**Computers and Concrete**, *Vol. 12, No. 6 (2013) 739-754*
DOI: http://dx.doi.org/10.12989/cac.2013.12.6.739


# Effects of cyclic loading on the long-term deflection of prestressed concrete beams

Lihai Zhang*[1], Priyan Mendis[1], Wong Chon Hon[1], Sam Fragomeni[2], Nelson Lam[1] and Yilun Song[1]

[1]*Department of Infrastructure Engineering, The University of Melbourne, VIC 3010, Australia*
[2]*Department of Civil Engineering, Victoria University, VIC 3010, Australia*

(*Received November 2, 2011, Revised May 21, 2013, Accepted July 30, 2013*)

**Abstract.** Creep and shrinkage have pronounced effects on the long-term deflection of prestressed concrete members. Under repeated loading, the rate of creep in prestressed concrete members is often accelerated. In this paper, an iterative computational procedure based on the well known Model B3 for creep and shrinkage was developed to predict the time-dependent deflection of partially prestressed concrete members. The developed model was validated using the experimental observed deflection behavior of a simply supported partially prestressed concrete beam under repeated loading. The validated model was then employed to make predictions of the long-term deflection of the prestressed beams under a variety of conditions (e.g., water cement ratio, relatively humidity and time at drying). The simulation results demonstrate that ignoring creep and shrinkage could lead to significant underestimation of the long-term deflection of a prestressed concrete member. The model will prove useful in reducing the long-term deflection of the prestressed concrete members via the optimal selection of a concrete mix and prestressing forces.

**Keywords:** shrinkage; creep; prestressed concrete beams; cyclic loading

## 1. Introduction

Prestressing is a technique which is widely used in design and construction practice. It is well known that creep induced long-term deflection of a concrete member could be several times higher than elastic deformation (Bažant and Panula 1980), and thereby can lead to an increased rate of loss of tendon forces, excessive structure deformation and crack development in prestressed concrete beams. Most importantly, when subjected to long-term cyclic loading (e.g., traffic loading on bridges and vibrations of heavy equipments), the rate of creep can be significantly accelerated (Au and Si 2011, Bažant and Kim 1992c, Li *et al*. 2011).

Creep of concrete under dynamic loading is a complex phenomenon (Islam *et al*. 2011, Khora *et al*. 2001). With the continuous accumulation of vast amount of experimental data over recent decades as well as recent advances in computer based data fitting and optimization techniques, significant achievement in the understanding of this phenomenon has been made. In 2000, Bažant

*Corresponding author, Ph.D., E-mail: lihzhang@unimelb.edu.au


http://www.techno-press.org/?journal=cac&subpage=8 ISSN: 1598-8198 (Print), 1598-818X (Online)

and co-researchers proposed the so called Model B3 for concrete creep and shrinkage prediction (Bažant and Baweja 2000). Derived as a significant improved version of the BP model developed by Bažant and Panula in the early 1990s (Bažant and Kim 1991, Bažant and Kim 1992a, Bažant and Kim 1992b, Bažant and Kim 1992c, Bažant *et al*. 1991, Bažant *et al*. 1993, Bažant *et al*. 1992), Model B3 represents an important step towards the understanding of the fundamental chemical and physical processes that govern the creep and shrinkage of concrete. The prediction results from Model B3 are generally more accurate than other available models (e.g., CEB-PIF 1990 and ACI 209, of variation of the deviations (Bažant and Baweja 2000). Most importantly, Model B3 takes into consideration a range of key factors that significantly influence the development of creep, such as relatively humidity, concrete water/cement ratio, age at loading and so on. A simplified model without consideration of these key factors, could lead to predictions errors. For example, a recent study on service load behaviour of low rise composite frames demonstrated that relative humidity and concrete age at loading cannot be neglected in the prediction of time-dependent deformations of the structure (Chaudhary *et al*. 2008). Moreover, the increase of creep caused by the cyclic component of the load is effectively captured in the Model B3 (Bažant and Kim 1992c). Therefore, Model B3 can better reflect the long-term serviceability and durability of concrete in practice.

Over recent decades, numerous studies have both experimentally and theoretically investigated the long-term effect of dynamic loading on the deformation behaviour of prestressed RC structures (Bazant *et al*. 2010). Kazuo *et al.* experimentally studied the deflection and crack development of partially prestressed RC beams under cyclic loading (Kazuo *et al*. 1986). Their research results showed that the deflection of beams increased largely during the initial several loading cycles, and then linearly increased on a semi-logarithmic scale. In addition, the increase in deflection and crack width was accompanied with an increase in steel strain and a reduction in stiffness of concrete in the compression zone.

The theoretical prediction of creep deflection of the RC member under dynamic loading is not a recent idea. Balaguru (Balaguru 1991) proposed a deflection analysis method for cyclic loaded partially prestressed concrete beams. The analytical model considers the cyclic creep of concrete in the compression zone, and the reduction in flexural stiffness due to reduction in contribution of tension zone based on the design philosophy of the ACI design code (Kenneth 1991). Equations derived by Naaman and Siriaksorn (Naaman and Siriakson 1979) have been used to calculate the stresses for both uncracked and cracked sections of partially prestressed beams. However, the model of Balaguru adopted a much earlier version of concrete creep model by Whaley and Neville (Whaley and Neville 1973). Further, the empirical parameters used in developing the predictive equations were based on a limited number of data and so may not be applicable to beams subjected to very high-cycle loading.

Koh *et al.* developed a computational procedure to investigate the time-dependent deformation behaviour of normal RC structures (Koh *et al*. 1997). Using a numerical iterative procedure, the model described the experimental data reasonably well, and can be used to predict long-term deformation behaviour of RC members under cyclic loading with consideration of the influence of some critical parameters (e.g., water/cement ratio, relative humidity and loading frequency). However, as the model was developed based on an earlier version of creep and shrinkage model by Bažant and Panula (i.e., BP-2 model) (Bažant and Panula 1978), further development work needs to be done in order to make the model suitable for the prediction of long-term deflection of a prestressed concrete member.

