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New strut-and-tie-models for shear strength prediction and design of RC deep beams

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Abstract. Reinforced concrete deep beams are structural beams with low shear span-to-depth ratio, and hence in which the strain distribution is significantly nonlinear and the conventional beam theory is not applicable. A strut-and-tie model is considered one of the most rational and simplest methods available for shear strength prediction and design of deep beams. The strut-and-tie model approach describes the shear failure of a deep beam using diagonal strut and truss mechanism: The diagonal strut mechanism represents compression stress fields that develop in the concrete web between diagonal cracks of the concrete while the truss mechanism accounts for the contributions of the horizontal and vertical web reinforcements. Based on a database of 406 experimental observations, this paper proposes a new strut-and-tie-model for accurate prediction of shear strength of reinforced concrete deep beams, and further improves the model by correcting the bias and quantifying the scatter using a Bayesian parameter estimation method. Seven existing deterministic models from design codes and the literature are compared with the proposed method. Finally, a limit-state design formula and the corresponding reduction factor are developed for the proposed strut-and-tie model.

Keywords: Bayesian parameter estimation; deep beam; experimental observations; probabilistic model; strut-and-tie model

1. Introduction

Reinforced concrete deep beams are useful structural components often used for the construction of buildings, bridges and many other infrastructures. Because of their relatively low shear span-to-depth ratio, the structural behavior of deep beams differs greatly from those of slender beams. In particular, the response of deep beams is characterized by nonlinear strain distribution that occurs even in the elastic load range. In addition, the strength of deep beams with

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a normal amount of longitudinal reinforcement is usually controlled by shear instead of flexure. Consequently, developing methods for accurate prediction of shear strengths of deep beams has become an important research topic in the field.

To design a deep beam, a whole member design approach is usually adopted instead of a sectional design approach that is often used for slender beams because of the different behaviors described above. In particular, the whole member design approach based on the strut-and-tie model (STM) is currently recognized as the most rational and simplest method for designing deep beams. Additionally, the STM has been applied to predict the capacity of other discontinuity region members such as corbel (e.g., Lu and Lin 2009), dapped-end beams (e.g., Lin *et al*. 2003), or joint in decked bulb-tee bridge (e.g., Li *et al*. 2013). The STM idealizes the complex flow of stresses in a structural member as truss-like members. The flow of concentrated compressive stresses in the concrete can be represented by diagonal struts while the induced concentrated tensile stresses can be represented by tension ties, which are resisted by longitudinal steel reinforcement. The regions where struts and ties intersect each other are called nodal zones.

In general, the STMs for deep beams are derived from various deterministic models, namely, analytical methods (Kotsovos 1988; Siao 1993; Ashour 2000; Hwang *et al*. 2000a; Tang and Tan, 2004; Zhang and Tan 2007; Park and Kuchma 2007), and semi-empirical formula (Matamoros and Wong 2003; Russo *et al*. 2005). These deterministic models often exhibit unpredictable biases and uncertain errors that prevent reliable and robust prediction of the strengths of deep beams. This is due to imperfect formulation, missing parameters, or insufficient numbers of the existing experimental data. For this reason, there exist strong research needs for developing methods that can correct the bias of STM-based prediction and quantify the uncertain errors for reliability-based design of deep beams.

For this purpose, in this paper, a new STM is proposed and further improved by use of a Bayesian parameter estimation method (Gardoni *et al*. 2002; Kim *et al*. 2007; Kim *et al*. 2009; Song *et al*. 2010), which corrects the bias of the STM and quantify the uncertain error based on an experimental database. A step-wise deletion process is employed to identify important explanatory functions that make significant contribution to the bias of the deterministic STM. A database of 406 experimental results compiled from several technical literatures is used for this analysis. All of specimens are simply supported deep beams subjected to symmetrically point loads and tested to fail in shear modes. In addition to the STM proposed in this paper, seven additional existing models (ACI 318-08 2008; AASHTO LRFO-2008; Siao 1993; Foster and Gilbert 1996; Matamoros and Wong 2003; Tang and Tan 2004; and Russo *et al*. 2005) were used for the same process to demonstrate the capability of the proposed probabilistic method through comparison. Additionally, the accuracy of the proposed model is tested for different ranges of important parameters in the database to confirm the consistent performance of the model. Finally, using the proposed probabilistic STM, a limit-state design formula and the corresponding reduction factor are developed by uncertainty analysis.

2. Strut-and-tie model for reinforced concrete deep beams

2.1 Proposed STM-based model

It is widely recognized that the main parameter affecting the behavior of reinforced concrete beams is the shear span-to-depth ratio (a/d) where a is the shear span, and d is the effective

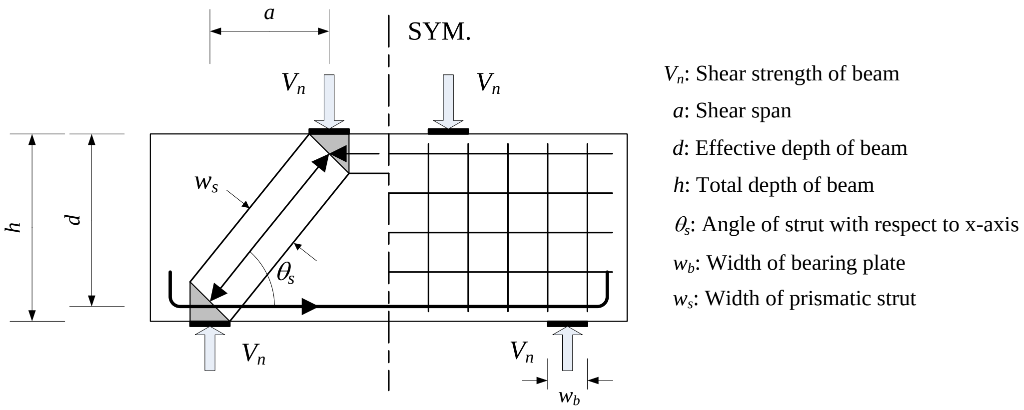


Fig. 1 Strut-and-tie model for simply supported deep beam

depth of the beam (see Fig. 1). For deep beams, the ratio has been long recognized as a major factor contributing to the load transfer characteristics (Ferguson *et al*. 1988; Wight and MacGregor 2012). Typically, a beam with a ratio less than 2.0 to 2.5 is considered to behave as a deep beam whereas a beam with a greater ratio is assumed to behave as a slender beam. Fig. 1 illustrates an STM for a simply supported deep beam subjected to the two pointed loads applied at the top of the beam. The approach assumes that the failure mechanism of deep beam consists of three components: 1) the diagonal strut, 2) horizontal mechanism, and 3) vertical mechanism (Matamoros and Wong 2003). As a result, by summing up the contributions from the three components, one can predict the shear strength V_n as

$$V_n = \nu f_c' \sin\theta_s b_w w_s + A_h f_{yh} \tan\theta_s + A_v f_{yv} \tag{1}$$

where ν is the concrete efficiency factor which will be discussed later; f_c', f_{yh}, and f_{yv} are the concrete compressive strength, and the yield strengths of horizontal and vertical reinforcement respectively; b_w is the width of beam section; and A_h and A_v are the areas of horizontal and vertical web reinforcement embedded within the effective widths respectively. Fig. 1 illustrates θ_s (angle between the concrete compression strut and horizontal direction) and w_s (width of prismatic strut).

