Computers and Concrete, *Vol. 17, No. 1 (2016) 141-156*
DOI: http://dx.doi.org/10.12989/cac.2016.17.1.141

Topological optimization procedure considering nonlinear material behavior for reinforced concrete designs

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(Received December 23, 2014, Revised January 16, 2016, Accepted January 20, 2016)

Abstract. The search for new structural systems capable of associating performance and safety requires deeper knowledge regarding the mechanical behavior of structures subject to different loading conditions. The Strut-and-Tie Model is commonly used to structurally designing some reinforced concrete elements and for the regions where geometrical modifications and stress concentrations are observed, called “regions D”. This method allows a better structural behavior representation for strength mechanisms in the concrete structures. Nonetheless, the topological model choice depends on the designer’s experience regarding compatibility between internal flux of loads, geometry and boundary/initial conditions. Thus, there is some difficulty in its applications, once the model conception presents some uncertainty. In this context, the present work aims to apply the Strut-and-Tie Model to nonlinear structural elements together with a topological optimization method. The topological optimization method adopted considers the progressive stiffness reduction of finite elements with low stress values. The analyses performed could help the structural designer to better understand structural conceptions, guaranteeing the safety and the reliability in the solution of complex problems involving structural concrete.

Keywords: reinforced concrete; strut-and-tie; Abaqus; FEM; topological optimization

1. Introduction

Nowadays, structural engineering commonly designs concrete linear elements in a simplified way, by adopting the Bernoulli-Euler hypothesis. However, it is important to note that this

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http://www.techno-press.org/?journal=cac&subpage=8 ISSN: 1598-8198 (Print), 1598-818X (Online)

hypothesis is valid for regions where the influence of strains due to shear effort is negligible. The application of this hypothesis to the entire structural element could lead to overestimate or to underestimate stress for some regions of the structure. Such regions with discontinuity are called "Regions D" in the literature therein shear stresses are significant and the distribution of strains in the cross sections is not linear. Therefore, for some of the assumptions used in Bernoulli-Euler bending procedure may not appropriately represent the stress distributions, or the structural behavior of the part.

The Strut-and-Tie Model is a simple method used to perform more realistic analyzes concerning the physical behavior of the regions of discontinuities. However, the designer needs great experience regarding the choice and distribution of the elements of the model to better represent the stress flow and the actual behavior of the discontinuous region. To guarantee that the model is reliable, it is necessary to use tools that automate and simplify the process.

The present work aims to contribute to the research about Strut-and-Tie Models applicable to concrete structures with nonlinear behavior using topology optimization techniques.

2. Constitutive model of concrete rupture

According to Chen and Han (1988), the existing models of rupture may be classified according to the number of material constants appearing in terms of the criteria for failure. Some researchers adopt the criterion of Drucker-Prager as a way to represent the brittle behavior of concrete, which presents two material constants, as can be seen in Eq. (1).

$$f(I_1, J_2) = \alpha\, I_1 + \sqrt{J_2} - k = 0 \tag{1}$$

where
k and α=constants of material. The term I_1 is the first invariant of the stress tensor and the term J_2 is the second principal invariant of the stress deviator tensor.

According to Chen and Han (1988), the Drucker-Prager criterion was established in 1952, has a simpler rupture surface and dependent on hydrostatic pressure. This criterion is considered a modification of the von Mises criterion by the introduction of an element of hydrostatic stress at fracture. Eq. (2) presented below corresponds to the von Mises rupture surface writing due to invariant stress.

$$f(J_2) = J_2 - k^2 = 0 \tag{2}$$

where
k = yield stress in pure shear.
And J_2=1/6·[$(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2$]

The Fig. 1 shows the three-dimensional view for the rupture surface of both criteria.

The Drucker-Prager criterion, however, according to Chen and Han (1988) presents two basic problems associated with the concrete modeling. The first problem is the linear relationship between the octahedral stress and the octahedral shear stress. The second problem is the independence in relation to the angle of similarity. The rupture surface shows to be more regular, as can be seen in Fig. 1(b); thus, there would be no consistency with the physical behavior of the concrete.

According to Chen and Han (1988), sections of the concrete curves, in stress deviator plane,

(a) von Mises criterion (b) Drucker-Prager criterion

Fig. 1 Three-dimensional view for the rupture surface

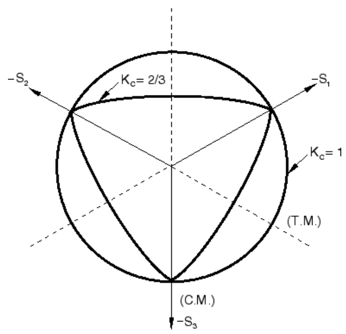


Fig. 2 Corresponding values for the parameter K_c (Abaqus® 2010)

show changes in the shape of its traces, being almost triangular for the tensile regions and small compressive stresses. Moreover, the concrete curves have a convex shape, approximated by circular geometries for high values of compressions stress, i.e., high hydrostatic pressures.

