Computers and Concrete, *Vol. 12, No. 6 (2013) 819-829*
DOI: http://dx.doi.org/10.12989/cac.2013.12.6.819

Load capacity of high-strength reinforced concrete slabs by yield line theory

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(*Received February 4, 2013, Revised July 16, 2013, Accepted July 30, 2013*)

Abstract. The objective of this study is to determine whether or not the yield line theory, an effective method widely used for slabs made of ordinary concrete, can be used also for the reinforced concrete slabs made of high-strength concrete. Flexural behavior of simply supported slabs in three different sizes were investigated under concentrated load at mid-span. Additionally, behavior of high strength reinforced concrete slabs with 50 mm and 150 mm reinforcement spacings also studied. Failure loads, deflections, experimental and theoretical failure mechanisms were evaluated. The difference between the moments based on yield line theory and experimental moments varied between 1% to 3%. Experimental and analysis results revealed that yield line analysis could conveniently be employed in the analysis of high strength reinforced concrete slabs.

Keywords: yield line theory; high strength concrete; reinforced concrete slab; collapse mechanism

1. Introduction

Concrete slabs are planar construction members providing large and smooth surfaces, able to transmit the loads over them to beams or columns. Slabs not only bear vertical loads but also transmit horizontal loads from one member to another (Ersoy 2010). The thicknesses of slabs are significantly lower than the other two dimensions. They can be casted in several different geometries and their geometrical properties are defined based on mid-surface separating the slab into two equal pieces parallel to plate surfaces. Working principle of slab plate is generally depend on orthogonality or eccentricity of the load over the slab. Ortogonal loads over the center line indicate bending stresses over slab (Uğural 1999).

There are three different methods to analyze a plate as of elastic, plastic and non-linear analysis. Irregular geometries of slab plates and loading conditions makes the elastic analysis difficult and time-consuming. Similar difficulties are also valid for non-linear analysis taking the real response of the slab into consideration. Limit analysis may also not hinder the difficulties in some cases. For instance, a limit analysis in a relatively long concrete continuous beam with several spans requires the investigation of collapse mechanisms, calculation of rotations in plastic hinges constituting the mechanism, evaluation of rotation capacities of hinges and their sufficiency against the impacts. In

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http://www.techno-press.org/?journal=cac&subpage=8 ISSN: 1598-8198 (Print), 1598-818X (Online)

such cases, there exists relatively simplified approaches taking re-distribution of plastic moment into consideration and making the calculations easier. Yield line method is one of these approaches (Nilson 2000, Herbert *et al*. 2011).

With yield line method, collapse mechanisms are investigated under actual loading and support conditions and ultimate bearing capacity. Collapse loads are determined in a more realistic approach (Johansen 1962, Kennedy and Goodchild 2000, Theodossopoulos 2012).

Structural analysis on slabs with yield line method is based on equilibrium equations for "n" number of rigid members constituting the mechanism. In this way, "3n" numbers of equilibrium equations are obtained for the most general case (Szilard 2004, Jones and Wood 1967).

Worldwide, concrete with a strength over a value specified in relevant standards is defined as "high strength concrete" and this standard value is specified as 50 MPa in Turkish Standard of TS 500 (2000). CEB/FIP (2010) specifies such values as minimum 60MPa and maximum 130 MPa. ACI 318-11 (2011) specifies 41 MPa as the ultimate strength limit for standard concrete. Effective design criteria for reinforced concrete are based on maximum compressive strength of 50 MPa. Therefore, compressive strengths over 50 MPa is referred to as "high strength" and validity of design criteria of ordinary concrete for high strength concrete should be investigated (Pul *et al*. 2002). Behavior of high strength concrete under load is more brittle than ordinary concrete. Therefore, applicability of relationships observed for ordinary concrete into high strength reinforced concrete slabs is always open for discussion.

Objective of this study is to investigate whether or not the yield line method commonly used for ordinary reinforced concrete slabs can also be applied for high-strength reinforced concrete slabs.