Matamoros and Wong (2003) suggested that the effective widths for the horizontal and vertical ties should be defined as $a/3$ and $d/3$, respectively. The contribution of the reinforcement outside these two bounds is neglected. To provide more realistic models, in this study, the effective widths are determined as those minimizing the coefficient of variation (COV) of the ratio of the shear strength observed in a test to the shear strength calculated by Eq. (1). Through a nonlinear optimization technique such as conjugate gradient method, the optimal horizontal and vertical effective widths were found to be $a/8$ and $d/8$, respectively. These effective widths can significantly affect the accuracy and consistency of the proposed STM. Thus, A_h and A_v are determined as:

$$A_h = \rho_h b_w d/8 \tag{2a}$$

$$A_v = \rho_v b_w a/8 \tag{2b}$$

where ρ_h and ρ_v are the horizontal and vertical reinforcement ratios, respectively; and θ_s can be obtained from:

$$\tan\theta_s = \frac{jd}{a} \tag{3}$$

where j is derived from the classical bending theory for a single reinforced beam section as follows:

$$j = 1 - \frac{k}{3} \tag{4a}$$

$$k = \sqrt{(n\rho)^2 + 2(n\rho)} - (n\rho) \tag{4b}$$

where n is the modular ratio and ρ is the longitudinal reinforcement ratio. This study uses the concrete efficiency factor ν by Zwicky and Vogel (2006), i.e.

$$\nu = (1.8 - 38\varepsilon_1)\cdot(f_c')^{-1/3} \tag{5a}$$

$$0.85\cdot(f_c')^{-1/3} \le \nu \le 1.6\cdot(f_c')^{-1/3} \tag{5b}$$

where ε_1 is the principal tensile strain of concrete, approximated as:

$$\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002)/\tan^2\theta_s \tag{6a}$$

where ε_s is the tensile strain in the concrete in the direction of the tension tie. For simplicity, the principal tensile strain is approximated as

$$\varepsilon_1 \approx \varepsilon_{cr} = 0.00008 \tag{6b}$$

where ε_{cr} is the strain at cracking of concrete (Hsu 2000).

In general, it may be difficult to determine the true geometry of the struts accurately. However, it can be assumed that the struts have a uniform cross section over their length (Fig. 1), which is called the “prismatic struts.” There exist various assumptions to determine the width of strut, w_s. In this paper, the following strut width definitions by Hwang *et al*. (2000a; Eq. 7a), Hwang *et al*. (2000b; Eq. 7b), and ACI 318-08 (Eq. 7c) were considered:

$$w_s = \sqrt{(kd)^2 + w_b^2} \tag{7a}$$

$$w_s = kd \tag{7b}$$

$$w_s = \min(w_t\cos\theta_s + w_b\sin\theta_s, kd\cos\theta_s + w_b\sin\theta_s) \tag{7c}$$

where w_b and w_t are the width of bearing plate and the effective width of tie, respectively. In the present study, the expressions of ε_1 and w_s in Eqs. (6) and (7) were validated against the experimental database in the same manner as in Eq. (2), and it was found that the expressions in Eqs. (6b) and (7a) were the most reliable ones resulting in the lowest COV of the shear strength ratio.

2.2 Experimental database

An experimental database of 406 reinforced concrete deep beams, summarized in Table 1, is used in this study for the purpose of finding optimal parameters in the proposed STM (Section 2.1) and developing probabilistic STM using Bayesian parameter estimation (Section 3). The data were

Table 1 Database of deep beam experiments used for this study

Reference	No. of tested samples	Concrete strength (MPa)	Shear span-to-depth ratio *a/d*	Shear strength (kN)
Clark (1951)	37	14-48	1.16-2.35	190-437
Kong *et al*. (1970, 1972)	25	19-25	0.35-1.18	78-276
Smith and Vantsiotis (1982)	52	16-23	1.00-2.08	73-184
Subedi *et al*. (1986)	8	23-33	0.42-1.53	150-485
Sarsam and Musawi (1992)	10	39-80	2.50	189-247
Tan *et al*.(1995)	18	41-59	0.27-2.16	150-675
Fang *et al*. (1995)	19	29-86	0.50-1.50	344-1,399
Tan *et al*. (1997a)	11	63-80	0.85-1.69	335-775
Tan *et al*. (1997b)	10	65-72	0.28-1.67	250-925
Foster and Gilbert (1998)	11	77-120	0.76-1.88	512-1,303
Tan *et al*. (1999)	6	31-49	0.56-1.13	570-1,636
Oh and Shin (2001)	52	24-74	0.50-2.00	165-746
Aguilar *et al*. (2002)	4	28-32	1.14-1.27	1,134-1,357
Yang *et al*. (2003)	21	31-79	0.53-1.13	192-1,029
Salamy*et al*. (2005)	19	23-38	0.50-1.50	308-4,198
Tanimura and Sato (2005)	37	21-98	0.50-2.50	327-2,624
Zhang and Tan(2007)	12	25-32	1.10	85-775
Garay and Lubell (2008)	9	23-48	1.18-2.39	539-1,371
Kunopas (2008)	6	22-28	1.00	195-342
Roy and Brena (2008)	12	27-36	1.05-2.10	133-372
Amornpinyo (2010)	6	22-24	1.00	178-280
Sagaseta and Vollum (2010)	8	68-80	1.04-1.12	326-707
Arabzadeh *et al*.(2011)	13	58-65	1.08	185-300
	Total 406			
	Min	14	0.27	73
	Max	120	2.50	4,198
	AVG	42	1.22	475
	COV	0.54	0.43	0.90

collected from studies reported in the literature that provide the shear strengths of the specimens, which are simply supported and subjected to either one or two point loads acting symmetrically with respect to the centerline of the beam span. The deep beams considered in this study are casted with concrete strength varied from 14 to 120 MPa which cover the practical ranges of normal and high strength concrete. The beams have the overall depth from 200 to 2,000 mm, and a/d ratio from 0.27 to 2.5. The longitudinal main reinforcement ratios of those beams are from 0.27% to 4.34%, and vertical and horizontal web reinforcement ratios are from 0 to 2.86%, and 0 to 3.17%, respectively. The observed shear strengths are from 73 to 4,198 kN.

3. Probabilistic prediction of shear strengths by STM

3.1 Bayesian parameter estimation for improving shear strength prediction by STM

Gardoni *et al*. (2002) introduced a Bayesian methodology for constructing probabilistic shear capacity models of reinforced concrete columns based on experimental observations. The probabilistic models were constructed by correcting the biases in existing deterministic models and by quantifying the remaining errors. In this paper, this methodology is adopted for improving the STM-based prediction proposed above. The same methodology is also applied to other existing STMs in Table 2, namely, ACI 318-08 (2008), AASHTO LRFD-2008 (2008), Siao (1993), Foster and Gilbert (1996), Matamoros and Wong (2003), Tang and Tan (2004), and Russo *et al*. (2005) for comparison purpose.