The present work uses an Evolutionary Topological Optimization script run on Abaqus® software. The adopted constitutive model available in the software, and used for analyzing some elements in reinforced concrete by Finite Element Method (FEM), is called Concrete Damaged Plasticity (CDP). This model is a modification of the Drucker-Prager criterion based on the considerations made according to Lubliner *et al*. (1989) and Lee and Fenves (1998).

The CDP model takes into account the degradation of the elastic stiffness induced by plastic deformation, both for tensile stresses and for compressive stresses.

According to Kmiecik and Kamiński (2011), the modification with respect to the cross section of the failure surface in the anti-spherical plane presents a circular shape and the model can be described by a parameter called K_c, as shown in Fig. 2. For the classical Drucker-Prager criterion, K_c=1. Thus, the CDP model adopted allows obtaining a good representation of the rupture behavior of concrete, consistent with the behavior obtained from experimental tests available in the literature.

3. Strut-and-Tie Models

Ritter and Mörsch were the first researchers to conduct studies that present the model called "Strut-and-Tie Model" for concrete design, in the early twentieth century, by conducting an analogy of the truss model for beams.

This analogy associates the stress distribution in a reinforced concrete beam to an equivalent truss structure, where the discrete elements (bars) represent the fields of tensile stresses (called ties) and compression (called strut), both jointed by nodes.

In structural engineering, the linear elements in concrete are generally designed in a simplified way adopting the Bernoulli-Euler hypothesis. This facilitates the design, because for all the load steps, the distribution of the strains is considered linear in the cross sections. In regions where there are no interferences, where the influence of deformations due to shear is negligible, this hypothesis is taken into consideration. These are called “Regions B”.

In structural elements presenting special regions such as openings in beams, beam-column connections between, corbels, foundation blocks and geometric discontinuities, among others, the Bernoulli-Euler hypothesis does not represent the distributions stress or the structural behavior of the region considered.

The application of simple structural elements for any region may lead to oversized or undersized parts of the structure. These regions present discontinuities, “Regions D”. In “Regions D”, shear stresses are significant and the distribution of strains in the cross section is not linear.

The Strut-and-Tie Model is adopted to make a better representation of the structural behavior and resistant mechanisms of these structural elements and regions where there are geometric changes and stress concentrations.

The main recommendations existing for the use of Strut-and-Tie Models in elements with discontinuities are prescribed in technical standards such as the CEB-FIP Model Code (2010), CSA-A23.3-04 (2004), EHE (2008) and ACI-318 (1995, 2005), EUROCODE 2 (2002)

There is a dependence on regions “B” and “D” with respect to their distances of support and points of application of concentrated loads on the element. This dependence can be explained by the “Principle of Saint Venant”.

According to Schlaich *et al.* (1987), the “Principle of Saint Venant” establishing that the stress trajectories are quite smooth in regions called “B-regions” as compared to their turbulent pattern near discontinuities. The stresses decrease rapidly with the distance from the origin point of the stress concentration. This behavior is considered to identify the “B regions” and “D regions” in a structure. The Fig. 3 shows examples of the identification elements with their respective “B regions” and “D regions”.

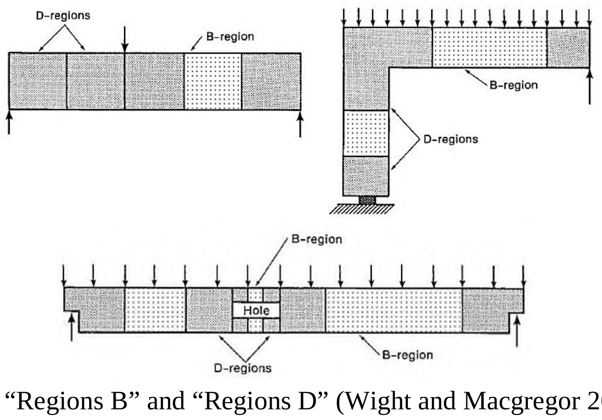


Fig. 3 “Regions B” and “Regions D” (Wight and Macgregor 2012)

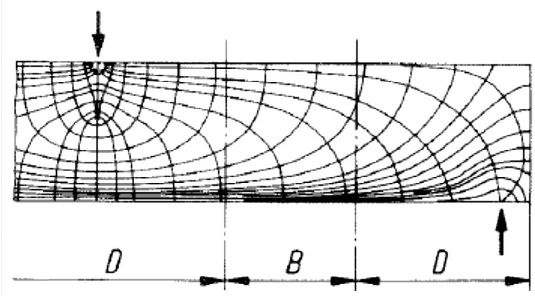


Fig. 4 Stress paths and regions B and D (Schlaich *et al.* 1987)

According Schlaich *et al.* (1987), from the stress trajectories, it is also possible to identify the difference between "Regions B", in which Hooke's Law applies, and "Regions D" discontinuity, given that near the concentrated load distributions, the stress fields are more complex, as shown in Fig. 4.

