Genetic algorithm optimization of precast hollow core slabs

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Abstract. Precast hollow core slabs (HCS) are technically advanced products in the precast concrete industry, widely used in the last years due to their versatility, their multipurpose potential and their low cost. Using three dimensional FEM (Finite Element Method) elements, this study focuses on the stresses induced by the prestressing of steel. In particular the investigation of the spalling crack formation that takes place during prestressing is carried out, since it is important to assure the appropriate necessary margins concerning such stresses. In fact, spalling cracks may spread rapidly towards the web, leading to the detachment of the lower part of the slab. A parametric study takes place, capable of evaluating the influence of the tendon position and of the web width on the spalling stress. Consequently, after an extensive literature review on the topic of soft computing, an optimization of the HCS is performed by means of Genetic Algorithms coupled with 3-D FEM models.

Keywords: hollow core slabs; genetic algorithms; finite element method; optimization; soft computing; spalling; rotating crack model; prestressed reinforced concrete structures

1. Introduction

Precast hollow core slabs (HCS), also known as void slabs or simply hollow slabs, are technically advanced products in the precast concrete industry, widely used in the last years due to their versatility, their multipurpose potential and their low cost.

These are a particular kind of concrete slabs that can be prestressed or conventionally reinforced. In the first case, the prestressing steel, which is the only reinforcement of the HCS, is inserted in the form of strands or wires, placed in the lower part of the vertical concrete part between two adjacent cores (webs), where the concrete segment allows the optimal wire or strand bonding. The steel is prestressed before the insitu concrete topping is poured, and transfers the prestressing force to the concrete in thetransfer zone. The bonding of the prestressed reinforcement is heavily influenced by the condition of the concrete in the transmission zone (along a distance equal to the transmission length). The degree of bonding is influenced by several parameters (e.g.

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concrete type), but it is mostly influenced by the presence or absence of cracking in the transmission zone (Buettner and Becker 1998). Capuano *et al*. (2002) deal with the HCS design requirements and production technique. Fig. 1 shows the principal cracks due to the tensile stresses at the slab head in a prestressed precast shallow core slab (adapted from Capuano *et al*. 2002):

- splitting cracks: caused by stresses resulting from the development of prestressing in the anchorage zone, that may generate traction stresses in the concrete.
- bursting cracks: a local effect, generated by the strand slippage into the slab end while the former widens slightly on being cut.
- spalling cracks: occurring above the axis of the strands in the HCS end zone, caused also by the development of prestressing in the concrete at the slab ends where only the lower part holding the strands begins to be prestressed.

Splitting cracks can occur only in elements prestressed with pretensioned tendons, while bursting and spalling cracks can occur both in elements with pretensioned or post-tenioned tendons. Stress checks for spalling are particularly important. The spalling stress has its peak at the very end of the slab and if the stress is large enough it develops into horizontal cracks which are commonly known as "crocodile mouths", normally located where the web has a minimum thickness. Such cracks start at the end of the slab and run rapidly as long as needed until the equilibrium is found. This distance is usually rather small and never leads to large delamination.

Since a HCS has to go through additional stages that can damage its load bearing capacity (i.e., demoulding, transportation storage and erection), it is important to assure the appropriate necessary margins concerning the stresses that take place during prestressing (EN 1168 + A2 2010).

The tensile strength of HCS has been studied extensively in literature. In an early study, Pisanty and Regan (1991) carried out extensive experimental tests on prestressed HCS, and obtained useful results on the tensile strength of the web as a function of the web geometry and for different specimen locations. An analytical investigation on the above is found in Pisanty (1992).

More recently, for what concerns their structural behavior in general, Girhammar and Pajari (2008), after providing a literature review on the topic, focus, both analytically and experimentally, on the shear strength, with specific reference on the influence of the concrete topping. Gálvez *et al*. (2009), study with an experimental procedure the splitting failure of the concrete, induced by the action of the indented wire, in hollow core slabs and prestressed joists. They focus on the influence of the distance between wires, the thickness of concrete cover and the depth of the wire indentations in the bond-splitting. On the later, the authors propose also a bond model that takes into account the possible failure of concrete by splitting. Benítez and Gálvez (2011), present an analytical model for simulating the bond between steel and concrete, in precast prestressed concrete elements, which allows evaluating the bond stress in the transmission zone where bond stress is not constant, during the prestressing force release. Araujo *et al*. (2011) perform numerical non-linear modeling on HCS, and propose a methodology for the shear design. Chang *et al*. (2008) focus on the structural behavior of HCS subjected to fire, proposing more realistic numerical models that account for the membrane action of the RC topping layer.

The shear resistance of HCS is also investigated for HCS connected to steel beams (Lam *et al*. 2000, Lam 2002, Hegger *et al*. 2009) and wall panels (Wilson *et al*. 2008), HCS strengthened with carbon fiber polymers (Elgabbas *et al*. 2010) and using fiber reinforced concrete (Cuenca and Serna 2013).

In this study, the stresses arising from prestressing are analyzed by means of three dimensional FEM (Finite Element Method) elements. A parametric study is carried out, capable of evaluating the influence of the tendon position and of the web thickness on the spalling stress. This is of
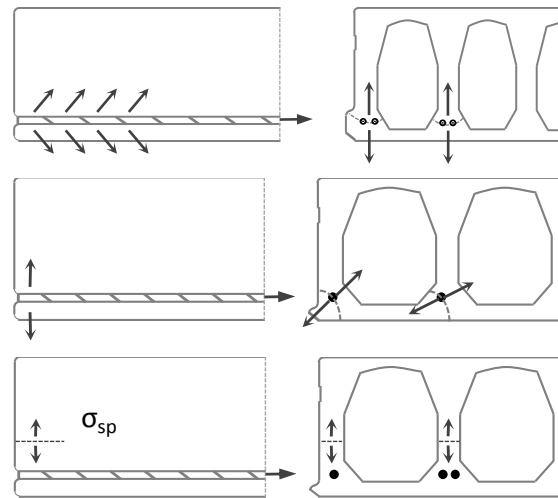
Fig. 1 Different tensile crack phenomena in HCS (splitting, bursting, spalling)

particular importance since the spalling stress relates directly with the structural robustness, intended as the ability of a structure to maintain localized an initial damage and avoid the propagation of failures in the system (for some of the latest achievements on the topic see Starossek and Haberland 2011 and Giuliani 2012). A comparison between the spalling stresses obtained with the FE analysis and those from European Standards is also performed.

