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A method for the non-linear and failure load analysis of reinforced concrete frames

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Abstract. Modern trend in structural design is to use smaller elements in order to ensure several purposes such as economy, functionality and aesthetic in appearance. However, because of decreasing rigidity of the structural elements, the system displacements increases and displacements become an important subject in this kind of structures takes into account both geometrical changes and the carrying capacity of the material after linear-elastic boundary. In this study, a method is proposed to calculate the failure loads and to analyse the reinforced concrete space frame systems. The numerical examples gathered from the literature survey are solved with this method utilising the prepared computer program and the comparable results are presented. The results show that the method is sufficiently accurate.

Keywords: Non-linear analysis; RC frames; collapse loads; compensating loads; second order effect

1. Introduction

In the last quarter of the last century, rapidly evolving computer technology has been enabled enough to develop nonlinear calculation methods and their usage in application. RC structural members show a linear behavior under increasing loads and it is possible to build slender structures more economical by using elasto-plastic analysis methods considering the bearing capacity after linear elastic boundary. Economical and sufficient reliable solutions can be obtained with the use of nonlinear analysis methods in order to determine the actual behaviour of structures subjected to external influences.

A number of researches (Wen and Farhoomand 1970, Otani 1980, Argyris *et al*. 1982, Mo and Yuh 1989, Pagnoni *et al*. 1992, Izzuddin and Smith 2000, Mwafy and Elnashai 2001, Barros and Almeida 2005, Habibi and Moharrami 2010, Quaranta *et al*. 2012, Stramandinoli *et al*. 2012) have been carried out to investigate the nonlinear analysis of RC frame systems. Habibi and Moharrami (2010) studied nonlinear sensitivity analysis of reinforced concrete frames. It was proposed that an analytical sensitivity technique for reinforcement concrete moment resisting frames (RCMRF) accounts for both material nonlinearity and geometric effects under pushover analysis. In another study, Mwafy and Elnashai (2001) conducted a static pushover versus dynamic collapse analysis

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of RC buildings, where the applicability and accuracy of inelastic static pushover analysis in predicting the seismic response of RC buildings were investigated. A meshfree method for nonlinear analysis of two-dimensional reinforced concrete structures subjected to monotonic static loading was presented by Quaranta *et al*. (2012). Here, the concrete model was implemented in the context of the smeared rotating crack approach. The proposed meshfree methodology was used to study the nonlinear behavior of reinforced concrete shear walls. A new study by Stramandinoli *et al*. (2012) illustrated the development of a FE model for nonlinear analysis of reinforced concrete beams considering shear deformation. The model was implemented into a computer program named as ANALEST, developed by the authors, which allows for material and geometrical nonlinear analysis of RC beams and frames. In an earlier work presented by Pagnoni *et al*. (1992), a nonlinear three-dimensional analysis of reinforced concrete based structures on a bounding surface model was performed. The work addressed the formulation of finite element procedure for the nonlinear analysis of general three-dimensional reinforced concrete structures. Similarly, a finite element analysis of two- and three-dimensional elasto-plastic frames was investigated by Argyris *et al*. (1982). In the study, limit state functions applied to different loading conditions and frame geometries were implemented into a computer program and a number of test examples illustrated practical applications.

Another study exemplifying the nonlinear analysis of RC frames carried out by Izzuddin and Smith (2000) showed the use of adaptive analysis concepts. The first two components of the proposed adaptive method, namely the elastic and elasto-plastic beam–column formulations, were described in details.