Garber *et al.* (2014), however, conducted studies involving wall beams containing regions considered discontinuous and analyze the flow of tensions in specimens tested experimentally and in their results, the rupture load exceeds the nominal resistance project, demonstrating the conservatism of the Strut-and-Tie Model and the dependence on stress concentrations presented in the elastic analysis. But modified Strut-and-Tie Models that consider material nonlinearity, such as the works published by Chetchotisak *et al*. (2014), Shah *et al.* (2011), are still rare in the literature.

3.1 Model definition

The main aspects to be considered for defining the Strut-and-Tie Model geometry refer to the types of actions in the element, the angle formed by the connecting strut-and-tie and boundary conditions. Moreover, the layers number of reinforcement and their coverings are also important.

The literature has indicated some criteria commonly adopted for choosing the Strut-and-Tie Model, among which it is possible to mention the prescribed standards codes for the criterion adopted by the path of the loads process ("Load Path Approach"), from elastic analysis by FEM using nonlinear analysis with consideration of concrete cracking, through experimental tests and by choosing an automation model.

Criteria used in analysis by FEM are considered herein, through nonlinear analysis taking into account the concrete damage and the choice of automation model by adopting a technique of Evolutionary Topology Optimization.

According to Schäfer and Schlaich (1991), in the case of elastic analysis, the direction of the struts is determined by considering the average direction of the principal compressive stress. Alternatively, the Strut-and-Tie Models are determined from the gravity center of the stress diagram in the typical sections structural element.

Considering the direction of failures in the cracked concrete element, since the flow of tensile stresses is parallel to the flow of compressive stresses, it is also possible to define the paths in the model.

Shah *et al.* (2011) reported a study comparing the theoretical and experimental ruptures loads considered in structural elements where it is possible to determine the arrangement of struts and ties from the cracked regions.

In a recent study, Najafian and Vollum (2013) point out the need to consider the nonlinearity of the material for defining the Strut-and-Tie Model. The distribution of stresses for linear analysis shows a significant difference compared with the nonlinear analysis by FEM.

Finally, it points to the importance of the choice of the model due to the lack of standardization of the ideal method for choosing the proper arrangement of the Strut-and-Tie in structural concrete components. The goal is to assist in the determination of this configuration, which provides higher safety in the design and in simplifying the completion of the structural design for special elements.

Analyses are conducted herein taking into account the technical ESO (Evolutionary Structural Optimization) topology optimization to automate the chosen Strut-and-Tie Model.

4. Topological optimization

During the 1980s, with the emergence of software for structural analyses, several studies were developed involving the structural optimization with the aid of the FEM. Important works such as Cheng and Olhoff (1982), Rozvany *et al.* (1982) and Kohon and Strang (1986) can be mentioned. In the same decade, the first studies related to topology optimization emerged, due to the need to solve problems with updated finite element meshes.

Emerging concepts related to different kinds of optimization arose, classified as Parametric Optimization, Shape Optimization and Topology Optimization.

Bendsøe and Kikuchi (1988) conducted one of the first studies to improve the method of shape optimization allowing topologies and obtaining optimal shapes of structures. In later years, several studies were conducted to combine the methods of topological, parametric and shape optimization. The study by Olhoff *et al.* (1991) can be mentioned. Subsequently, there was an extension of studies related to topological optimization and several methods have been developed.

The present study adopts a technique of evolutionary optimization called ESO (Evolutionary Structural Optimization). The ESO is a technique in which the shape and topology are determined simultaneously and has originated in a procedure called “Hard-Kill” which is the permanent removal of elements that do not meet the criteria for rejection. That is, the ESO technique is a heuristic (approximated) optimization process for elements removal, made gradually and iteratively considering predefined rejection criteria. It is a simple technique for structural topology modification. Through FEM, the finite element mesh corresponding to regions that do not effectively contribute to the good performance of the structure is gradually removed. The purpose

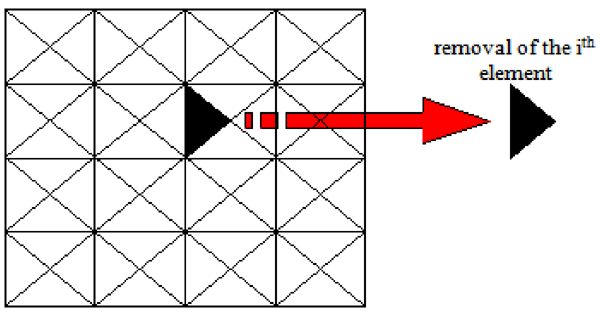


Fig. 5 Removal of the mesh element by the evolutionary optimization method (adapted from Almeida *et al.* 2013b)

of the procedure is to find the best distribution of material in a fixed design domain satisfying the imposed restrictions.