After this, an optimization of the slab dimensions is carried out using a soft computing method (Sgambi 2008 provides an overview of artificial intelligence methods in the civil engineering field) in particular using a genetic algorithm approach. Topology optimization problems (that is to optimize for a specific performance the material layout within a given design space, for a given set of loads and boundary conditions) are common in civil engineering. Literature is vast, also focusing on the specific topic (concrete and non-linear structures). Usually, when distributing concrete, steel and voids, the optimized design resembles a strut-and-tie model, thus, principal applications of topology optimization in reinforced concrete design is in generating strut-and-tie models. Guan (2005) applies topology optimization on reinforced concrete deep beams with web openings under stress and displacement constraints. Bogomolny and Amir (2012) optimize the design of reinforced concrete structures and provide examples, discussing also issues arising from nonlinear analysis. In their formulation, focusing on the distribution of concrete, steel and void of simple structures, the objective is to maximize the end compliance. Fairbairn *et al*. (2004) focus on cracking of massive concrete structures (a small concrete dam) by means of a coupled thermo-chemo-mechanical model in FEM code and genetic algorithm optimization. Tomás and Martí (2010) apply optimization methods for the optimum geometric design of concrete shells using different objective functions (strain energy, weight or tensile stress). Del Coz Díaz *et al*. (2011) minimize the weight of a hollow concrete masonry block. In their problem the global stiffness (objective function) is maximized subject to a volume reduction constraint. More recently, Amir (2013) proposes a methodology and applies structural optimization to different concrete structures (a deep beam, a corbel and a wall with openings), optimizing the distribution of both concrete and reinforcement bars. Luo and Kang (2013) focus on the layout optimization problem of reinforced concrete structures, implementing a hybrid constraint-reduction strategy, thus minimizing the

structural mean compliance under yield stress constraints on concrete elements and a volume constraint on the steel elements. As stated earlier, in this study, coupled FEM models (micro- and meso-scale) are used together with genetic algorithm optimization, for minimizing the HCS volume subject to resistance and spalling stress criteria.

To this point, it is useful to provide a brief overview of SC methods and their applications in the civil engineering field. The concept of Soft Computing (SC), nowadays a formal computer science area of study, was formally introduced by Zadeh in 1981 (Zadeh 1997): it is a whole methodology directed to exploit the tolerance to the imprecision and to the uncertainty to achieve tractability and robustness with little computational cost, being the human mind the model of reference. These methods of analysis are therefore suitable for the treatment of problems in which an exact solution does not exist or if exists it is a very complex one. This issue is important for complex structural systems (Bontempi *et al*. 2008), with the complexity characterized by the fact that every system includes physical parts (hardware), logical aspects (software) and is designed and operated by people (humanware).

There are three principal constituents of SC: artificial neural networks, fuzzy systems and evolutionary algorithms.

Artificial Neural Networks (ANN) are systems of elaboration of the information inspired by the biological neural network. They are characterized for their robustness and flexibility and for their ability to generalize. In the last years ANN are applied in the civil engineering field, in structural design (Adeli and Park, 1995) reliability engineering and structural identification (Ceravolo *et al*. 1995, De Stefano *et al*. 1999, Kim *et al*. 2000, Jiang and Adeli 2005, Arangio and Bontempi 2010, Arangio and Beck, 2011), in transportation engineering (Vlahogianni *et al*. 2007, Boto-Giralda *et al*. 2010, Graf *et al*. 2010), foundation engineering (Reuter and Möller 2010), and for pattern recognition (Adeli and Samant 2000).

Fuzzy Systems (FS) allow reproducing the approximate reasoning of the human mind. The term derives from the fuzzy sets theory (Zadeh 1965). In civil engineering, FS are applied in problems that require analysis in presence of uncertainty: some noteworthy applications concern the structural design (Biondini *et al*. 2004a, Malekly *et al*. 2009), structural reliability (Dordoni *et al*. 2010), structural control (Kim *et al*. 2010), risk analysis (Sadeghi *et al*. 2010) and the treatment of uncertainties (Möller *et al*. 2003, Sgambi 2004, Baitsch and Hartmann 2006, Tagherouit *et al*. 2011, Wang and Li 2011).

A Genetic Algorithm (GA) is an example of a stochastic evolutionary process where an initial random population evolves in order to maximize a fitness function. It is important to observe that, (a.) during its evolution the GA acquires knowledge about the problem characterization, and (b.) GA's are adaptive, in the sense that they are able to interact with a mutable environment. In the structural engineering field, GA applications were pioneered in early 1990's (see for example Adeli and Kumar 1995 and Bontempi *et al*. 2000).

More recently, they have been used in design optimization (Descamps *et al*. 2011, BelHadj Ali *et al*. 2010), in maintenance (Orcesi and Frangopol 2011, Bocchini and Frangopol 2011), in structural health monitoring and control (Gaudenzi *et al*. 1998, Koh and Dyke 2007, Liu *et al*. 2008), in damage detection (Jafarkhani and Masri 2011), for the identification of moving loads (Deng and Cai 2009), for forecasting of freeway accident duration times (Lee and Wei 2010), for crew allocation in the precast industry (Al-Bazi and Dawood 2010) and in the design of composite structures (Rohani *et al*. 2013).

