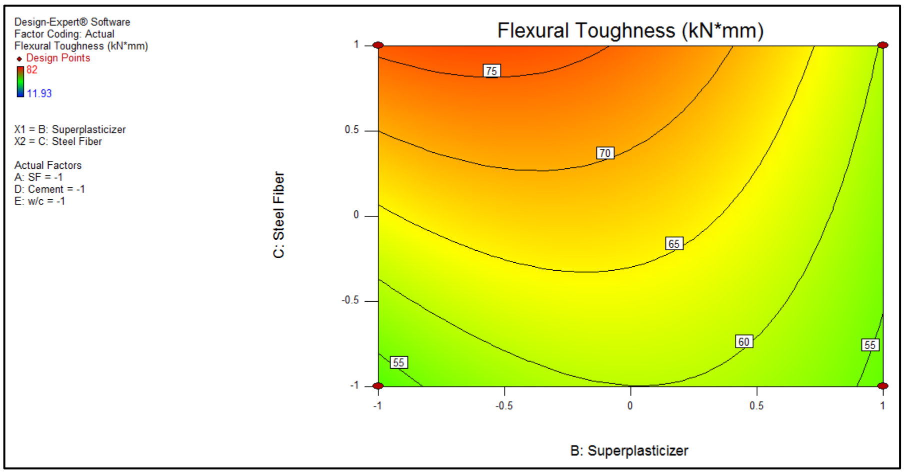

Fig. 6 Contour plot of flexural toughness changes, X1=superplasticizer amount and X2=steel fiber

were eliminated and the final regressions for each were performed. Consequently, the quadratic model was selected for response. The quality of prediction models were determined by coefficient of multiple determination R^2, which shows the total deviation of the variables from the prediction model. The p-value (probability of errors) with 95% confidence level and statistical significant test at 5% and also lack of fit with p-value greater than 0.05 was performed for model validations.

Table 4 shows, the ANOVA results for the response parameters. The result illustrate that the model was significant at the 5% confidence level because the P values was less than 0.05. Furthermore, the large p-value of 0.59 for lack of fit (>0.05) of response demonstrates that the F-

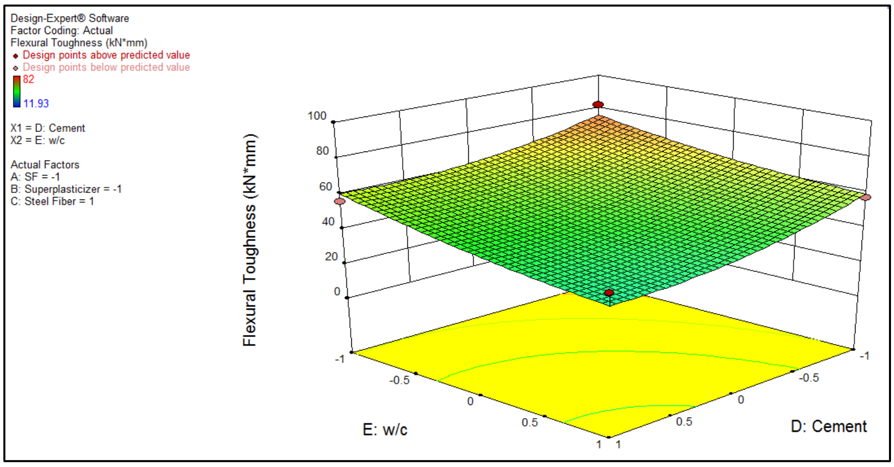

Fig. 7 Response surface plot of X1 = Cement amount, X2 = w/c, SF = -1, Superplasticizer = -1 and steel fiber= 1 on flexural toughness

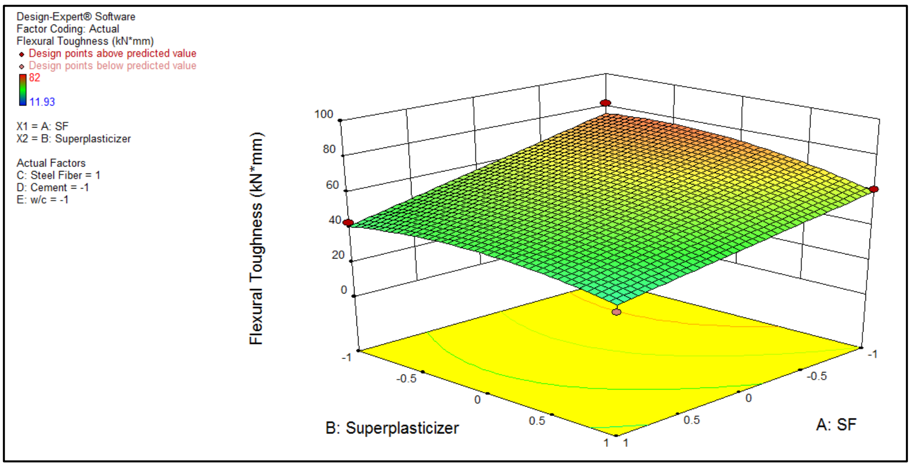

Fig. 8 Response surface plot of X1 = SF, X2 = Superplasticizer amount, Steel fiber = 1, Cement amount = -1 and w/c = -1 on flexural toughness.