The implementation of Model B3 in the prediction of deflection for a prestressed RC structure is relatively new. Using a short form of Model B3, Robertson *et al.* (Robertson 2005) developed a numerical procedure for prediction of vertical deflections of a long-span prestressed RC bridge. The model results were compared with the nine year data collected. It showed that model predictions using a simple form of Model B3 provided a more reasonable prediction than other creep and shrinkage models, such as ACI 209 committee model (ACI 2002), CEB-90 model (CEB and Chiorino 1993) and Gardner model (Gardner and Zhao 1993). However, the model is greatly simplified by ignoring the concrete cyclic creep induced by the daily traffic loading, which could play an important role in long-term deformation behaviour of the bridges.

In summary, there is a clear need for the development of a comprehensive model to describe the long-term deflection behaviour of prestressed RC structures under cyclic loading. In the framework of the Model B3, the objective of this paper is to construct a computational procedure to predict the deflection of a partially prestressed concrete beam under cyclic loading. The model was firstly validated by using the available experimental data, and then a series of parametric studies were carried out to identify the key model parameters.

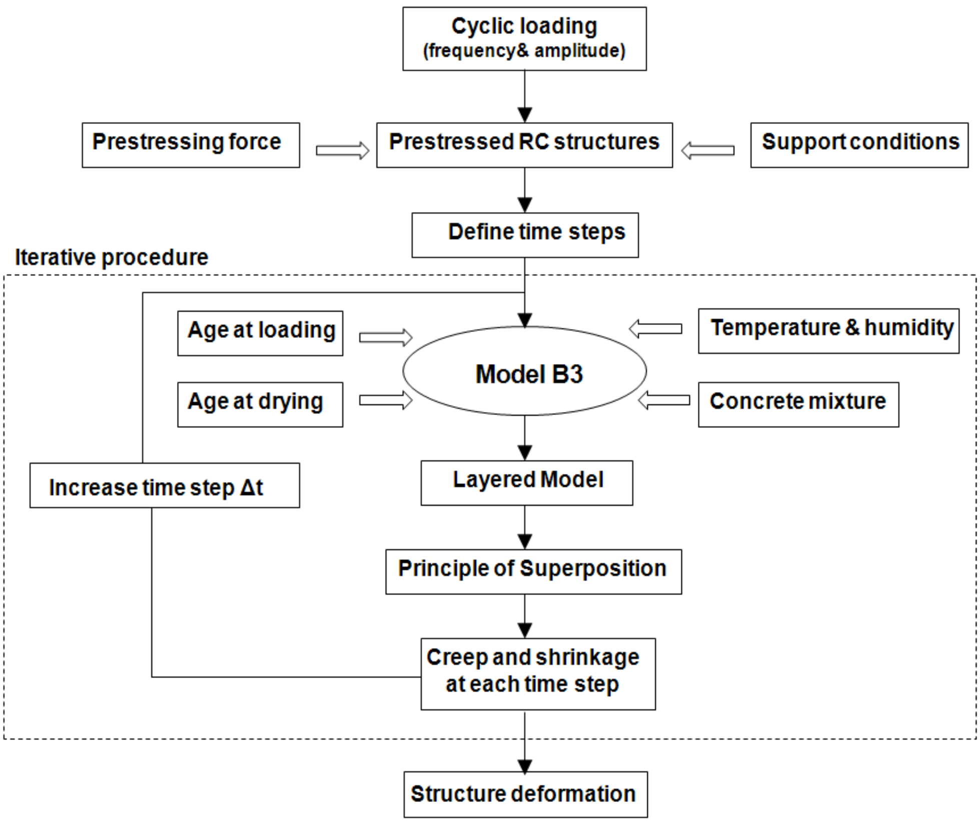



Fig. 1 Flow chart for proposed computational procedure. To determine the deformation of prestressed RC structures under long term cyclic loading, the overall stress-time history is divided into some time steps. An iterative procedure is implemented to calculate structure deformation by using Model B3, a layered model and principal of superposition

## 2. Computational model

The flow chart for the proposed computational procedure is shown in Fig. 1. To determine the long-term deflection of prestressed concrete members under cyclic loading, the overall stress-time history is divided into several time steps. The deflection of the prestressed concrete member is to be estimated at each time step. As it is rather difficult to solve the problem analytically, an iterative procedure is presented in this study involving Model B3 for creep and shrinkage, using a layered model and principal of superposition. The creep and shrinkage in each layer of cross section of a prestressed concrete member are computed at each time step. For a prestressed concrete beam, the Gauss quadrature integration was used to calculate the time-dependent beam deflection numerically. In this study, a MATLAB computer program was developed to simulate this computational procedure.

### *2.1 Model B3 for shrinkage and cyclic creep of concrete*

In the framework of Model B3, the total strain $\varepsilon(t)$ of concrete at time $t$ (in days) for a constant stress ($\sigma$) applied at age $t'$ (in days) is defined as

$$\varepsilon(t) = J(t,t',\sigma)\sigma + \varepsilon_{sh}(t) \tag{1}$$

where $J(t,t',\sigma)$ is the compliance function, and $\varepsilon_{sh}(t)$ the shrinkage strain.

Consider a uniaxial cyclic stress ($\sigma$) is given by

$$\sigma = \sigma_0 + \frac{1}{2}\Delta \sin(2\pi\omega t) \tag{2}$$

where $\sigma_0$ is mean stress, $\Delta$ peak-to-peak cyclic stress amplitude, and $\omega$ circular frequency. The cyclic creep resulting from cyclic stress in Eq. (2.2) can be considered as the creep in excess of the creep caused by mean stress $\sigma_0$ (Bažant and Kim 1992c).

The average compliance function, $J(t,t',\sigma)$, representing $\varepsilon/\sigma_0$ can be expressed as

$$J(t,t',\sigma) = q_1 + F(\sigma)\left[C_0(t,t') + \kappa C_d(t_{dc},t',t_0) + C_p(t_{dc},t',t_0)\right] \tag{3}$$

where

$$t_{dc} = (t-t')\left[1 + 10\omega^{1/4}\Delta^2 F^3(\sigma_{\max})\right] + t' \tag{4}$$

In Eq. (2.3), $C_0(t,t')$ is the basic creep compliance which occurs at constant moisture content, $C_d(t_{dc},t',t_0)$ is the drying creep caused by the variations of moisture content, and $C_p(t_{dc},t',t_0)$ represents a further increase of creep due to predrying. The $C_0(t,t')$, $C_d(t_{dc},t',t_0)$, and $C_p(t_{dc},t',t_0)$ are determined by the current age of concrete ($t$), age at loading ($t'$) and age at the start of drying (t0). The details could be found in the study of Bažant and Baweja (Bažant and Baweja 2000). $\kappa$ is a parameter describing the cyclic component of environmental humidity, and Eq. (2.4) represents the acceleration of the creep due to drying caused by the cyclic stress component (Bažant and Kim 1992c). Further, $q_1$ in Eq. (2.3) represents the instantaneous strain (i.e., $q_1 = 1/E_0$). Different from the conventional elastic modulus of concrete ($E_{28}$) which is