Mathakari *et al*. 2007 use GA's coupled with FEM modeling for the optimal design of steel truss structures. In Cheng (2007), a hybrid GA and FEM algorithm is proposed for the optimum

design of truss arch bridges, focusing both on the safety (in terms of stress) and the serviceability (in terms of deflection). Srinivas and Ramanjaneyulu (2007) implement a GA approach aided by Artificial Neural Networks to get the optimum sectional configuration of a T-girder bridge deck by minimizing the cost. More recently, Sgambi *et al*. 2012 apply GA optimization for the serviceability performance assessment of a long span suspension bridge, within a dependability framework.

Several research papers are found on GA applications on HCS, most of them dealing with the integrated production cost minimization (de Castilho *et al*. 2005, de Castilho *et al*. 2007, de Albuquerque *et al*. 2012).

Conceptually, the rest of this paper is organized in the following manner: Section 2 describes the numerical analyses for the investigation of the spalling crack formation on HCS and provides a comparison between the spalling stresses obtained with the FE analysis and those from European Standards. Section 3 deals with the GA optimization of the HCS form. Finally, Section 4 provides some generic conclusions.

2. Numerical modeling

A realistic numerical analysis is fundamental for the proper structural design since a numerical model has the purpose to faithfully reproduce the actual mechanical behavior of the structure. However, the numerical representation does not always represent reality since a series of hypothesis are made. These hypotheses influence the accuracy of the numerical response.

In this sense, in the analysis it is appropriate to use models with different levels of accuracy (micro-, meso - and macro - level) as indicated in Petrini *et al*. 2010. A micro or meso-level model can be built with a refined discretization and more accurate mathematical formulations. It is able to represent in a proper way the local behavior of the element but, geometrically, does not represent the whole structure, so no information can be obtained for the global behavior of the structure. On the contrary, a macro-level model is able to describe the general behavior of the structure, but it is not able to provide information on the local behavior of the single element or substructure. The
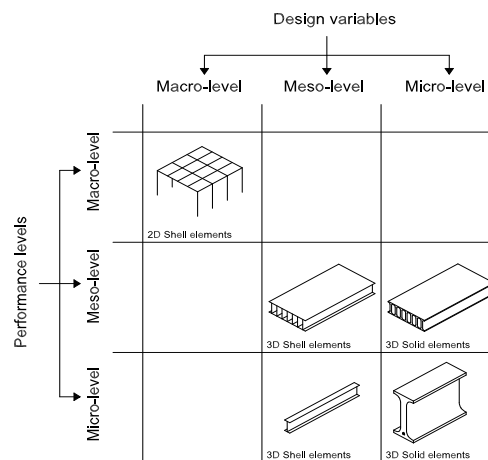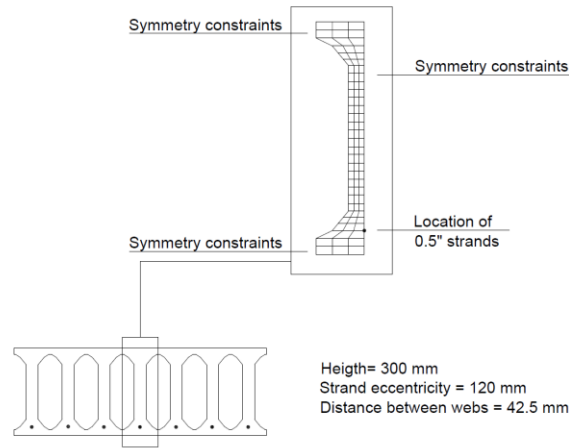

Fig. 2 Performance and modeling levels

Fig. 3 Cross-section of the reference HCS and corresponding numerical model

various model types result appropriate or not according to the quantity of information that can be extracted (e.g. a model with shell elements provides a more accurate representation than a beam model).

Considering the above, the structural analysis can be subdivided, from an operational point of view, in coordinated phases, as shown in Fig. 2 (adapted from Petrini and Bontempi 2011). On the x-axis are reported the design variables and on the y-axis the different performance levels. The link is established by an efficient modeling at different structural scale, maintaining a global coherence, being necessary that the results of a model at one level are the input for another model at a different scale as dictated by the sub-structuring technique (Przemieniecki 1968).

In the next paragraph, the above concept is applied in the specific case of HCS, where, for the spalling crack investigation a micro-scale model is implemented, while, in Section 3, an additional meso-scale model is implemented for the assessment of the global bending behavior of the HCS.

2.1 Description of the numerical model

HCS are produced with webs of different form and height. In this study, analyses are carried out on a slab having the web of Fig. 3. The reference slab, taken from a commercial HCS technical data catalogue, has a length of 8 meters and a load bearing capacity (excluding self-weight) of 8 kN/m^2. As stated in Section 1, in order to evaluate the internal stress state of the entire slab and it dependence on the geometric characteristics of the cross section, a parametric study is carried out, by varying the strand position and the rib width.

Focusing on the evaluation of the spalling stress, a 3-D micro-scale model is implemented, and the slab is modeled using 8-node hexahedral FEM elements, considering a single section consisting of a web and the two adjacent semi-voids and fixing the boundary conditions in such a way to reproduce the three dimensional behavior of the slab. Since the micro-scale model is implemented only for investigating the diffusion of the prestress force, and not the bending behavior, the self-weight is not considered in the analysis. Symmetry constraints are inserted along both the flanges and the lateral profile, in such a way to reproduce the continuity of the slab (Fig.