Various researchers developed models, methods and/or algorithms for advanced analysis of RC frames (Uzgider 1980, Arnesen *et al*. 1980, Saka and Ulker 1992, Dinno and Mekha 1993, 1995, Thanoon *et al*. 2004, Krätzig and Pölling 2004, Arslan 2007, Lepage *et al*. 2010, Akhaveissy and Desai 2012, Birely *et al*. 2012). Thanoon *et al*. (2004) investigated the infuence of torsion on the inelastic response of three-dimensional RC frames. In this study, a three-dimensional reinforced concrete framed building was modelled by using finite element method. Recently, Akhaveissy and Desai (2012) made an application of the DSC (disturbed state concept) model for nonlinear analysis of reinforced concrete frames, in which a nonlinear finite element method with eight-noded isoparametric quadrilateral elements for concrete and two-noded elements for reinforcement was used to predict the behavior of reinforced concrete frame structures. Similar to this study, Krätzig and Pölling (2004) developed an elasto-plastic damage model for reinforced concrete with minimum number of material parameters, where the material behavior of all reinforced concrete components -concrete, reinforcement and bond-, based on a fully 3-D elasto-plastic damage theory, was realistically modelled for biaxial loading including cyclic action. Different models for nonlinear seismic response of reinforced concrete frames were presented by Lepage *et al*. (2010). The proposed models can accurately identify the optimal combination of hysteresis-modeling and damping parameters for use in practical nonlinear dynamic analysis to obtain satisfactory correlations in both amplitude and waveform between the calculated and measured seismic response of reinforced concrete frames. Additionally, a new model was developed by Birely *et al*. (2012) to simulate the nonlinear response of planar reinforced-concrete frames including all sources of flexibility. In the study, conventional modeling approaches consider only beam and column flexibility by using concentrated plasticity or springs to model this response. A sensitivity of the Drucker–Prager modeling parameters in the prediction of the nonlinear response of reinforced concrete structures was investigated by Arslan (2007). The Drucker–Prager yield criterion was evaluated by using finite element simulation with optimum mesh size for reinforced concrete beams having different geometrical and material properties. Saka and Ulker (1992) developed a structural optimization algorithm for geometrically nonlinear three-dimensional trusses subjected to displacement, stress and cross-sectional area constraints. This method was obtained by coupling the nonlinear analysis technique with the optimality criteria approach. The advantage of the proposed algorithm is that it takes into account the realistic behaviour of the structure, where an optimum design might lead to erroneous result without it. In a similar way, Dinno and Mekha (1995) developed an algorithm for the inelastic analysis of reinforced concrete frames. In the study, a typical reinforced concrete frame was analyzed under different load combinations to show the features of the developed algorithm. They presented (1993) elsewhere a new algorithm in performing inelastic analysis for the optimal design of RC frames. In another work, Arnesen *et al*. (1980) developed two different computer programs for nonlinear analysis of reinforced concrete structures. The first program handles plane stress problems. Flow theory of plasticity was used in modelling of concrete and reinforcement. The second program was developed for analysis of plates and shells. Endochronic theory was used in the constitutive law for concrete whereas an overlay model was utilized for the reinforcement. The plane stress program was used for analysis of a beam and two different corbels, while the shell program has been applied to a square plate and a shell with geometric nonlinearities. Meanwhile, Uzgider (1980) developed a numerical method for the analysis of three-dimensional frames loaded dynamically into the inelastic range. The elasto-plastic force-deformation behaviour at the ends of the frame members was represented by an equation corresponding essentially to the inverse of the Ramberg-Osgood representation. The system stiffness matrix was formulated as a tangent stiffness by taking into account P-Δ effect. The inelastic interaction of biaxial end moments and axial force were included for the elasto-plastic behaviour of the frame members.

In this study, a method is proposed to calculate 3D RC frame systems under increasing loads and to determine failure load by considering nonlinear moment-curvature relationship and second order effects.

2. Material and method

2.1 Equation of bending moment-curvature in RC cross-sections

Experimental studies conducted by researchers indicated equation of bending moment-curvature of RC cross sections as shown in Fig. 1 (Çelik 1977 and Deeble 1973). This diagram shows a linear feature in region *OA*. Outside fibers under the tensile stress initiate cracking around point *A*. When these cracks evolve, bending stiffness is reduced till to the middle point *B* and shows a variation close to a line. Scaled sections made based on elastic calculations are seen in this region. When the outermost tension reinforcement reaches the limit of yield stress, bending stiffness reduces rapidly making M-φ curve between *B*-*C* close to a horizontal shape. In this region, the reason of a slight increase in moment is due to a shift towards neutral axis to compression zone which leads an increase in moment arm and hardening in tension reinforcement. Leading cause of the collapse of the section which is in the limit positions is the concrete crush in compression zone (point *D*). In the meantime, a small amount of unloading is also observed between *C* and *D* points.

Fig. 1 shows that the bending moment-curvature diagram for RC cross sections can be idealized in three line segments as in Fig. 2.