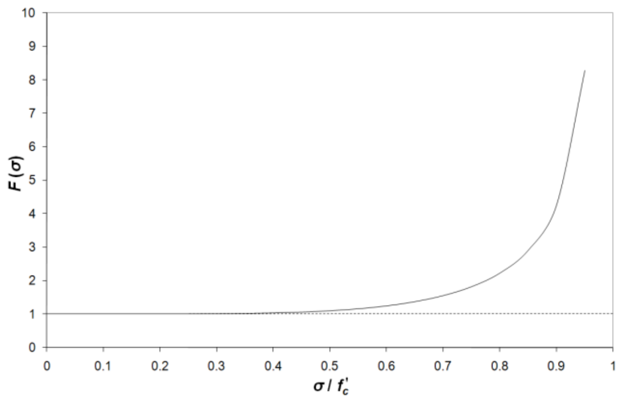



Fig. 2 The empirical function $F(\sigma)$ as a function of stress ratio $\sigma / f_c'$. When applied stresses ($\sigma$) are within service range (i.e., $0 \le \sigma / f_c' \le 0.45$), the non-linearity of the creep is insignificant (i.e., $F(\sigma) \approx 1$)

normally obtained via a slow rate of loading test, $E_0$ is the age dependent asymptotic modulus for extremely fast loading (as a rough estimate, $E_0 = 1.5E_{28}$ (Bažant 1995)). The empirical function $F(\sigma)$ describes the non-linear dependence of creep on stress. As shown in Fig. 2, when stresses are within service range (i.e., $0 \le \sigma \le 0.45 f_c'$, where $f_c'$ is 28-day compressive strength of concrete), the non-linearity of $F(\sigma)$ is insignificant, and so it is reasonable to assume $F(\sigma) \approx 1$.

The shrinkage strain $\varepsilon_{sh}$ at time *t* (in days, representing age of concrete) measured from the commencement of drying $t_0$ (in days) is given by

$$\varepsilon_{sh}(t, t_0) = -\varepsilon_{sh\infty} k_h S(t) \tag{5}$$

where $\varepsilon_{sh\infty}$ is ultimate shrinkage strain, $k_h$ humidity dependence coefficient, and $S(t)$ time function for shrinkage. $C_0(t, t')$, $C_d(t_{dc}, t', t_0)$, $C_p(t_{dc}, t', t_0)$ and $S(t)$ have been described in detail by Bažant and Baweja (Bažant and Baweja 2000).

### *2.2 Iterative procedure*

In the present study, an iterative computational procedure is developed to predict the long-term deflection behaviour of the prestressed concrete members. To simplify the problem, the stress relaxation of prestressing tendons is ignored in this study. As shown in Fig. 3, the cross-section of a prestressed concrete member (e.g., a simply supported partially prestressed RC beam with rectangle cross-section) is divided into several layers with equal thickness (*d*). The total strain of each layer ($\varepsilon_{ci}$) in a cross section may be expressed as the sum of elastic strain ($\varepsilon_{ei}$) creep strain ($\varepsilon_{cri}$) and shrinkage strain ($\varepsilon_{shi}$). That is

$$\varepsilon_{ci} = \varepsilon_{ei} + \varepsilon_{cri} + \varepsilon_{shi} \tag{6}$$

Under service range stresses, it is reasonable to assume a linear total strain profile across the beam depth and a perfect bond existing between the concrete and steel bars (both normal and prestressed). The total strain in each concrete layer and strain in each steel bar can be expressed as

$$\varepsilon_{ci} = \varepsilon_{c1} - \kappa\, y_{ci} \tag{7}$$

$$\varepsilon_{sj} = \varepsilon_{c1} - \kappa\, y_{sj} \tag{8}$$

$$\varepsilon_{psk} = \varepsilon_{c1} - \kappa\, y_{psk} \tag{9}$$

where

$$\kappa = \varepsilon_{c1} / y_{na} \tag{10}$$

Here $\varepsilon_{c1}$ is the total strain in the upper most layer, and $\kappa$ the curvature of the cross-section. $y_{ci}$, $y_{si}$ and $y_{psi}$ are the coordinate of *i* - th concrete layer, *j* - th normal steel bar and *k* - th prestressed steel bar, respectively. $y_{na}$ is the coordinate of neutral axis.

The resultant axial force (N) and bending moment (M) on the cross-section of a prestressed beam are, respectively

$$N = \sum_{i=1}^{n} \sigma_{ci} A_{ci} + \sum_{j=1}^{m} \sigma_{sj} A_{sj} + \sum_{k=1}^{q} \sigma_{psk} A_{pk} \tag{11}$$

$$M = \sum_{i=1}^{n} \sigma_{ci} A_{ci} \left(y_{ci} - y_{na}\right) + \sum_{j=1}^{m} \sigma_{sj} A_{sj} \left(y_{sj} - y_{na}\right) + \sum_{k=1}^{q} \sigma_{psk} A_{psk} \left(y_{psk} - y_{na}\right) \tag{12}$$

where $A_{ci}$, $A_{sj}$ and $A_{psk}$ are areas of *i* - th concrete layer, *j* - th normal steel bar and *k* - th prestressed steel bar respectively. $\sigma_{ci}$, $\sigma_{sj}$ and $\sigma_{psk}$ are stresses in *i* - th concrete layer, *j* - th normal steel bar and *k* - th prestressed steel bar respectively,

$$\sigma_{ci} = E_c \varepsilon_{ei} \tag{13}$$

$$\sigma_{sj} = E_s \varepsilon_{sj} \tag{14}$$

$$\sigma_{psk} = E_{ps} \left( \varepsilon_{psk} + \frac{F_k}{E_{ps} A_{psk}} \right) \tag{15}$$

where $E_c$ is elastic modulus of concrete ($E_0$ for compression and $E_t$ for tension). $E_s$ and $E_{ps}$ are Young's modulus of normal and prestressed steel bars, respectively. $F_k$ is the initial prestressing force of the *k* - th prestressed steel bar.