3). Furthermore, horizontal restraints are used to simulate the continuity of the structure. This is a big approximation, however it can be accepted since the micro-FEM model is used to study the prestressing effect only in the slab head. The cable prestress in the micro-model is modeled as an equivalent force. In order to better simulate the spalling effect, the prestressing force is not considered only in correspondence to the slab head, but instead is set along the line of the prestressing cable using a parabolic relationship, for a length equal to the transmission length, evaluated according to Eurocode 2 (EN 1992-1-1 1994). In fact, even if guidelines allow using a linear relationship (Capuano *et al.* 2002), this is not actually on the safe side for what regards the spalling stresses, since it underestimates the development of the prestressing force on the slab head, and thus, underestimates the vertical stresses developed in the ribs. The rib cross-section corresponds to the section of Fig. 3, while the model length is 100 cm, sufficient to allow a complete diffusion of the pre-stressing forces inside the rib.

Even though deformations are not very high and the spalling tensions should not create cracks, the analyses carried out are non-linear, using custom FORTRAN software. In this way, it is possible to account for the slight non-linearity of the concrete behavior under compression stress, and evaluate the spalling crack formation by means of the non-convergence of the iteration solution process.FEM nonlinear concrete modeling has been addressed in Garavaglia *et al.* 2013, while, the general importance of non-linear analysis for prestressed concrete structures is explained in Biondini *et al.* 2004b.

The constitutive relation for the concrete is assumed a generalization in three dimensions of well-known uniaxial constitutive relations that consider Reinforced Concrete (RC) elements as assemblies of two-dimensional membrane elements subjected to in-plane shear and normal stress:

• The Compression Field Theory (CFT), developed by Collins and Mitchell (1980), which assumes that the direction of the cracks (inclined compression field) and the principal compressive stress coincide.

• The Modified Compression Field Theory (MCFT), a modified theory based on the CFT which accounts for the contribution of tensile stresses in the cracked concrete (Vecchio and Collins 1986); and,

• The Rotating-Angle Softened Truss Model (RA-STM) where the shear stresses along crack plane are not considered and the direction of cracks is assumed to coincide with the direction of principal compressive stress after cracking (Hsu, 1988; Pang and Hsu, 1995).

The resulting constitutive model is a rotating crack model (RCM), (see for example Foster *et al.* 1996) with distributed cracking inside the finite element. The following procedure is carried out to calculate the stiffness matrix of the material in a global reference system.

• From the linear elastic solution obtained from the FEM analysis, the three principal deformations and the three principal directions are obtained.

• The stiffness matrix in the principal reference system is obtained by correlating at each principal direction a uniaxial constitutive relation. The Poisson's ratio is taken as null, thus, the stiffness matrix in this reference system results to be diagonal. The following constitutive relations are assumed for the stressed concrete in the principal reference system

$$\sigma_d = \zeta \cdot f'_c \left[2\left(\frac{\varepsilon_d}{\zeta \cdot \varepsilon_0}\right) - \left(\frac{\varepsilon_d}{\zeta \cdot \varepsilon_0}\right)^2 \right] \qquad \text{if}\left(\varepsilon_d \leq \zeta \cdot \varepsilon_0\right) \qquad (1)$$

$$\sigma_d = \zeta \cdot f'_c \left[1 - \left(\frac{\frac{\varepsilon_d}{\zeta \varepsilon_0} - 1}{\frac{2}{\zeta} - 1} \right)^2 \right] \qquad \text{if} \left(\varepsilon_d > \zeta \cdot \varepsilon_0 \right) \tag{2}$$

where σ_d is the compression strength, ε_d the compression strain, ζ the softening parameter, f'_c the uniaxial compressive strength, ε_0 the corresponding deformation.

In Eq. (1) and Eq. (2) the softening parameter $\zeta = \dfrac{1}{\sqrt{1 + 400 \cdot \varepsilon_{eq}}}$ helps reducing the compression resistance of the concrete in presence of transversal tensile deformation. The corresponding tensile deformation is taken equal to $\varepsilon_{eq} = \sqrt{\langle \varepsilon_1 \rangle^2 + \langle \varepsilon_2 \rangle^2 + \langle \varepsilon_3 \rangle^2}$, where $\langle \varepsilon_i \rangle$ is 0 if the deformation is due to compressive stress, and ε_i if it is due to tensile stress.

For concrete under tensile strain the following constitutive relation is used

$$\sigma_r = E_c \cdot \varepsilon_r \text{ if } (\varepsilon_r \leq \varepsilon_{cr}) \tag{3}$$

$$\sigma_r = f_{cr} \left(\frac{\varepsilon_{cr}}{\varepsilon_r} \right)^{0.4} \text{if} (\varepsilon_r > \varepsilon_{cr}) \tag{4}$$

where σ_r is the tensile stress, ε_r the tensile strain, f_{cr} the uniaxial tensile strength and ε_{cr} the corresponding deformation.

The value of the concrete elastic modulus is taken equal to $E_c = 9500 \cdot \sqrt[3]{f_c}$ [MPa] and the concrete cracking stress $f_{cr} = 0.25 \cdot \sqrt[3]{f_c^2}$ [MPa]. The shear modulus of the unckracked concrete G is evaluated by Eq. (5)

$$G_{ij} = \frac{E_i \cdot E_j}{E_i + E_j} \tag{5}$$

After the stiffness matrix $\underline{\underline{D}}$ is obtained in the principal reference system, the material properties in the global reference system are obtained by means of Eq. (6)

$$\underline{\underline{D}}_{XYZ} = \underline{\underline{T}}^T \cdot \underline{\underline{D}}_{123} \cdot \underline{\underline{T}} \tag{6}$$

where, $\underline{\underline{T}}$ is a rotation matrix. In addition, it is well known that the nonlinear analysis of reinforced concrete structures can suffer problem of non-objectivity and localization: it means that special attention must be given to the mesh size and control. In this case, the lack of objectivity of the mesh problem has been treated using a non-local approach (Bontempi and Malerba 1997).