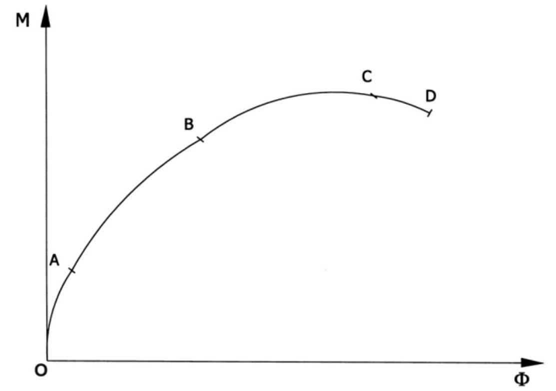

Fig. 1 The diagram of bending moment-curvature (*M*-φ) of RC cross sections

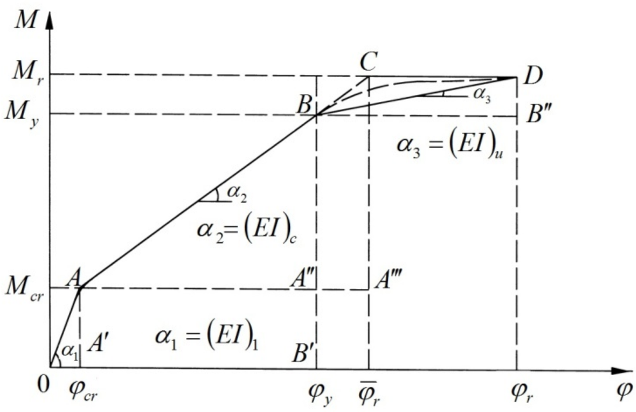

Fig. 2 Diagram (*M*-φ) idealized for RC cross-sections

When it comes to defining the parameters specified in the idealized diagram in Fig. 2; moment and curvature values when the first crack occurs in cross-section are donated as M_{cr}, φ_{cr} (the coordinates of point *A)*; bending stiffness of uncracked cross-section is $(EI)_1$; bending moment and curvature values when outmost tension reinforcement reaches yield stress are M_y, φ_y (the coordinates of point *B)*; bending stiffness of cracked cross-section is $(EI)_c$ (gradient of AB line segment); in addition, limit value of bending moment and curvature are M_r, φ_r (the coordinates of point *D)*; bending stiffness of outermost tension reinforcement after it starts yielding is $(EI)_u$ (gradient of *BD* line segment)*;* and notional curvature at point *C* is $\bar{\varphi}_r$ (Çelik 1980).

Assumptions considering an analysis made for obtaining failure loads of RC frame;

- Concrete in the compression zone is elastic till it reaches to the yield stress of outmost tension bar.
- Till cracks occur, bending stiffness of uncracked cross-sections $(EI)_1$ remains constant.
- Cracks begin to occur when the concrete reaches to tensile strength.
- Curvature is merely the function of bending moment; and normal force is assumed to be zero (only to be valid for beams).

2.1.1 Uncracked situation

Bending stiffness of uncracked cross-section $(EI)_1$ is obtained by multiplying modulus of elasticity of concrete E_c with total moment of inertia (steel bar included) of the section, I_G.

$$(EI)_1 = E_c\, I_G \tag{1}$$

When the first crack occurs, the bending moment is;

$$M_{cr} = f_{ctk}\, w_e \tag{2}$$

In these expressions, f_{ctk} represents tension strength of the concrete, w_e represents the moment of resistance of uncracked cross-sections. Curvature at the time of cracking is;

$$\varphi_{cr} = \frac{M_{cr}}{(EI)_1} \tag{3}$$

2.1.2 Cracked situation

Due to presense of uncracked areas in cracked areas, the behavior of structural members is quite complex (Fig. 3). Bending stiffness varies from point to point with a sudden change which makes exact calculation impossible in theory. To simplify the solution of the problem and to allow practical application, an empirical formula offered by Monnier (1970) is suitable to calculate bending stiffness value $(EI)_c$ in this region. In the study conducted by Monnier, value of $(EI)_c$ is expressed as follows; effective height of beam is (d_1), steel bar ratio is (ρ) and section thickness is (b) and finally the equation is denoted as;

$$(EI)_c = (-2.5\,\rho^2 - 19.9\,\rho - 1.1)\, b\, d_1^{\,3}\; 10^6\;\; Nmm^2 \tag{4}$$

Eq.4 indicates that steel bar diameter and quality of concrete have a slight effect on $(EI)_c$ hence it can be neglected. For normal RC cross-sections, effective height d_1 may be taken as the distance between tension bar and compression hedge. For multi-row rebar sections, effective height d_1 may be taken as the distance between the centre of gravity of tension forces and the compression hedge. Cracked position continues till point *B* as in Fig. 2. This point symbolizes the time when the outmost tension reinforcement begins yielding. As it can be seen from Eq.4, the behavior of this section in *AB* region is accepted as linear.