2. Experimental study

2.1 Materials used in test samples

Reinforced concrete test samples were produced from high-strength concrete. Several mixtures were tried to achieve the desired compressive strength by using different aggregate rates, cement doses, water/cement ratios (W/C), silica fume or substitute additive rates and ultimate mixture was

Table 1 Aggregate physical characteristics

Size	Loose unit weight (kg/m^3)	Specific gravity (kg/m^3)		Water absorption (%)
		Dry	Saturated	
Coarse (> 4mm)	1435	2712	2692	0,49
Fine (< 4mm)	1486	2668	2685	0,55

Table 2 Concrete composition

Concrete	Cement type	Cement dose (kg/m^3)	W/C	Aggregate (kg/m^3)	Silica fume (kg/m^3)	Additive %	Saturation water %
High Strength	Cem I 42,5 R	500	0,30	1737	50	2	1,52

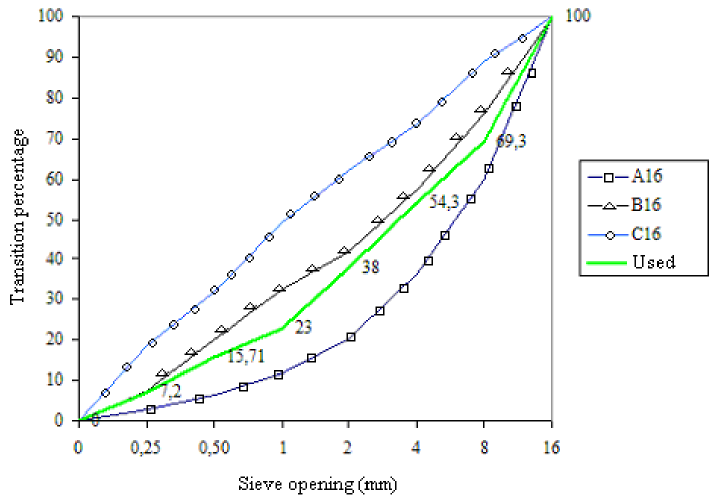

Fig. 1 Grain size distribution of aggregate

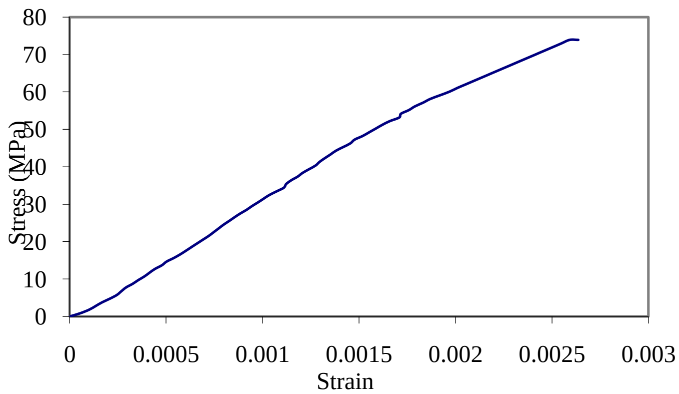

Fig. 2 Stress-strain curve for high-strength concrete

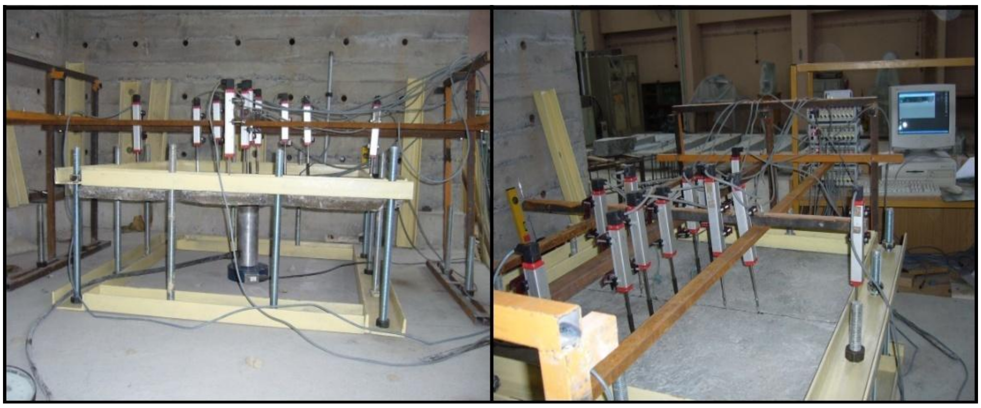

Fig. 3 Experimental setup

Table 3 Mechanical chracteristics of slab concrete

	High-strength concrete	Modules of elasticity (MPa)	Poisson ratio
f_{cm} (MPa)	74,68		
Std. Dev.	6,51	34000	0,236
f_{ck} (MPa)	66,4		

f_{cm} : average compressive strength

f_{ck} :characteristic compressive strength

Table 4 Mechanical characteristics of reinforcement

Diameter (mm)	Average tension strength (N/mm^2)	Average yield strength (N/mm^2)	Rupture strain (%)
8	619	430	21

Table 5 Characteristics of reinforced concrete slabs

Slab	Dimensions (mm)	Free-span (mm)	Reinforcement
B4	900 × 900 × 40	660 × 660	Φ8/100
B4-V	900 × 900 × 40	660 × 660	Φ8/50
B4-XV	900 × 900 × 40	660 × 660	Φ8/150
BD4	900 × 1300 × 40	660 × 1060	Φ8/100
BB4	1300 × 1300 × 40	1060 × 1060	Φ8/100

determined (TS 802 1985). Aggregate grain size distribution curve was presented in Fig. 1 and physical characteristics were provided in Table 1. Maximum aggregate size was 16 mm. Compositions of concretes produced by using Cem I 42,5R cement were provided in Table 2.