The steel is modeled as a perfectly elastoplastic material, however, the performed analysis involve only the elastic branch.

Finally, the FEM model implemented in the analysis has 8000 degrees of freedom and consists of 1800 hexahedral elements.

2.2 Comments on the spalling stress

Fig. 4 shows the analyses outcome in terms of tensile deformations in the vertical directions that lead to the spalling effect. It is possible to observe that the only substantial vertical tensile deformations are actually along the slab head.

By assigning different values to some of the parameters of Table 1, nine analyses are carried out, aiming at the identification of the influence the rib thickness and the strand eccentricity on the rib vertical tensile stress. Figs 5, 6 and 7 show the spalling stress along the rib height, respectively for a web thickness of a 42.5, 52.5 e 62.5 mm and for different values of the strand eccentricity. It can be observed that a variation of merely one centimeter in the strand position or in the rib thickness leads up to a 30% variation to the vertical tensile stress on the rib. Furthermore, the sensitivity to the variation of the strand position is higher with respect to the variation of the web thickness.

The obtained results are also reported in the 3D graph of Fig. 8, which shows the spalling stress for the different values of the web thickness and the strand eccentricity. It can be observed that for the considered values, the spalling stress is approximately linear. This leads to the possibility to obtain (for the specific HCS) a sensitivity trend of the spalling stress to the strand eccentricity of 0.6 MPa/cm and to the web thickness of 0.20 Mpa/cm.

Table 1 The considered variables and their values.

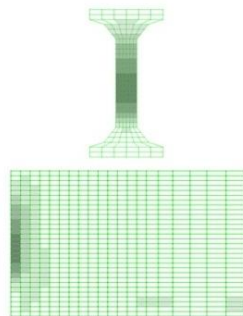| Variable	Symbol	Value
Slab height	h	300 mm
Web Thickness	c	42 mm
Distance between webs	b	425 mm
Concrete Grade	---	C30/37
S_{p0j}	S_{p0j}	1250 MPa
Prestressing strand area	A_p	93 mm^2 (one 0.5" strand)
Prestressing force	P_0	116250 N
Strand eccentricity	E_0	120 mm
Transmission length	l_{bp}	875 mm (estimated from the EN 1168+A2, 2010)

Fig. 4 Areas of maximum concentration of the vertical tensile deformations at the head (above) and at the central cross section (below)
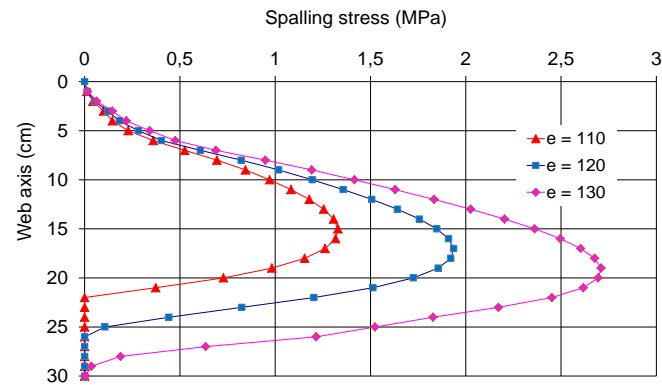
Fig. 5 Spalling stress along the web height, for a web thickness of 42.5 mm and for different values of the strand eccentricity
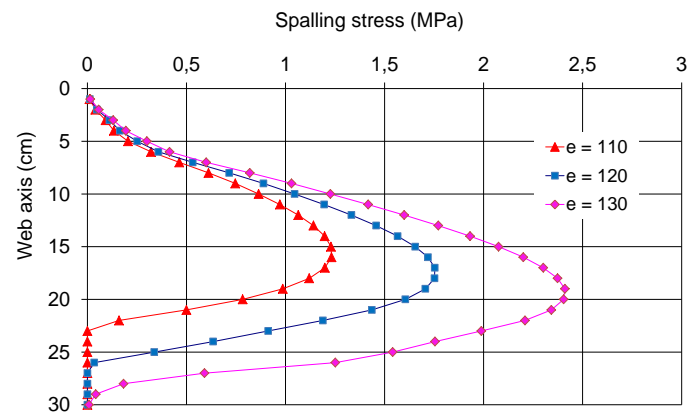

Fig. 6 Spalling stress along the web height, for a web thickness of 52.5 mm and for different values of the strand eccentricity
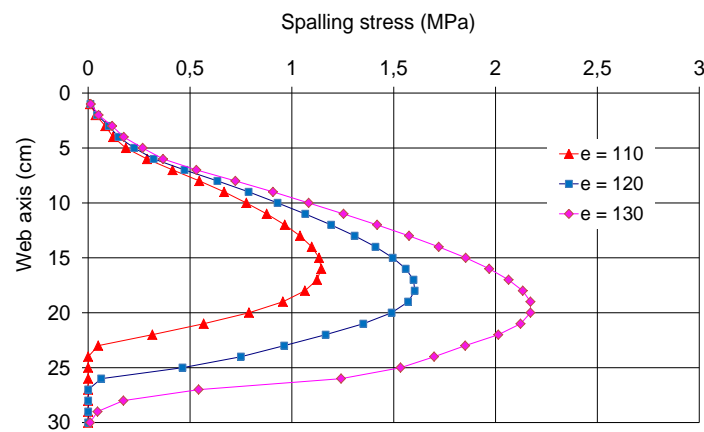