The Bayesian methodology constructs a probabilistic model in the form:

$$C(\mathbf{x},\mathbf{\Theta}) = C_d(\mathbf{x}) + \gamma(\mathbf{x},\boldsymbol{\theta}) + \sigma\varepsilon \tag{8}$$

where $C(\mathbf{x},\mathbf{\Theta})$ denotes the prediction by the probabilistic model, e.g. the shear strength of a deep beam for this study, $\mathbf{x}$ is the vector of input parameters measured during tests on deep beams, e.g., f'_c, b_w, ρ_v, ρ_h, a, and d; $\mathbf{\Theta} = (\boldsymbol{\theta},\sigma)$ is a set of model parameters introduced to fit the model to the test results; $C_d(\mathbf{x})$ is an existing deterministic model, e.g. the proposed model in Eq. (1) and other existing models listed in Table 2; $\gamma(\mathbf{x},\boldsymbol{\theta})$ is the term introduced to correct the bias inherent in the deterministic model that is expressed as a function of the input parameter $\mathbf{x}$ and uncertain model parameters $\boldsymbol{\theta} = [\theta_1,\theta_2,\ldots,\theta_p]^T$; ε is the normal random variable with zero mean and unit variance; and σ is a model parameter that represents the standard deviation of the model error $\sigma\varepsilon$ that remains after the bias-correction. To satisfy the homoskedasticity assumption, i.e. the assumption that the magnitude of the model error does not depend on $\mathbf{x}$, and the assumption that the model error follows a normal distribution, suitable nonlinear transformation of $\mathbf{x}$ and $C(\mathbf{x},\mathbf{\Theta})$ might be needed (Gardoni *et al*. 2002; Song *et al*. 2010).

Since the true form of the bias-correction term is unknown, the correction term $\gamma(\mathbf{x},\boldsymbol{\theta})$ needs to be expressed using a suitable set of "explanatory terms" $h_i(\mathbf{x})$, $i = 1,\ldots,p$, in the form given below.

$$\gamma(\mathbf{x},\boldsymbol{\theta}) = \sum_{i=1}^{p} \theta_i h_i(\mathbf{x}) \tag{9}$$

Table 2 Selected existing shear strength models of reinforced concrete deep beams

Model	Shear strength model
ACI 318-08	$V_n = f_{ce}\sin\theta_s b_w w_s$ where $f_{ce} = 0.85\beta_s f_c'$, $w_s = \min[(w_t\cos\theta_s + w_b\sin\theta_s),(h_c\cos\theta_s + w_b\sin\theta_s)]$, $\beta_s = 0.75$ for bottle-shaped struts with reinforcement satisfying Section A.3.3 of ACI 318-08; $\beta_s = 0.60$ for bottle-shaped struts without reinforcement satisfying Section A.3.3 of ACI 318-08
AASHTO-LRFD 2008	$V_n = f_{cu}\sin\theta_s b_w w_s$ where $f_{cu} = \frac{f_c'}{0.8+170\varepsilon_1} \le 0.85 f_c'$, $\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002)/\tan^2\theta_s$, $w_s = w_t\cos\theta_s + w_b\sin\theta_s$
Siao (1993)	$V_n = 1.8 f_t bd$ where $f_t = \max\left[0.58\sqrt{f_c'}\left(1+n\left(\rho_h\sin^2\alpha + \rho_v\cos^2\alpha\right)\right), f_y\left(\rho_h\sin^2\alpha + \rho_v\cos^2\alpha\right)\right]$
Foster and Gilbert (1996)	$V_n = \nu f_c'\sin\theta_s b_w w_s$ where $w_s = w_b/\sin\theta_s$, $\tan\theta_s = \frac{d-\Omega/2}{a}$, $\Omega = d - \sqrt{d^2 - 2a w_b} \le 2(h-d)$, $\nu = 1.25 - \frac{f_c'}{500} - 0.72\frac{a}{d} + 0.18\left(\frac{a}{d}\right)^2 \le 1$ for $a/d < 2$ and $\nu = 0.53 - \frac{f_c'}{500}$ for $a/d \ge 2$
Matamoros and Wong (2003)	$V_n = \frac{0.30}{a/d} f_c' b w_{st} + A_v f_{yv} + 3(1-a/d)A_h f_{yh}$ where $w_{st} = w_t\cos\theta_s + w_b\sin\theta_s$, $A_h = \rho_h bd/3$ and $A_v = \rho_v ba/3$
Tang and Tan (2004)	$\frac{1}{V_n} = \frac{1}{V_{dc}} + \frac{1}{V_{ds}}$ where $V_{dc} = f_c' A_{str}\sin\theta_s$, $A_{str} = b_w(w_t\cos\theta_s + w_b\sin\theta_s)$, $V_{ds} = \frac{f_{ct}A_{ct} + f_{yv}A_v\cos\theta_s + f_{yh}A_h\sin\theta_s + 2 f_y A_s\sin\theta_s}{2\cos\theta_s}$, $f_{ct} = 0.5\sqrt{f_c'}$, $A_h = \rho_h bd$, and $A_v = \rho_v ba$
Russo *et al*. (2005)	$V_n = 0.545\left(k\chi f_c'\cos\alpha_s + 0.25\rho_h f_{yh}\cot\alpha_s + 0.35\frac{a}{d}\rho_v f_{yv}\right)b_w d$ where $\chi = 0.74\left(\frac{f_c'}{105}\right)^3 - 1.28\left(\frac{f_c'}{105}\right)^2 + 0.22\left(\frac{f_c'}{105}\right) + 0.87$, and $\tan\alpha_s = \frac{a}{0.9d}$

Notations: V_n (kN): shear strength; f_c' (MPa): concrete compressive strength; b_w (mm): web width; d (mm): effective depth; $\rho = A_s/b_w d$: longitudinal reinforcement ratio in which A_s is the amount (area) of longitudinal reinforcement; $E_s = 2\times10^5$ (MPa): elastic modulus of reinforcement; $E_c = 4700\sqrt{f_c'}$ (MPa): elastic modulus of concrete; a (mm): shear span length.

In this study, to satisfy the homoskedasticity assumption, a transformation by natural logarithm is utilized, i.e.

$$\ln[C(\mathbf{x},\mathbf{\Theta})] = \ln[C_d(\mathbf{x})] + \sum_{i=1}^{p}\theta_i h_i(\mathbf{x}) + \sigma\varepsilon \tag{10}$$

and the explanatory terms are also defined as the natural logarithms of important parameters. A probabilistic model is constructed by finding the model parameter $\mathbf{\Theta} = (\boldsymbol{\theta}, \sigma)$ that makes the best fit of Eq. (10) to the test results. This process can be done using the well-known Bayesian updating rule (Box and Tiao 1992) which produces the posterior distribution of the unknown model parameter, $f(\mathbf{\Theta})$ by updating a prior distribution based on observations as follows.

$$f(\mathbf{\Theta}) = \kappa L(\mathbf{\Theta}) p(\mathbf{\Theta}) \tag{11}$$

where $L(\mathbf{\Theta})$ is the "likelihood" function that represents the likelihood of the test results, $\kappa = \left[\int L(\mathbf{\Theta}) p(\mathbf{\Theta}) d\mathbf{\Theta}\right]^{-1}$ is the normalizing factor, and $p(\mathbf{\Theta})$ is the joint probability density function (PDF) of a "prior" distribution reflecting the knowledge about Θ prior to obtaining the objective data such as experimental results. Eq. (11) thus updates the "prior" distributions based on subjective information to the "posterior" distribution using the objective information gained from the tests. The details about the selection of prior distributions, the formulation of likelihood functions, and the computational method for obtaining $f(\mathbf{\Theta})$ are given in Gardoni *et al*. (2002). The posterior means of $\mathbf{\Theta} = (\boldsymbol{\theta}, \sigma)$ obtained from the Bayesian parameter estimation are substituted to the corresponding model parameters in Eq. (10) to complete the model construction. It is noted that the computer implementation of the Bayesian parameter estimation in this study was performed by the computer codes developed by Dr. Seung-Yong Ok at Hankyong National University for a course taught by Dr. Junho Song at the University of Illinois at Urbana-Champaign.