The stress time history is divided into several time steps as

$$t_n = t_{n-1} + \Delta t_n \tag{16}$$

Where $\Delta t_n$ is time increment of *n* – th time step which is constant on a logarithmic time scale. Similarly, the stress in *i* - th concrete layer of a cross section at *n* - th time step can be defined as

$$\left(\sigma_i\right)_n = \left(\sigma_i\right)_{n-1} + \left(\Delta\sigma_i\right)_n \tag{17}$$

where $(\Delta\sigma_i)_n$ is the stress increment of *n* - th time step.

The present study is restricted to the service stress range (i.e., $\sigma \le 0.45 f_c^{'}$). Under this assumption, the accumulation of creep strain caused by various stresses can be quantified by using the principal of superposition. By assuming a linear variation of a unit stress increment, the total creep strain in *i* - th concrete layer at *n* - th time step is defined as

$$(\varepsilon_{cri})_n = (\bar{\varepsilon}_{cri})_n + \overline{C}_n (\Delta\sigma_i)_n \tag{18}$$

where

$$(\bar{\varepsilon}_{cri})_n = \sum_{j=0}^{n-1} \left[ C_0(t_n, t_j) + \kappa C_d(t_{dcn}, t_j, t_0) + C_p(t_{dcn}, t_j, t_0) \right] \Delta\sigma_j \tag{19}$$

$$\overline{C}_n = C_0(t_n, t_{n-1}) + \kappa C_d(t_{dcn}, t_{n-1}, t_0) + C_p(t_{dcn}, t_{n-1}, t_0) \tag{20}$$

$$t_{dcn} = (t_n - t_j)(1 + 10\omega^{1/4}\Delta^2) + t_j \tag{21}$$

Substituting Eq. (2.18) into Eq. (2.17) gives

$$(\Delta\sigma_i)_n = \frac{E_c\left[(\varepsilon_{ci})_n - (\bar{\varepsilon}_{cri})_n - (\varepsilon_{shi})_n\right] - (\sigma_i)_{n-1}}{1 + E_c \overline{C}_n} \tag{22}$$

The Eq. (2.22) cannot be solved analytically as $(\Delta\sigma_i)_n$ and $(\varepsilon_{ci})_n$ are two unknowns at the *n* - th time step. Thus, the following iterative procedure is developed to estimate these two parameters

•Step 1: Compute shrinkage strain $(\varepsilon_{shi})_n$ and $\overline{C}_n$ using Eqs. (2.5) and (2.20) respectively.

•Step 2: Compute $(\Delta\sigma_i)_n$ by assuming a value of the total strain $(\varepsilon_{ci})_n$. Then, determine creep strain $(\varepsilon_{cri})_n$ by using Eq. (2.18).

•Step 3: Assume a value of the coordinate of neutral axis ($y_{na}$) and curvature of the cross-section ($\kappa$).

•Step 4: Compute the internal resisting axial force (*N*) and bending moment (*M*) by using Eqs. (2.11) and (2.12) respectively. The actual $y_{na}$ and $\kappa$ are obtained by using the bisection method iteratively until an equilibrium is reached, i.e., *N* equals applied axial force $N_a$ and *M* equals the applied moment $M_a$. Calculate the new value of total strain $(\varepsilon_{ci})_n$.

•Step 5: If the difference between the total strain $(\varepsilon_{ci})_n$ obtained in Step 4 and the trial value assumed in Step 3 is small enough (i.e. within the specified tolerance $1 \times 10^{-6}$), go to Step 6. Otherwise, go back to Step 2.

•Step 6: Increase time step using Eq. (2.16) and repeat Steps 1-5.

A computer program using MATALB (R2010a) language was developed to simulate the above mentioned iterative procedure. To determine the deflection of a partially prestressed simply supported concrete beam Fig. 3, a normal computer run for 10 Gaussian points along the beam and 50 layers in beam cross section takes less than 30 seconds.

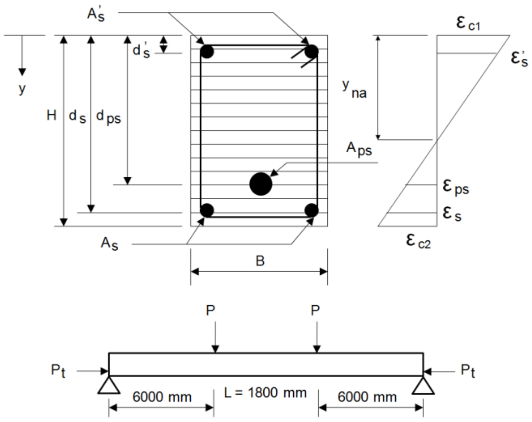



Fig. 3 Proposed Layered Model. The cross-section of the partially prestressed structure member is divided into several layers with equal thickness. The total strain of each layer is expressed as the sum of elastic strain, creep strain and shrinkage strain. In addition, the resultant stress acting on the cross-section is equal to the sum of stress acting on each layer

## 3. Results and discussions

### *3.1 Model validation*

The present method and Balaguru method (Balaguru 1991) are verified against experimental test data available in the literature (Kazuo *et al*. (Kazuo *et al*. 1986)) to check their accuracy. In the experimental test of Kazuo *et al.*, a series of simply supported prestressed concrete beams under high cycles repeated loading were investigated (Kazuo *et al*. 1986). Nevertheless, only one beam of which deals with bonded prestressing steel and cyclic loading (i.e., beam PPC-BR) is used for comparison. The cyclic loading was performed by using a servo fatigue testing machine while deflections of beams were measured by dial gages. The loading frequency was 3Hz, the maximum was defined as the load corresponding to the steel stress about 1000 kgf/cm2 and the minimum load was 20% of it. The experimental setup of beam PPC-BR and test data required for the present analysis is shown in Fig. 3 and Table 1 respectively.

Fig. 4 and Table 2 show the deflection histories of beam PPC-BR predicted by the present method and Balaguru method (1991) respectively. The results from both methods are compared to the experimental results of Kazuo *et al.* (1986). The comparison results demonstrate that the present method agrees remarkably well with the experimental data and gives much better predictions than Balaguru method (1991), with an average of error 3.4% and standard deviation 2.69%. It also shows that Balaguru method (1991) significantly underestimates the deflection of a partially prestressed beam with increasing of the loading cycles (an average of error 21.4% and standard deviation 7.0%).