Fig. 7 Spalling stress along the web height, for a web thickness of 62.5 mm and for different values of the strand eccentricity
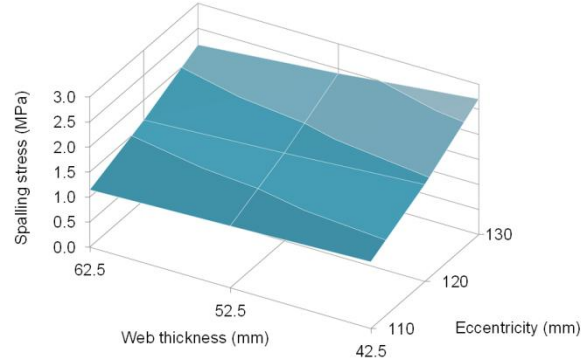
Fig. 8 Spalling stress for different values of the web thickness and the strand eccentricity

2.3 Comparison with the prescriptions of european standards

In the HCS design, the spalling stress in the webs is regulated by EN 1168+A2 (2010), Art. 4.3.3.2.1 "Resistance to spalling for prestressed hollow core slabs". The resistance to spalling has to be verified for the web in which the highest spalling stress will be generated, or, for the whole section if the strands or wires are essentially well distributed over the width of the element, the spalling stress σ_{sp} shall satisfy the following condition

$$\sigma_{sp} \leq f_{ct} \tag{7}$$

with

$$\sigma_{sp} = \frac{P_0}{b_w \cdot e_0} \times \frac{15 \cdot \alpha_e^{2,3} + 0,007}{1 + \left(\dfrac{l_{pt1}}{e_0}\right)^{1,5} \cdot \left(1,3 \cdot \alpha_e + 0,1\right)} \tag{8}$$

and

$$\alpha_e = \frac{(e_0 - k)}{h} \geq 0 \tag{9}$$

where
- f_{ct} is the value of the tensile strength of the concrete deduced at the time that the prestress is released on the basis of tests;
- P_o is the initial prestressing force just after release in the considered web;
- b_w is the thickness of an individual web;
- e_o is the eccentricity of the prestressing steel;
- l_{pt1} is the lower design value of the transmission length;
- k is the core radius taken equal to the ratio of the section modulus of the bottom fiber and the net area of the cross section (W_b/A_c);

The spalling stress has been calculated for all nine different ribs. The spalling stress calculated
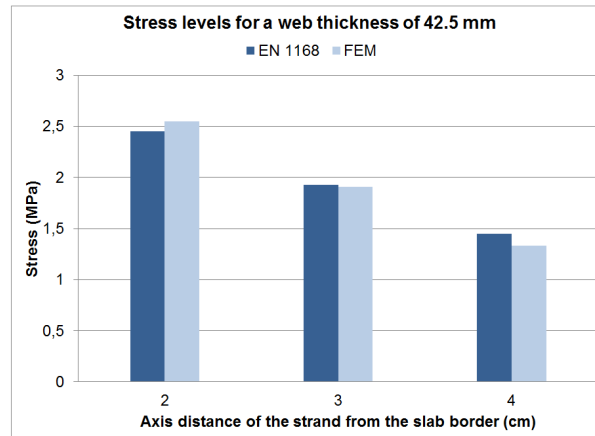
Fig. 9 Comparison of the spalling stresses from EN 1168+A2 (2010) and those obtained from the FEM analyses for a web thickness of 42.5 mm
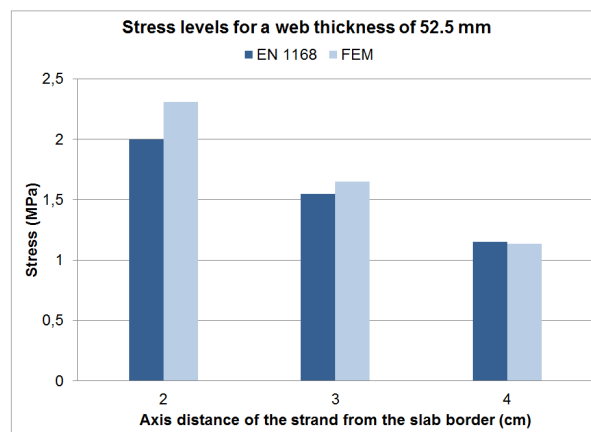

Fig. 10 Comparison of the spalling stresses from EN 1168+A2 (2010) and those obtained from the FEM analyses for a web thickness of 52.5 mm
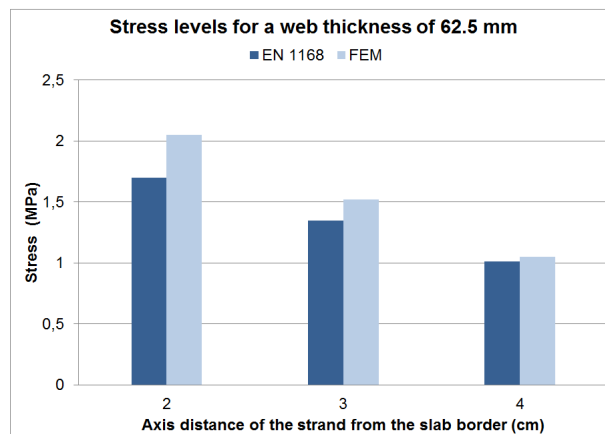

Fig. 11 Comparison of the spalling stresses from EN 1168+A2 (2010) and those obtained from the FEM analyses for a web thickness of 62.5 mm

by means of FEM analyses values are compared with those obtained with the indications provided in EN 1168 + A2 (2010) for different web thicknesses (Figs. 9-11).