Assuming that concrete in compression hedge is still elastic and that M_y and φ_y values determine point *B* and if $n = \dfrac{E_s}{E_c}$ and $\rho = \dfrac{A_s}{bd}$, then bending moment when tension reinforcement reaches yield stress is obtained as;

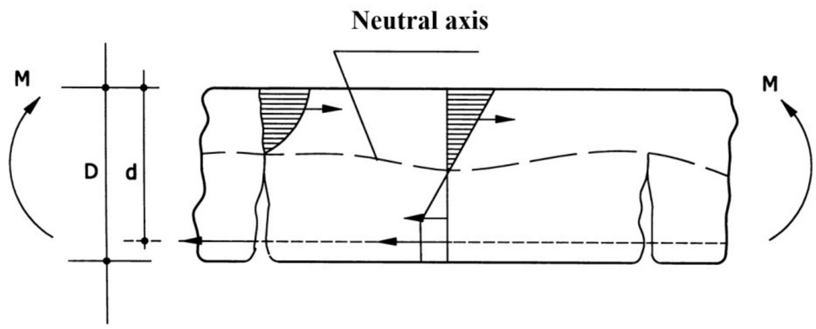

Fig. 3 General view of a cracked RC element

$$M_y = f_y \rho b d^2 \left\{ 1 - n\rho / 3 \left(\sqrt{1 + \frac{2}{n\rho}} - 1 \right) \right\} \tag{5}$$

and from Fig. 2, curvature is obtained as;

$$\varphi_y = \varphi_{cr} + \frac{M_y - M_{cr}}{(EI)_c} \tag{6}$$

Here A_s and f_y refer to area of tension bar and yield stress of tension bar, respectively.

2.1.3 Limit position

Limit position infers the position of RC cross section reaching bearing capacity. Maximum moment that a section can bear is obtained with the help of M_r and limit curvature, φ_r iteratively equivalence and compatibility conditions. In due course of these processes distribution of compression stresses in concrete, resultant of these stresses and the place of the resultant are determined by β_1, β_2 parameters. These parameters and ultimate strain of concrete can be calculated in terms of concrete cube strength with the help of emprical formulas provided by Hognestad (1955);

$$\beta_1 = 0.5 - \frac{f_{cu}}{700} \tag{7a}$$

$$\beta_2 = \frac{27 + 0.27 f_{cu}}{28.3 - f_{cu}} \tag{7b}$$

$$\varepsilon_{cu} = 0.004 - \frac{f_{cu}}{57000} \tag{7c}$$

in which, it is taken into account that $\sigma - \varepsilon$ diagram of the bar is not linear.

When an equation of equilibrium is written along rod axis, it will be as;

$$C = T \rightarrow \quad \beta_2\, f_{cu}\, b\, x = \rho\, b\, d\, f_y \rightarrow \quad x = \frac{\rho\, d\, f_y}{\beta_2\, f_{cu}} \tag{8}$$

From the the general similarity of triangle in strain diagram, compatibility requirement is obtained as;

$$\frac{\varepsilon_{cu}}{x} = \frac{\varepsilon_s}{d - x} \quad \rightarrow \quad \varepsilon_s = \varepsilon_{cu}\, \frac{d - x}{x} \tag{9}$$

If the location of neutral axis location is x, then it can be determined by iterative approach as follows;

1. β_1, β_2 and ε_{cu} calculated from Eqs. (7a), (7b), (7c) and it is assumed $x = x_0$
2. ε_s is computed from Eq.9.
3. New value of x is computed as x_n from Eq.8.
4. New and old values of x are compared. If the difference is less than a certain tolerance, iteration is terminated. Otherwise, it is assumed that $x = x_0$ and the iteration is continued starting from the second step. After the calculation of x, moment bearing capacity of the section, M_r is calculated from moment equivalance requirement as;

$$M_r = \rho\, b\, d\, f_y\, (d_1 - \beta_1\, x) \tag{10}$$

Limit curvature is obtained from $\sigma - \varepsilon$ diagram of the steel as;

$$\varphi_r = (\varepsilon_{cu} - \varepsilon_s) / d \tag{11}$$

When $M - \varphi$ diagram in Fig.2 is idealized as *OACD*, then $\overline{\varphi}_r$ at the against point of *C* is obtained in the same way as φ_y obtained (Eq.6) which is calculated by the equation below;

$$\overline{\varphi}_r = \varphi_{cr} + \frac{M_r - M_{cr}}{(EI)_c} \tag{12}$$

2.2 Compensating loads

A change of stiffness of an element in isostatic structural systems does not have impact on either internal force of that element or other elements but on deformations just because internal force in this type of structures can be calculated only by equation of equilibrium. If the structure system is not determined statically (hyperstatic system), a change in any element's stiffness leads to internal force deformation changes both in that element and other elements of structure. In this kind of systems if it is not required to change internal force and deformation positions of other elements in the structure, although rod stiffness is changed, 'Compensating Loads' are used at joint points of the rod to eliminate the effect of changing stiffness (Çelik 1982). It is possible to define compensation loads as the equivalent force of rod *I*'s stiffness variation.