The finite elements stress values are considered as a criterion of optimization, i.e., elements with lower values of stress throughout the structure will be gradually selected and removed from the mesh during the evolutionary optimization steps, from the rejection criterion adopted. Fig. 5, presented in Almeida *et al*. (2013b), illustrates the removal of the element by the removal criterion adopted.

This removal criterion, originally proposed by Xie and Steven in 1993, is identical to that followed in Lanes and Greco (2013) and is described by Inequality 3 below

$$\sigma_e^{vM} < RR_i \cdot \sigma_{MAX}^{vM} \tag{3}$$

where

σ_e^{vM} = von Mises stress of the analyzed element;

RRi= Rejection Ratio adopted to slow down the removal process

σ_{MAX}^{vM} = Maximum von Mises stress of the iteration.

Therefore, the cycle of removing elements can be said to occur until it is not possible to remove any more elements in a given value of RRi. When that level of element balance removal is reached, without the optimal configuration being obtained, however, the evolutionary process is redefined by adding an Evolutionary Ratio (ER) to the RRi. A new evolutionary cycle begins until there are no more elements to be eliminated with this new rejection ratio. Reaching the state of element balance removal, but not reaching the stopping criterion of the iterative process (optimal configuration by ESO method), the evolutionary process is restarted by adding an evolution ratio ER to the RRi. Eq. (4) describes this process.

$$RR_{i+1} = RR_i + ER \qquad i = 0, 1, 2, 3, ... \tag{4}$$

The RR is updated to obtain an optimized configuration.

It is worth highlighting that the von Mises stress is adopted only as a material removal criterion in the evolutionary process. During the analysis of the nonlinear constitutive model, the material model CDP is considered. It was deemed appropriate to adopt the equivalent von Mises stress as a criterion for removal, since both models, von Mises and CDP (rupture criterion considered for analysis behavior nonlinear of concrete available at Abaqus® software), in their equations have in common a magnitude, the second stress invariant (J_2) in plan deviator tensions.

The Abaqus® software provides an object-oriented platform that enables the development of routines to automate specific operations or interventions in the data output programming environment, according to the user's need.

This feature is called Abaqus® Scripting and its main advantage is the possibility of developing scripts using a language that has an open source code, i.e., Python programming language.

Lanes and Greco (2013) implemented an algorithm considering the ESO topology optimization method for implementation in Abaqus® software. This algorithm enabled the development of a script used in this scientific study as an aid to automate the strut-and-tie method for topology optimization in the present work.

The evolutionary process is summarized by the following steps

• Step 1: discretization of the initial domain structure, using a fine mesh of finite elements, and applying the boundary conditions and prescribed actions;

• Step 2: analyze the structure finite element (both for elastic and nonlinear behaviors);
• Step 3: remove the elements that satisfy Inequality (3);
• Step 4: increase the rejection rate according to Eq. (4) until the equilibrium is reached, otherwise repeat steps 2 and 3;
• Step 5: repeat steps 2-4 until the optimal design is achieved.

For each kind of analysis, the equation system related to the specific problem must be solved at Step 2. For nonlinear (physical or geometrical) problems, an appropriate algorithm must be used.

After identifying a region considered inefficient, Lanes and Greco (2013) propose that the mechanical properties of this region are modified to a section or a material with negligible structural characteristics when compared with its initial mechanical properties.

Thus, the script requires the user to set such physical characteristics (such as low modulus of elasticity and density, among others) for the structural deactivation of the elements of the domain.

Another recent study is being conducted by Zhang *et al.* (2014). This paper also discusses the automation to obtain the Strut-and-Tie Model through topology optimization. However, a probabilistic optimization technique is adopted, called GESO, which would be a probabilistic technical modification of the ESO. The nonlinearity of the material is taken into account and the results point to the need not to exclude this hypothesis in the analysis. The analysis includes the nonlinearity of the material indicating the need of a smaller amount of steel in the structure, when compared to the linear analysis.

Results and conclusions of the analysis performed in this study adopting the ESO topology optimization method are presented in the following section.

5. Numerical examples

In order to validate the proposed method for automating the choice of the topology Strut-and-Tie Model from the application of the ESO method optimization topology, comparative analyses will be performed in the next items with examples available in the literature.

5.1 Example1 - deep beam with hole

The first structural element analyzed is a simply supported beam with an opening. Its geometry, applied load and boundary conditions are shown in Fig. 6.