Fig. 5 shows the elastic, shrinkage, creep and total strain as a function of number of loading cycles in the upper most layer of the cross section that is located at the midspan of the beam PPC-BR. It can be seen that, at the initial stage of loading (e.g., < 1000 loading cycles), the total strain

in the concrete layer is primarily dominated by the elastic strain. However, with the increase of loading cycles, the creep strain gradually overtakes elastic and shrinkage, eventually becomes the most dominant strain when number of loading cycles reaches a certain threshold (e.g., > 100,000 loading cycles). This is because creep is both stress and time-dependent. The elastic strain decreases over time due to the stress redistribution, while there is a little change of shrinkage which is stress independent.

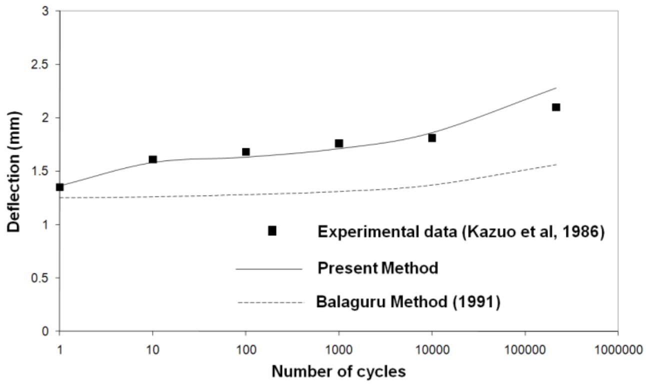



Fig. 4 Comparison of numerical solution to Kazuo *et al*.'s experimental data (Kazuo *et al*. 1986) and Balaguru method (1991)

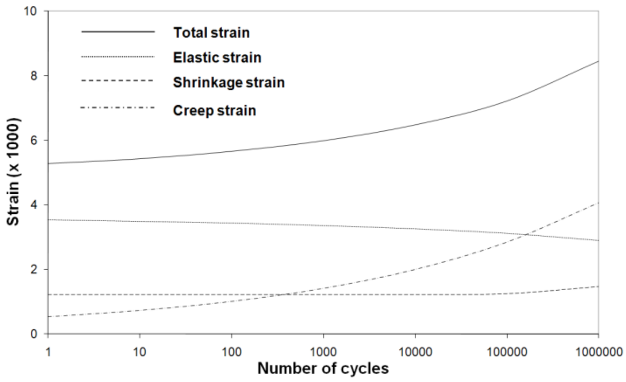



Fig. 5 Elastic, shrinkage, creep and total strains in the upper most layer of the cross section located at the midspan of the beam PPC-BR

Table 1 Material parameters used for comparison with Kazuo *et al.* (1986)'s experimental data of beam PPC-BR (Kazuo *et al*. 1986)

| Parameter | Value |
|---|---|
| Concrete age at loading | 23 days |
| Concrete age at start of drying | 15 days |
| Relative humidity | 60% |
| Temperature | 25° |
| Cement : sand : gravel ratio | 1 : 2.4 : 3.6 |
| Water/cement ratio | 0.6 : 1 |
| Width of beam | 100 mm |
| Depth of beam | 240 mm |

Table 1 Continued

| | |
|---|---|
| Compressive strength of concrete ( $f_c^{'}$ ) | 36.2 MPa |
| Tensile strength of concrete ($f_t$) | 3.3 MPa |
| Conventional Young's modulus of concrete ($E_c$) | 23442 MPa |
| Area of normal steel | 63 $mm^2$ |
| Young's modulus of normal steel | $2 \times 10^5$ MPa |
| Yield strength of normal steel | 414 MPa |
| Area of prestressing steel | 66 $mm^2$ |
| Young's modulus of prestressing steel | $2 \times 10^5$ MPa |
| Yield strength of prestressing steel | 1372 MPa |
| Prestressing force ($P_t$) | 55 kN |
| Prestressing steel bar location ($d_{ps}$) | 160 mm |

Table 2 Comparison of deflection of partially prestressed concrete beam PPC-BR

| Cycles | Measured by Kazuo *et al.* (1986) (mm) | Present method (mm) | Error (%) | Balaguru method (1991) (mm) | Error (%) |
|---|---|---|---|---|---|
| 1 | 1.35 | 1.36 | 0.92 | 1.25 | 7.41 |
| 10 | 1.61 | 1.58 | 1.86 | 1.26 | 21.74 |
| $10^2$ | 1.68 | 1.63 | 3.21 | 1.28 | 23.81 |
| $10^3$ | 1.76 | 1.71 | 2.84 | 1.31 | 25.57 |
| $10^4$ | 1.81 | 1.86 | 2.95 | 1.37 | 24.31 |
| $2.15 \times 10^5$ | 2.10 | 2.28 | 8.61 | 1.56 | 25.71 |
| Mean | — | — | 3.40 | — | 21.43 |
| Std. Dev. | | | 2.69 | | 7.02 |

### *3.2 Parametric study*

After model validation, a series of parametric studies were carried out by using the developed model to identify the key parameters which determine the creep behaviour of prestressed concrete members. It can be seen from Fig. 6(a) that prestressing force helps reduce deflection as eccentrically applied prestressing force produces a negative moment, which is favourable for prestressed members under cyclic loading. However, under a high prestressing force, the rate of creep could accelerate. As shown in Fig. 6(b), the prediction results under various prestressing forces obtained previously in Fig. 6(a) are normalized to their initial values (i.e., $t$ = 1 day) respectively. The results suggest that a higher prestressing force leads to an increase rate of deflection.

The effects of shrinkage and creep on the deflection behaviour of the beam PPC-BR are investigated in Fig. 7. The current model predicts that both shrinkage and creep could lead to a significant increase of the beam deflection. While time-dependent shrinkage reaches its maximum effect at around 1000 days, creep strain would continue to increase deflection over a much longer period as creep is both time and stress-dependent. Mostly importantly, the simulation results indicate that ignoring the effects of creep and shrinkage significantly underestimates the deflection of the beam PPC-BR by approximately 4.8-fold, 5.9-fold and 6.8-fold over a loading period of 100 days, 1000 days and 10000 days respectively. In summary, the model results theoretically demonstrate that the influence of creep and shrinkage cannot be ignored in prestressing concrete design and construction.

