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Statistical division of compressive strength results on the aspect of concrete family concept

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Abstract. The article presents the statistical method of grouping the results of the compressive strength of concrete in continuous production. It describes the method of dividing the series of compressive strength results into batches of statistically stable strength parameters at specific time intervals, based on the standardized concept of "concrete family". The article presents the examples of calculations made for two series of concrete strength results, from which sets of decreased strength parameters were separated. When assessing the quality of concrete elements and concrete road surfaces, the principal issue is the control of the compressive strength parameters of concrete. Large quantities of concrete mix manufactured in a continuous way should be subject to continuous control. Standardized approach to assessing the concrete strength proves to be insufficient because it does not allow for the detection of subsets of the decreased strength results, which in turn makes it impossible to make adjustments to the concrete manufacturing process and to identify particular product or area on site with decreased concrete strength. In this article two independent methods of grouping the test results of concrete with statistically stable strength parameters were proposed, involving verification of statistical hypothesis based on statistical tests: Student's t-test and Mann – Whitney – U test.

Keywords: division of compressive strength results of concrete; family of concrete; compressive strength of concrete; reliability of structure

1. Introduction

The continuous production of concrete mix requires ongoing monitoring. The main parameter subject to testing is the compressive strength of concrete. When evaluating the quality of concrete structures (reinforced concrete, compressed concrete, road surfaces) special attention is paid to strength parameters of structural elements made of a specific amount of concrete mix Day (2006), Konkol and Prokopski (2007). This specific amount of the mixture is called a concrete family and is subject to strict control Harrison (1999), Caspeele and Taerwe (2007), Ping *et al.* (2010), Taerwe (1999). Assigning concrete to a certain family is based on the relationship between compressive

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strength and technological considerations, based on a set of test results from production control, meeting the compliance criteria Czarnecki *et al*. (2004), Brunarski (2008).

The determination of separate concrete families is related to the division of the results of the compressive strength of concrete into groups with statistically stable strength parameters at specific time intervals, and in linear objects it is related to the length of the structure (e.g. concrete motorway).

In the continuous production of large quantities of concrete mix, proper estimation of concrete family is crucial also from the point of view of further reliability of structures in use Sarja (2000), [ISO 2394:2000, **EN 1992**:2004]. In Annex E to the standard [ISO 2394:2000] “General principles on reliability of structures” it is clearly stated that identification and division of statistical populations should be strictly followed, since it is important for the determination of serviceability of limit states.

The standard specifies the level of serviceability $\mu\,(0 \le \mu \le 1)$ and serviceability parameters λ, e.g. the strength of concrete, for which in the range of $\lambda:\ \lambda_1$ a structure is fully serviceable as the requirements for the class of concrete are met, outside the range $\lambda_1:\ \lambda_2$ - the structure is completely unserviceable. Therefore the limit state for bearing capacity controlled by the function $g(\mathbf{X})$ of basic variable $\mathbf{X}$ relating to concrete strength such that: $g(\mathbf{X}) < 0$ is not met. In the case analyzed the basic variable is dependent on time, in this case it depends on the dates of production of concrete mix. The probability of meeting reliability criteria, i.e. achieving the value of characteristic resistance above the projected level satisfies the relation:

$$P_s = 1 - P_f \ge 0.95, \tag{1}$$

where P_s - reliability – probability of result meeting the criterion of achieving the value characteristic for the compressive strength of concrete, P_f - the above criterion is not satisfied. Such an approach, the ISO standard [ISO 2394:2000] defines as the analysis of element, which is consistently applied in this article.

The division into concrete families is quite complicated and requires the use of appropriate statistical calculation procedures based on verification of the assumed statistical hypothesis. However, it is crucial for regulating the manufacturing process of concrete mix and its optimization in order to achieve production stability, which will result in obtaining the required strength parameters of the concrete, and also the desired economic results.

The above requirements and assumptions are satisfied in the definition contained in study Jasiczak (1992), Jasiczak (2011) which even though refers to the concept of batch, it is perfectly suited to define the term of concrete family: “The concrete batch is the amount of concrete with probabilistically stabilized strength parameters, subject to systematic assessment in the way enabling the division of results, manufactured in a time interval set by the statistical constancy of parameters”.

With reference to this definition, two sample divisions were made for the test results of concrete strength into individual families of concretes. Two series of results were used, 6 results per day during 31 days (one month), for concrete of projected characteristic compressive strength $f_{ck} = 45$ MPa. Concrete samples were subject to compression after 28 days of curing.

Characteristic compressive strength of concrete is denoted on cylindrical samples as $f_{ck,cyl}$ or on cubic samples as $f_{ck,cube}$, which corresponds, according to the European standard [EN 206-1:2000], to the strength class C35/45 analysed in this article. In the further parts of the article the authors consistently use the alphanumeric symbols to achieve unequivocal record. Such a record is widely accepted in Europe as it is difficult to use the output formula included in the standard (table 14):

$$f_{ck} = f_{cm} - 1.48\sigma, \text{ [MPa]} \tag{2}$$

where: f_{ck} – characteristic compressive strength of concrete; f_{cm} - average compressive strength of concrete; σ – standard deviation of the population.

In the American standards [ACI 214R-11], [ACI 318-08, Chapter 5 – Concrete Quality Mixing and Placing, table 5.3.2.1] a similar rule applies:

$$f_c' = f_c'{}_r - 1.34 s_s, \text{ [psi]} \tag{3}$$

where s_s – sample standard deviation; f_c' – specified compressive strength; $f_c'{}_r$ – average compressive strength denoted on cylindrical samples.

In the analyzed calculation examples we based on the series of results of the compressive strength of concrete, composed of small subsets $n = 6$. This has its advantages and disadvantages. On the one hand, it is convenient to carry out calculations on a limited number of results, which means a small amount of concrete samples needed for the tests. On the other hand, however, there is a risk of wrong and costly strategies of concrete production, to manufacture concrete with increased average strength and high volatility of properties analyzed Brunarski (2008) and Woliński (2006). To minimize this risk, a detailed analysis of variations of concrete strength parameters should be carried out.