Control samples taken from produced slab concretes were cured in accordance with relevant standards (TS 1247 1984) and concrete strengths were determined. Concrete mechanical characteristics were given in Table 3 and average stress-strain curve was presented in Fig. 2. Slabs were also cured until the test date by proving continuous wetting.

8 mm reinforcement was used in test samples. The reinforcement arrangements were made as 50, 100 and 150 mm spacings in two perpendicular directions. Mechanical characteristics of the reinforcement were provided in Table 4.

2.2 Reinforced concrete slabs and testing assembly

Test slabs were casted in three different sizes with three different reinforcements and supported at four edges with simple supports. A concentrated load was applied at mid-span. Testing assembly was constructed by using U140 (h = 140 mm) steel girders. For better observation of collapse mechanism, loading was applied from the bottom to top with a loading piston pushing toward 100 × 100 × 20 mm steel plate. Geometrical characteristics of test slabs were provided in Table 5. Experimental setup with a test slab and measurement system was presented in Fig. 3.

2.3 Experimental behaviour and collapse mechanisms of slabs

Mid-span deflection graphs for load-span interval for each test were presented in Fig. 4. Some collapse mechanisms were presented in Figs. 5 to 7.

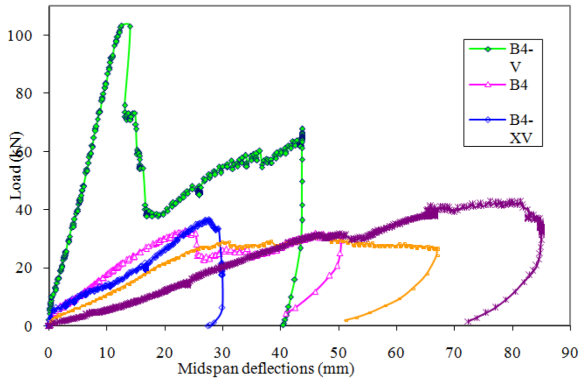

Fig. 4 Load-midspan deflections

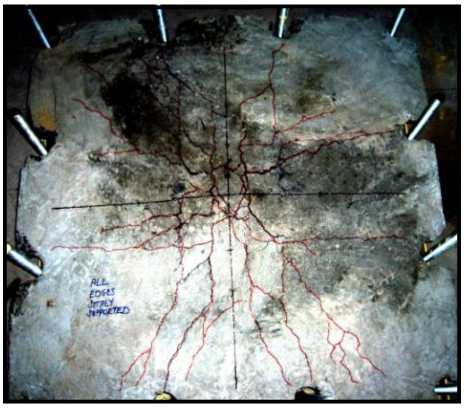

Fig. 5 Collapse mechanism sample-1 (BB4)

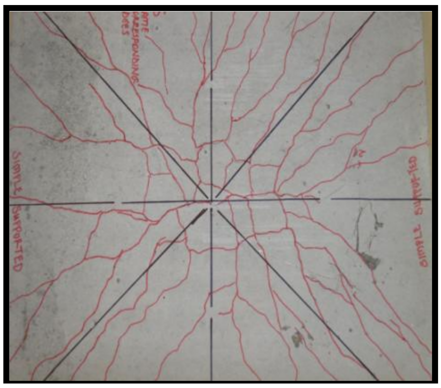

Fig. 6 Collapse mechanism sample-2 (B4)

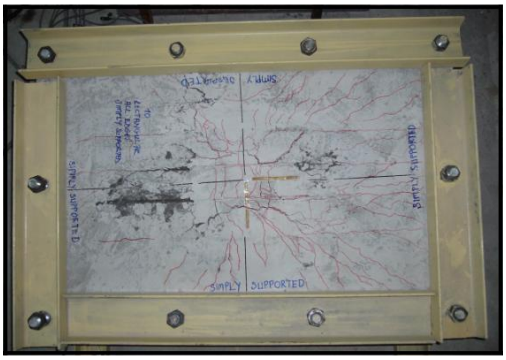

Fig. 7 Collapse mechanism sample-3 (BD4)

2.4 Evaluation of slab behaviors with yield line method

Theoretical collapse mechanisms determined by using experimental collapse mechanisms were presented in Fig. 8. The parameters “a” and “b” were used to determine the precise locations of collapse mechanisms.