3.2 Overall performance of selected existing deterministic shear strength models of deep beams

Before developing a probabilistic model in Eq. (10) using Bayesian parameter estimation, the overall performances of the aforementioned existing deterministic shear strength models and the proposed model in Eq. (1) are investigated by using the Bayesian methodology with a constant bias-correction term (Song *et al*. 2010), i.e.,

$$\ln[C(\mathbf{x}, \mathbf{\Theta})] = \ln[C_d(\mathbf{x})] + \theta + \sigma\varepsilon \tag{12}$$

Bayesian parameter estimation using Eq. (12) based on the database provides the posterior means of θ and σ, which are the mean and standard deviation of the error measure $\ln[C(\mathbf{x}, \mathbf{\Theta})] - \ln[C_d(\mathbf{x})]$, respectively, and can be used as measures for overall bias (θ) and scatter (σ) of the model.

Table 3 provides these measures for the aforementioned deterministic formulations of deep beam shear strengths and the proposed STM. The STM using U.S. code provisions namely ACI 318-08 and AASHTO-LRFD 2008 results in very large scatter and bias (conservatism), which is confirmed by the large absolute values of the posterior means. The model proposed by Siao (1993) exhibits unconservative estimation with large scatters while Foster and Gilbert (1996) gives small bias, but still has large scatters. Tang and Tan (2004) and Russo *et al*. (2005) provide reasonably good prediction when compared with the first five models. By contrast, the STM

Table 3 Overall bias and scatter of base models

Model	Posterior means	
	θ (bias)	σ (scatter)
ACI318-08	0.391	0.417
AASHTO LRFD-2008	0.286	0.363
Siao (1993)	–0.182	0.349
Foster and Gilbert (1996)	–0.049	0.384
Matamoros and Wong (2003)	0.155	0.288
Tang and Tan (2004)	0.179	0.223
Russo *et al*. (2005)	0.252	0.196
The proposed STM in Eq. (1)	0.008	0.179

proposed in Eq. (1) of this paper results in smallest bias and scatter. It is also interesting to note that, although the multi-parameter and compression softening models are expected to provide the least bias and scatter as reported by Foster and Malik (2002), some multi-parameter models such as Siao (1993), Foster and Gilbert (1996) and Matamoros and Wong (2003) are not showing superior performance for a large size of the database covering wide ranges of parameters such as f_c', a/d, ρ, ρ_v, and ρ_h.

In the next section, the aforementioned Bayesian parameter estimation methodology is used to reduce the bias and uncertainties of these STMs using a general bias-correction function, as shown in Eq. (10).

3.3 Selection of explanatory function for bias-correction

For effective bias correction, it is necessary to identify parameters influencing the shear strength of deep beams. Based on the force transfer mechanism in STM illustrated in Fig. 1 and the existing experimental results, some important parameters relevant to the behaviors of deep beams are identified as follows: 1) the concrete compressive strength, 2) the shear span-to-depth ratio, 3) the amount of longitudinal reinforcement, 4) the amount of vertical web reinforcement, 5) the amount of horizontal web reinforcement, 6) the size of bearing plate, and 7) axial forces in normal reinforcement or in prestressing strands. However, since most of available databases of deep beams do not consider the effect of axial forces, in this paper, explanatory functions with the axial force parameters were not considered. Kim *et al*. (2007) and Song *et al*. (2010) experienced that Bayesian parameter estimation using the form in Eq. (10) produces better results when the natural logarithms are applied to the normalized parameters. Therefore, this study chose

$$\mathbf{h}(\mathbf{x}) = \{\ln 2,\ \ln E_s/E_c,\ \ln a/d,\ \ln d/h, \ln d/w_b,\ \ln\rho, \ln\rho_v, \ln\rho_h\} \tag{13}$$

where the constant term $h_1(\mathbf{x}) = \ln 2$ is to detect the constant bias that is independent of the parameters in $\mathbf{x}$ .The other explanatory terms are selected based on experimental evidences and known influences on the shear strength of deep beam governed by the crushing of bottle shape strut and shear compression failure modes. In addition, all explanatory terms are dimensionless so that the bias-corrected prediction has consistent dimension with that of the deterministic shear strengths. From Eqs. (9) and (13), the bias-correction term can be rewritten in the form:

$$\gamma(\mathbf{x},\boldsymbol{\theta}) = \ln\left[2^{\theta_1}\cdot\left(\frac{E_s}{E_c}\right)^{\theta_2}\cdot\left(\frac{a}{d}\right)^{\theta_3}\cdot\left(\frac{d}{h}\right)^{\theta_4}\cdot\left(\frac{d}{w_b}\right)^{\theta_5}\cdot\rho^{\theta_6}\cdot\rho_v^{\theta_7}\cdot\rho_h^{\theta_8}\right] \tag{14}$$

3.4 Improvement of STM-based shear strength models

Applying exponential functions to Eq. (10), the probabilistic shear strength model takes the form (Song *et al*. 2010)

$$C(\mathbf{x}) = C_d(\mathbf{x})\cdot\exp[\gamma_s(\mathbf{x},\mu_\theta)]\cdot\exp(\mu_\sigma\varepsilon) = C_d(\mathbf{x})\cdot\tilde{\gamma}_s\cdot\exp(\mu_\sigma\varepsilon) \tag{15}$$

where $\gamma_s(\mathbf{x},\boldsymbol{\mu}_\theta)$ is the bias-correction term with the explanatory terms that survive the stepwise process to remove unexplantory terms during the Bayesian parameter estimation (Gardoni *et al*. 2002). The term with the highest posterior COV is removed at each parameter estimation round and the removal is repeated until the posterior mean of σ starts showing a significant increase due to the removal. For example, Table 4 illustrates the stepwise removal process for the proposed STM. The removal process stops after the fourth removal (σ = 0.160) because the fifth removal (σ = 0.166) causes a significant increase in the posterior mean. Additionally, it can be seen from Table 4 that the uncertainty may be reduced for the model using less term of parameters. This is because unnecessary parameter (s) may prevent the model from describing the behavior effectively. On the contrary, removing an important parameter also makes the model perform poorly and thus increase the uncertainty, as seen in the fifth removal. If the uncertainties of the model parameters in $\boldsymbol{\Theta}$ are neglected, i.e. the uncertain parameters in $\boldsymbol{\Theta}=(\boldsymbol{\theta},\sigma)$ are replaced by the corresponding posterior mean values, the normal random variable ε is the only random variable in the model. Hence, the shear strength predicted by Eq. (15) follows the lognormal distribution, and the mean and COV of the strength are derived as $C_d(\mathbf{x})\cdot\tilde{\gamma}_s\cdot\exp(\mu_\sigma^2/2)$ and $\sqrt{\exp(\mu_\sigma^2)-1}$, respectively (Song *et al*., 2010; Ang and Tang, 2006). If $\mu_\sigma << 1$, the mean and COV of the predicted strength are closely approximated by $C_d(\mathbf{x})\cdot\tilde{\gamma}_s$ and μ_σ, respectively.