To analyze strength parameters volatility, control cards are also used as the graphical method for statistical control of production Gebler (1990) and Kanoniczak (2011). The rules for creating control cards are specified in the standard [ISO 8258:1996]. Individual results of the compressive strength, average or standard deviation values as well as calculated control lines are recorded on the cards. This allows for the supervision of the stability of manufacturing process of concrete mix, however, the cards do not allow for the assignment of individual strength results to the concrete family of probabilistically stabilized strength parameters.

This article presents the procedure for verification of the hypothesis on the attachment of result series to a particular concrete family using the Student's t-test and Mann – Whitney U test.

2. Defining parameters of the entire set of compressive strength results of concrete before division into concrete families

In order to determine parameters (standard deviation, average compressive strength of concrete) of the whole set of strength test results, histograms of the numerosity for both classes of concrete were made.

On the basis of the classes selected (strength parameter ranges), for each of them were determined their numerosities, average strength and arithmetic mean of the stem-and-leaf plot.

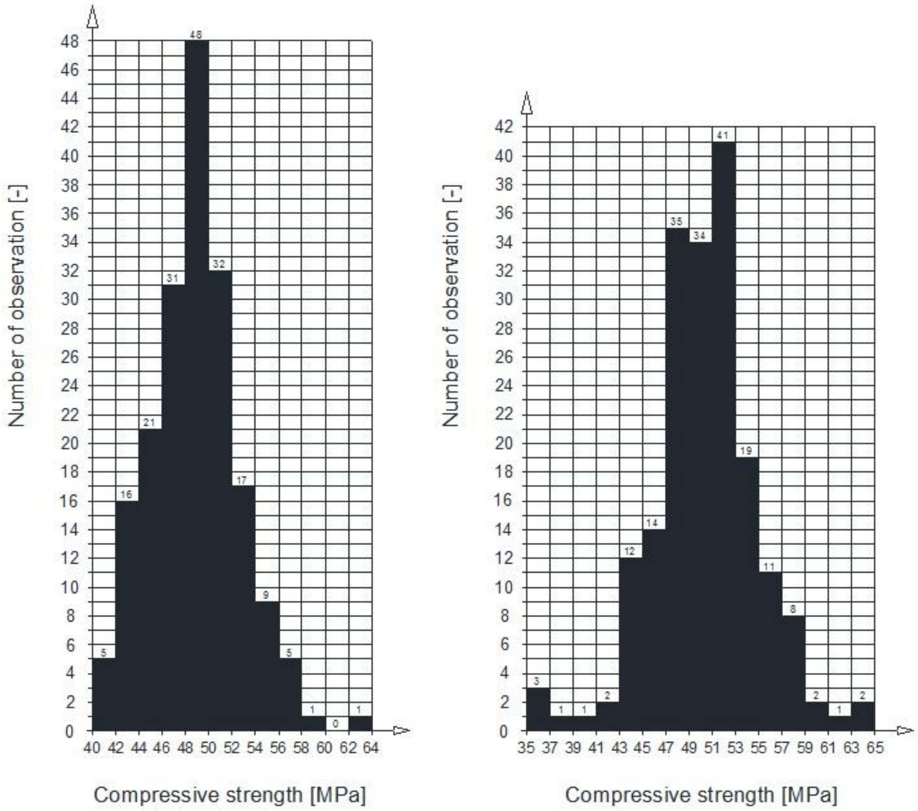

Fig. 1 Histogram of numerosity for the first and second series of compressive strength results

Table 1 Results of calculations of standard deviations, average strength, projected characteristic strength and actual characteristic strength as well as projected concrete class and concrete class according to standard [EN 206-1:2000]

Parameters	First series of data	Second series of data
	MPa	
Standard deviation	3.81	4.56
Average strength	48.83	50.38
Projected characteristic strength	45.00	45.00
Actual characteristic strength	42.56	42.88
Projected concrete class	C35/45	C35/45
Concrete class according to EN 206-1 standard	C30/37	C30/37

Standard deviation was calculated for the entire set of results.

Fig. 1 presents histograms of numerosity for initially assumed class of concrete, respectively for the first and second series of results for the compressive strength of concrete.

In order to determine the characteristic strength based on all the strength test results (31 days x 6 results per day = 186 results), and thus the class of concrete corresponding to this strength, definition included in standard [EN 206-1:2000] was used, which states that the characteristic

strength of concrete is the value below which 5% of the population of all concrete test results can be found. This is the distribution quantile of order 0.05 for one-sided tolerance. This means that 5% of all the results of the concrete tests (5% of 186 results = 9.3, i.e. 9 results after rounding down) can be found below the strength value characteristic for this class of concrete.
After analyzing the whole series of data for concrete of the assumed class C35/45 it was found that the value of 10 results (in order from the lowest to the highest strength) was 43.0 MPa. The closest strength class corresponding to this value according to [EN 206-1:2000] is C30/37. As it can be seen, it differs from the initially assumed class C35/45.

For the second series of data for assumed concrete class C35/45 it was found that the value 9 equaled value 10 of the result (in order from the lowest to the highest strength) and is 44.4 MPa. Thus, the limit value not exceeding 5% of the total results is value 8 totaling 44.3 MPa. However, the closest class of concrete strength corresponding to 44.4 MPa according to [EN 206-1:2000] is C30/37. Also in this case it differs from the initially assumed class C35/45.

The results of calculations of the standard deviation, average strength, characteristic strength projected and actual and also projected concrete class and the concrete class according to [EN 206-1:2000] are presented in Table 1.

The results in Table 1 referred to the construction of linear structures of the city bypass in Poznan. The project assumed concrete class C35/45, however, the results of statistical calculations for the entire set proved that it was impossible to obtain the assumed concrete strength. The total results were classified as concrete class C30/37.

3. Preliminary assumptions concerning division of series of concrete strength test results

As the subject of study, series n with numbered working plots was assumed. To each working plot a series $\mathbf{x}_i = (x_{i,1}, x_{i,2}, \ldots, x_{i,n_i}), i = 1, 2, \ldots, n,$ of the compressive strength of concrete was assigned.

In order to divide the series of concrete strength test results into concrete families, the authors proposed two alternative methods involving the statistical verification of two generally different series of hypotheses systems.