The comparison shows a good fitting between the values obtained by the FEM analyses and those obtained by the EN 1168 standard. However, for lower stress values, the EN 1168 standard provides values almost equal or slightly higher than those obtained by the FEM analyses, a trend that inverts when the stress values come close to limit tensile strength of the concrete. For example, with reference to the case of Fig. 11 (web thickness of 62.5 mm), when the strand is placed 2 cm from the inferior border of the slab, the code check underestimates about 17% the spalling stresses calculated with the numerical method.

3. Optimization of the cross section

The slab dimensioning takes place accounting for both stresses due to service loads and those due to the prestressing force transfer. The stresses due to service loads depend on the global behavior of the slab under the vertical service loads, while the tensile stresses during prestressing, are local stresses that have an influence only on the transmission length. In order to take into account the prestressing stresses it is necessary to adopt a refined numerical model of the slab near the support, such as the 3D model implemented in Section 2 that accounts for the diffusion of the prestressing forces. The stresses due to service loads on the other hand, can be calculated with simpler numerical models that account for the global behavior of the slab subjected to flexure and shear.

Considering the above, in the adopted optimization procedure, two different numerical models have been adopted. A micro-scale model consisting in hexahedral FEM elements as the one described in Section 2, and a meso-scale model obtained by the discretization of the slab with shell elements, using appropriate boundary conditions that account for the translation and reflection symmetries (Fig. 12).

More precisely, the model represents only half of the rib (consisting in one web and the corresponding lower and upper parts of the slab). In order to reproduce the entire rib, reflection symmetry constraints are introduced in the opposed section of the reference system (with origins at the edge of the half rib). In order to reproduce the behavior of the entire slab starting from the single rib, translation symmetry constraints are introduced at flanges along the entire length of the rib.

In the meso-scale model, equivalent forces are inserted in correspondence to the slab head for modeling the pre-stress of the cables, while both concrete and steel are considered with linear elastic constitutive relations.

Fig. 13 provides an overview of the five geometric features considered as variables during the automatic sizing of the cross section, while the strand prestressing is considered as a sixth variable.

Table 2 provides a description of the variables. These variables are optimized with respect to the total slab weight, accounting for the verification of global and local stress states. Using different values for the geometric features, it is possible to consider hollow core slabs totally different in form and size during the optimization process.

As stated in Section 1, a GA is an example of a stochastic evolutionary process where an initial random population evolves in order to maximize a fitness function. The GA initiates by considering an initial population of vectors $\underline{x}$ created assigning random values to the unknown variables. This represents a specific configuration of the problem under consideration.

Table 2 Description of the considered variables

VARIABLE	DESCRIPTION
X1	Upper flange thickness
X2	Web thickness
X3	Lower flange thickness
X4	Slab height
X5	Distance between webs
X6	Strand prestressing force

Table 3 Original values and values obtained for the variables after the optimization process

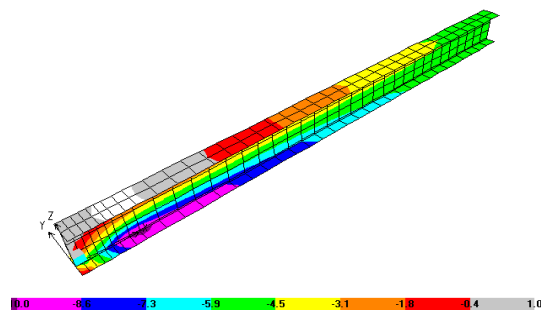| Variable	Initial value	Value after the optimization
Upper flange thickness	30 mm	25 mm
Web thickness	42 mm	35 mm
Lower flange thickness	30 mm	25 mm
Slab height	300 mm	320 mm
Distance between webs	425 mm	350 mm
Strand prestressing force	116 KN	106 kN

Fig. 12 Longitudinal stress in the global numerical model of the as a consequence of both the prestressing and the vertical loads [MPa]
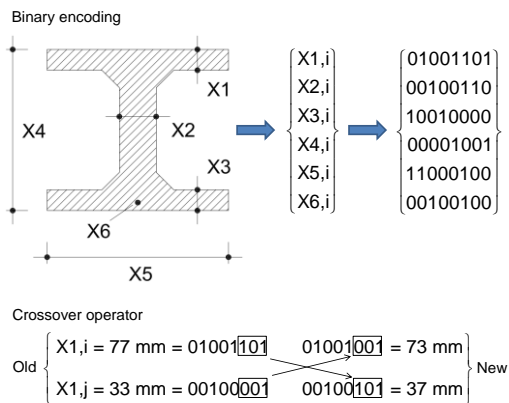

Fig. 13 The binary genetic code matching the geometry characteristics of the section (above) and example of crossover operation (below)
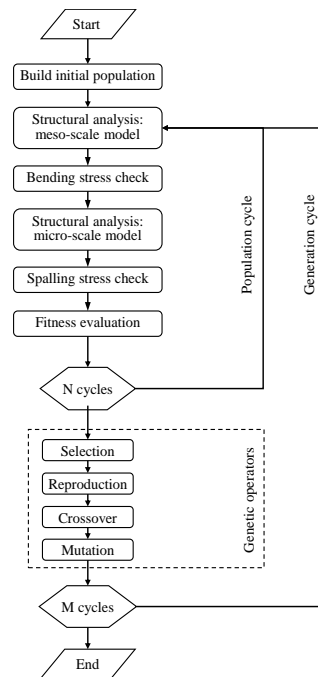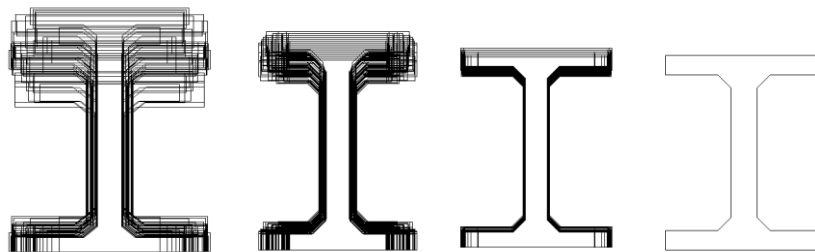
Fig. 14 Flowchart of the optimization process

Fig. 15 Evolution of the cross section during the optimization: (a) initial population; (b) population after 100 regenerations; (c) population after 150 regenerations; (d) best individual

In the case under study for example, each population represents a section with different geometric features. In this sense, the values adopted for the optimization problem variables, characterize a specific solution (individual). These variables, appropriately decoded in binary code, represent the genetic code of the individual.