When a space frame rod is taken into consideration, the rod has three rotation freedoms and three drift freedoms at each end. These freedoms which are 12 in total for two members end correspond to 12 member end forces. Assuming that *i* rod section, which is under the influence of external forces and belongs to a system whose member end force and displacement position is known, is changed. With the change of rod section, characteristics of the section (A^1 cross-section area, I_1^1, I_2^1 moment of inertia, J^1 torsional moment of inertia) will get their new values.

If the section is assumed to become smaller, expressions below are obtained.

$$A^1 = A - \delta A; \quad \alpha_1 = -\delta A / A; \quad A^1 = (1+\alpha_1) A \tag{13}$$

$$I_1^1 = I_1 - \delta I_1; \quad \alpha_2 = -\delta I_1 / I_1; \quad I_1^1 = (1+\alpha_2) I_1 \tag{14}$$

$$I_2^1 = I_2 - \delta I_2; \quad \alpha_3 = -\delta I_2 / I_2; \quad I_2^1 = (1+\alpha_3) I_2 \tag{15}$$

$$J^1 = J - \delta J; \quad \alpha_4 = -\delta J / J; \quad J^1 = (1+\alpha_4) J \tag{16}$$

In these expressions $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ rates can be given as change ratio of the section characteristics. If the characteristic values of new section are replaced in rod's stiffness equations provided that member end displacements remain constant, new member end forces of the rod can be obtained in terms of old member forces and change ratio of the section characteristics. A change in forces of the rod will change the balance of joint. When joint points make additional displacements with other rods combined to them as well as rod *i* there will be moment increment providing balance. This event also means that system's status of internal forces and deformations will change. It is obvious that compensating loads will be the variant between old members and new end forces of rod *i*. In this case, it is possible to obtain compensating load vector by subtracting new member end forces vector, which is computed after section characteristic change, from old member end forces vector.

If the vectors below are defined as;

$\underline{P}_i$: Unchanged rod's end forces vector

$\underline{P}_{Yi}$: Changed rod's end forces vector

Then it can be written as;

$$\underline{P}_i = \underline{P}_{Yi} + \underline{P}_{Ti} \tag{17}$$

From this equation, compensating load vector $\underline{P}_{Ti}$ is obtained as;

$$\underline{P}_{Ti} = \underline{P}_i - \underline{P}_{Yi} \tag{18}$$

Besides, if $\underline{P}_i$ and $\underline{P}_{Yi}$ are substituted in Eq.18, it is simply;

$$\underline{P}_{Ti} = -\alpha_i \, \underline{P}_i \tag{19}$$

where equations obtained for $\underline{P}_{Ti}$ is for local axis of rod i and by multiplying this equation with a transformation matrix should be expressed in global coordinate system in application.

3. Addition of compensating loads to computer program

Compensating loads should be added to the computer program (Çoşgun, 2001) which is developed to obtain computation under increasing loads and failure loads of space frame systems considering their nonlinear moment-curvature relationship and second order effects. For this, if it is assumed that at a certain stage ($\lambda = \lambda_{cri}$ status) loading which has impact on structure systems has reached critic moment M_{cr} on any rod, then bending stiffness changes from $(EI)_c$ to $(EI)_y$. Solution at $\lambda = \lambda_{cri}$ should be equal to other solutions at $\lambda = \lambda_{cri} - \delta\lambda$. However, solutions which are found at new and old stiffness will be different. This problem can be solved when the load equivalent of difference between $(EI)_c$ and $(EI)_y$ is acted to the joint points where the rod combines.