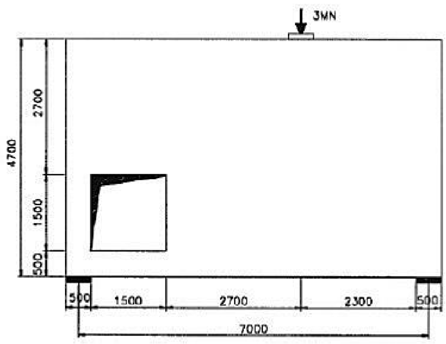
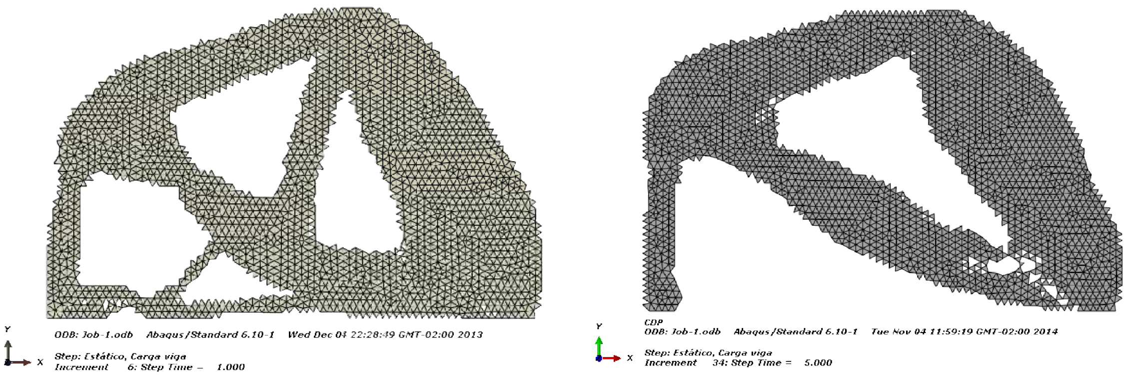


Fig. 6 Example 1 - Simply supported beam with opening (Adapted from Schlaich *et al.* 1987)



(a) Solution considering the elastic linear behavior of the material

(b) Solution considering the nonlinear behavior of the material

Fig. 7 Optimal topologies obtained

The material properties adopted were the same as presented in Almeida *et al.* (2013b); Young's modulus equal to E=20820 MPa and Poisson ratio equal to v=0.15.

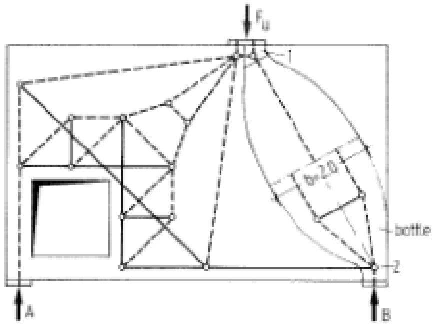
For modeling the element in Abaqus® software, a finite element mesh of the simple triangular type CPS3 (Continuum / Plane-Stress / 3 Node Element) was considered. The structure was represented by 6693 elements and 3499 nodes.

The parameters used for optimization via ESO were: Rejection Rate (RR)=4.0% and Evolutionary Rate (ER)=2.0%. The optimal topology is obtained after analyzing the problem through a routine developed in Lanes and Greco (2013), to a volume of approximately 50% of the initial volume.

The solutions considering the elastic and nonlinear behavior of the material are presented in Figs. 7(a)-(b).

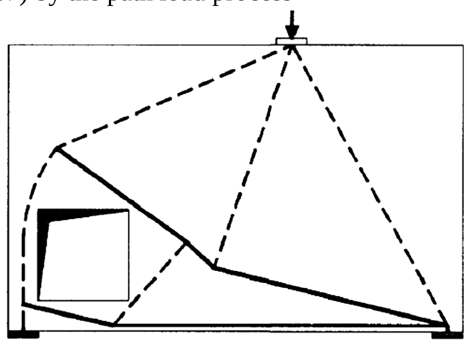
In order to validate the application of the implemented algorithm, for the purpose proposed herein, Fig. 8(a) presents the linear Strut-and-Tie Model obtained for the same structure in the original work, Schlaich *et al.* (1987), who adopted the criterion of the load path for designing the Strut-and-Tie Model. The optimal topologies in Figs. 8(b)-(e) were obtained in other studies, developed with the same purpose of automating the design of the linear Strut-and-Tie Model by of different methods of topology optimization. In Fig. 8(d) red elements are in regions of compression and grey elements are in regions of tension. In Fig. 8(e), dashed lines represent struts elements of compression and continuum lines represent tie elements of tension. Fig. 8(f) presents the proposed Strut-and-Tie Model and Fig. 8(g) presents the proposed reinforced disposition. The angles shown in Fig. 8(f) are equal to 44°, 62° and 23° for nonlinear analysis. For linear analysis the respective angles are equal to 45°, 70° and 15°, as presented in Fig. 8(e). For the nonlinear analysis, the engineering problem demands a minimum armature at the inferior horizontal tensioned region related to the tie T1. Proposed reinforcements areas for nonlinear analysis are equal to 2×2 ϕ 20 mm (T1), 2×7 ϕ 20 mm (T2), 2×2 ϕ 20 mm (T3) and 2×7 ϕ 20 mm (T4). Proposed reinforcements areas for linear analysis are equal to 2×5 ϕ 20 mm (T1), 2×7 ϕ 20 mm (T2), 2×2 ϕ 20 mm (T3) and 2×5 ϕ 20 mm (T4).

