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Reliability-based modeling of punching shear capacity of FRP-reinforced two-way slabs

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Abstract. This paper deals with the reliability analysis of design formulations derived for predicting the punching shear capacity of FRP-reinforced two-way slabs. Firstly, a new design code formulation was derived by means of gene expression programming. This formulation differs from the existing ones as the slab length (L) was introduced in the equation. Next, the proposed formulation was tested for its generalization capability by a parametric study. Then, the stochastic analyses of derived and existing formulations were performed by Monte Carlo simulation. Finally, the reliability analyses of these equations were carried out based on the results of stochastic analysis and the ultimate state function of ASCE-7 and ACI-318 (2011). The results indicate that the prediction performance of new formulation is significantly higher as compared to available design equations and its reliability index is within acceptable limits.

Keywords: reliability; fiber reinforced polymer; RC slab; punching shear; structural safety; Monte Carlo simulation

1. Introduction

The design of engineering structures after 1970 has turned into a new direction regarding reliability analysis and its concept developed by (Ang and Cornell 1974). The fundamental concept states that the uncertainties of material and geometry parameters should be considered in the design where reliability analysis is applied to RC structures together with the ultimate limit state design. Furthermore, Ellingwood and Ang (1974) studied the evaluation of structural safety in conjunction with the design procedure (Ellingwood and Ang 1974). Recently, reliability analysis has been applied successfully to various areas of structural engineering (Albegmprli *et al.* 2015,

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Gulsan *et al*. 2015).

Recent advance in data modeling technology has drawn great attention from various engineering practices. As a result, empirical data modeling is becoming more significant to researchers and engineers in practice. Thus, a process of data training is used to create a model of a system for the purpose of obtaining predictions for the cases that are yet to be observed. The model performance, therefore, is largely dependent on the quantity and the accuracy of experimental findings used for training the model.

For the construction industry, service life and durability of concrete structures are substantial issues. The corrosion risk of steel reinforcement is considered to be one of the main concerns that should be solved. Additionally, steel reinforced structures that are built, particularly, in wet environments generally require a comprehensive care. Recently, fiber-reinforced polymer (FRP) materials that are free of these problems appear to offer useful solutions as a substitute material for reinforcement. The user-friendly and nonmetallic nature, high tensile strength and low density make FRPs advantageous (ACI 440 2006).

During the past decade, it is confirmed by numerous practical applications around the globe that FRP composite reinforcing bars can be successfully used in construction industry. To use this technology in a professional manner, the suitable application of FRP reinforcement and awareness of its limits as mentioned in various resources is required (ACI 440 2006). Thus, demand for educated professionals has risen by wide-ranging use of composites in several applications (industrial, commercial, domestic, defense, medical and construction) in addition to specific literature devoted to extend the theoretical knowledge in order to provide engineering principles for structural applications (GangaRao *et al*. 2006).

This study focuses on the reliability analysis of a new punching shear capacity formulation as well as the existing formulations for FRP reinforced two-way slabs. A new design equation for punching shear capacity of FRP reinforced two-way slabs is presented and reliability analysis of this formulation is carried out.

2. FRP reinforcement in RC structures

Over the last two decades, significant improvement on the use of fiber reinforced polymer (FRP) composites was observed. Many structures are exposed to corrosion risk and deicing salts and they require high cost maintenance during their service life. Being free of these problems, the use of FRP composites has emerged as an alternative solution to be used as a replacement of steel reinforcement.

The use of FRP bars as an alternative of the traditional mild-steel reinforcement provides some main advantages such as high resistance for corrosion, high tensile strength, lightweight and low conductivity. Yet, the applications of FRP reinforcements have restrictions due to unwanted characteristics such as brittle nature, low elastic modulus, low transverse strength, low shear strength and suspicion in fire resistance. (GangaRao *et al*. 2006, Mahroug *et al*. 2014)

Elastic modulus of a FRP bar might vary depending on its constituents. Carbon FRP bars have a higher modulus of elasticity than those made with glass fibers. FRP bars made with carbon can be made with an elastic modulus that is nearly the same as that of mild steel, but this will be very costly. Commercially available FRP bars have a modulus of elasticity that is lower than that of steel. The type of fiber used, fiber-matrix ratio, adhesion, manufacturing process, etc. are important factors for the magnitude of the modulus of elasticity (Hussein *et al*. 2004).

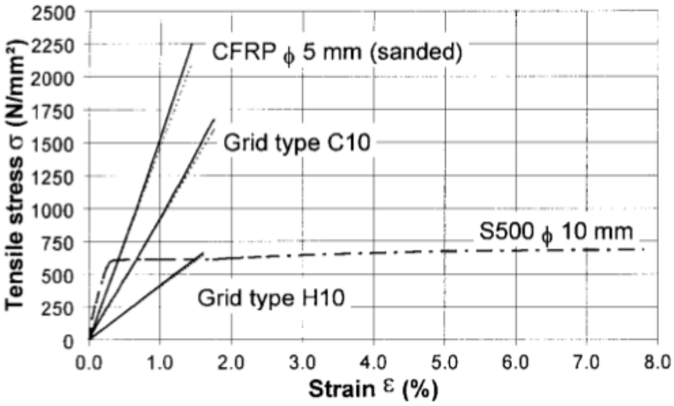


Fig. 1 Stress strain behavior of FRP bars (Matthys and Taerwe 2000)

Table 1 Literature summary on punching shear tests on FRP reinforced two-way slabs

Source	FRP bar type	Number of specimens	Remarks
Ahmed *et al*. 1994	3-D CFRP grids	4	Concentrated loading and simply supported
Banthia *et al*. 1995	FRP grids	2	Concentrated loading and simply supported
Matthys and Taerwe 2000	FRP grids (carbon and hybrid)	13	Concentrated loading and simply supported
Ospina 2003	GFRP bar and grids	3	Concentrated loading and simply supported
El-Ghandour *et al*. 2003	CFRP and GFRP	5	Loaded from 8 points and simply supported
Hussein *et al*. 2004	GFRP	4	Concentrated loading and simply supported
Zhang 2006	GFRP	1	Concentrated loading and simply supported
Lee *et al*. 2009	GFRP	3	Concentrated loading and simply supported
Nguyen-Minh and Rovňák 2012	GFRP	3	Concentrated loading and simply supported
Hassan *et al*. 2013	GFRP	15	Concentrated loading and simply supported

FRP bars show elastic behaviors up to rupture and fail in brittle nature. Typical stress-strain relations of FRP and steel reinforcement are shown in Fig. 1 to demonstrate the different properties (Hussein *et al*. 2004). FRP bars can be produced by means of various techniques such as braiding, pultrusion and weaving and are anisotropic in nature (ACI 2001).