Applying the first method, systems of hypotheses concerning the expected values were verified:

$$\begin{cases} H_0 : \mu_{\{\mathbf{x}_m, \mathbf{x}_{m+1}, \ldots, \mathbf{x}_{m+r}\}} = \mu_{\{\mathbf{x}_{m+r+1}\}} \\ H_1 : \mu_{\{\mathbf{x}_m, \mathbf{x}_{m+1}, \ldots, \mathbf{x}_{m+r}\}} \neq \mu_{\{\mathbf{x}_{m+r+1}\}}, \end{cases} \tag{4}$$

where $\mu_{\{\mathbf{x}_m, \mathbf{x}_{m+1}, \ldots, \mathbf{x}_{m+r}\}}$ marks the expected value of probability distribution of "total" sample $\{\mathbf{x}_m, \mathbf{x}_{m+1}, \ldots, \mathbf{x}_{m+r}\}$. It was assumed that such expected value exists. The series of hypothesis systems of the form Eq. (4) was verified according to the following procedure:

Step 1. Assumption that $m = 1, r = 0$ and verification of hypothesis system in Eq. (4) for such values. If null hypothesis is rejected, then we proceed to Step 2, otherwise to Step 3.

Step 2. Let $m = m + r + 1, r = 0$ and verification of hypothesis system in Eq. (4) for such values. If null hypothesis is rejected, then procedure is continued in Step 2, otherwise we proceed to Step 3.

Step 3. Let $m = m, r = r + 1$ and verification for such values of hypothesis system in Eq. (4). If null hypothesis is rejected, then we proceed to Step 2, otherwise Step 3 is continued.

In case of rejection of the null hypothesis it is concluded that samples come from different concrete families, and if there is no reason to reject H_0, it is assumed that the samples are from the same family. To verify the systems of hypotheses for expected values, t-test, described later in this paper, is used for two independent samples (as measurements of the compressive strength of concrete are made separately on each working plot). The second method verifies systems of hypotheses concerning the distribution of two samples, namely the hypothesis of equality of the distribution functions:

$$\begin{cases} H_0 : F_{\{\mathbf{x}_m, \mathbf{x}_{m+1}, \dots, \mathbf{x}_{m+r}\}} = F_{\{\mathbf{x}_{m+r+1}\}} \\ H_1 : F_{\{\mathbf{x}_m, \mathbf{x}_{m+1}, \dots, \mathbf{x}_{m+r}\}} \neq F_{\{\mathbf{x}_{m+r+1}\}}, \end{cases} \tag{5}$$

where $F_{\{\mathbf{x}_m, \mathbf{x}_{m+1}, \dots, \mathbf{x}_{m+r}\}}$ means distribution function of the probability distribution of the "total" sample $\{\mathbf{x}_m, \mathbf{x}_{m+1}, \dots, \mathbf{x}_{m+r}\}$. The series of hypotheses system in Eq. (5) is verified according to the procedure described for the system of hypotheses in Eq. (4). The division of the series of the compressive strength results into batches is made similarly as in case of hypotheses system Eq. (4), but the systems of hypotheses in Eq. (5) are subject to Mann-Whitney U test.

4. Verification of hypothesis on attachement of series of results to defined concrete family using t-test for two independent samples

The results from two random samples were compared. Two independent random samples $\mathbf{X} = (X_1, X_2, \dots, X_n)$ and $\mathbf{Y} = (Y_1, Y_2, \dots, Y_m)$ were established from the populations with continuous distribution.

The mean and the variance from sample $\mathbf{X}$ were marked $\overline{X}$ and $S_{\mathbf{X}}^2$, respectively. Similarly, the mean and the variance from sample $\mathbf{Y}$ were marked $\overline{Y}, S_{\mathbf{Y}}^2$. The defined significance level of a test was marked as α. It was assumed that samples $\mathbf{X}$ and $\mathbf{Y}$ come from the normal distribution $N(\mu_{\mathbf{X}}, \sigma_{\mathbf{X}}^2)$ and $N(\mu_{\mathbf{Y}}, \sigma_{\mathbf{Y}}^2)$ of unknown parameters. Null hypothesis $H_0 : \mu_{\mathbf{X}} = \mu_{\mathbf{Y}}$ was verified with two-sided alternative hypothesis $H_1 : \mu_{\mathbf{X}} \neq \mu_{\mathbf{Y}}$. If $\sigma_{\mathbf{X}}^2 = \sigma_{\mathbf{Y}}^2$, then hypothesis H_0, is to be rejected when

$$T(\mathbf{X}, \mathbf{Y}) = \frac{\left|\overline{X} - \overline{Y}\right|}{\sqrt{\frac{(n-1)S_{\mathbf{X}}^2 + (m-1)S_{\mathbf{Y}}^2}{n+m-2} \cdot \frac{n+m}{nm}}} \geq t(1-\alpha/2, n+m-2), \tag{6}$$

where $t(1-\alpha/2, n+m-2)$ is the quantile of order $1-\alpha/2$ from Student's *t*-distribution with $n+m-2$ degrees of freedom. However, when $\sigma_{\mathbf{X}}^2 \neq \sigma_{\mathbf{Y}}^2$, then null hypothesis H_0, is to be rejected if:

$$T_*(\mathbf{X},\mathbf{Y}) = \frac{\left|\bar{X} - \bar{Y}\right|}{\sqrt{\frac{S_{\mathbf{X}}^2}{n} + \frac{S_{\mathbf{Y}}^2}{m}}} \geq t(1-\alpha/2,\beta), \tag{7}$$

Table 2 Results of testing hypothesis on the attachement of the first series of concrete strength results to defined concrete family using the Student's *t*–test for the projected concrete class C35/45