Comparing the solutions presented in Figs. 7(a)-(b) and Figs. 8(a)-(e), the results from applying the routine by ESO technique are verified to be consistent with those in the literature. Results



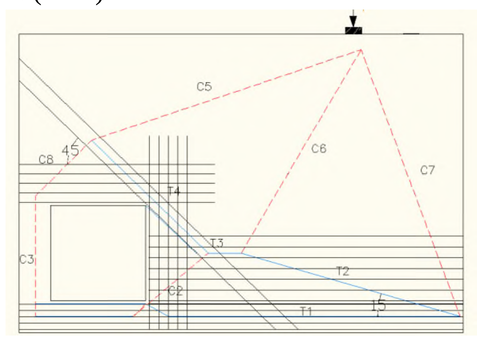
(a) obtained in Schlaich *et al.* (1987) by the path load process

(b) Optimal topology obtained in Liang *et al.* (2000) by the ESO optimization method

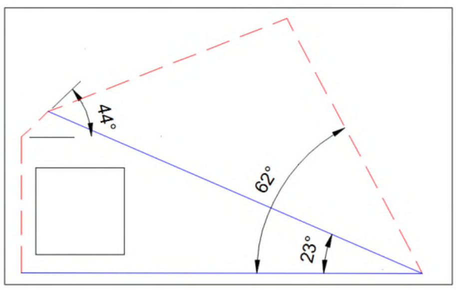


(c) Strut-and-tie Model obtained in Liang *et al.* (2000)

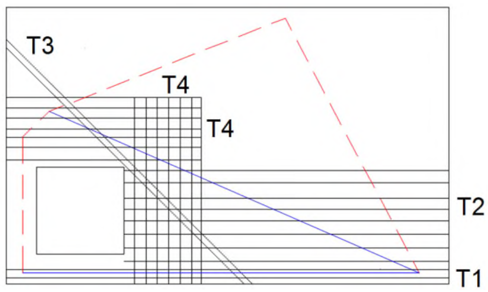
(d) Optimal topology obtained in Almeida *et al.* (2013b) by the SESO optimization method



(e) Strut-and-tie Model and proposed disposition of reinforcement in Almeida *et al.* (2013b)

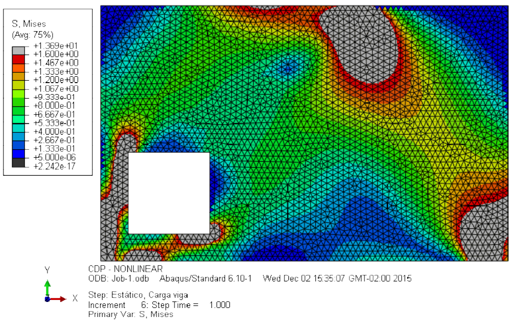


(f) Proposed strut-and tie models for nonlinear behavior

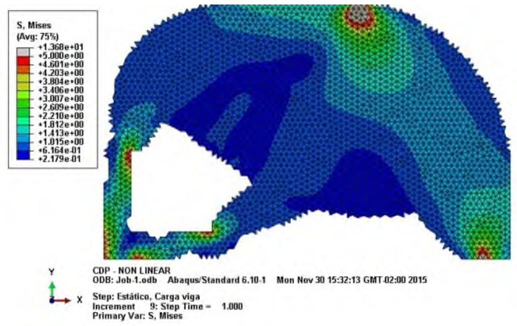


(g) Proposed reinforcement disposition

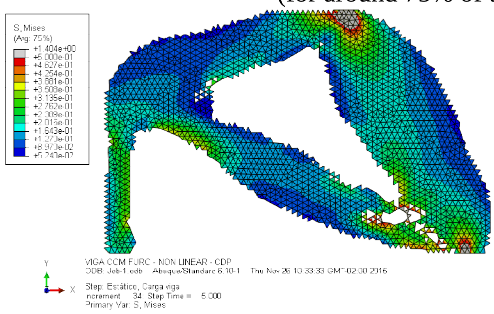
Fig. 8 Solutions presented in the literature



(a) Solution before topological optimization considering nonlinear behavior of the material



(b) Solution after topological optimization considering nonlinear behavior of the material (for around 75% of the initial volume)



(c) Solution after topological optimization considering nonlinear behavior of the material (for around 50% of the initial volume)

Fig. 9 Stress distributions for the nonlinear deep beam

considering the linear elastic behavior and nonlinear material behavior showed considerable differences in their optimal topologies. The stress distributions considering the nonlinear behavior of the structural system are shown in Fig. 9, before and after topology optimization. Just considering the initial stress distribution is not sufficient to characterize the most suitable Strut-and-Tie Model to be used.