3. FRP-reinforced two-way slabs

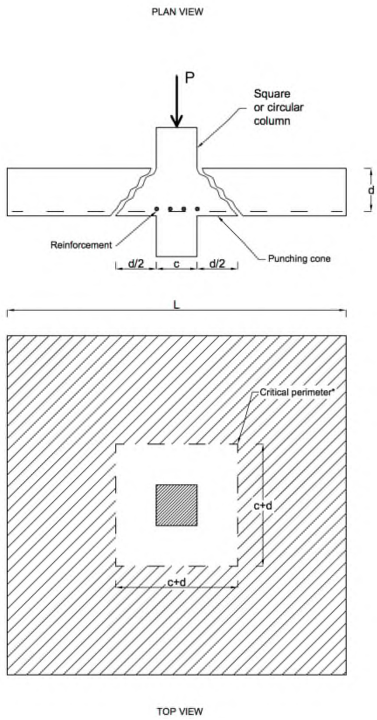


Fig. 2 Typical geometry configuration of FRP-reinforced two-way slabs

It is essential to comprehend the advantages and limitations of FRPs as well as which materials work and which shapes or forms suit best in order to successfully apply FRP reinforcement in slab construction. It has been observed that FRPs display a brittle-elastic behavior under direct tension and are much more flexible than steel in most cases. FRP reinforced concrete slabs are also likely to exhibit greater deflections and crack widths at the serviceability limit state level, owing to the low stiffness of FRP reinforcement (Ospina *et al*. 2003). Consequently, design of concrete members with FRP bars is generally controlled by serviceability limit states (Kara *et al*. 2015). Available test details on punching shear capacity of FRP-reinforced two-way slabs are summarized in Table 1. Typical geometry of FRP-reinforced two-way slabs in Table 1 is illustrated in Fig. 2.

As can be observed from Fig. 2, a punching cone is formed within the critical perimeter (*b0*) in a test setup where loading is applied from the top face of upper column. This critical perimeter is calculated at a distance of 0.5*d* (according to ACI 440) or 1.5*d* (according to BS8110) from the column face. Table 2 summarizes the design formulations by ACI440, BS8110 and some researchers (ACI440 2006, El-Ghandour *et al*. 1999, Matthys and Taerwe 2000, Ospina *et al*. 2003, Zaghloul 2003, Metwally 2013).

4. Modeling of punching shear strength

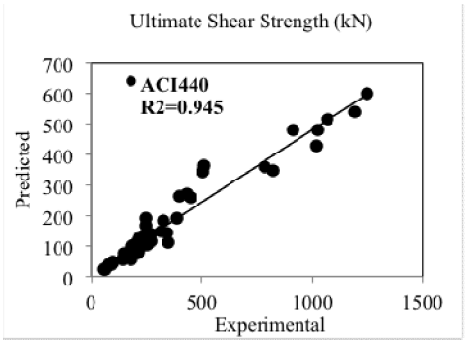
In this study, gene expression programming (GEP) software -namely as GeneXproTools- is used to derive the formulation for punching shear capacity of FRP-reinforced two-way slabs (Ferreira 2001). Genetic algorithm and gene expression programming techniques have been

Table 2 Existing formulas for estimation of punching shear resistance of FRP reinforced two-way slabs

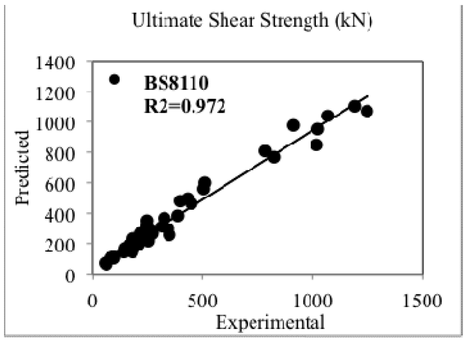
Source	Formula	Remarks
(ACI 440 2006) Eq. (1)	$P=\frac{4}{5}\sqrt{f_c}\,b_{0,0.5d}\,c$ c = kd $k=\sqrt{2\rho_f n_f+(\rho_f n_f)^2}-\rho_f n_f$	$b_{0,0.5d}=4(c+d)$ $E_s=210GPa$ *nf* is the modular ratio *ρf* is the reinforcement ratio
(British Standards Institution 1997) Eq. (2)	$P=0.79\left[100\rho(1.8)\frac{E_f}{E_s}\right]^{1/3}\left[\frac{400}{d}\right]^{1/4}\left[\frac{f_{ck}}{25}\right]^{1/3}b_{0,1.5d}\,d$	$b_{0,1.5d}=4(c+3d)$ $E_s=210GPa$
(El-Ghandour *et al*. 2003) Eq. (3)	$P=0.33\sqrt{f_c}\left[\frac{E_f}{E_s}\right]^{1/3}b_{0,0.5d}\,d$	$b_{0,0.5d}=4(c+d)$ $E_s=210GPa$
(Matthys and Taerwe 2000) Eq. (4)	$P=1.36\frac{\left[100\rho\frac{E_f}{E_S}f_{cm}\right]^{1/3}}{d^{1/4}}b_{0,1.5d}\,d$	$b_{0,1.5d}=4(c+3d)$ $E_s=210GPa$ *fcm* = mean concrete compressive strength of cylinder specimens at 28 days
(Ospina *et al*. 2003) Eq. (5)	$P=2.77(\rho f_c)^{1/3}\left[\frac{E_f}{E_s}\right]^{1/2}b_{0,1.5d}\,d$	$b_{0,1.5d}=4(c+3d)$ $E_s=210GPa$
(Zaghloul 2003) Eq. (6)	$P=0.07(\rho f_c E_f)^{1/3}b_{0,0.5d}\,d$	$b_{0,0.5d}=4(c+d)$
(Metwally 2013) Eq. (7)	$P=0.368\sqrt{f_c}\,b_0 d\alpha(1.2)^N$	$\alpha=0.62*\sqrt[3]{\rho_f}E_f\left(1+\frac{8d}{b_{0,0.5d}}\right)$ $b_{0,0.5d}=4(c+d)$ N = 0 for one span slab in both directions (simply supported)

successfully implemented to several civil engineering problems (Cevik 2007, Cevik 2007, Gomes *et al*. 2011, Sonebi and Cevik 2009, Tanyildizi and Çevik 2010, Köroğlu *et al*. 2011, Cevik *et al*. 2010). Using this software, computer programs with different sizes and shapes are evolved and coded in chromosomes of predetermined length. These linear chromosomes include several genes and each gene encodes a smaller subprogram. Additionally, the linear chromosomes allow the operation of essential genetic operators such as recombination, transposition and mutation. GEP approach has two main strengths: the first is that, since the genetic operators function at the chromosome level, a significant simplification for the formation of genetic diversity is provided. The second strength of GEP approach is that, thanks to its unique functionality, more complex programs containing several subprograms can be evolved. The following formulation has been derived using gene expression programming

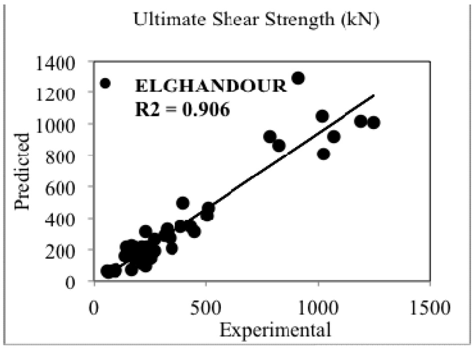
$$P=\frac{0.0862c^{0.391}d^{1.6}f_c^{0.2}E_f^{0.2}\rho^{0.354}}{L^{0.2}} \qquad \text{SI Units} \qquad (8)$$



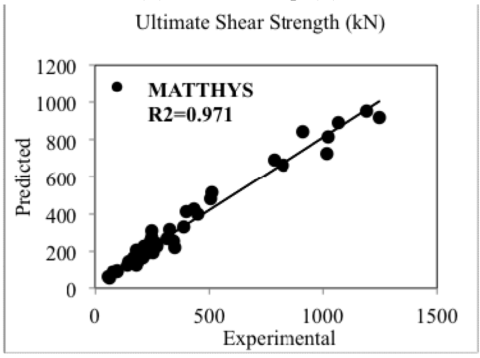
(a) ACI 440 Eq. (1)