First series of compressive strength results For projected concrete class C35/45			
Value calculated for test statistics	Comparison of values	Quantile value *t*(1-α/2;n+m-2) from t-Student distribution	Attachment of two independent samples to concrete families
T(1,2) = 0.68	<	*t*(0.975;10)=2.23	Belonging to concrete family
T([1,2],3) = 0.20	<	*t*(0.975;16)=2.12	Belonging to concrete family
T([1,2,3],4) = 0.65	<	*t*(0.975;22)=2.07	Belonging to concrete family
T([1,2,3,4],5) = 1.33	<	*t*(0.975;28)=2.05	Belonging to concrete family
T([1,2,3,4,5],6) = 0.03	<	*t*(0.975;34)=2.03	Belonging to concrete family
T([1,2,3,4,5,6],7) = 0.79	<	*t*(0.975;40)=2.02	Belonging to concrete family
T([1,2,3,4,5,6,7],8) = 3.10	>	*t*(0.975;46)=2.01	Not belonging to concrete family
T(8,9) = 5.54	>	*t*(0.975;10)=2.23	Not belonging to concrete family
T(9,10) = 0.58	<	*t*(0.975;10)=2.23	Belonging to concrete family
T([9,10],11) = 2.69	>	*t*(0.975;16)=2.12	Not belonging to concrete family
T(11,12) = 1.93	<	*t*(0.975;10)=2.23	Belonging to concrete family
T([11,12],13) = 0.72	<	*t*(0.975;16)=2.12	Belonging to concrete family
T([11,12,13],14) = 1.96	<	*t*(0.975;22)=2.07	Belonging to concrete family
T([11,12,13,14],15) = 0.67	<	*t*(0.975;28)=2.05	Belonging to concrete family
T([11,12,13,14,15],16) = 3.12	>	*t*(0.975;34)=2.03	Not belonging to concrete family
T(16,17) = 2.69	>	*t*(0.975;10)=2.23	Not belonging to concrete family
T(17,18) = 1.59	<	*t*(0.975;10)=2.23	Belonging to concrete family
T([17,18],19) = 1.90	<	*t*(0.975;16)=2.12	Belonging to concrete family
T([17,18,19],20) = 0.94	<	*t*(0.975;22)=2.07	Belonging to concrete family
T([17,18,19,20],21) = 2.02	<	*t*(0.975;28)=2.05	Belonging to concrete family
T([17,18,19,20,21],22) = 2.93	>	*t*(0.975;34)=2.03	Not belonging to concrete family
T(22,23) = 2.28	>	*t*(0.975;10)=2.23	Not belonging to concrete family
T(23,24) = 0.90	<	*t*(0.975;10)=2.23	Belonging to concrete family
T([23,24],25) = 2.79	>	*t*(0.975;16)=2.12	Not belonging to concrete family
T(25,26) = 0.31	<	*t*(0.975;10)=2.23	Belonging to concrete family
T([25,26],27) = 1.59	<	*t*(0.975;16)=2.12	Belonging to concrete family
T([25,26,27],28) = 0.15	<	*t*(0.975;22)=2.07	Belonging to concrete family
T([25,26,27,28],29) = 0.56	<	*t*(0.975;28)=2.05	Belonging to concrete family
T([25,26,27,28,29],30) = 5.07	>	*t*(0.975;34)=2.03	Not belonging to concrete family
T(30,31) = 4.64	>	*t*(0.975;10)=2.23	Not belonging to concrete family

where

$$\beta = \frac{\left(\frac{S_{\mathbf{X}}^2}{n} + \frac{S_{\mathbf{Y}}^2}{m}\right)^2}{\frac{1}{n-1}\left(\frac{S_{\mathbf{X}}^2}{n}\right)^2 + \frac{1}{m-1}\left(\frac{S_{\mathbf{Y}}^2}{m}\right)^2} \quad (8)$$

and $t(1-\alpha/2,\beta)$ is the quantile of order $1-\alpha/2$ from Student's *t*-distribution with β degrees of freedom (in this case this test is called Welch's test). Quantiles from Student's *t*-distribution can be found in the statistical tables or are calculated using statistical packages, e.g. in "R" programme, where the quantile $t(\lambda,\delta)$ of order λ from Student's *t*-distribution with δ degrees of freedom are calculated with command $qt(\lambda,\delta)$. In order to check whether variances $\sigma_{\mathbf{X}}^2$, $\sigma_{\mathbf{Y}}^2$ can or cannot be regarded as equal we use F-test which was described, for instance, in Górecki (2011). The results of testing hypothesis on the attachment of the series of the compressive strength results for concrete to defined concrete family using Student's t – test are presented in table 2 or 3, where for example $T([1, 2], 3)$ means the value of test statistics T for "total" sample of results from day 1 and 2 and for sample from day 3, similarly we reason about statistics T_*. Using Student's t – test it was assumed that samples have normal distribution. Such an assumption was verified with Shapiro-Wilk test (see, for example, Górecki 2011) and, with two exceptions, there was no ground for rejecting null hypothesis on normal distribution of samples at the significance level of 0.01.

5. Verification of hypothesis on attachment of series of results to defined family of concretes using Mann-Whitney U test for two independent samples

In null hypohesis H_0 it was assumed that samples $\mathbf{X}$ and $\mathbf{Y}$ were taken from the same distribution, however, in the alternative hypothesis H_1 it was assumed that they were not. The null hypothesis was assumed to be rejected if the value of test statistics of:

$$U(\mathbf{X},\mathbf{Y}) = \sum_{j=1}^{m}\sum_{i=1}^{n} I(X_i < Y_j), \quad (9)$$

where

$$I(X_i < Y_j) = \begin{cases} 1 & \text{when } X_i < Y_j \\ 0 & \text{when } X_i \geq Y_j, \end{cases} \quad (10)$$

belongs to the critical area $C = [0, u(n, m, \alpha/2)] \cup [u(n, m, 1-\alpha/2), \infty)$, where $u(n,m,\phi)$ is the quantile of order ϕ from distribution of U, which can be found in statistical tables or is calculated in statistical packages, e.g. in "R" package with a command $qwilcox(\phi, n, m)$. In this example the values of quantile were calculated in "R" programme. In

Table 3 Results of testing hypothesis on the attachement of the second series of concrete strength results to defined concrete family using the Student's t – test for the projected concrete class C35/45