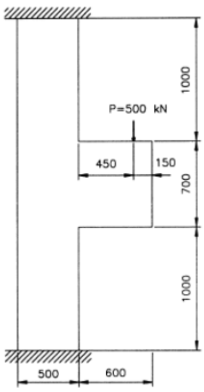
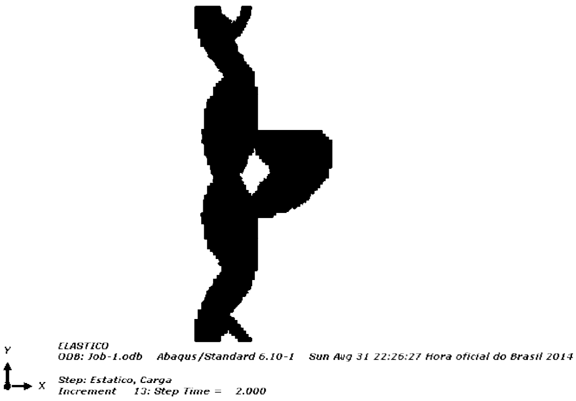
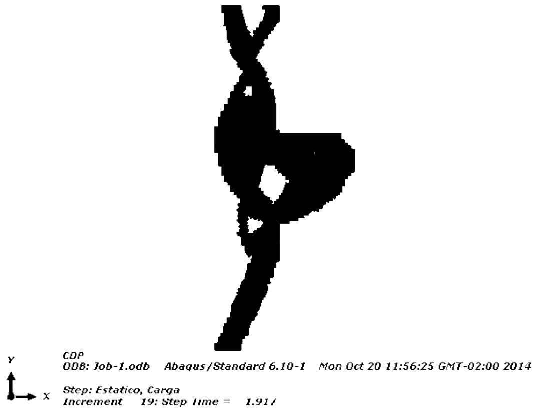


Fig. 10 Corbel with a column (Adapted from Liang *et al.* 2000)



(a) Solution considering the elastic linear behavior of the material



(b) Solution considering nonlinear material behavior

Fig. 11 Optimal topologies obtained

5.2 Example 2 - corbel jointed with column

The second structural element analyzed is a corbel jointed with a column taking the structure as a whole. Its geometry, applied load and boundary conditions are shown in Fig. 10.

The material properties adopted were the same as presented in Liang *et al.* (2000), Young's modulus equal to E=28567 MPa and Poisson ratio equal to ν=0.15.

For modeling the element in Abaqus® software, a finite element mesh linear quadrilateral type CPS4R (Continuum / Plane-Stress / Shell elements / 4 Node Element) was considered. The structure was represented by 3317 elements and 3470 nodes.

The parameters used for optimization via ESO were: Rejection Rate (RR)=4.0% and Evolutionary Rate (ER)=2.0%. The optimal topology is obtained after analyzing the problem through a routine developed in Lanes and Greco (2013), to a volume of approximately 50% of the initial volume.

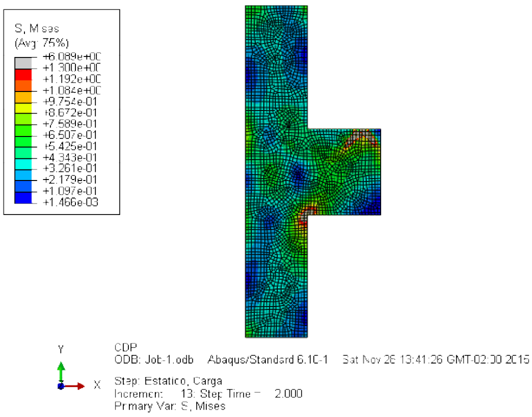
The solutions considering the elastic linear behavior and nonlinear behavior of the material are presented in Figs. 11(a)-(b).

The optimal topology shown in Fig. 12(a) was obtained in another study developed with the same purpose of automating the design of linear Strut-and-Tie Model, also by a method of evolutionary topology optimization (ESO). And Fig. 12(b) shows the elastic linear solution obtained by Almeida *et al*. (2013a) from a variation in the ESO technique called SESO.