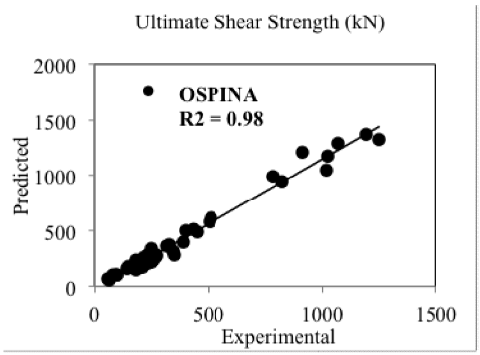
(b) BS8110 Eq. (2)



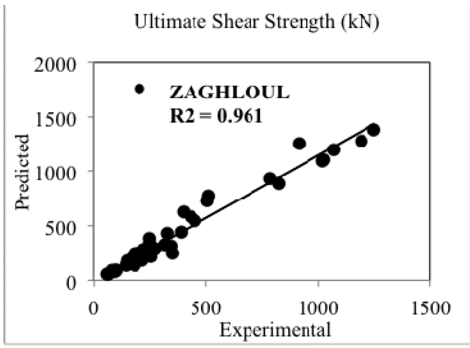
(c) El-Ghandour *et al.* Eq. (3)



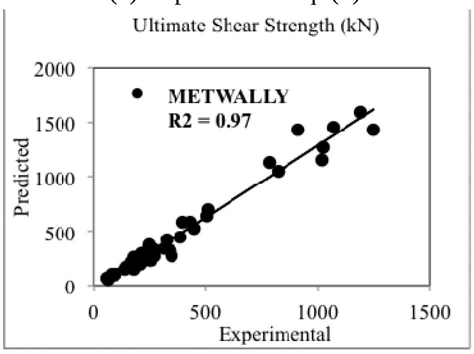
(d) Matthys *et al.* Eq. (4)



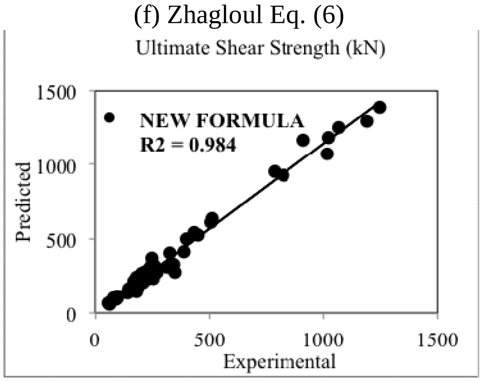
(e) Ospina *et al.* Eq. (5)



(f) Zhagloul Eq. (6)



(g) Metwally Eq. (7)



(h) New formula Eq. (8)

Fig. 3 Prediction performances of existing equations and proposed equation where P is ultimate punching load (kN), ρ is reinforcement ratio (%), f_c is compressive strength of concrete (MPa), c is dimension of column section (mm), L is span length of slab (mm), E_f (GPa) is elastic modulus of FRP bars and d is effective flexural slab depth (mm). Fig. 3 (a)-(h) illustrates the performances of Eq. (1)-(8) in predicting the ultimate punching shear capacity. There are two known limitations of Eq. (8): (i) The equation addresses only the square slabs with square columns as it also involves the span length (L) parameter; (ii) The parameters should be within the range of those provided in Table A.1.

5. Reliability analysis

As properties of most construction materials exhibit complex random variation, it is generally difficult to model the real behavior by deterministic analysis. Probabilistic models are needed to quantify the uncertainties of these properties to develop realistic representations of the output and failure state of these systems and to obtain a rational and safe design. Also, the geometries of the members have uncertain values. In this study the Monte Carlo simulation technique is used to generate the samples. Then, the reliability indices of each function are calculated with variance value of strength reduction factor.

5.1 Monte Carlo simulation

The Monte Carlo method is widely used to generate numerical results without needing to perform physical experiments. Firstly, probability distributions are derived by using the results of previous experiments. Then, numerical data samples are generated using this distribution information (Nowak and Collins 2012).

In this paper, Monte Carlo simulation was used to generate random samples of punching shear capacity. The value of punching shear capacity is calculated by deterministic models represented by the formulas. The probabilistic values for punching shear capacity are calculated by the implementation of following steps

• The values for each design parameter (material properties and geometries) were generated randomly based on statistical distribution method,

• The punching shear capacity was calculated N times from the formulas,

• Finally, the mean value and the standard deviation of punching shear capacity were calculated for N values.

5.2 Uncertainty of design parameters

In this study, the uncertainties of the parameters are modeled as random variables described by the probability distribution functions (PDF). Then, in order to select the most appropriate PDF (e.g., Gaussian, Lognormal, Gumbel, Weibull), a statistical assessment of such experimental data should be done. Also, it is possible to work directly with measured histograms (raw data) without mathematical model.

The parameters of material properties for the concrete and FRP are taken randomly. Table 3 summarizes the statistical parameters of the material properties, member geometries and load factors. The uncertainty of parameters is represented by mean value, coefficient of variation and probability distribution type. The coefficient of variation of each parameter is taken from previous

Table 3 Statistical distributions used during reliability analysis

Variable	Statistical distribution	Mean	CoV	Source
Dimensions (*c*, *L*)	Normal	Nominal	0.03	Okeil *et al*. (2002)
Effective depth (*d*)	Normal	Nominal	0.03	Okeil *et al*. (2002)
Reinforcement ratio (ρ)	Normal	Nominal	0.05	Strauss *et al*. (2006)
Concrete compressive strength (*fc*)	Normal	Nominal	0.18	Mirza and MacGregor (1976)
FRP modulus (*Ef*)	Lognormal	Nominal	0.1	Atadero and Karbhari (2008)

Table 4 Coefficient of variation values for modeling (VP) and material – geometry (VMF) factors

Factor	ACI 440 Eq. (1)	BS8110 Eq. (2)	El-Ghandour *et al.* Eq. (3)	Matthys *et al.* Eq. (4)	Ospina *et al.* Eq. (5)	Zaghloul Eq. (6)	Metwally (Eq. 7)	New Formula Eq. (8)
VP	0.19	0.13	0.29	0.13	0.14	0.18	0.13	0.13
VMF	0.084	0.083	0.106	0.083	0.095	0.084	0.025	0.076

studies available in the literature (Atadero and Karbhari 2008, Mirza and MacGregor 1976, Okeil *et al*. 2002, Strauss *et al*. 2006).

5.3 Statistical parameters

The most important parameters that reflect the uncertainty of resistance are bias factor (λ) and coefficient of variation (V). The values of λ and V are determined by the following equations

$$\lambda_R = \lambda_M \times \lambda_P \times \lambda_F \tag{9}$$

$$V_R = (V_M^2 + V_P^2 + V_F^2)^{0.5} \tag{10}$$

where R indicates the resistance, M indicates the material, P indicates the professionalism (modeling) and F indicates the fabrication (geometry).

The statistical parameters are necessary to perform the reliability analysis. In this study, the coefficient of variation values (Table 4) represent the average CoV of all existing slabs. The existing experimentally tested slabs were analyzed rapidly 1000 times with random values of design parameters. The modeling factor was determined from the deterministic analysis whereas the material and geometry factors were found using the results of stochastic analysis.