If stiffness value is assumed to become smaller as $\alpha = \dfrac{EI_c - EI_y}{EI_c}$ as (20)

Then new stiffness becomes;

$$(EI)_y = (1-\alpha)(EI)_c \tag{21}$$

As $\underline{Z}$ defines end deflection vector, $\underline{k}$ defines stiffness matrix and $\underline{P}$ defines member end force vector, then before change EI is like;

$$\underline{P} = \underline{k} \cdot \underline{Z} \tag{22}$$

And after change EI, when $\underline{Z}$ is constant, it will be;

$$\underline{P}' = (1-\alpha)\underline{k}\,\underline{Z} \tag{23}$$

From Eq.21 and Eq.22, it can be written as,

$$\underline{P}' = (1-\alpha)\underline{P} \tag{24}$$

To protect the values of other parts of the structure from this change, the loads below is;

$$-(\underline{P} - \underline{P}') = -\alpha\,\underline{P} = -\alpha\left\{Q \; M_{ij} \; M_{ji}\right\} \tag{25}$$

or, in global coordinates loads below it should be acted with external loads to joint points where i

and j combines with the rod as follows;

$$-(\underline{P}-\underline{P}')=-\alpha\,\underline{P}=\left\{-\alpha m_p Q_{ji} \quad \alpha l_p Q_{ij} \quad \alpha\, M_{ij} \quad \alpha m_p Q_{ij} \quad -\alpha l_p Q_{ij} \quad \alpha\, M_{ji}\right\} \tag{26}$$

When compensating loads are taken into consideration, solution at $\lambda = \lambda_{cr}$ which is found by using changed EI coincides with the solution found by using unchanged EI.

3.1 A proposed algorithm for failure load analysis of reinforced concrete frames

The principle of the proposed method is to obtain frame's entire load-deflection relation passing from one critical point to another on moment-curvature diagram until failure occurs for a spesific load factor. The method is similar to elastic-fully plastic analysis method applied to steel frame, but plastic hinge used for the method in question is changed with critical point concept in presented method. The critical point is defined as the intersection point of two linear segments at moment-curvature diagram. When a critical point of any member on the frame is reached, bending stiffness of that rod, EI, is applied to gradient of next linear segment of rod's M-φ diagram. The lowest load factor necessary to reach next point on any rod is obtained by extrapolation. Loads in this load factor are calculated and they are attiributed to the frame. Finally, accurate value of this load factor is computed by iteration. Similar procedure is carried out for the next critical point. Algorithm based on computer program made for this kind of analysis is as follows;

Step 1) At the beginning, frame is considered to be elastic. The stiffness matrix is obtained by using EI values of the rods and it is assumed that normal forces are zero.

Step 2) Joint equation of equilibrium, $\underline{L}=\underline{K}\cdot\underline{X}$, is solved for a given load factor λ_1 and $\underline{X}$ is obtained as joint displacement. Then, rod end forces are obtained from $\underline{P}=\underline{k}\cdot A\cdot\underline{X}$. Effective moments are derived for each rod by means of end moments. Here, $\underline{A}$ denotes transformation matrix, $\underline{P}$ denotes end forces vector.

Step 3) Load factor, necessary for effective moment of each rod to reach next critical point on M-φ diagram of that rod, is computed by linear extrapolation. The smallest load factor (λ) is the load factor expected to be obtained first on the frame. This load factor is applied to frame as new load factor and load acting on the frame, $\underline{L}$ is computed.

Step 4) Normal forces having effect on rods under λ are obtained by extrapolation of values at Step 2. Newly found values are used at calculation of stability functions ϕ of each rod and new stiffness matrix, $\underline{K}$ is obtained with the help of stability functions and current EI values.

Step 5) $\underline{L}=\underline{K}\cdot\underline{X}$ equation set calculated for λ is solved. The calculated joint displacements $\underline{X}$ are used in calculations of new moments and normal forces. These obtained moments will be used to compute new effective moments and normal forces will be used in computing new stability functions.

Step 6) The process is repeated starting from Step 3. When λ_j is calculated from iteration j and the relative difference between this value and the value of iteration calculated previously is lower than a certain tolerance, which means that the critical point at any rod is reached, for instance rod k.

Step 7) As soon as effective moment on rod k reaches a critical point on M-φ diagram of the rod, bending stiffness EI_k of the rod is equal to the curve of linear region which is right after that point. Making use of new and old values of EI_k change ratio is calculated as follows;

$$\alpha_k = \frac{(EI)_{new} - (EI)_{old}}{(EI)_{old}} \tag{27}$$

Besides, compensating loads which correspond to this ratio is obtained from Eq.24.

To determine the new critical point, the load factor is increased slightly. External loads corresponding to this are calculated. While new compensating loads are added to external loads, old ones are removed. Normal forces are kept constant for the first iteration and the search of new critical point continues starting from third step. Whenever the critical point is reached, determinants of global stiffness matrix are calculated by using new bending stiffness of rod k. If the mentioned determinant is negative, the iteration is stopped. This situation determines that the structure is collapsed. Calculation can be made for any load factor, as well as collapse. This load factor is the multiplier of constant external loads acting on structure. Internal forces and displacements occur when the iterations brought to a halt are the solution of structure under given loads.