value was not significant, implying significant model correlation between the variables and process response. The model coefficient of determination R^2 has a reliable confidence with 0.85.

The predicted R^2 of 0.63 in reasonable agreement with adjust R^2 of 0.78, whereas, the difference is less than 0.2.

Fig. 4 shows the normal plot of the residual value of flexural toughness, which was used to help determine the model satisfactoriness. Based on the adequacy of the model, the residuals from the least square fit were important, as shown in Fig. 4. The constructed plot of the studentized residual versus the normal percentage of probability was satisfied because flexural toughness residual plot agreed well with the straight line, as shown in Fig. 3. Consequently, it could be mentioned that this model is enough reliable.

Table 5 listed the finalized prediction model to reach the desired performance of flexural toughness of UHPC. Probability factor is given for each parameter, in Table 5, it is clear that linear

B with a high *P*-value are not statistically significant factors at the stipulated level of 5%. Moreover, linear A , B, C and E are statistically significant factors Table 5. The significant of some two-way interaction terms are given in Table 5. A significant two-way interactions explain that the simple effect of a variable is not the same at all levels of other variables. The 2-ways interaction of A with B, C, D (AB, AC, AD), B with C (BC), and D with E (DE) are statistically significant factors at the stipulated level of 10% for Flexural strength. The quadratic value for A, D, E are significantly important.

4. Discussion

The effect of five concrete mix design parameters (amount of silica fume, amount of superplasticizer, amount of cement, amount of steel fiber, and w/c ratio) on flexural toughness has been considered employing response surface methodology. The effects of variables on response can be presented graphically by 3D plotting of response value versus variables.

Fig. 5 and Fig. 6 show the contours effect of SF and steel fiber amount and also effect of SP amount and steel fibers at fixed variables on flexural toughness, respectively. As it is clearly shown, increasing rate of silica fume from 0.15 to 0.3 of fine aggregate mass decreases the flexural toughness where an the amount of SF was changed from 15% to 43% (by weight of cement). Effect of steel fiber with SF is given in Fig. 8 that shows that the effect of steel fiber is very significant for increasing the energy absorption (Pyo *et al.* 2015a, b). Effect of SP is variable, by increasing the SP amount the toughness is increased until it reaches to a code of around 0, then, the toughness which is going downward. Alsadey (2012) reported that this phenomenon happen since over dosage of SP will cause segregation and bleeding, which is effectting the uniformity and cohessiveness of the UHPC mixture. Therefore, flexural toughness will reduce if the used dosage is beyond the optimum dosage. Consequently, the energy absorption will be decreased by separation of concrete particle and the concentration of fiber and aggregate in one side of beam.

The combination effects of Cement and SF ratio is given in Fig. 7. Decreasing the w/c ratio and amount of silica fume and cement causes an increase the energy absorption significantely (Wille and Boisvert-Cotulio 2015). The effect of SF with superplsticizer on toughness are inversly corrolated which is shown in Fig. 8. The effect of only superplasticizer is not very significant on toughness as shown in Figs. 7 and 8.

5. Conclusions

The effects of five independent variables (amount of silica fume, amount of steel fibers, amount of cement, amount of superplasticizer, and w/c ratio) on flexural of UHPC with local materials were investigated by using central composition response surface methodology) and the quadratic model for responses was performed. In this experimental study, the interaction and correlation of five variables were analyzed. The validity and significance of the model and factors were analyzed by ANOVA. A total of 45 batches were used for the production of the model. The most important findings of the study are as given below

• Quadratic model with R^2 of above 0.85 was obtained for flexural toughness.

• Increasing amount of cement paste will not necessarily increase the energy absorption of UHPC.

• The increase in silica fume content (15% to 43% of cement mass) had negative effect of flexural toughness.

• The effect of w/c (0.18 to 0.32) on flexural toughness was the most highlighted effect on flexural toughness.

• Increasing the fiber (10% to 20% of aggregate mass) increased the energy absorption.

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