5.4 Reliability analysis

5.4.1 Concepts

Structural reliability is the ability of a structure or a structural member to fulfill the specified requirements for which it has been designed (EN1990 2002) i.e., the element fails if the applied load (Q) exceeds the resistance of the member (R). The corresponding limit state function can be simplified as follows

$$g = R - Q \tag{11}$$

$$R = \phi R_n(X_1, X_2, \ldots\ldots, X_n) \tag{12}$$

$$Q = \lambda_D D + \lambda_L L \tag{13}$$

where X_1 represents the random parameters, D dead load, L live load, φ reduction factor and λ bias factor. The performance of the structure is assessed by the failure probability of limit state function, which is given explicitly as follows

$$P_f = P(g < 0) \tag{14}$$

Fig. 4 illustrates the PDF of load, resistance and safety margin.

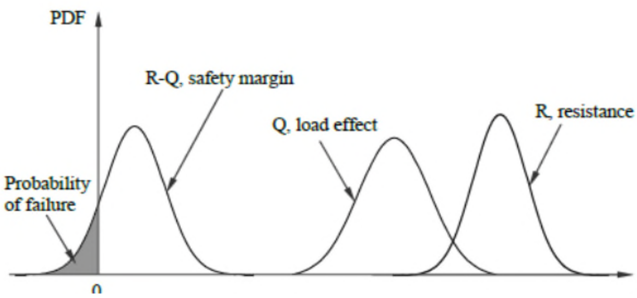


Fig. 4 Probability density function of load, resistance and safety margin. (Nowak and Collins 2012)

5.4.2 Reliability index

The reliability index is defined as the shortest distance from the origin to the failure surface, line $g(Z_R,Z_Q)=0$ as shown in Fig. 5, where Z_R is the reduced variable for resistance and Z_Q is the reduced variable for the load as introduced and defined by (Hasofer and Lind 1974). The relation between the reliability index β and the probability of failure, P_f, is described by

$$\beta = -\phi^{-1}(P_f) \tag{15}$$

where $\varphi-1$ is the inverse of the probabilistic distribution function, P_f is the failure probability and β is the reliability index. The expression of the reliability index is expressed in Eq. (16)

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \tag{16}$$

Eq. (15) is used to calculate the reliability indices of punching shear capacity formulas. The ultimate limit state load case Eq. (17) is specified by (ACI 2008, ASCE 1998)

$$1.4D < \phi R \qquad \text{L=0} \tag{17}$$

$$1.2D + 1.6L < \phi R \qquad \text{L>0} \tag{18}$$

where D is the dead load, L is the live load and φ is the strength reduction factor.

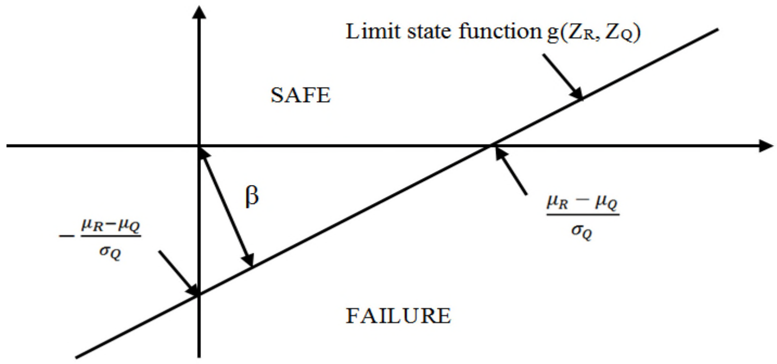
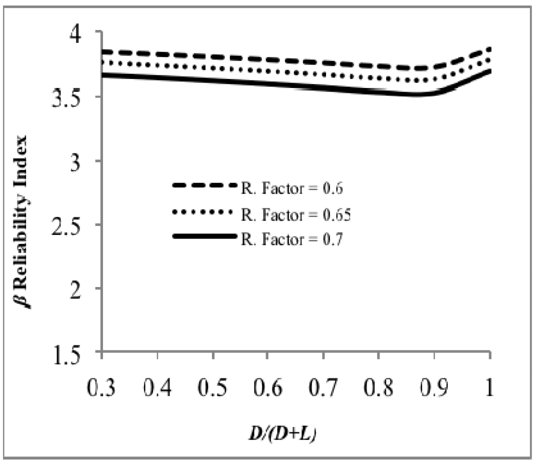


Fig. 5 Reliability index definition (Nowak and Collins 2012)

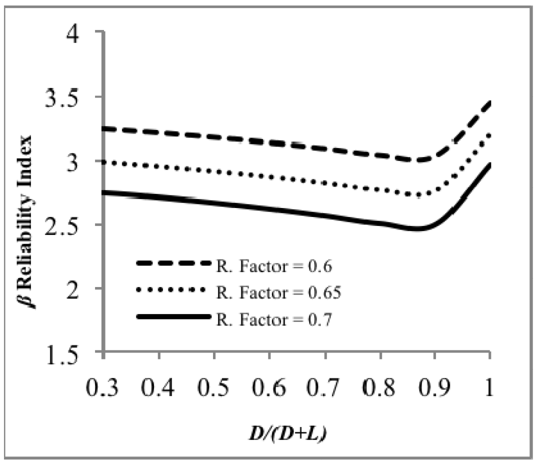
Table 5 Target reliability indices with load ratio *D/D+L*=0.5 and statistical results of formulations

	φ	Eq. (1)	Eq. (2)	Eq. (3)	Eq. (4)	Eq. (5)	Eq. (6)	Eq. (7)	Eq. (8)
Reliability Indices	0.6	3.8	3.17	2.03	3.6	2.91	2.48	2.82	3.97
	0.65	3.71	2.92	1.93	3.38	2.67	2.26	2.6	3.71
	0.7	3.62	2.67	1.82	3.16	2.43	2.05	2.35	3.45

The target reliability index of the member depends on the consequence of failure, cost and feasibility of structural use. The load ratio *D/*(*D+L*) is usually varies from 0.3 to 0.7. Table 5 shows the calculated average values of the reliability index for a typical *D/D+L* ratio for different values of strength reduction factor. The reduction factor is selected as 0.65 for GFRP and CFRP as recommended by ACI 440. Reliability indices of the Eq. (1)-(8) based on ASCE7 and ACI318-11 ultimate limit state loading case is also illustrated in Fig. 6.