4. Numerical example

By means of computer program developed by considering presented calculation steps, failure load of six-storey RC space frame system under the influence of instant vertical loads and increasing horizontal loads will be determined by elasto-plastic calculation analysis according to second-order theory. Floor formwork plan of observed structure is given in Fig. 4, dimensions of beam and column cross-section are given in Tables 1 and 2, and properties of used cross section are given in Table 3 (Girgin, 1996).

Longitudinal reinforcement of beam and column of RC space frame system at X ve Y directions are given in Tables 4-6. It is adopted that all beams have 2ϕ12 web reinforcement. It is taken into consideration that there are three different regions at placement of beam reinforcement. Diameter/spacing of transverse reinforcement at beams and columns is taken as ϕ10/200 (at densification regions ϕ10/100).

Vertical load values acting on beams in both directions of structure are given in Table 7. Loads act on the system *1m*. intervals and the horizontal loads acting on the structure in two directions are given in Fig. 5.

By means of the proposed method, failure load of reinforced concrete space frame system under the influence of constant vertical loads and increasing horizontal loads is obtained as $\lambda=1.562$ by elasto-plastic calculation analysis according to second-order theory. The locations of 50 sections with plastic hinges occurred before the failure are illustrated in Fig. 6 (due to overlap, all plasticization couldn't be demonstrated in the figure).

Failure load factor with vertical load factor where the initial plastic section occurs is obtained as a result of analysis of the system of structure under the influence of vertical loads and horizontal loads acting in both directions and it is compared with calculations made by Girgin (1996) formerly which is presented in Table 8.

Load factor-horizontal deflection curve (λ-δ) which is obtained for X direction under the influence of constant vertical and increasing horizontal loads is given in Fig. 7. The difference between value of failure load (curve number II) calculated by Girgin (1996) and value obtained from presented study (curve number I) remains at the order of 1.7%.

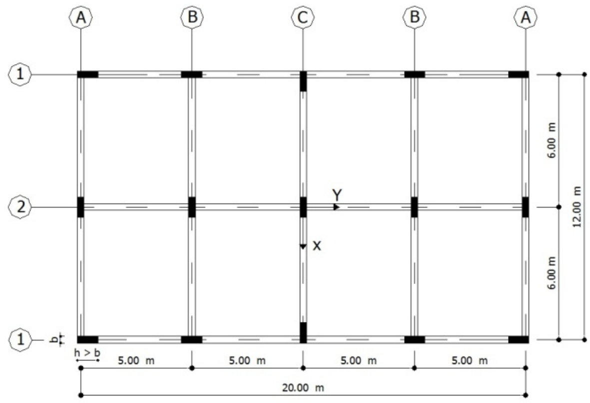

Fig. 4 Floor formwork plan of six-storey RC system

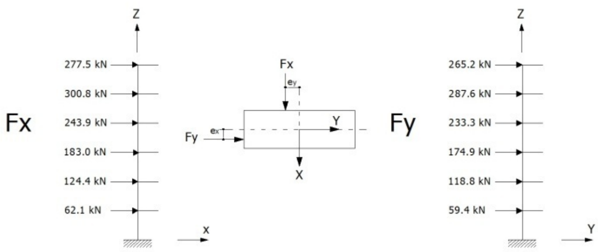

Fig. 5 Horizontal loads acting on the structure

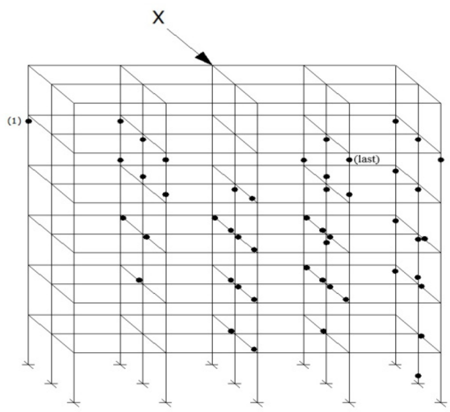

Fig. 6 Locations of plastic sections

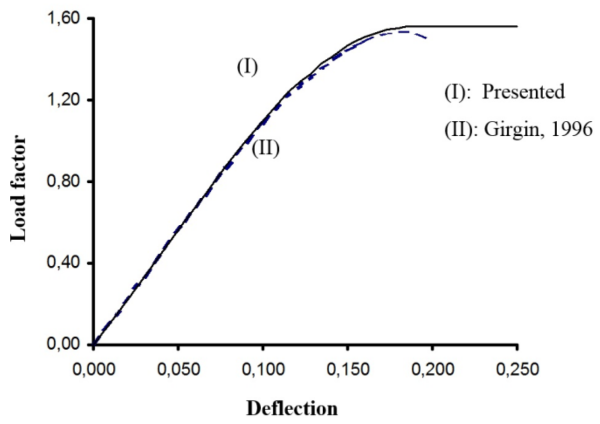

Fig. 7 Horizontal load factor–horizontal deflection curve (λ-δ)