Second series of compressive strength results For projected concrete class			
Value calculated for test statistics	Comparison of values	Quantile value t(1-α/2;n+m-2) from t-Student distribution	Attachment of two independent samples to concrete families
T(1,2) = 6.49	>	*t*(0.975;10)=2.23	Not belonging to concrete family
T(2,3) = 7.03	>	*t*(0.975;10)=2.23	Not belonging to concrete family
T(3,4) = 1.71	<	*t*(0.975;10)=2.23	Belonging to concrete family
T([3,4],5) = 1.61	<	*t*(0.975;16)=2.12	Belonging to concrete family
T([3,4,5],6) = 3.54	>	*t*(0.975;22)=2.07	Not belonging to concrete family
T(6,7) = 0.17	<	*t*(0.975;10)=2.23	Belonging to concrete family
T([6,7],8) = 3.12	>	*t*(0.975;16)=2.12	Not belonging to concrete family
T(8,9) = 4.14	>	*t*(0.975;10)=2.23	Not belonging to concrete family
T(9,10) = 1.63	<	*t*(0.975;10)=2.23	Belonging to concrete family
T([9,10],11) = 0.39	<	*t*(0.975;16)=2.12	Belonging to concrete family
T([9,10,11],12) = 0.93	<	*t*(0.975;22)=2.07	Belonging to concrete family
T([9,10,11,12],13) = 1.96	<	*t*(0.975;28)=2.05	Belonging to concrete family
T([9,10,11,12,13],14) = 3.07	>	*t*(0.975;34)=2.03	Not belonging to concrete family
T(14,15) = 2.03	<	*t*(0.975;10)=2.23	Belonging to concrete family
T([14,15],16) = 2.87	>	*t*(0.975;16)=2.12	Not belonging to concrete family
T(16,17) = 2.55	>	*t*(0.975;10)=2.23	Not belonging to concrete family
T(17,18) = 1.80	<	*t*(0.975;10)=2.23	Belonging to concrete family
T*([17,18],19) = 2.97	>	*t*(0.975;16)=2.12	Not belonging to concrete family
T(19,20) = 3.21	>	*t*(0.975;10)=2.23	Not belonging to concrete family
T(20,21) = 1.10	<	*t*(0.975;10)=2.23	Belonging to concrete family
T([20,21],22) = 1.40	<	*t*(0.975;16)=2.12	Belonging to concrete family
T([20,21,22],23) = 1.67	<	*t*(0.975;22)=2.07	Belonging to concrete family
T([20,21,22,23],24) = 0.73	<	*t*(0.975;28)=2.05	Belonging to concrete family
T([20,21,22,23,24],25) = 0.02	<	*t*(0.975;34)=2.03	Belonging to concrete family
T([20,21,22,23,24,25],26) = 2.57	>	*t*(0.975;40)=2.02	Not belonging to concrete family
T(26,27) = 0.12	<	*t*(0.975;10)=2.23	Belonging to concrete family
T([26,27],28) = 1.25	<	*t*(0.975;16)=2.12	Belonging to concrete family
T([26,27,28],29) = 0.04	<	*t*(0.975;22)=2.07	Belonging to concrete family
T([26,27,28,29],30) = 1.56	<	*t*(0.975;28)=2.05	Belonging to concrete family
T([26,27,28,29,30],31) = 3.25	>	*t*(0.975;34)=2.03	Not belonging to concrete family

Table 4 Results of testing hypothesis on the attachement of the first series of concrete strength results to defined concrete family using Mann – Whitney U test for the projected concrete class C35/45

First series of compressive strength results for projected concrete class C35/45			
Value calculated for test statistics for two independent samples	Quantile value u(n;6;α) from U statistics distribution	Critical area	Attachment of two independent samples to concrete families
$U(1,2)=25$	*u*(6;6;0.025) = 6 *u*(6;6;0.975) = 30	< 0;6> u <30;+∞)	Belonging to concrete family
$U([1,2],3)=31$	*u*(12;6;0.025) = 15 *u*(12;6;0.975) = 57	< 0;15> u <57;+∞)	Belonging to concrete family
$U([1,2,3],4)=39.5$	*u*(18;6;0.025) = 25 *u*(18;6;0.975) = 83	< 0;25> u <83;+∞)	Belonging to concrete family
$U([1,2,3,4],5)=87$	*u*(24;6;0.025) = 34 *u*(24;6;0.975) = 110	< 0;34> u <110;+∞)	Belonging to concrete family
$U([1,2,3,4,5],6)=93$	*u*(30;6;0.025) = 44 *u*(30;6;0.975) = 136	< 0;44> u <136;+∞)	Belonging to concrete family
$U([1,2,3,4,5,6],7)=130.5$	*u*(36;6;0.025) = 54 *u*(36;6;0.975) = 162	< 0;54> u <162; +∞)	Belonging to concrete family
$U([1,2,3,4,5,6,7],8)=31$	*u*(42;6;0.025) = 64 *u*(42;6;0.975) = 188	< 0;64> u < 188; +∞)	Not belonging to concrete family
$U(8,9)=36$	*u*(6;6;0.025) = 6 *u*(6;6;0.975) = 30	< 0;6> u <30;+∞)	Not belonging to concrete family
$U(9,10)=12$	*u*(6;6;0.025) = 6 *u*(6;6;0.975) = 30	< 0;6> u <30;+∞)	Belonging to concrete family
$U([9,10],11)=10$	*u*(12;6;0.025) = 15 *u*(12;6;0.975) = 57	< 0;15> u <57;+∞)	Not belonging to concrete family
$U(11,12)=9$	*u*(6;6;0.025) = 6 *u*(6;6;0.975) = 30	< 0;6> u <30;+∞)	Belonging to concrete family
$U([11,12],13)=29$	*u*(12;6;0.025) = 15 *u*(12;6;0.975) = 57	< 0;15> u <57;+∞)	Belonging to concrete family
$U([11,12,13],14)=80$	*u*(18;6;0.025) = 25 *u*(18;6;0.975) = 83	< 0;25> u <83;+∞)	Belonging to concrete family
$U([11,12,13,14],15)=58.5$	*u*(24;6;0.025) = 34 *u*(24;6;0.975) = 110	< 0;34> u <110;+∞)	Belonging to concrete family
$U([11,12,13,14,15],16)=153$	*u*(30;6;0.025) = 44 *u*(30;6;0.975) = 136	< 0;44> u <136;+∞)	Not belonging to concrete family
$U(16,17)=5.5$	*u*(6;6;0.025) = 6 *u*(6;6;0.975) = 30	< 0;6> u <30;+∞)	Not belonging to concrete family
$U(17,18)=9.5$	*u*(6;6;0.025) = 6 *u*(6;6;0.975) = 30	< 0;6> u <30;+∞)	Belonging to concrete family
$U([17,18],19)=53$	*u*(12;6;0.025) = 15 *u*(12;6;0.975) = 57	< 0;15> u <57;+∞)	Belonging to concrete family
$U([17,18,19],20)=68$	*u*(18;6;0.025) = 25 *u*(18;6;0.975) = 83	< 0;25> u <83;+∞)	Belonging to concrete family
$U([17,18,19,20],21)=109.5$	*u*(24;6;0.025) = 34 *u*(24;6;0.975) = 110	< 0;34> u <110;+∞)	Belonging to concrete family
$U([17,18,19,20,21],22)=26.5$	*u*(30;6;0.025) = 44 *u*(30;0.975) = 136	< 0;44> u <136;+∞)	Not belonging to concrete family