The model was then used to predict the influence of compressive strength of concrete (at age of 28 day) on time-dependent midspan deflection of a PPC-BR type beam (Fig. 8). Consistent with the research investigation of Ngab *et al*. (Ngab *et al*. 1981), it can be seen that, the use of a higher compressive strength concrete can generally reduce the long-term deflection of the beam. The model can be further used to investigate the effects of concrete water/cement ratio on creep and shrinkage behaviour of a PPC-BR type beam. As shown in Fig. 9, the long-term deflection of the beam PPC-BR is more sensitive to the water/cement ratio than the compressive strength of concrete. For example, after a loading period of 1000 days, a higher water/cement ratio (i.e., w/c = 0.6) could increase PPC-BR deflection by around 15% compared to $w/c$ = 0.4. The implication of

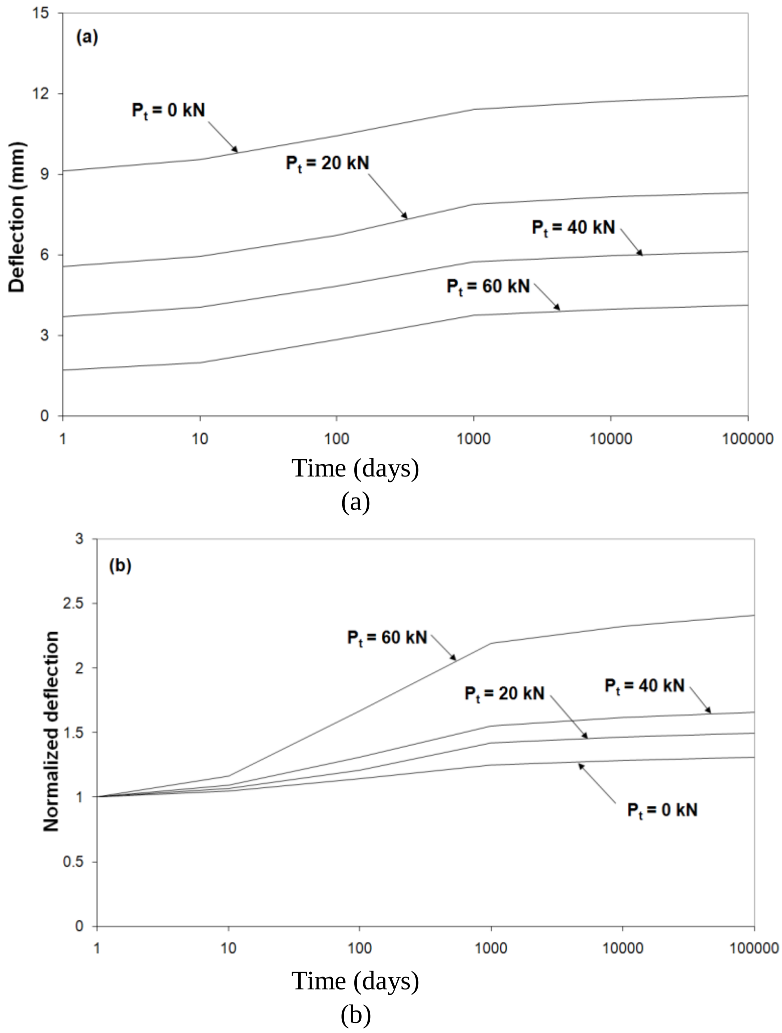
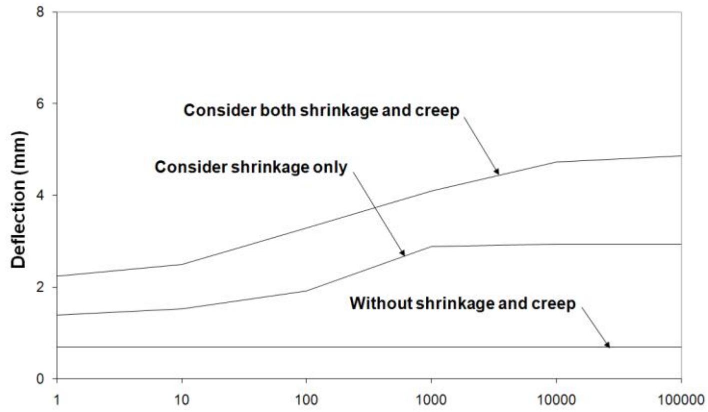



Fig. 6 Midspan deflection of beam PPC-BR as a function of time (days). (a) Time dependent deflections under various prestressing forces; (b) prediction results under various prestressing forces obtained in (a) are normalized to their initial values (i.e., $t$ = 1 day) respectively




Time (days)

Fig. 7 The effect of shrinkage and creep on the deflection of beam PPC-BR

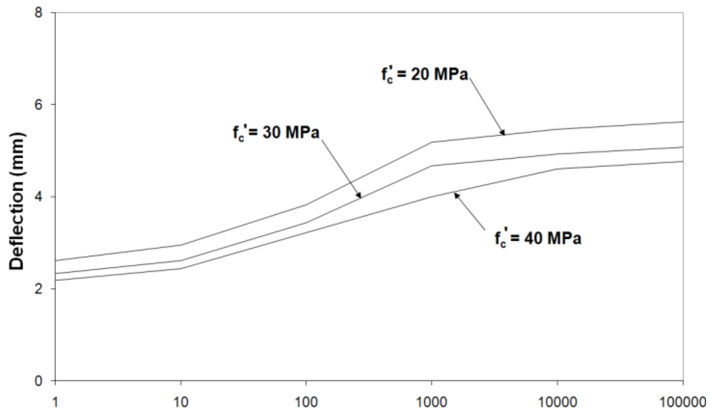
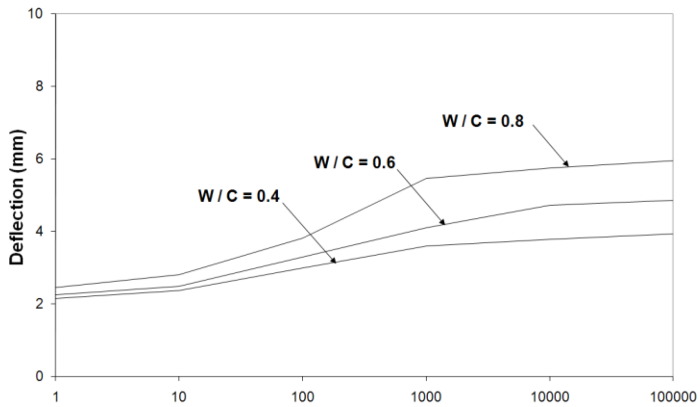