Cracks extended up to edges and the cracks around the loading plate started just by the plate then extended toward edges either with an angle or along the plate direction. Collapse mechanism of the slab was similar to mechanism of a flat slab and resting over a square column at span (Imam and Collins 2013, Mokhatar *et al*. 2013, Muthu *et al*. 2007, Maunder *et al*. 2012, Jones and Wood 1967, Jones 1962) The distance “b” shows the furcation formed by corner lever at yield line. In case of simply supported edges, main cracks over the slab do not reach to corners, they end up at the edge with an angle (Chang *et al*. 2010, Wüst and Wagner 2008, Jones 1962, Inglersev 1921). Similar cracking patters were observed in current slabs. Cracking mechanism was composed of triangular cracks (I), rectangular cracks parallel to loading plate (II) and the cracks within the regions formed by furcation of yield line at corners (III). All these regions constitute total form of the collapse mechanism of the slab.

Yield lines formed in simply supported slabs reach up to slab edges which are also the rotation axis of the slab. Since the corners are not fixed, they separate out from the supports and set on with increasing loading. Yield lines either are not formed at corners due to corner lever or left as minor cracks (Soudki *et al*. 2012, Esfahani *et al*. 2009, Foster *et al*. 2004, Johansen 1962, Jones 1962, Inglersev 1921). A corner lever formation observed in a test was presented in Fig. 9.

660 × 660 mm free-span simply supported square slabs were analyzed according to virtual work principles by using the collapse mechanism in Fig. 8. Several analyses were performed to determine the parameters of the best collapse mechanism. Subsequent analysis revealed that the corner lever region was distinctive parameter to find out the best collapse mechanism of the slab. Following relationship was obtained for collapse load by using virtual work principle for each section of the mechanism (I, II and III);

$$P\delta = [8mL\theta]_I + [4mL\theta]_{II} + [4mL\theta]_{III} \tag{1}$$

Where; θ is rotation angle of each section, L is the length of yield line over rotation axis, δ is vertical deflection at load point. Since the virtual work method is an upper bound analysis, the analysis yields lower limits of collapse loads. “The traditional hand calculation approach to yield line analysis (Johansen 1962) requires the initial specification of a potential mechanism for which collapse load is calculated. This estimate will be an upper bound on the true collapse load. Subsequent trial mechanisms may than be investigated and the lowest collapse load found is taken as the exact value” (Johnson 2006). Collapse loads determined by this method were presented in Fig. 10.

The load of P = 8,0565 m obtained for the best collapse mechanism was considered as bearing capacity of simply supported 660 × 660 mm slab with isotropic reinforcement and loaded at center with a square steel plate. Collapse mechanisms and collapse loads were also determined for the other simply supported slabs. Experimental ultimate moments were calculated with following equations by taking into consideration the compression and tension force, calculation principles for rectangular reinforced concrete sections with single reinforcement (Ersoy 2010, Doğangün 2010) and by using relevant stress block parameter for high-strength concrete (Ö ztekin *et al*. 2003)