Table 1 Beam cross-section dimensions

Storeys	Axle	h (*mm*)	b (*mm*)	b_w (*mm*)	h_f (*mm*)
1-6	A	600	660	300	150
	B,C	600	1020	300	150
	1	600	600	300	150
	2	600	900	300	150

Table 2 Column cross-section dimensions

Storeys	*mm/mm*	A1	B1	C1	A2	B2	C2
6-5	h/b	400/300	400/300	400/300	400/300	400/300	400/300
4-3	h/b	400/300	500/300	500/300	500/300	600/300	600/300
2-1	h/b	400/300	600/300	600/300	600/300	800/300	800/300

Table 3 Properties of cross-section

Material	f_{yk} (*N/mm²*)	E_s (*N/mm²*)	ε_{su}	f_{ck} (*N/mm²*)	E_c (*N/mm²*)	ε_{co}	ε_{cu}
Steel	420	$2x10^5$	0.010				
Concrete				20	$2.85x10^4$	0.002	0.006

Table 4 Longitudinal bars in beams (Direction-*X*)

Storeys	in support areas (*up*)	in support areas (*down*)	in span areas (*up*)	in span areas (*down*)
1-4	3ϕ14+2ϕ20	4ϕ16	3ϕ14	4ϕ16
5-6	2ϕ12+2ϕ18	3ϕ16	2ϕ12	3ϕ16

Table 5 Longitudinal bars in beams (Direction-*Y*)

Storeys	in support areas (*up*)	in support areas (*down*)	in span areas (*up*)	in span areas (*down*)
1-4	3ϕ14+3ϕ22	4ϕ16	3ϕ14	4ϕ16
5-6	2ϕ12+2ϕ18	3ϕ16	2ϕ12	3ϕ16

Table 6 Longitudinal bars in columns (Direction-*X*)

Storeys	in support areas (*up*)	in support areas (*down*)	in span areas (*up*)	in span areas (*down*)
1-4	3ϕ14+2ϕ20	4ϕ16	3ϕ14	4ϕ16
5-6	2ϕ12+2ϕ18	3ϕ16	2ϕ12	3ϕ16

Table 7 Vertical loads acting on beams

Location	Axle	P_1 (*kN*)	P_2 (*kN*)	P_3 (*kN*)	P_4 (*kN*)
Normal storeys	A	2.82	10.13	16.74	19.97
	B,C	3.92	16.88	30.10	36.56
	1	2.82	10.13	16.60	-
	2	3.95	16.88	29.82	-
Roof storey	A	7.15	20.13	28.70	32.89
	B,C	4.61	20.88	38.02	46.40
	1	7.15	20.13	28.52	-
	2	4.61	20.88	37.65	-

Table 8 Formation of plastic sections, load factors and failure loads

Direction	Load factor (λ)	Number of plastic section	Load factor for initial plastic section
X	1.562 (1.535)	52 (50)	1.171 (1.163)
Y	1.581 (1.553)	83 (81)	1.192 (1.189)

5. Conclusions

In the present study conducted by using the proposed method, it is articulated that sizing the structure systems on the basis of failure load for both safety and economic aspects is a necessity. As a result of analysis of structural systems with second order effects, it is determined that there is 10% reduction at bearing capacity of the system and it is also indicated that if the capacity of rotation in plastic sections is exceeded, the structure can come to a state of collapse. After the analysis, it comes into foreground once again that strong column and weak beam approach should be taken into consideration. Consequently, it is observed that while sizing structures, an adequate safety and the economy can be made providing a sizing method based on failure load as well as proposed method and other similar methods considering real effects and behavior.

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CC

Appendix : Notation

A	Cross section area	l	Rod length
$\underline{A}$	Displacement transformation matrix	M	Bending moment
E	Modulus of elasticity	M_y	Yield moment
I	Moment of inertia	M_r	Moment bearing capacity of section
$\underline{K}$	Overall stiffness matrix	$\underline{P}$	Member end forces vector
$\underline{k}$	Member stiffness matrix	$\underline{X}$	Displacement matrix
$\underline{L}$	Load vector	λ	Load factor