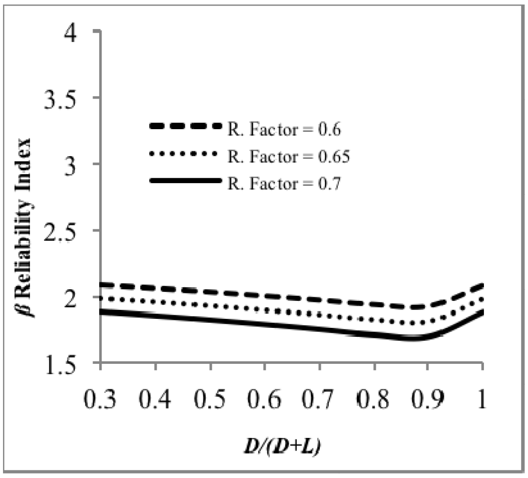


(a) ACI 440 Eq. (1)

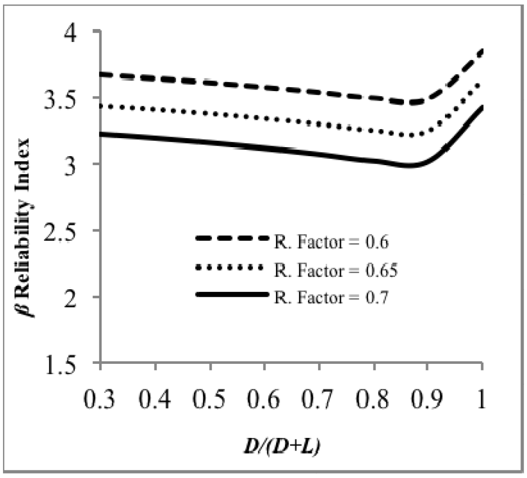


(b) BS8110 Eq. (2)

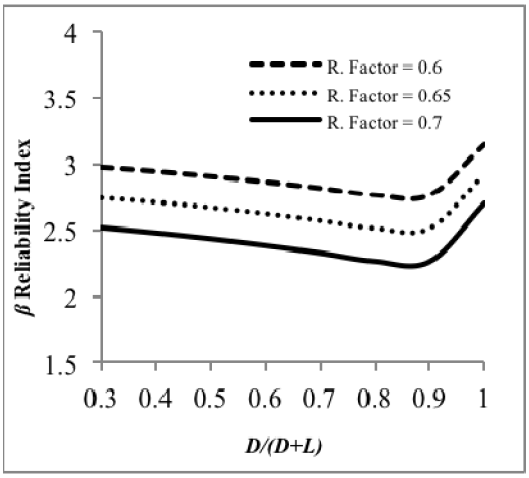
Fig. 6 Reliability indices for the formulas based on ASCE7 and ACI318-11 ultimate limit state loading case



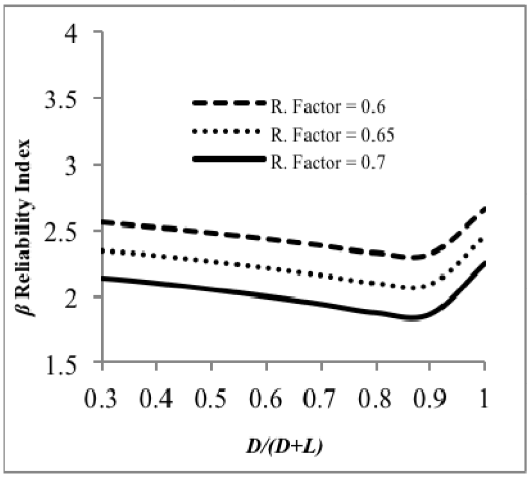
(c) El-Ghandour *et al*. Eq. (3)



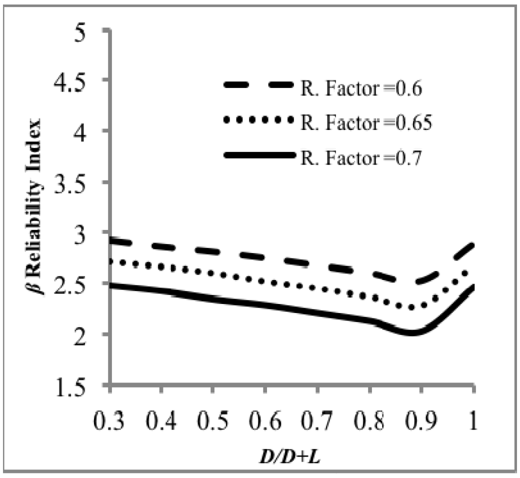
(d) Matthys *et al*. Eq. (4)



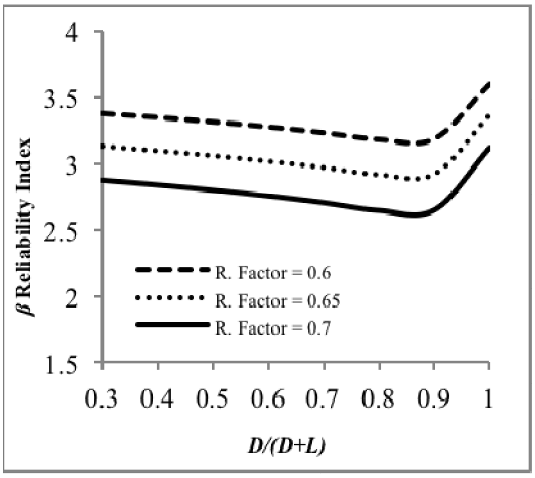
(e) Ospina *et al*. Eq. (5)



(f) Zhagloul Eq. (6)



(g) Metwally Eq. (7)



(h) New formula Eq. (8)

Fig. 6 Continued

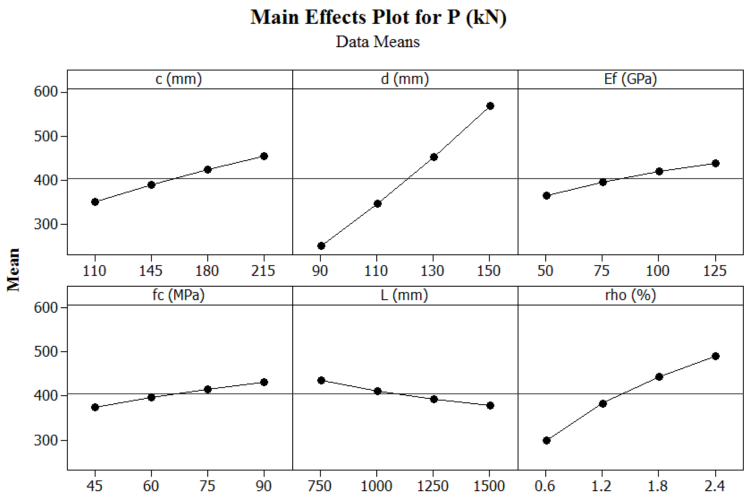


Fig. 7 Main effect trends for the parametric study on punching shear capacity (P)