Fig. 8 Midspan deflection of beam PPC-BR as a function of time (days) under various concrete compressive strengths at 28 days of age ( $f_c^{'}$ )




Time (days)

Fig. 9 Midspan deflection of beam PPC-BR as a function of time (days) under various water/cement ratios

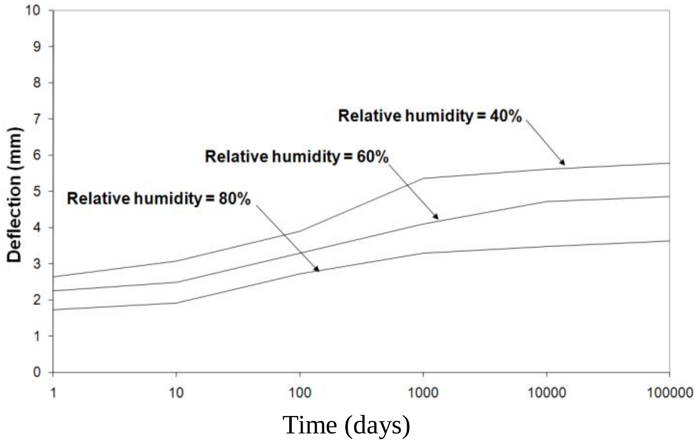



Fig. 10 Midspan deflection of beam PPC-BR as a function of time (days) under various relative humidity

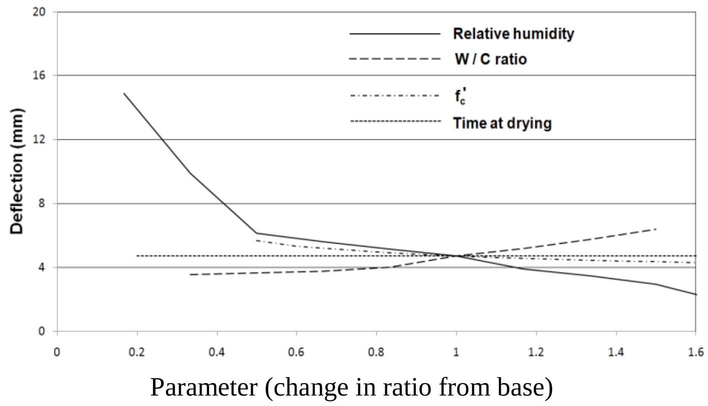



Fig. 11 Sensitivity analysis of model parameters on midspan deflection at 10,000 days. The base values of model parameters (i.e., relative humidity, W/C ratio, concrete compressive strength at 28 days of age ( $f_c'$ ), time at drying) are obtained from beam PPC-BR

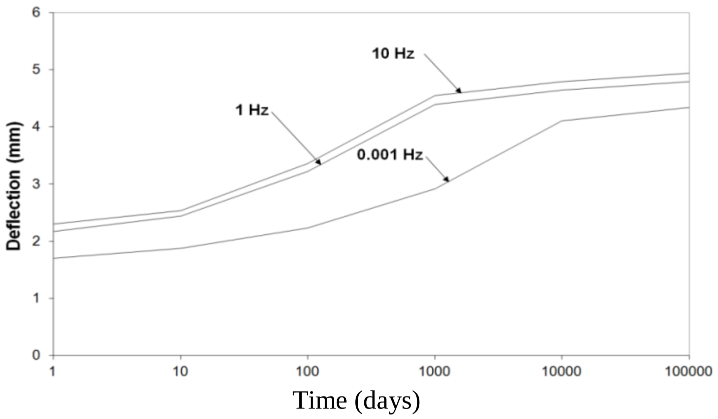



Fig. 12 Midspan deflection of beam PPC-BR as a function of time (days) under various loading frequencies

these results is that concrete water/cement ratio has to be carefully chosen in order to reduce the excessive deflection of a prestressed concrete member caused by creep and shrinkage. The developed model enables optimal selection of concrete water/cement ratio.

Further, creep and shrinkage are also shown to be affected by the relative humidity which strongly affects the rate of concrete hydration (Bažant and Chern 1985, Li *et al*. 2011). As shown in Fig. 10, relative humidity was found to be one of the important key factors that influence concrete shrinkage and concrete to a great extent. Exposure to an external environment with low humidity, a gradual decrease of concrete pore humidity causes an additional shrinkage and creep. Moreover, identifying the parameters to which the developed model is most sensitive helps the designer to focus future creep prevention. As shown Fig. 11, we systematically varied one parameter (e.g., relative humidity, w/c ratio, $f_c'$, time at drying) at a time and observed the corresponding change in the model output of interest (i.e., midspan deflection at 10,000 days). The base values of model parameters are obtained from the beam PPC-BR (Table 1). The results clearly demonstrate that relative humidity is one of the most critical parameters governing the long-term creep deflection the beam. For example, a five-fold decrease of the base value of relative humidity increases midspan deflection at 10,000 days by around 200%.

Fig. 12 investigates the midspan deflection of beam PPC-BR as a function of time (days) under various loading frequencies. It can be seen that a higher frequency would generally lead to a larger deflection. Most importantly, in comparison to a very low loading frequency (e.g., 0.001Hz) which is close to a static loading condition, cyclic loading with a higher frequency(e.g., 1-10 Hz) could significantly reduce time for deflection to reach steady-state (i.e., 1000 days for 1-10Hz, and 10000 days for 0.001Hz) and result in an approximately 50% increase of deflection at 1000 days. The implication of this numerical outcome is that cyclic loading with a higher frequency could potentially lead to a significant increase of creep and consequently a larger deflection of a structure member.