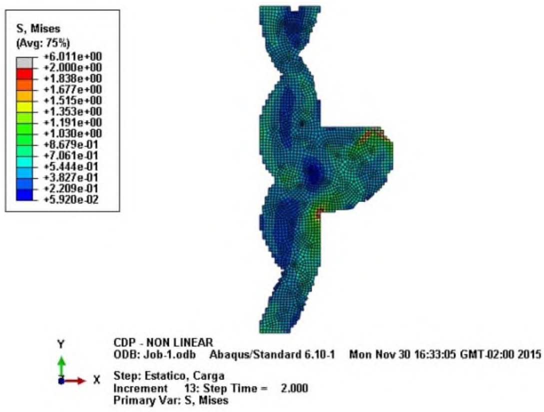
For this example, comparing the solutions presented in Figs. 11(a)-(b) and Fig. 12(a)-(b), the results from applying the routine via ESO were observed to be consistent with those obtained in the literature. In Fig. 12(b), red elements are in regions of compression and blue elements are in regions of tension. The results considering the elastic linear and the nonlinear material behavior also showed considerable differences in their optimal topologies. The stress distributions considering the nonlinear behavior of the structural system are shown in Fig. 13, before and after topology optimization. As occurred for the first numerical example, just considering the initial stress distribution is not sufficient to characterize the most suitable Strut-and-Tie Model to be used.

(a) Solution presented in Liang *et al.* (2000) (b) Solution presented in Almeida *et al.* (2013a)

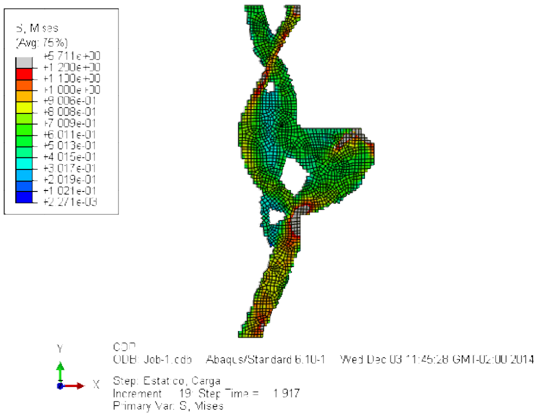
Fig. 12 Solutions presented in the literature



(a) Solution before topological optimization considering nonlinear behavior of the material



(b) Solution after topological optimization considering nonlinear behavior of the material (for around 75% of the initial volume)



(c) Solution after topological optimization considering nonlinear behavior of the material (for around 50% of the initial volume)

Fig. 13 Stress distributions for corbel jointed with column

For each optimization cycle, the Concrete Damaged Plasticity (CDP) evaluates the structural stiffness and the element removal criterion is activated. The removed elements are excluded from the analysis domain and no longer contribute to the global structural stiffness (blue regions in Figs. 9 and 13). The equivalent von Mises stress distributions presented in Figs. 9 and 13 do not allow the direct identification of compressive regions. To identify the compressive regions to be used in the Strut-and-Tie Model, an analysis of the stress flux must be performed. The software Abaqus® features these distributions. An application of the linear ESO, used to obtain the Strut-and-Tie Model of a "T" bridge pier, can be found in Liang *et al*. (2002).

6. Conclusions

Considering the numerical simulations, it is possible to say that the solutions obtained for the purpose of this study are relevant for structural designers, since they can provide better understanding of the strut-and-tie functioning of the method, improving the design of the model and the resolution of complex problems involving special structures with nonlinear behavior.

The evolutionary optimization method (ESO) used to develop the Strut-and-Tie Model for reinforced concrete structures has not been used yet for this purpose, considering the nonlinear constitutive model that represents material damage. Previous works represent material behavior as elastic linear. As far as the authors know, there is no previous work employing the used optimization technique with nonlinear material behavior to obtain the Strut-and-Tie Model. To design concrete behavior in Stage II (after the main tension stress reaches the strength of concrete and crack propagation is developed), it is necessary to consider a suitable nonlinear constitutive model that can represent the design domain.

Concerning the comparative analysis between the results for linear and nonlinear material behavior, it is possible to say that the solutions showed considerable differences in optimal topology in both examples discussed.

Finally, it is emphasized that although there are many algorithms available in some commercial software, the topology optimization is very dependent on the parameters adopted and the research about the topic enables applications and to choose optimization parameters more consistently. Moreover, even for simple constitutive models, as the elastoplastic model, there are few works addressing the structural optimization problem considering the nonlinear behavior available.

Acknowledgments

The authors acknowledge the financial support granted by CNPq (National Council for Scientific and Technological Development), CAPES (Coordination for the Improvement of Higher Education Personnel), FAPEMIG (Foundation for Research Support of Minas Gerais State) and Federal University of Minas Gerais (UFMG). The authors would also thank to the criterious revision work provided by the reviewers that have improved the quality of the paper.

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