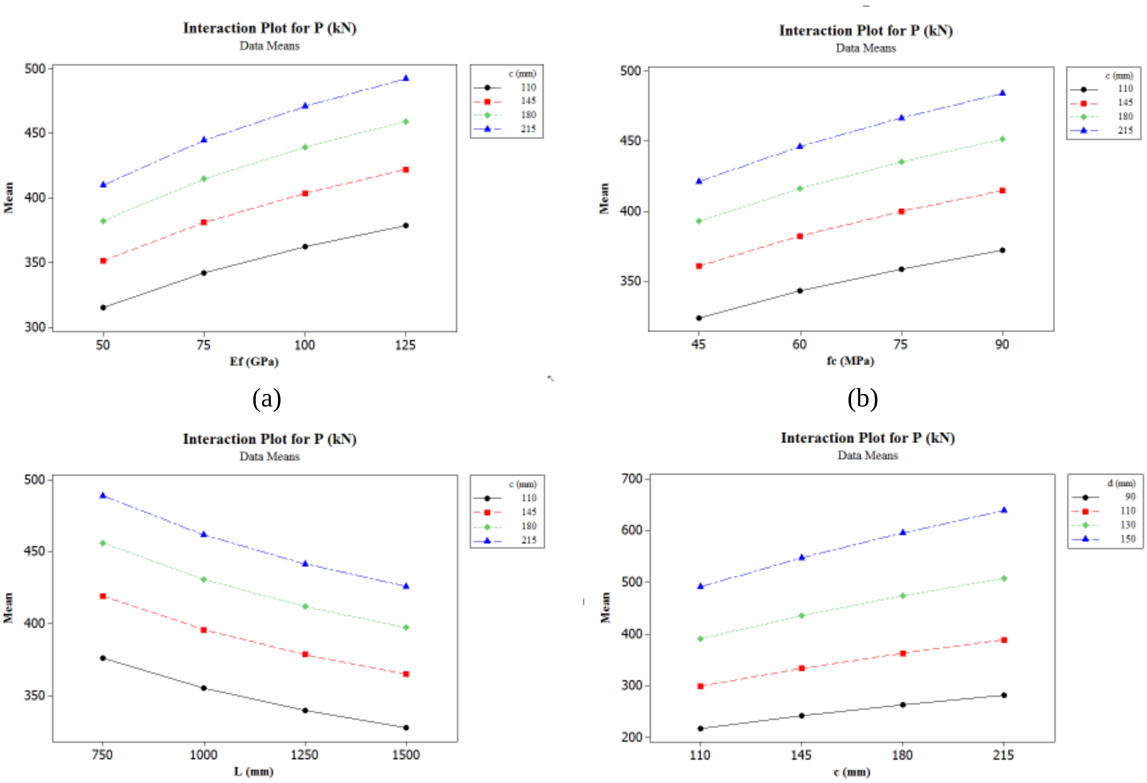
6. Discussion and results

In this study, reliability-based modeling of a new design code equation was carried out. After deriving the equation, a parametric study was performed. The prediction performance of proposed equation is found to be quite high with a correlation coefficient (R^2) of 0.984. Additionally, generalization capability of the proposed design code formulation was investigated by means of a wide range of parametric study. For this parametric study, a new dataset has been generated for each variable and the values were kept within the range of variables in experimental dataset. The main effect (Fig. 7) and interaction plots (Fig. 8 (a)-(o) of each variable are obtained by implementing the generated data to Eq. (8). The main effect plot is a significant tool to display the effect of each variable on punching shear capacity. This graphical tool allows viewing the overall importance of variable effects on the output and provides a general snapshot. On the other hand, the interaction plot is another essential tool as to observe the effect of each input parameter in conjunction with other inputs. This tool allows viewing the effect of each variable in detail. The same parametric dataset, which is used for main effect plot, has been utilized in order to obtain interaction plots provided in Fig.8 (a)-(o).

Main effect (Fig. 7) and interaction plots (Fig. 8 (a)-(o) indicate that punching shear capacity of FRP reinforced two-way slabs is influenced significantly by all input parameters. In particular, c (column section), d (effective flexural depth of slab), E_f (elastic modulus of FRP), f_c (compressive strength of concrete) and ρ (reinforcement ratio) have an increasing effect on punching shear capacity whereas increasing L (span length of slab) leads to lower punching shear capacity.

It is found that the derived formulation has high generalization capability for the range of variables in experimental dataset. It can be observed from the interaction plot (Fig. 8(d)) that the dimension of column section (c) has a significant effect (directly proportional) on ultimate punching load and this effect is pronounced more together with the influence of effective depth (d). The effect of elastic modulus of FRP reinforcements (E_f) is observed to be higher with the contribution of reinforcement ratio (ρ) and effective depth (d) (Fig. 8(e) and 8(m)). Reinforcement ratio and effective depth also have significant influence on the effect of compressive strength of concrete (f_c) on ultimate punching load (Fig. 8(f) and 8(n)). As far as the main effect and interaction plots considered, the span length (L) has inversely proportional effect on punching load and its effect is influenced significantly with the contribution of effective depth. It can be seen that the slab length is an effective parameter on punching shear, although punching can be considered local failure (Fig. 8(c), (g), (i) and (o)). Finally, the proportionality of reinforcement ratio (ρ) can be considered as parabolic and its effect on punching load is pronounced more with the contribution of effective depth (d) (Fig. 8(l)).

On the other hand, the reliability indices of each design formulation and proposed formulation was found. The target reliability index of the member depends on the consequence of failure, cost and feasibility of structural use. The load ratio $D/(D+L)$ is usually varies from 0.3 to 0.7. Table 5 shows the calculated average reliability index values for a typical $D/D+L$ ratio of 0.5 and different values of strength reduction factor (φ=0.6, 0.65 and 0.7). The calculated reliability indices for the formulas gave different results due to uncertainty of material, geometry and modeling factors. The reliability index for the proposed formula is found to be greater than 3.0 at strength reduction factor 0.65. According to the EN (1990) and (ISO 1998), the value of reliability index (β=3.0) is acceptable because the slab is an un-sequential failure member and its cost is high compared to traditional building system. Fig. 6 (a)-(h) presents the influence of the dead load ratio to the reliability index of analyzed



(a) (b)

(c) (d)

Fig. 8 Interaction plots for the parametric study on punching shear capacity (P)