Table 4 Continued

First series of compressive strength results for projected concrete class C35/45			
Value calculated for test statistics for two independent samples	Quantile value u(n;6;α) from U statistics distribution	Critical area	Attachment of two independent samples to concrete families
$U(22, 23) = 30$	u(6;6;0.025) = 6 u(6;6;0.975) = 30	< 0;6> u <30;+∞)	Not belonging to concrete family
$U(23, 24) = 26$	u(6;6;0.025) = 6 u(6;6;0.975) = 30	< 0;6> u <30;+∞)	Belonging to concrete family
$U([23, 24], 25) = 62$	u(12;6;0.025) = 15 u(12;6;0.975) = 57	< 0;15> u <57;+∞)	Not belonging to concrete family
$U(25, 26) = 18$	u(6;6;0.025) = 6 u(6;6;0.975) = 30	< 0;6> u <30;+∞)	Belonging to concrete family
$U([25, 26], 27) = 22$	u(12;6;0.025) = 15 u(12;6;0.975) = 57	< 0;15> u <57;+∞)	Belonging to concrete family
$U([25, 26, 27], 28) = 52$	u(18;6;0.025) = 25 u(18;6;0.975) = 83	< 0;25> u <83;+∞)	Belonging to concrete family
$U([25, 26, 27, 28], 29) = 55.5$	u(24;6;0.025) = 34 u(24;6;0.975) = 110	< 0;34> u <110;+∞)	Belonging to concrete family
$U([25, 26, 27, 28, 29], 30) = 4$	u(30;0.025) = 44 u(3;0.975) = 136	< 0;44> u <136;+∞)	Not belonging to concrete family
$U(30, 31) = 36$	u(6;6;0.025) = 6 u(6;6;0.975) = 30	< 0;6> u <30;+∞)	Not belonging to concrete family

Table 5 Results of testing hypothesis on the attachement of the second series of concrete strength results to defined concrete family using Mann – Whitney U test for the projected concrete class C35/45

Second series of compressive strength results For projected concrete class C35/45			
Value calculated for test statistics for two independent samples	Quantile value u(n;6;α) from U statistics distribution	Critical area	Attachment of two independent samples to concrete families
$U(1, 2) = 0$	u(6;6;0.025) = 6 u(6;6;0.975) = 30	< 0;6> u <30;+∞)	Not belonging to concrete family
$U(2, 3) = 36$	u(6;6;0.025) = 6 u(6;6;0.975) = 30	< 0;6> u <30;+∞)	Not belonging to concrete family
$U(3, 4) = 9.5$	u(6;6;0.025) = 6 u(6;6;0.975) = 30	< 0;6> u <30;+∞)	Belonging to concrete family
$U([3, 4], 5) = 19$	u(12;6;0.025) = 15 u(12;6;0.975) = 57	< 0;15> u <57;+∞)	Belonging to concrete family
$U([3, 4, 5], 6) = 96$	u(18;6;0.025) = 25 u(18;6;0.975) = 83	< 0;25> u <83;+∞)	Not belonging to concrete family
$U(6, 7) = 18$	u(6;6;0.025) = 6 u(6;6;0.975) = 30	< 0;6> u <30;+∞)	Belonging to concrete family
$U([6, 7], 8) = 61$	u(12;6;0.025) = 15 u(12;6;0.975) = 57	< 0;15> u <57;+∞)	Not belonging to concrete family

Table 5 Continued

Second series of compressive strength results For projected concrete class C35/45			
Value calculated for test statistics for two independent samples	Quantile value u(n;6;α) from U statistics distribution	Critical area	Attachment of two independent samples to concrete families
$U(8,9)=1$	*u*(6;6;0.025) = 6 *u*(6;6;0.975) = 30	< 0;6> *u* <30;+∞)	Not belonging to concrete family
$U(9,10)=25$	*u*(6;6;0.025) = 6 *u*(6;6;0.975) = 30	< 0;6> *u* <30;+∞)	Belonging to concrete family
$U([9,10],11)=38.5$	*u*(12;6;0.025) = 15 *u*(12;6;0.975) = 57	< 0;15> *u* <57;+∞)	Belonging to concrete family
$U([9,10,11],12)=65.5$	*u*(18;6;0.025) = 25 *u*(18;6;0.975) = 83	< 0;25> *u* <83;+∞)	Belonging to concrete family
$U([9,10,11,12],13)=36$	*u*(24;6;0.025) = 34 *u*(24;6;0.975) = 110	< 0;34> *u* <110;+∞)	Belonging to concrete family
$U([9,10,11,12,13],14)=31.5$	*u*(30;6;0.025) = 44 *u*(30;6;0.975) = 136	< 0;44> *u* <136;+∞)	Not belonging to concrete family
$U(14,15)=28.5$	*u*(6;6;0.025) = 6 *u*(6;6;0.975) = 30	< 0;6> *u* <30;+∞)	Belonging to concrete family
$U([14,15],16)=59$	*u*(12;6;0.025) = 15 *u*(12;6;0.975) = 57	< 0;15> *u* <57;+∞)	Not belonging to concrete family
$U(16,17)=4$	*u*(6;6;0.025) = 6 *u*(6;6;0.975) = 30	< 0;6> *u* <30;+∞)	Not belonging to concrete family
$U(17,18)=28.5$	*u*(6;6;0.025) = 6 *u*(6;6;0.975) = 30	< 0;6> *u* <30;+∞)	Belonging to concrete family
$U([17,18],19)=65$	*u*(12;6;0.025) = 15 *u*(12;6;0.975) = 57	< 0;15> *u* <57;+∞)	Not belonging to concrete family
$U(19,20)=3$	*u*(6;6;0.025) = 6 *u*(6;6;0.975) = 30	< 0;6> *u* <30;+∞)	Not belonging to concrete family
$U(20,21)=24.5$	*u*(6;6;0.025) = 6 *u*(6;6;0.975) = 30	< 0;6> *u* <30;+∞)	Belonging to concrete family
$U([20,21],22)=23.5$	*u*(12;6;0.025) = 15 *u*(12;6;0.975) = 57	< 0;15> u *u* <57;+∞)	Belonging to concrete family
$U([20,21,22],23)=79$	*u*(18;6;0.025) = 25 *u*(18;6;0.975) = 83	< 0;25> *u* <83;+∞)	Belonging to concrete family
$U([20,21,22,23],24)=85$	*u*(24;6;0.025) = 34 *u*(24;6;0.975) = 110	< 0;34> *u* <110;+∞)	Belonging to concrete family
$U([20,21,22,23,24],25)=87.5$	u(30;6;0.025) = 44 *u*(30;6;0.975) = 136	< 0;44> *u* <136;+∞)	Belonging to concrete family
$U([20,21,22,23,24,25],26)=177.5$	*u*(36;6;0.025) = 54 *u*(36;6;0.975) = 162	< 0;54> *u* <162;+∞)	Not belonging to concrete family
$U(26,27)=17$	*u*(6;6;0.025) = 6 *u*(6;6;0.975) = 30	< 0;6> *u* <30;+∞)	Belonging to concrete family
$U([26,27],28)=50$	*u*(12;6;0.025) = 15 *u*(12;6;0.975) = 57	< 0;15> *u* <57;+∞)	Belonging to concrete family
$U([26,27,28],29)=56$	*u*(18;6;0.025) = 25 *u*(18;6;0.975) = 83	< 0;25> *u* <83;+∞)	Belonging to concrete family