The initial population (representing cross sections of different geometry) is randomly chosen. On the basis of the genetic code of each individual, the global and local structural models are created. The structural analyses are carried out on this model, focusing on the evaluation of the maximum stress levels. The optimization aims to the cross section geometry with the minimum weight, subjected to the check of local and global stresses. The latter constitute the optimization problem constrains. From the genetic code of each individual it is possible to calculate the volume, and thus the weight. In this study the fitness function, the one that measures the quality of the represented solution, is taken as a function of the weight of each individual.

From an analytical point of view, the optimization problem is equivalent to the estimation of a minimum of a function $F(x)$ subject to certain constraints $g_i(\underline{x})$ and $h_i(\underline{x})$, as shown in Eq. 10.

$$\begin{cases} \min F(\underline{x}) \\ a_i \le g_i(\underline{x}) \le b_i \quad i = 1...m \\ h_j(\underline{x}) \le c_i \quad j = 1...k \end{cases} \tag{10}$$

The fitness function F includes terms to represent the weight of the slab as a function of the slab section geometry. The considered constraints in Eq. 10 are two kind of inequality constraints.

First, functions $g_i(x)$, representing the geometry constrains. In order to reduce the optimization space close to a meaningful design solution, $g_i(x)$ assume the values in the range indicated in Tab. 2. The above constraints are implicitly satisfied during the definition of the variable space within the optimization procedure.

Functions $h_i(\underline{x})$ instead, represent the structural safety constraints. In this study, two checks are carried out. The first one on the bending stress, carried out after the initial structural analyses on the meso-scale model. The second one, on the spalling stress, carried out on the micro-scale model. If both checks are positive, the individual is considered as fitting the constrain conditions, otherwise, it is discarded and a different element is introduced in the population.

After the structural analyses it is possible to order all individuals of the population that respect the stress constrains, on the basis of their fitness function. Consequently, by applying different genetic operators (reproduction, crossover, mutation), it is possible to come up with a new population with a generally higher value of the fitness function.

Fitness proportionate selection (roulette-wheel selection) is the implemented selection approach. In general, this is a very common selection approach, for which, if there is a big difference between the fitness function of individuals, a weaker individual has lower probability to survive through to the next generation. For this particular case, and considering the specific geometric limitations for the slab, this selection procedure is reliable since the research space is considered limited to solutions already valid from an engineering point of view. his procedure is repeated for an adequate number of cycles, until the population of the individuals reaches a adequately stable reproduction state, that is until a convergence criterion is satisfied. The entire procedure is shown in the flowchart of Fig. 14.

As discussed earlier, the six variables considered in the design are transformed in binary code and the string of 0 and 1's formed in this manner represents the genetic code of a specific individual. During the optimization process, the assumed variables are bounded in the intervals indicated in Table 2. The genetic analyses are performed on a population of 70 individuals that has been regenerated 200 times. The probability variables that govern the crossover and mutation are taken equal respectively to 0.85 and 0.02.

Table 3 shows the initial values and those obtained values at the end of the optimization process, while Fig. 15 reproduces the evolution of the cross section during the different population generations.

As it can be observed from the results of Table 3, comparing the obtained geometry with the original one from Table 1, the optimization leads to a slightly higher section, with webs placed closer to each other. Both upper and lower flanges are now thinner, while the webs are thicker. Also, the prestressing force is slightly lower. The total weight reduction, compared to the original design, is of approximately 8%.

4. Conclusions

This study focuses on the structural design and optimization of geometric and mechanical characteristics of hollow core slabs. The structural behavior of such structures requires the use of different modeling levels for the safety assessment. Indeed, if a meso-scale model is capable of reproducing the bending behavior of the structure at a global level, at a local level, the prestressing stress at the transfer zone can lead to splitting and spalling that may compromise the safety of the entire slab.

Considering the above, the spalling crack formation has been investigated in detail, using both three dimensional FEM elements and simplified formulas found in codes and standards. More precisely, the following points are made:

- The spalling stress value is very sensitive to the web thickness and the strand eccentricity. For the specific cases considered, the vertical tensile stress has a sensitivity trend to the web thickness of 0.20 Mpa/cm and to the strand eccentricity of 0.6 MPa/cm.

- The assumption of a linear diffusion of the prestressing, usually accepted in codes, leads to an underestimation of the spalling stress of up to 20%. This underestimation is higher for slabs that have a greater strand eccentricity.

These points lead to the conclusion that the form optimization of a HCS cannot be implemented by considering the purely shell behavior of the slab and the structural modeling in different scales has to be considered. More precisely, a macro and meso-scale model, capable to swiftly portray the bending characteristics of the slab and a micro-scale model, capable of reproducing the diffusion of the cracks due to the prestressing forces at the slab head. In fact, the optimization was performed by coupling a commercial FEM code, able to represent the meso-scale model, and a numerical model in FORTRAN, able to represent the micro-scale model and to conduct the genetic optimization.

With this in mind, it is important to point out the way an optimization process can envisage different models of the same structure, each one of which optimized for a particular aspect of the structural behavior.

Acknowledgements

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