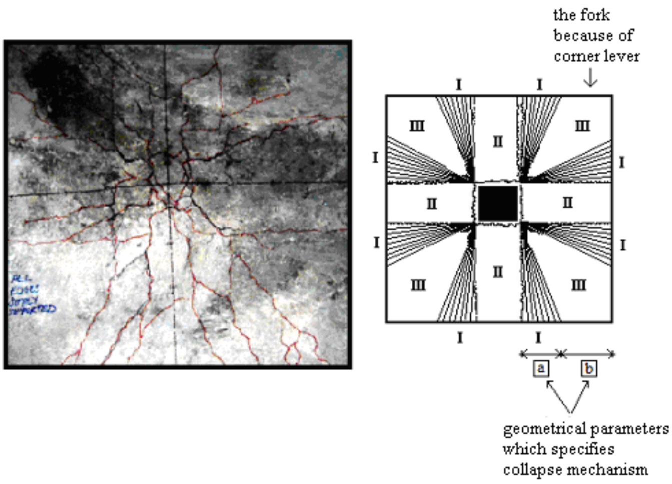

Fig. 8 Collapse mechanism of simply-supported slabs

Fig. 9 Corner lever at free corners

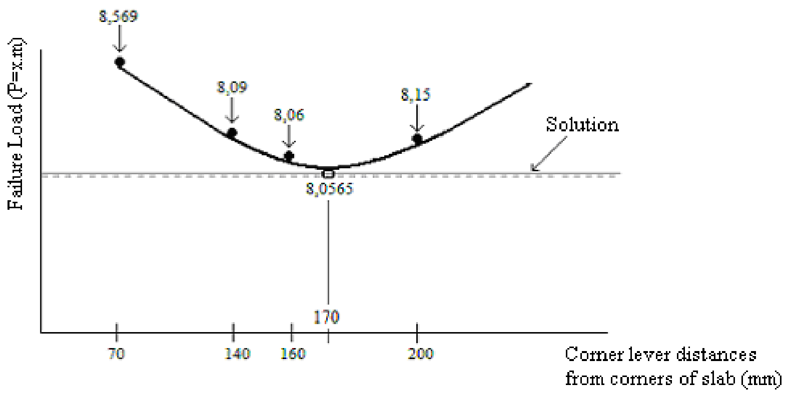

Fig. 10 Collapse load by virtual work method

Table 6 Comparison of simply supported slabs

Slab	Collapse mechanism parameters		Yield line analysis	Experimental load (kN)	Moment by yield lines method , m (kNm/m)	Experimental resistance moment, m_r (kNm/m)	m/m_r
	a (mm)	b (mm)					
B4	110	170	$P = 8{,}0565m$	31,79	3,94	3,90	1,01
B4-V	110	170	$P = 8{,}0565m$	65,49	8,13	8,02	1,01
B4-XV	110	170	$P = 8{,}0565m$	22,96	2,84	2,81	1,01
BD4	Long edge		$P = 7{,}86m$	31,22	3,97	3,91	1,02
	164,2	315,8					
	Short edge						
	95,7	184,3					
BB4	188,5	291,4	$P = 7{,}54m$	30,59	4,05	3,92	1,03

$$F_c = 0{,}83 f_{ck} ba \tag{2}$$

$$F_s = A_s f_y \tag{3}$$

When these equations were put into equilibrium, depth of rectangular stress block will become

$$a = \frac{A_s f_y}{0{,}83 f_{ck} b} \tag{4}$$

Then the moment arm (z) will be

$$z = d - a/2 \tag{5}$$

Finally, the moment bearing capacity of the slab is calculated by

$$m_r = m = A_s f_y z \tag{6}$$

Resistance moment (unit moment) for high-strength slabs with $\phi 8/100$ reinforcement, A_s =

5,02 cm^2/m, and f_y = 430 MPa calculated for 8 mm ribbed bars was calculated as m = 3,90 KNm/m. Calculations and experimental values were provided in Table 6 to compare the slabs. The m_r moments in table were calculated by using f_{ck} values of the control samples of the slabs.

3. Conclusions

Yield line method was used in this study to analyze reinforced concrete slabs made of high-strength concrete. Slabs were loaded with a concentrated load applied through 100 × 100 mm steel plate at mid-span. Load-midspan deflections and collapse loads were determined experimentally. Moving from experiments, theoretical failure mechanisms of slabs were determined. Corner lever was found to be the distinctive parameter for calculations in simply supported slabs. Corner lever was limiting the yield lines and determining the yield line locations. Such locations and cracked parts of slabs (denoted as I, II and III in Fig. 8) were used together with virtual work principles to determine the moment value specifying the lower limits of collapse loads for tested slabs (Görkem 2009). Several analyses were performed to determine the parameters of the best collapse mechanism according to lower bound solution. Calculations and experimental values were examined, comparatively. The difference between the moments to be realized based on yield lines theory and experimental moments were mostly 3%. It was concluded herein that yield lines method could be used for high-strength reinforced concrete slabs.

Symbols

I, II, III : Regions of collapse mechanism
a, b : Corner lever distances, depth of concrete compressive region, slab unit width
A_s : Reinforcement area
$B4$: 900 × 900 × 40 mm square slab with Φ8/100 reinforcement
$BB4$: 1300 × 1300 × 40 mm square slab with Φ8/100 reinforcement
$BD4$: 900 × 1300 × 40 mm rectangular slab with Φ8/100 reinforcement
$B4-V$: 900 ×900 × 40 mm square slab with Φ8/50 reinforcement
$B4-XV$: 900 × 900 × 40 mm square slab with Φ8/150 reinforcement
F_c : Total concrete compressive force at cross-section
F_s : Total tension force beared by steel reinforcement at cross-section
f_{ck} : 28-day characteristic compressive strength of concrete
f_y : Yield strength of steel
L : Projection length of rotation axis of yield lines (support length)
m : Unit moment
m_r : Resistance moment
P : Experimental collapse load
z : Moment arm in single reinforcement
θ : Rotation angle
δ : Deflection at loading point

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