Table 5 compares the posterior means of σ, i.e. the approximate COVs of the predictions by the constant bias correction in Eq. (12) with those by the model with general bias-correction term in Eq. (10). In general, the mean values are significantly reduced by using the selected explanatory terms. The posterior mean of the proposed STM (0.160) is still smaller than all the other models after the bias-correction. From Eqs. (1), (14), (15) and the removal process summarized in Table 4, the approximate mean prediction of the proposed model improved by the Bayesian approach is written in the form:

$$V_n = (\nu f_c'\sin\theta_s b_w w_s + A_h f_{yh}\tan\theta_s + A_v f_{yv})\cdot\tilde{\gamma}_s \tag{16}$$

where

$$\tilde{\gamma}_s = (0.66)\cdot\left(\frac{E_s}{E_c}\right)^{0.137}\cdot\left(\frac{a}{d}\right)^{-0.141}\cdot\left(\frac{d}{h}\right)^{-1.368} \tag{17}$$

Table 6 lists the bias-correction term $\tilde{\gamma}_s$ derived for the proposed STM and the other existing models along with the approximate COV values. As explained above, the product of the bias-correction term in Table 6 and the corresponding deterministic model provides the approximate mean prediction after the bias-correction. In addition, it is observed that the bias-correction terms are different depending on the base STMs used. For instance, some of the explanatory terms such as ρ do not exist in some bias-correction terms while others still include the term. This is because the corresponding original STMs already describe the effect successfully and therefore do not require improvement in that aspect. Finally, the results in the table confirm that E_s/E_c, a/d and ρ are principal parameters used for the bias corrections.

Table 4 Step-wise removal process for the proposed STM

Step	Posterior COV of the corresponding θ_i								
	θ_1 ($\ln 2$)	θ_2 ($\ln E_s/E_c$)	θ_3 ($\ln a/d$)	θ_4 ($\ln d/h$)	θ_5 ($\ln d/w_b$)	θ_6 ($\ln \rho$)	θ_7 ($\ln \rho_v$)	θ_8 ($\ln \rho_h$)	σ
Initial	-0.330	0.285	-0.129	-0.242	1.315	4.117	0.427	-0.955	0.035
1st removal	-0.191	0.276	-0.125	-0.171	1.322	X	0.402	-0.842	0.035
2nd removal	-0.201	0.237	-0.128	-0.194	X	X	0.441	-0.701	0.033
3rd removal	-0.210	0.243	-0.119	-0.165	X	X	0.476	X	0.036
4th removal	-0.140	0.192	-0.088	-0.125	X	X	X	X	0.029
5th removal	-0.168	X	-0.120	-0.141	X	X	X	X	0.030
6th removal	0.711	X	-0.177	X	X	X	X	X	0.036
Step	Posterior means of the corresponding θ_i								
	θ_1 ($\ln 2$)	θ_2 ($\ln E_s/E_c$)	θ_3 ($\ln a/d$)	θ_4 ($\ln d/h$)	θ_5 ($\ln d/w_b$)	θ_6 ($\ln \rho$)	θ_7 ($\ln \rho_v$)	θ_8 ($\ln \rho_h$)	σ
Initial	-0.565	0.128	-0.147	-1.434	0.023	0.005	0.008	-0.003	0.162
1st removal	-0.600	0.126	-0.147	-1.482	0.023	X	0.009	-0.003	0.161
2nd removal	-0.585	0.137	-0.147	-1.396	X	X	0.008	-0.004	0.161
3rd removal	-0.522	0.132	-0.144	-1.358	X	X	0.007	X	0.162
4th removal	-0.610	0.137	-0.141	-1.368	X	X	X	X	0.160
5th removal	-0.254	X	-0.142	-1.556	X	X	X	X	0.166
6th removal	0.017	X	-0.095	X	X	X	X	X	0.173

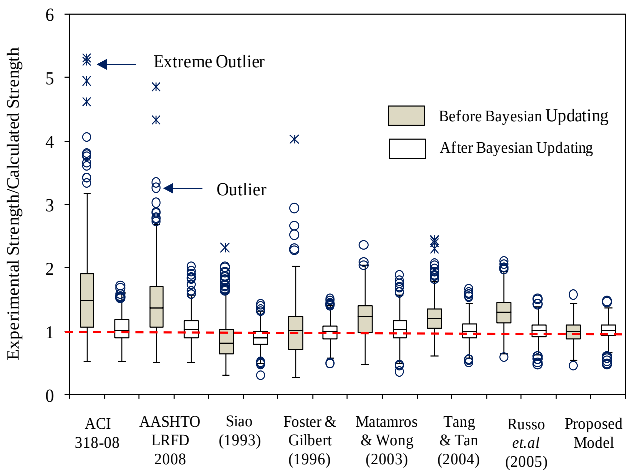
The terms in the ( ) mean the related explanatory terms

Table 5 Comparison of posterior means of σ by constant bias correction term (Table 3) and general explanatory terms

Model	Posterior means of σ	
	Constant bias (Table 3)	Explanatory terms
ACI318-08	0.417	0.216
AASHTO LRFD-2008	0.363	0.230
Siao (1993)	0.349	0.188
Foster and Gilbert (1996)	0.384	0.188
Matamoros and Wong (2003)	0.288	0.229
Tang and Tan (2004)	0.223	0.184
Russo *et al*. (2005)	0.196	0.171
The proposed STM in Eq. (1)	0.179	0.160

Table 6 Bias-correction term $\tilde{\gamma}_s$ developed for existing shear strength models

Model	Bias-correction term	COV
ACI318-08	$(0.32)\cdot\left(\frac{E_s}{E_c}\right)^{0.919}\cdot\left(\frac{d}{w_b}\right)^{0.707}\cdot\rho^{0.298}$	0.216
AASHTO-LRFD 2008	$(0.39)\cdot\left(\frac{E_s}{E_c}\right)^{0.630}\cdot\left(\frac{a}{d}\right)^{0.450}$	0.230
Siao (1993)	$(2.92)\cdot\left(\frac{a}{d}\right)^{-0.596}\cdot\left(\frac{d}{h}\right)^{-1.267}\cdot\left(\frac{d}{w_b}\right)^{-0.298}\cdot\rho^{0.213}$	0.188
Foster and Gilbert (1996)	$(0.22)\cdot\left(\frac{E_s}{E_c}\right)^{0.909}\cdot\left(\frac{a}{d}\right)^{-0.146}\cdot\left(\frac{d}{w_b}\right)^{0.651}\cdot\rho^{0.289}$	0.188
Matamoros and Wong (2003)	$(0.83)\cdot\left(\frac{E_s}{E_c}\right)^{0.650}\cdot\left(\frac{a}{d}\right)^{0.141}\cdot\left(\frac{d}{h}\right)^{2.231}\cdot\rho^{0.159}$	0.229
Tang and Tan (2004)	$(0.60)\cdot\left(\frac{E_s}{E_c}\right)^{0.444}\cdot\left(\frac{d}{h}\right)^{1.418}$	0.184
Russo *et al*. (2005)	$(0.76)\cdot\left(\frac{a}{d}\right)^{-0.167}\cdot\left(\frac{d}{h}\right)^{-1.704}\cdot\left(\frac{d}{w_b}\right)^{-0.196}\cdot\rho^{0.141}$	0.171
The proposed STM in Eq. (1)	$(0.66)\cdot\left(\frac{E_s}{E_c}\right)^{0.137}\cdot\left(\frac{a}{d}\right)^{-0.141}\cdot\left(\frac{d}{h}\right)^{-1.368}$	0.160



Note: Points beyond 1.5 interquartile ranges (IQR) but within 3 IQR from the corresponding box edge are considered as "*outliers*" in the box plot and are denoted by open circles. Points more than 3 IQR away from the box edge are considered as "*extreme outliers*" denoted by asterisk markers.