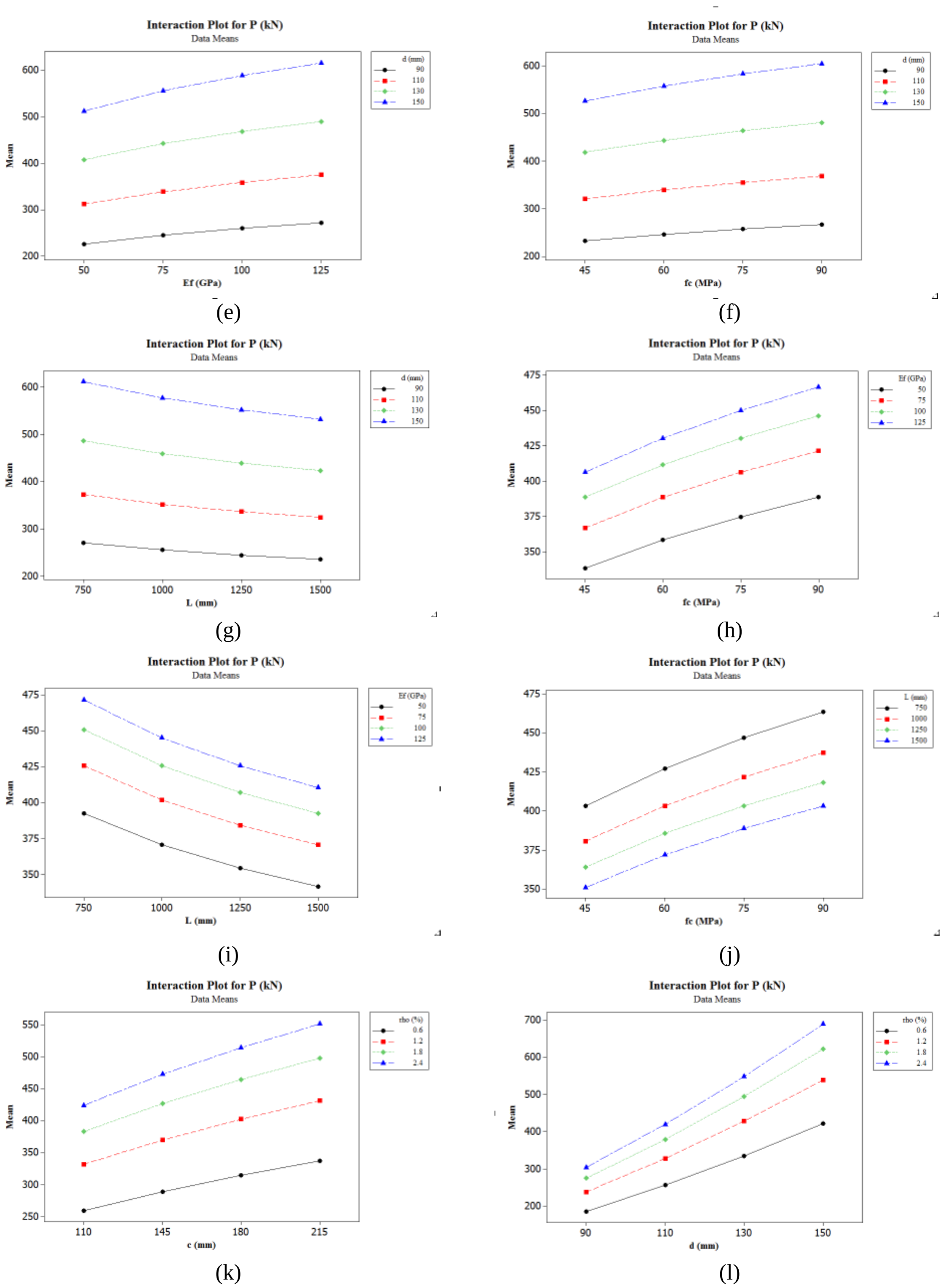
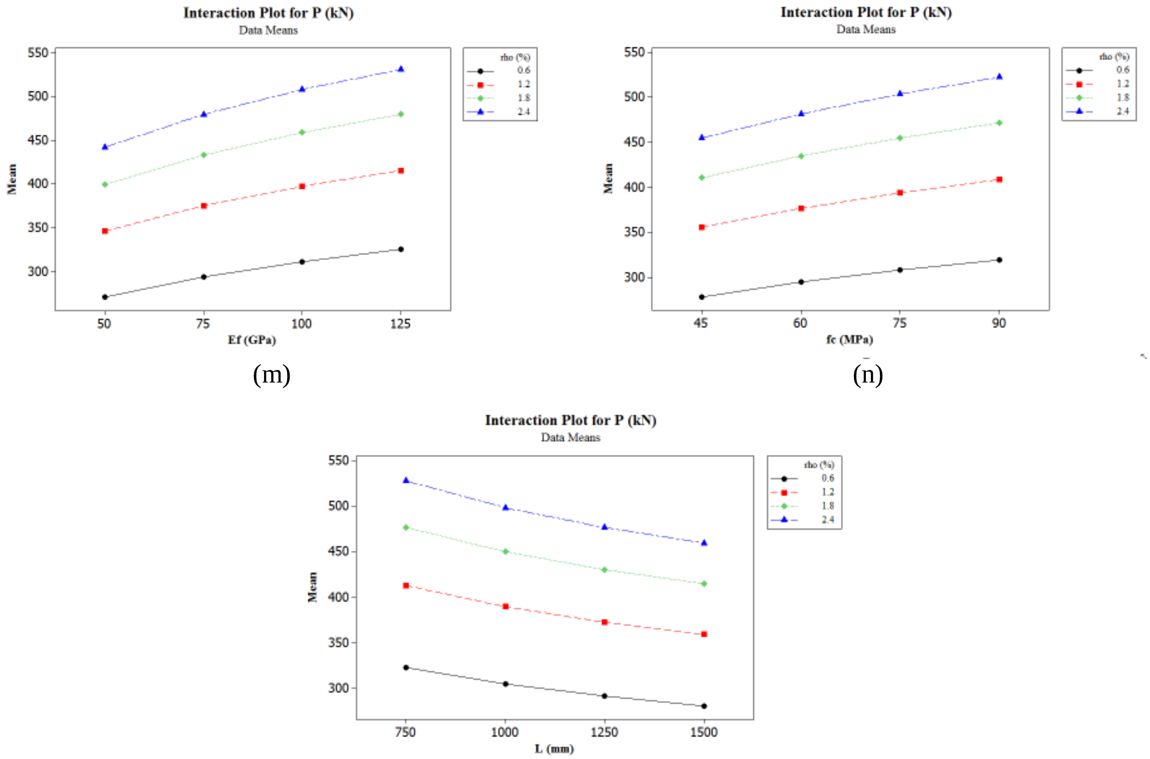


Fig. 8 Continued



(m) (n)

(o)

Fig. 8 Continued

formulations Eq. (1)-(8) for three levels of strength reduction factors (φ=0.6, 0.65 and 0.7). Fig. 6 (a)-(h) indicates that the live load is riskier than the dead load and the influence of strength reduction factors on the proposed formula Eq. (8) is more sensitive compared to ACI 440's formula.

7. Summary and conclusions

Material and geometry uncertainties in structural members lead to various difficulties in modeling the actual structural behavior by deterministic analysis. To obtain a rational and safe design and to develop realistic models, it is required to quantify the uncertainties of these properties. This study concentrates on the reliability analysis of design formulations that are derived for predicting the punching shear capacity of FRP-reinforced two-way slabs. Prior to the reliability analysis, a new formulation was derived by means of gene expression programming. Next, the proposed formulation was tested for its generalization capability by a parametric study. Then, the stochastic analyses of existing formulations were performed by means of Monte Carlo simulation. Finally, the reliability analyses of these equations were carried out based on stochastic analysis results and the ultimate state function of ASCE-7 and ACI-318 (2011).

The performance of the proposed and other existing formulations was tested by reliability analysis, which is represented in the reliability index value. Firstly, the statistical parameters for FRP reinforced concrete slabs are determined by stochastic analysis. Then, each slab was analyzed rapidly 1000 times for each formula with random values of material properties and geometries using Monte Carlo simulation sampling. The parameters that are considered uncertain include the properties of concrete and reinforcement in addition to the geometry parameters.

The last part of this study is based on the reliability analysis of punching shear capacity of FRP-reinforced two-way slabs for each formula using the ultimate state function of ASCE-7 and ACI-318 (2011).

It should be noted that the dataset taken from the literature are based on the test specimens generally subjected to concentric loading. As the loading type can be a significant factor on punching shear capacity, full account must be taken on this parameter.

In this study, the following conclusions can be drawn

1. The proposed formulation has high generalization capability and performs better than formulations available in the literature in terms of correlation coefficient (R^2) and root mean squared error (*RMSE*).
2. As opposed to the fact that punching shear failure is considered a local failure, the effect of span length (L) on punching shear resistance should be taken into account in design.
3. Uncertainty in the material and geometry parameters substantially influences the ultimate shear capacity.
4. Safety margin of the new formulation is within the acceptable limits according to EN (1990) and ISO 2348.
5. Reliability of the new formulation is largely dependent on the strength reduction factor and recommended value for strength reduction factor is 0.65.