Table 5 Continued

Second series of compressive strength results For projected concrete class C35/45			
Value calculated for test statistics for two independent samples	Quantile value u(n;6;α) from U statistics distribution	Critical area	Attachment of two independent samples to concrete families
$U([26, 27, 28, 29], 30) = 99$	$u(24;6;0.025) = 34$ $u(24;6;0.975) = 110$	< 0;34> *u* <110;+∞)	Belonging to concrete family
$U([26, 27, 28, 29, 30], 31) = 28.5$	$u(30;6;0.025) = 44$ $u(30;6;0.975) = 136$	< 0;44> *u* <136;+∞)	Not belonging to concrete family

Table 6 Strength characteristics of the entire set of the compressive strength results of concrete and identified concrete families for the first series of results (projected concrete class C35/45)

Projected concrete class (C35/45)	The entire set of results for the compre-ssive strength of concrete	Concrete families identified										
		1	2	3	4	5	6	7	8	9	10	11
Number of individual observations in set	186	42	6	12	30	6	30	6	12	30	6	6
Average strength	48.83	49.3	44.7	52.0	47.3	51.0	47.1	43.3	48.8	52.4	46.0	50.3
Standard deviations	3.81	3.6	2.3	2.8	2.2	1.7	3.1	2.3	3.6	3.0	1.8	1.6
Actual characteri-stic strength for individual sets	42.56	43.4	40.9	47.4	43.7	48.2	42.0	39.5	42.9	47.5	43.0	47.7
Projected characteri-stic strength for concrete class C35/45	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0
Comparison of actual and projected data	42.56 < 45.0	43.4 < 45.0	40.9 < 45.0	47.4 > 45.0	43.7 < 45.0	48.2 > 45.0	42.0 < 45.0	39.5 < 45.0	42.9 < 45.0	47.5 > 45.0	43.0 < 45.0	47.7 > 45.0
Defining concrete class for the entire set of results or individual concrete family	C 30/37	C 30/37	C 30/37	**C 35/45**	C 30/37	**C 35/45**	C 30/37	C 30/37	C 30/37	**C 35/45**	C 30/37	**C 35/45**

Table 7 Strength characteristics for the entire set of the compressive strength results of concrete and identified concrete families for the second series of results (projected concrete class C35/45)

Projected concrete class (C35/45)	The entire set of results for the compressive strength of concrete	Concrete families identified												
		1	2	3	4	5	6	7	8	9	10	11	12	13
Number of individual observations in set	186	6	6	18	12	6	30	12	6	12	6	36	30	6
Average strength	50.38	46.7	38.0	47.9	52.2	56.0	50.1	48.7	53.7	50.0	58.7	49.4	54.1	49.3
Standard deviations	4.56	1.5	2.5	2.4	2.2	2.5	2.5	3.2	3.7	1.9	6.6	3.1	3.0	3.3
Actual characteristic strength for individual sets	42.88	44.2	33.9	44.0	48.6	51.9	46.0	43.4	47.6	46.9	47.8	44.3	49.2	43.9
Projected characteristic strength for concrete class C35/45	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0
Comparison of actual and projected data	42.88 < 45.0	44.2 < 45.0	33.9 < 45.0	44.0 < 45.0	48.6 > 45.0	51.9 > 45.0	46.0 > 45.0	43.4 < 45.0	47.6 > 45.0	46.9 > 45.0	47.8 > 45.0	44.3 < 45.0	49.2 > 45.0	43.9 < 45.0
Defining concrete class for the entire set of results or individual concrete family	C 30/37	C 30/37	C 25/30	C 30/37	**C 35/45**	**C 40/50**	**C 35/45**	C 30/37	**C 35/45**	**C 35/45**	**C 35/45**	C 30/37	**C 35/45**	C 30/37

case of identical observations in samples $\mathbf{X}$ and $\mathbf{Y}$, a correction must be applied by adding to value of U half of the number of pairs (x, y) such that $x = y$. The results of testing hypotheses on the attachment of the series of concrete compressive strength results to defined concrete family using Mann-Whitney U test are presented in table 4 and 5.

The division of the first series of tests results for the compressive strength of concrete into individual concrete families (assumed class C35/45) is shown in Fig. 2. Strength characteristics of the entire set of test results for the compressive strength of concrete and separated concrete families are presented in Table 6. In the first graph additional line of tolerance was applied on the level of assumed characteristic strength of concrete totaling $f_{ck} = 45.00$ MPa.