Fig. 2 Boxplots of ratio of observed experimental strength to strength calculated by different prediction models

4. Validation of the proposed shear strength model

4.1 Comparison with existing models

The boxplots of the ratio of the observed experimental strength to the strength calculated by different prediction models are given in Fig. 2. The boxplots provide statistical information such as the lower quartile (Q1), median (Q2), upper quartile (Q3), and outliers. As indicated in Table 3 and Table 5, Fig. 2 clearly shows that the proposed STM-based deterministic prediction by Eq. (1) has less bias (i.e. medians closer to 1) and scatter (i.e. shorter boxes) than the other deterministic models. Although the other deterministic models can be improved by the Bayesian approach in terms of their bias and uncertainties, those "bias-corrected" models still have larger scatters than the bias-corrected prediction of the proposed STM. Compared to the well-known Russo's equation, the proposed model adopts the equations by Zwicky and Vogel (2006) to compute the concrete efficiency factor that are much simpler than those of Russo's model, which are based on Zhang and Hsu's equations (Zhang and Hsu 1998). In summary, the proposed model is simple and easy to use while providing more accurate and robust predictions.

4.2 Comparison with existing models

To investigate the consistent performance of the probabilistic STM, the boxplots of the experiment-prediction ratios are made for different ranges of each of the following key explanatory terms: f_c' (Fig. 3), a/d (Fig. 4) and d (Fig. 5). The boxplots confirm that the proposed deterministic STM in Eq. (1) has consistent accuracy over different ranges of the parameters. The Bayesian updating further improves the consistency of the performance. This also demonstrates that the selections of explanatory terms were suitable to describe the behaviors of deep beams, e.g., concrete efficiency factor, the width of strut, concrete tensile strain in a consistent manner.

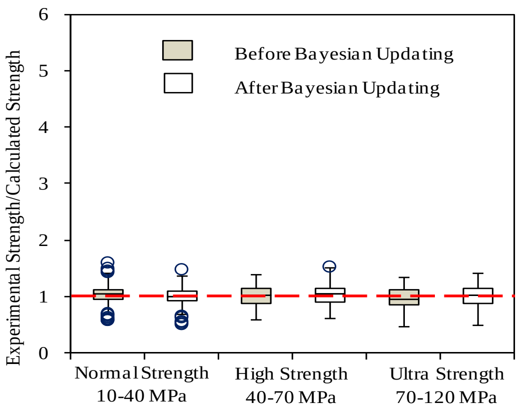


Fig. 3 Boxplots of the ratio of experimental strength to calculated strength of the proposed STM for different ranges of concrete compressive strength (f_c')

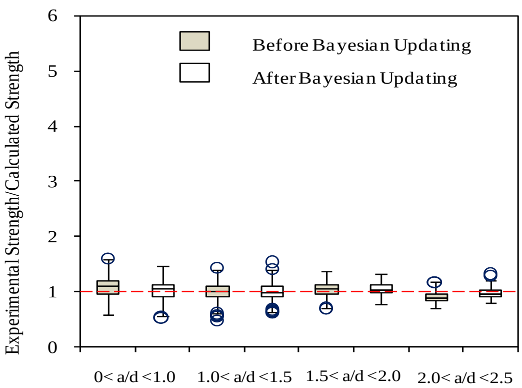


Fig. 4 Boxplots of the ratio of experimental strength to calculated strength of the proposed (STM) for different ranges of shear span-to-depth ratio (a/d)

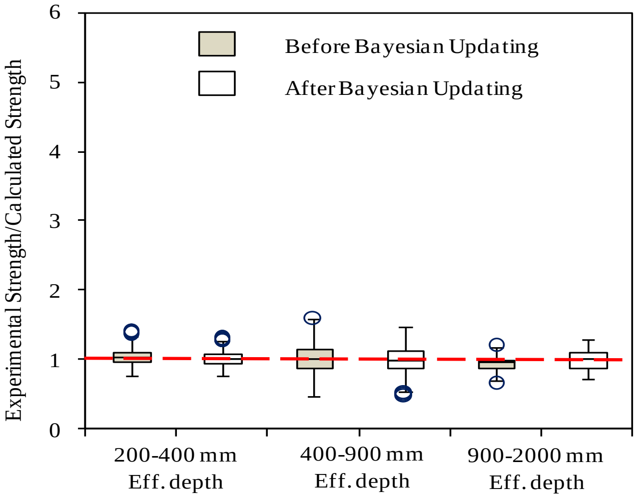


Fig. 5 Boxplots of the ratio of experimental strength to calculated strength of the proposed STM for different ranges of effective depth (*d*)

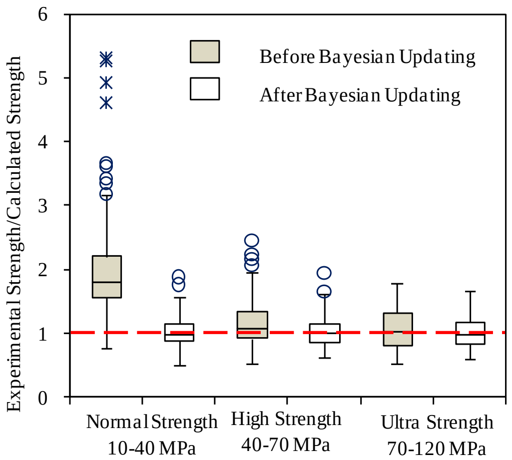


Fig. 6 Boxplots of the ratio of experimental strength to calculated strength of the ACI approach for different ranges of concrete compressive strength (f_c')

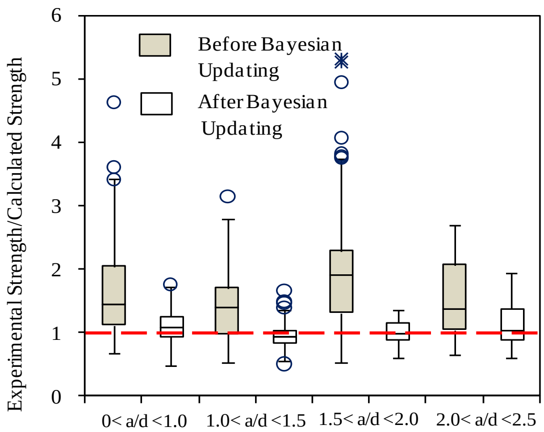


Fig. 7 Boxplots of the ratio of experimental strength to calculated strength of The ACI approach for different ranges of shear span-to-depth ratio (*a*/*d*)

For comparison, the boxplots for the ACI 318-08 approach are also made for the same ranges in Fig. 6 (f_c'), Fig. 7 (a/d) and Fig. 8 (d). As seen in Fig. 6, the strength ratio decreases as concrete strength increases, this seems to agree with Park and Kuchma (2007). Fig. 7 shows that the ACI 318-08 gives a very large scatter of prediction for all ranges of *a*/*d* and the bias is not uniform. Fig. 8 shows that ACI 318-08 fails to account for the size effect as significant biases are observed depending on the given effective depth. The probabilistic model developed based on ACI 318-08 corrects these biases and allows for more consistent performance over the considered ranges.