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Appendix

Table A.1 Experimental database of FRP-reinforced two-way slabs and comparison of models

	c	d	E_f	f_c	L	ρ	P_{test}	$P_{test}/P_{predicted}$							
Source	(mm)	(mm)	(GPa)	(MPa)	(mm)	(%)	(kN)	Eq. (1)	Eq. (2)	Eq. (3)	Eq. (4)	Eq. (5)	Eq. (6)	Eq. (7)	Eq. (8)
(Ahmed *et al.* 1994)	75	61	113	42.4	590	0.95	93	2.32	1.58	0.92	1.07	0.96	1.12	0.97	0.96
	75	61	113	44.6	590	0.95	78	1.92	1.29	0.76	0.88	0.79	0.92	0.79	0.80
	100	61	113	39	590	0.95	96	2.07	1.43	0.89	1.04	0.93	1.01	0.95	0.90
	100	61	113	36.6	590	0.95	99	2.17	1.53	0.94	1.09	0.98	1.06	1.01	0.94
(Banthia *et al.* 1995)	100	55	100	41	500	0.31	65	2.79	1.14	1.04	1.21	1.13	1.17	1.12	1.05
	100	55	100	52.9	500	0.31	61	2.44	0.94	0.89	1.04	0.98	1.01	0.93	0.94
(El-Ghandour *et al.* 2003)	200	142	45	33.3	1700	0.22	170	2.32	0.76	0.88	1.03	0.87	0.84	0.78	0.82
	200	142	110	34.7	1700	0.18	229	2.22	0.74	0.93	1.08	0.79	0.89	0.82	0.98
	200	142	45	46.6	1700	0.47	271	2.36	1.02	0.98	1.14	0.96	0.93	0.82	0.93
	200	142	45	30.3	1700	0.47	237	2.31	1.10	0.99	1.15	0.97	0.94	0.89	0.89
	200	142	110	29.6	1700	0.43	317	2.15	1.11	1.02	1.18	0.86	0.97	0.92	1.03
(Matthys and Taerwe 2000)	150	96	91.8	36.7	900	0.26	181	3.26	1.24	1.30	1.51	1.25	1.33	1.25	1.28
	230	96	91.8	37.3	900	0.26	189	2.56	0.97	1.14	1.33	1.10	1.04	1.09	1.13
	150	95	95	35.7	900	1.05	255	2.46	1.78	1.17	1.36	1.12	1.19	1.13	1.12
	230	95	95	36.3	900	1.05	273	1.97	1.42	1.05	1.22	1.01	0.95	1.02	1.01
	150	126	92	33.8	900	0.52	347	3.15	1.68	1.38	1.60	1.24	1.41	1.25	1.26
	230	126	92	34.3	900	0.52	343	2.40	1.28	1.18	1.37	1.06	1.07	1.07	1.05
	150	95	147.6	32.6	900	0.19	142	2.48	0.90	1.02	1.19	0.91	1.04	1.00	1.06
	230	95	147.6	33.2	900	0.19	150	1.97	0.71	0.91	1.05	0.81	0.82	0.89	0.95
	150	95	37.3	118	900	0.64	207	2.78	1.09	1.03	1.19	1.15	1.04	0.81	1.03
	150	89	40.7	35.8	900	3.78	231	2.02	2.34	1.00	1.16	1.13	1.02	0.99	0.85
	80	89	40.7	35.9	900	3.78	171	2.11	2.44	0.89	1.03	1.00	1.07	0.88	0.80
	150	122	44.8	32.1	900	1.21	237	2.16	1.57	0.96	1.12	0.98	0.98	0.89	0.79
	80	122	44.8	32.1	900	1.21	217	2.66	1.94	1.02	1.18	1.04	1.21	0.94	0.92
(Ospina *et al.* 2003)	250	120	34	29.5	1670	0.73	217	2.16	1.23	1.01	1.17	1.08	0.90	0.95	0.88
	250	120	34	28.9	1670	1.46	260	1.90	1.49	0.96	1.12	1.04	0.86	0.91	0.83
	250	120	28.4	37.5	1670	0.87	206	1.93	1.10	0.88	1.03	0.98	0.79	0.80	0.78
(Hussein *et al.* 2004)	250	100	42	40	1830	1.18	249	2.11	1.43	1.05	1.23	1.15	0.94	0.99	1.05
	250	100	42	35	1830	1.05	218	2.02	1.34	1.00	1.17	1.09	0.89	0.96	0.98
	250	100	42	29	1830	1.67	240	1.90	1.62	1.01	1.17	1.09	0.90	1.00	0.95
	250	100	42	26	1830	0.95	210	2.21	1.50	1.10	1.28	1.20	0.98	1.11	1.04
(Lee *et al.* 2009)	225	110	48.2	36.3	2000	1.17	222	1.73	1.22	0.86	1.00	0.89	0.79	0.80	0.85
	225	110	48.2	36.3	2000	2.14	246	1.47	1.35	0.78	0.90	0.81	0.71	0.73	0.76
	225	110	48.2	36.3	2000	3	248	1.28	1.36	0.70	0.81	0.73	0.64	0.65	0.68
(Zhang 2006)	250	100	42	35	1830	1.05	218	2.02	1.34	1.00	1.17	1.09	0.89	0.96	0.98

Table A.1 Continued

(Nguyen-Minh and Rovňák 2012)	200	129	48	39	2000	0.48	180	1.80	0.83	0.77	0.89	0.77	0.73	0.68	0.75
	200	129	48	39	2000	0.68	212	1.81	0.98	0.81	0.94	0.80	0.76	0.71	0.79
	200	129	48	39	2000	0.92	244	1.81	1.12	0.84	0.97	0.84	0.80	0.74	0.81
(Hassan *et al.* 2013)	300	150	48.2	34	2000	0.71	329	1.79	1.02	0.90	1.05	0.86	0.77	0.78	0.83
	300	150	48.2	39	2000	0.71	386	2.02	1.11	1.01	1.17	0.97	0.86	0.86	0.94
	300	150	48.1	39	2000	1.56	431	1.58	1.25	0.87	1.01	0.83	0.74	0.74	0.80
	300	150	48.1	32	2000	1.56	451	1.75	1.44	0.97	1.13	0.93	0.83	0.85	0.87
	450	150	48.2	45	2000	0.71	400	1.51	0.81	0.83	0.97	0.80	0.64	0.69	0.81
	450	150	48.1	32	2000	1.56	504	1.47	1.21	0.90	1.05	0.87	0.69	0.80	0.83
	450	150	48.1	39	2000	1.56	511	1.41	1.11	0.86	1.00	0.82	0.66	0.73	0.81
	300	300	48.2	34	2000	0.34	825	2.37	0.96	1.07	1.25	0.86	0.92	0.78	0.89
	300	300	48.2	39	2000	0.34	782	2.17	0.85	0.97	1.13	0.78	0.84	0.69	0.82
	300	300	48.1	39	2000	0.73	1071	2.08	1.16	1.03	1.20	0.83	0.89	0.74	0.86
	300	300	48.1	30	2000	0.73	1027	2.14	1.27	1.08	1.25	0.87	0.93	0.81	0.87
	300	300	48.1	47	2000	0.73	1195	2.21	1.18	1.08	1.26	0.87	0.93	0.75	0.92
	450	300	48.2	49	2000	0.34	911	1.90	0.70	0.93	1.08	0.75	0.72	0.64	0.78
	450	300	48.2	32	2000	0.34	1020	2.38	0.98	1.20	1.40	0.97	0.93	0.89	0.95
	450	300	48.1	30	2000	0.73	1248	2.08	1.23	1.16	1.35	0.94	0.90	0.87	0.90
Mean								2.11	1.25	0.98	1.14	0.95	0.92	0.88	0.91
Standard Dev.								0.39	0.36	0.13	0.15	0.13	0.16	0.14	0.12
CoV								0.19	0.29	0.13	0.13	0.14	0.18	0.16	0.13
R^2								0.945	0.906	0.972	0.971	0.98	0.961	0.97	0.984