The division of the second series of the results for the compressive strength of concrete into individual concrete families (projected class C35/45) is shown in Fig. 3. Strength characteristics for the entire set of test results for the compressive strength of concrete and concrete families obtained are shown in Table 7. In the first graph additional tolerance line was applied on the level of the assumed characteristic strength of concrete equaling $f_{ck} = 45.00$ MPa.

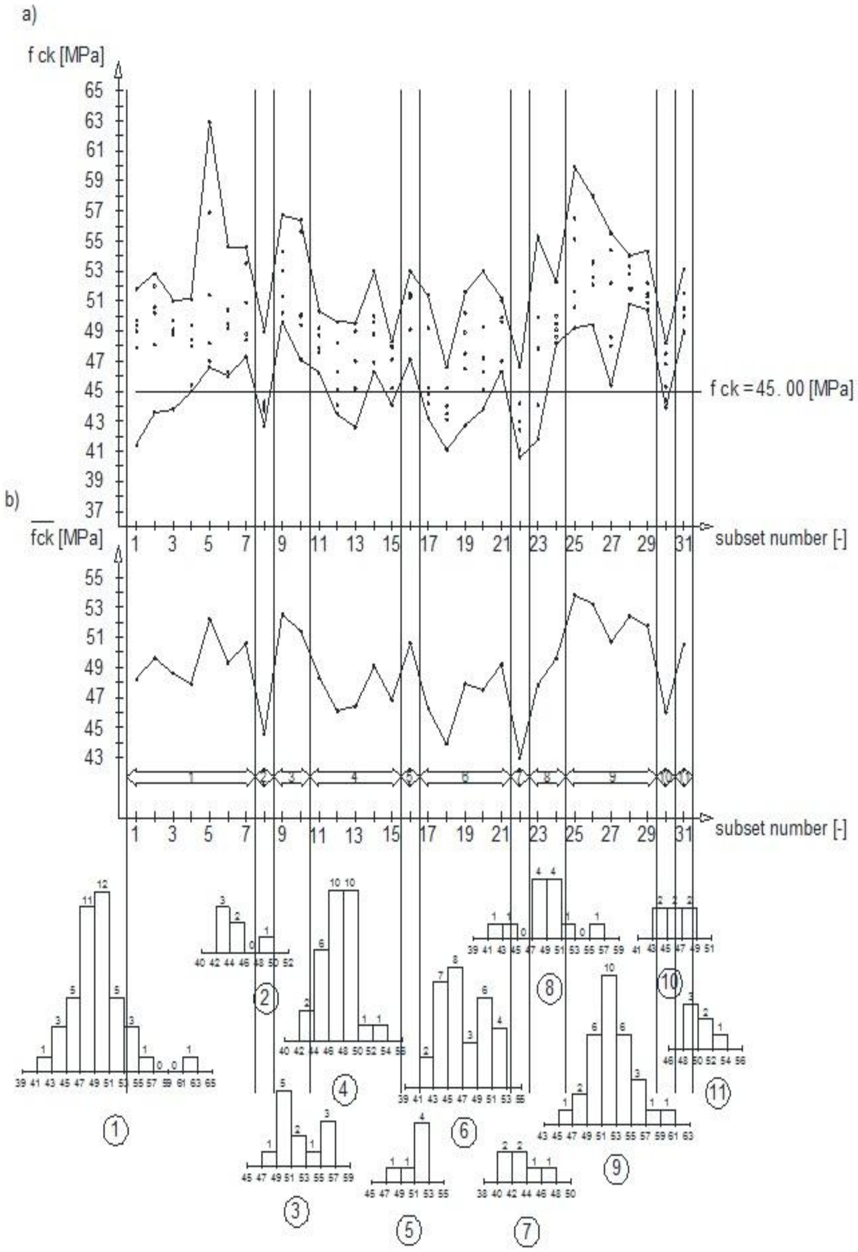

Fig. 2 Division of the first series of results for the compressive strength of concrete into individual concrete families (assumed class C35/45)

6. Conclusions

6.1. To determine the strength parameters of the entire set of results for the compressive strength of concrete, histograms for numerosities were used. Statistical assessment of the tested sample was achieved without reference to the time of production and parameters representing the entire data set were defined. The assignment of all the results to one class of concrete makes it impossible to present their variability over time. With this estimation it turned out that the whole set of results does not allow for qualifying the manufactured concrete to the strength class C35/45 but only to the lower class C30/37.

6.2. While analyzing the series of individual strength results it can be noticed that the strength is subject to significant volatility during the successive production processes. Thus specific concrete families were identified, i.e. the series of results for the compressive strength of concrete were divided into groups with statistically stabilized strength parameters. That is why there was a need to use appropriate calculation procedures based on verification of the assumed statistical hypotheses.

6.3. The conclusion from the statistical analysis of the two sample series of concrete strength test results is that in the tested period of 31 days significant volatility of the compressive strength can be observed. It is so clear (considerable hikes of average strength values for individual 6-element subsets) that there is a need to divide the series of results with focus on the attachment to one population. After verification of statistical hypotheses it turned out that the first set of results for the compressive strength of projected class C35/45 is composed of 11 independent concrete families, four of which fulfill the requirements of the assumed strength class and seven of which do not fulfill these requirements (meeting the requirements of a lower class C30/37). The second set of results for concrete of the same projected class C35/45 is composed of as many as 13 independent concrete families, seven of which meet the requirements of the projected strength class (fulfill even higher class requirements) and six of which do not meet these requirements. The statistical analysis performed showed that due to this the division of study results into concrete families is economically justified – a part of the batch fulfilling the strength requirements does not cause the disqualification of all structure elements as it had place in the case of the statistical evaluation of the whole set (conclusion 6.1). The need for time factor in determining concrete families is beyond dispute - the stabilization of concrete strength parameters should be sought in real-time interval in the assumed conditions of concrete production.

6.4. The division methods proposed by the authors of this article and applied in the presented examples for the series of concrete strength test results lead to very similar effects, even though they do not always identify identical batches. The statistical methods presented in the article can certainly be applied also in the case where the series of values for the compressive strength of concrete assigned to each of the working plots are not of the same numerosity.

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