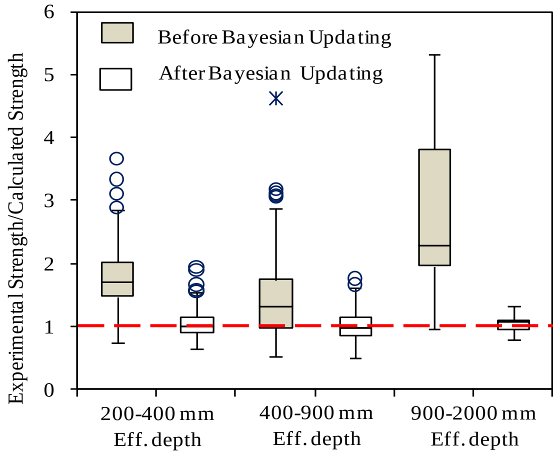


Fig. 8 Boxplots of the ratio of experimental strength to calculated strength of the ACI approach for different ranges of effective depth (*d*)

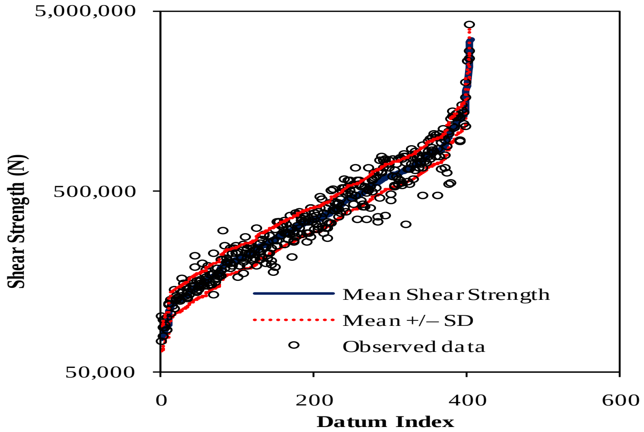


Fig. 9 Performance of the proposed probabilistic model at each experimental data

4.3 Comparison with experimental observations

Fig. 9 demonstrates the performance of the proposed probabilistic STM for shear strength prediction through comparison with each experimental data point. To make the plot, the test cases in the database are sorted in an increasing order of the mean shear strengths predicted by the probabilistic STM. The mean, and mean +/–one standard deviation (SD) curves are plotted based on the mean and COV of the shear strength estimated by the Bayesian method. The curve of the mean predicted shear strength mostly passes through the center of the experimental points, which confirms the probabilistic STM provides unbiased shear strength prediction of deep beam. The interval of mean +/– SD, which covers approximately 70% of the probability distribution of the strength for a normal random variable, is covering most of the experimentally observed shear strength throughout the whole range of strengths in the database. It is concluded that the predictions by the probabilistic STM are reliable and consistent with the experimental data.

5. Developing limit state design equation and factors based on the proposed STM

In this section, a limit state design equation is developed by using the proposed probabilistic STM. First, the shear strength of a beam is predicted based on the mean of strength prediction in Eq. (16) which can be written as:

$$\varphi \cdot V_n = \varphi \cdot \left(\nu\, f_c' \sin\theta_s b_w w_s + A_h f_{yh} \tan\theta_s + A_v f_{yv}\right) \cdot \tilde{\gamma}_s \tag{18}$$

where φ is a reduction factor introduced to achieve the performance at a reliable level despite various uncertainties in the strength prediction, which can be determined later by uncertainty analysis. The likelihood of having no shear failure for a given design is quantified by the reliability index or safety index, β. As the reliability index increases, the probability of failure decreases, indicating a higher level of reliability (more safe design).The reliability index accounts for the uncertainties inherent in the design parameters, such as the resistance and applied load. For the limit state function based on the safety margin concept, i.e. the resistance minus the applied load and the assumption of no significant correlation between the resistance and applied load, the reliability index β is derived as:

$$\beta = \frac{\mu_R - \mu_D - \mu_L}{\sqrt{\sigma_R^2 + \sigma_D^2 + \sigma_L^2}} \tag{19}$$

where μ_R, μ_D and μ_L are the mean value of resistance, applied dead and live loads, respectively; and σ_R, σ_D and σ_L are the corresponding standard deviations. Following ACI 318-08 code, the load and resistance should satisfy:

$$\varphi V_n = \varphi R_n \geq 1.2 D_n + 1.6 L_n \tag{20}$$

and

$$\varphi V_n = \varphi R_n \geq 1.4 D_n \tag{21}$$

where R_n, D_n and L_n are the nominal values of resistance, applied dead and live load, respectively. In a design code that accounts for uncertainties in the resistance and loads, the mean values in Eq. (19) are estimated by use of the nominal values and the bias factors as follow:

$$\mu_R = \lambda_R R_n \,,\, \mu_D = \lambda_D D_n \,,\, \mu_L = \lambda_L L_n \tag{22}$$

where λ_R, λ_D, and λ_L are the bias factors (i.e., ratios of mean to nominal value) of resistance, dead load and live load, respectively. The standard deviations in Eq. (19) are computed by the products of the means and the corresponding COVs. The bias factors and the corresponding COV used in this analysis are summarized in Table 7.

The professional factor (P-factor) is obtained from the mean and COV for the tested strength to the nominal strength predicted by Eq. (16). It was also found that the lognormal distribution provided the best fit according to a Kolmogorov-Smirnov goodness-of-fit test and the hypothesis was not rejected at the 5% significance level. The material and fabrication factors (M and F-factor) representing variation in material properties and variation in geometry are derived by Monte Carlo simulations based on the statistical parameters proposed by Nowak *et al*. (2011). Additionally, the nominal properties of the beam used in these simulations are as follows: (1) geometry: $a = d =$ 905 mm, h = 1000 mm, $b_w = w_b$ = 300 mm; (2) concrete strength: f_c' = 30 MPa (normal strength concrete), f_c' = 60 MPa (high strength concrete); (3) reinforcement ratios: ρ =1.0%, $\rho_v = \rho_h$ = 0.5%; and (4) yield strengths: $f_y = f_{yv} = f_{yh}$ = 420 MPa. For this investigation, 5,000,000 samples are chosen to provide reasonable estimates. Since the three factors, P, M and F are combined to construct the resistance (Nowak and Szerszen 2003), the bias factor λ_R in Eq. (22) and COV of resistance V_R are given by:

Table 7 Statistical parameters for resistance and loading variables

Parameter	Bias factor, λ	COV, V	Distribution
P-Factor	1.02	0.15	Lognormal
MF Factor (normal strength concrete)	1.12	0.07	Lognormal
MF Factor (high strength concrete)	1.07	0.07	Lognormal
Dead Load*	1.05	0.10	Normal
Live Load*	1.00	0.18	Normal

*Adopted from Szerszen and Nowak (2003)

Table 8 Proposed reduction factors φ for different STMs-Normal strength concrete

	Before bayesian updating			After bayesian updating		
Model	φ	λ_R	φ_{unbias}	φ	λ_R	φ_{unbias}
ACI318-08	0.75	2.22	0.34	0.65	1.06	0.61
AASHTO LRFD-2008*	0.75	1.89	0.40	0.65	1.12	0.59
Siao (1993)	0.40	0.99	0.41	0.70	1.13	0.62
Foster and Gilbert (1996)	0.55	1.50	0.37	0.70	1.13	0.62
Matamoros and Wong (2003)	0.85	1.69	0.50	0.65	1.16	0.56
Tang and Tan (2004)	0.90	1.57	0.57	0.70	1.14	0.61
Russo *et al*. (2005)	0.90	1.55	0.58	0.75	1.17	0.63
The proposed STM in Eq. (1)	0.70	1.15	0.61	0.75	1.14	0.66