## 4. Conclusions

In present study, an iterative numerical procedure based on Model B3 to predict the long-term deformation behaviour of prestressed RC structures was developed. The model was validated by using the available experimental data, and takes into account a range of critical factors that could potentially influence the creep and shrinkage behaviour of concrete, such as prestressing forces, concrete water/cement ratio, relative humidity and etc. The model results suggest that creep and shrinkage have significant effect on a cyclic loaded prestressed concrete beam. Furthermore, in comparison to static loading condition, a cyclic loading with high loading frequency (e.g., 1-10Hz) could lead to an obvious increase of the deflection (e.g., approximately 50% within the first three years), and significantly reduce the time for the deflection to reach its steady-state. Therefore, the excess creep resulting from cyclic loading should be taken into consideration in prestressed concrete design and construction practices.

## References

ACI (2002), “Committee 318, Building code requirements for structural concrete”, Farmington Hills, MI, American Concrete Institute.

Au, F.T.K. and Si, X.T. (2011), “Accurate time-dependent analysis of concrete bridges considering concrete creep, concrete shrinkage and cable relaxation”, *Eng. Struct.*, **33**(1), 118-126.

Balaguru, P. (1991), “Prediction of the effect of fatigue loading on the serviceability of reinforced and prestressed concrete members”, *International Symposium on Fatigue and Fracture in Steel and Concrete Structures*, 521-535.

Bažant, Z.P. (1995), “Creep and shrinkage prediction model for analysis and design of concrete structures - model B3”, *Mater. Struct.*, **28**, 357-365.

Bažant, Z.P. and Baweja, S. (1995), “Creep and shrinkage prediction model for analysis and design of concrete structures: Model B3”, *Adam Neville Symposium: Creep and Shrinkage-Structural Design Effects*, ACI SP194, A. A1-Manaseer, Farmington Hills, Michigan.

Bažant, Z.P. and Chern, J.C. (1985), “Concrete creep at variable humidity: Constitutive law and mechanism”, *Mater. Struct.*, **18**(1), 1-20.

Bažant, Z.P. and Kim, J.K. (1991), “Improved prediction model for time-dependent deformations of concrete: Part 2 - Basic cree”, *Mater. Struct.*, **24**(6), 409-421.

Bažant, Z.P. and Kim, J.K. (1992a), “Improved prediction model for time-dependent deformations of concrete: Part 3 - Creep at drying”, *Mater. Struct.*, **25**(1), 21-28.

Bažant, Z.P. and Kim, J.K. (1992b), “Improved prediction model for time-dependent deformations of concrete: Part 4 - Temperature effects”, *Mater. Struct.*, **25**(2), 84-94.

Bažant, Z.P. and Kim, J.K. (1992c), “Improved prediction model for time-dependent deformations of concrete: Part - load and cyclic humidity”, *Mater. Struct.*, **25**(3), 163-169.

Bažant, Z.P., Kim, J.K. and Panula, L. (1991), “Improved prediction model for time-dependent deformations of concrete: Part 1 - Shrinkage”, *Mater. Struct.*, **24**(5), 327-345.

Bažant, Z.P., Kim, J.K., Xi, Y. and Baweja, S. (1993), “Improved prediction model for time-dependent deformations of concrete: Part 7 - Short form of BP-KX model, statistics and extrapolation of short-time data”, *Mater. Struct.*, **26**(10), 567-574.

Bažant, Z.P. and Panula, L. (1978), “Practical prediction of time-dependent deformations of concrete (Part II: Basic creep)”, *Mater. Struct.*, **11**(5), 317-328.

Bažant, Z.P. and Panula, L. (1980), “Creep and shrinkage characterization for analyzing prestressed concrete structures”, *J. Prestressed Concrete Inst.*, **25**(3), 86-122.

Bažant, Z.P., Panula, L., Kim, J.K. and Xi, Y. (1992), “Improved prediction model for time-dependent deformations of concrete: Part 6 - Simplified code-type formulation”, *Mater. Struct.*, **25**(4), 219-223.

Bazant, Z. P., Yu, Q., Li, G.H., Klein, G.J. and Kristek, V. (2010), “Excessive deflections of record-span prestressed box girder”, *Concrete Int.*, **32**(6), 44-52.

CEB and Chiorino, M.A. (1993), “Revision of the design aids of the CEB design manual structural effects of time dependent behaviour of concrete, in accordance with the CEB-PIF Model Code 1990”, CEB-PIF Model Code 1990, Bulletion d’Information N. 215, 297.

Chaudhary, S., Pendharkar, U. and Nagpal, A.K. (2008), “Service load behavior of low rise composite frames considering creep, shrinkage and cracking”, *Latin Am. J. Sol. Struct.*, **5**(4), 237-258.

Gardner, N.J. and Zhao, J.W. (1993), “Creep and shrinkage revisited”, *Mater. J.*, **90**(3), 236-246.

Islam, M.J., Liu, Z. and Swaddiwudhipong, S. (2011), “Numerical study on concrete penetration/perforation under high velocity impact by ogive-nose steel projectile”, *Comput. Concr.*, **8**(1), 111-123.

Kazuo, S., Ohno, Y., Miyamaru, T. and Masaru, Y. (1986), “Flexural behaviour of partially prestressed concrete beams”,*Tran. Japan Concrete Institute*, **8**, 242-252.

Kenneth, L. (1991), Reinforced concrete design, McGraw-Hill, USA.

Khora, E.H., Rosowsky, D.V. and Stewartc, M.G. (2001), “Probabilistic analysis of time-dependent deflections of RC flexural members”, *Comput. Struct.*, **79**(16), 1461-1472.

Koh, C.G., Ang, K.K. and Zhang, L. (1997), “Effects of repeated loading on creep deflection of reinforced concrete beams”, *Eng. Struct.*, **19**(1), 2-18.

Li, G., Huang, Y. and Chen, C. (2011), “Effects of humidity field on creep of reinforced concrete beams”, *Adv. Mater. Res*., **250-253**, 1765-1768.

Naaman, A.E. and Siriakson, A. (1979), “Serviceability based design of partially presstressed beams (Part I:

Analytic formulation)", *J. Prestress. Concrete Ins.*, **24**(2), 64-89.
Ngab, A.S., Nilson, A.H. and Slate, F.O. (1981), "Shrinkage and creep of high strength concrete", *ACI J.*, **78**(4), 255-261.
Robertson, I.N. (2005), "Prediction of vertical deflections for a long-span prestressed concrete bridge structure", *Eng. Struct.*, **27**(12), 1820-1827.
Whaley, C.P. and Neville, A.M. (1973), "Non-elastic deformation of concrete under cyclic compression", *Concrete Res.*, **25**(84), 145-154.

*CC*