*Dead load factor = 1.25, Live load factor = 1.75

Table 9 Proposed reduction factors φ for different STMs-High strength concrete

	Before bayesian updating			After bayesian updating		
Model	φ	λ_R	φ_{unbias}	φ	λ_R	φ_{unbias}
ACI318-08	0.50	1.14	0.44	0.60	1.01	0.60
AASHTO LRFD-2008*	0.50	1.25	0.40	0.65	1.06	0.61
Siao (1993)	0.35	1.17	0.30	0.70	1.08	0.65
Foster and Gilbert (1996)	0.35	0.83	0.42	0.65	1.08	0.60
Matamoros and Wong (2003)	0.60	1.16	0.52	0.65	1.10	0.59
Tang and Tan (2004)	0.75	1.20	0.63	0.70	1.10	0.64
Russo *et al*. (2005)	0.80	1.34	0.60	0.70	1.08	0.65
The proposed STM in Eq. (1)	0.70	1.09	0.64	0.75	1.09	0.69

*Dead load factor = 1.25, Live load factor = 1.75

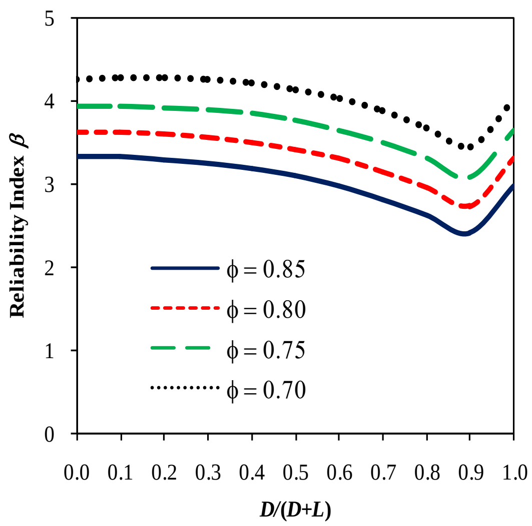
$$\lambda_R = \lambda_{MF}\lambda_P \tag{23}$$

$$V_R = \sqrt{V_{MF}^2 + V_P^2} \tag{24}$$

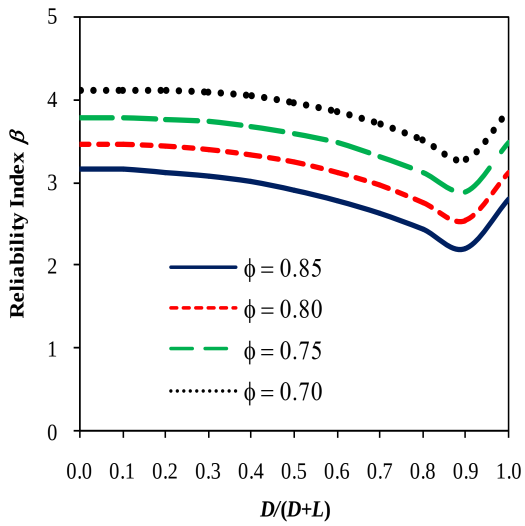
where λ_{MF} = bias factor of material-fabrication, λ_P = bias factor of professional factor, and V_{MF} and V_P are the corresponding COVs. As seen from Table 7, each of these bias factors follows different types of distributions, however, the equivalent normal distributions method described by Nowak and Collins (2000) can be used.

Now, the reduction factors φ are found so that the reliability index β in Eq. (19) calculated based on the prediction by the proposed STM and the statistical parameters described above, is close to the target value of 3.50 (Szerszen and Nowak, 2003). Fig. 10 shows the reliability index for four different reduction factor values (rounded to the nearest 0.05) for a range of dead load to dead load plus live load ratio from 0.3 to 0.7 (Szerszen and Nowak, 2003). The results suggest reduction factors φ = 0.75 for both normal and high strength concrete within the considered range of load ratio.

By using the same procedure, the reduction factors φ are obtained for the other STMs, as shown in Table 8 (normal strength concrete, $f_c' \leq$ 41 MPa) and Table 9 (high strength concrete, $f_c' >$ 41 MPa). First, it is seen from Table 8 that the reduction factor calculated for ACI 318-08 (φ = 0.75) has the same value as the one proposed in the ACI code, while the factor calculated for AASHTO (φ = 0.76) has a slightly different value (φ = 0.70 for AASHTO LRFD-2008). Next, although ACI code does not allow for the use of concrete strength greater than 6,000 psi (41 MPa), the reduction factors for different STMs with high strength concrete are calculated as shown in Table 9. It is seen that the reduction factors for AASHTO LRFD-2008 (φ = 0.52) has lower value than one provided in the code (φ = 0.70).



a) Normal Strength Concrete



b) High Strength Concrete

Fig. 10 Reliability index of the proposed model for different reduction factors vs. dead load to dead load plus live load ratio

For the purpose of measuring how much the actual resistance of a structural design needs to be discounted to achieve the target reliability using an STM, this paper introduces an unbiased reduction factor φ_{unbias}, i.e.

$$\varphi_{unbias} = \varphi / \lambda_R \tag{25}$$

Substituting $R_n = \mu_R / \lambda_R$ (derived from Eq. (22)) into Eqs. (18), (20) and (21) shows that the unbiased factor φ_{unbias} quantifies the reduction of the mean resistances of structural designs instead of nominal resistances predicted by STMs, which have different level of biases. A small value of φ_{unbias} therefore indicates that the resistance is discounted by a relatively large amount due to the bias and/or uncertainties of the model prediction. As observed in Table 8 and Table 9, for all STMs, φ_{unbias} is increased by the Bayesian parameter estimation due to the bias correction and model error reduction. It is also noteworthy that the proposed STM has the largest value of φ_{unbias} among all STMs both before and after the improvement by the Bayesian parameter estimation. The results suggest that the deterministic and probabilistic STM proposed in this paper have less bias and scatter, and thus helps achieve more economical designs while satisfying the target reliability.

6. Conclusions

This paper proposed a new strut-and-tie-model for deterministic and probabilistic prediction of the shear strength of reinforced concrete deep beams, and develops corresponding limit-state design formula and reduction factors for a reliable design of reinforced concrete deep beams. A Bayesian parameter estimation method was used to reduce the bias and uncertain errors of the proposed model based on a large database of experimental results. Seven different deterministic models which selected from code of practices and literatures were compared with the proposed models. The study has led to the following conclusions:

- The STM approaches of ACI 318-08 (2009) and AASHTO-LRFD (2008) resulted in large scatter and bias of prediction. Moreover, there still remained significant bias and uncertainties in the prediction even if multi-parameter models such as Siao (1993), Foster and Gilbert (1996), and Matamoros and Wong (2003) are used.
- Both before and after Bayesian updating processes, the proposed STM allowed for more accurate and robust prediction of shear strength of reinforced concrete deep beams than other deterministic methods.
- Three key parameters (E_s/E_c , a/d ,and ρ) were identified as the most effective parameters in capturing the errors and biases of the models using explanatory functions while ρ_v and ρ_h turn out to be relatively insignificant parameters for the improvement.
- Both the deterministic and probabilistic (i.e. Bayesian updating) STM proposed in this paper exhibited accurate predictions consistently over the ranges of key parameters. This demonstrated the selections of explanatory terms were suitable to describe the behaviors of deep beams e.g., concrete efficiency factor, the width of strut, concrete tensile strain in a consistent manner.

- A limit state design formula was presented for the proposed STM and the reduction factors were calibrated to achieve the target reliability index β of 3.5. The reduction factors of the proposed model were0.75 for normal strength concrete and 0.70 for high strength concrete.
- Finally, by using the same procedure, the unbiased reduction factors were computed. The results suggested that the deterministic and probabilistic STM proposed in this paper have less bias and scatter, and thus helps achieve more economical designs while satisfying the target reliability.

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