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Predicting diagonal cracking strength of RC slender beams without stirrups using ANNs

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Abstract. Numerous studies have been conducted to understand the shear behavior of reinforced concrete (RC) beams since it is a complex phenomenon. The diagonal cracking strength of a RC beam is critical since it is essential for determining the minimum amount of stirrups and the contribution of concrete to the shear strength of the beam. Most of the existing equations predicting the diagonal cracking strength of RC beams are based on experimental data. A powerful computational tool for analyzing experimental data is an artificial neural network (ANN). Its advantage over conventional methods for empirical modeling is that it does not require any functional form and it can be easily updated whenever additional data is available. An ANN model was developed for predicting the diagonal cracking strength of RC slender beams without stirrups. It is shown that the performance of the ANN model over the experimental data considered in this study is better than the performances of six design code equations and twelve equations proposed by various researchers. In addition, a parametric study was conducted to study the effects of various parameters on the diagonal cracking strength of RC slender beams without stirrups upon verifying the model.

Keywords: artificial neural networks; reinforced concrete; slender beams; diagonal cracking; shear strength

1. Introduction

Shear behavior of reinforced concrete (RC) beams has been studied extensively since the beginning of the last century. It has been a common practice to focus on RC beams without stirrups to acquire a better understanding of shear behavior of RC beams. A critical property of a RC beam is its diagonal cracking strength, which is essential for determining the contribution of concrete to the shear strength of the beam and the minimum amount of stirrups necessary for providing a shear strength exceeding diagonal cracking stress. The diagonal cracking strength of RC beams without stirrups is of interest to this research. The beams with the ratio of shear span (a) to effective depth (d) greater than 2.5, referred to as slender beams, are considered.

A review of approaches and related theories for designing RC members against shear is given by Joint ACI-ASCE Committee 445 (1998). Various equations, most of which are based on experimental data, have been derived to predict the diagonal cracking strength of RC slender

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beams without stirrups. A common approach used for empirical modeling is to conduct a regression analysis on experimental data. A regression analysis requires a functional form to be assumed a priori. The analysis delivers the unknown coefficients in the functional form. Therefore, the accuracy of the resulting equation depends strongly on the functional form. However, it is not easy to derive a functional form describing a complex phenomenon such as the shear behavior of RC beams. With increasing computational power, alternative methods have been developed for empirical modeling. A powerful computational tool is an artificial neural network (ANN), which is able to establish the relationships between the parameters involved without requiring any functional form. An ANN model is developed in two stages: training and testing. First, the network is trained through a learning algorithm, where the relationships between the parameters involved are determined using experimental data. Second, the model is tested with the data which is never presented to the network. Unlike conventional empirical models, an ANN model can be easily improved whenever additional experimental data is provided. The shortcoming of an ANN model is its inability to deliver an explicit expression of solution.

A number of researchers have studied the shear behavior of RC slender beams without stirrups using ANNs. Oreta (2004) developed an ANN model to predict the ultimate shear strength of RC slender beams without stirrups, and studied the influence of various parameters on the shear strength. Cladera and Mari (2004a) and El-Chabib *et al*. (2005) developed ANN models for predicting the ultimate shear strength of normal and high-strength RC slender beams without stirrups, and conducted sensitivity analyses to study the effects of design parameters on the shear strength. Jung and Kim (2008) developed two ANN models for RC slender beams without stirrups: one of which estimates shear strength and the other one provides a conservative estimate for design purpose. The ANN models developed by Oreta (2004), Cladera and Mari (2004a), El-Chabib *et al*. (2005) and Jung and Kim (2008) exhibit a better performance over the experimental data considered for developing the models than the equations of several design codes and researchers do. Also, several researches were conducted to develop ANN models focusing on RC beams with stirrups (Cladera and Mari 2004b, Mansour *et al*. 2004, El-Chabib *et al*. 2006, Abdalla *et al*. 2007), deep RC beams (Goh 1995, Sanad and Saka 2001) and RC beams strengthened with fiber reinforced polymers (Perera *et al*. 2010, Tanarslan *et al*. 2012).

There are also other computational techniques used for developing prediction models. Cevik and Ozturk (2009) and Choi *et al*. (2009) developed models based on fuzzy set theory to predict the shear strength of RC beams without stirrups. Amani and Moeini (2012) compared an ANN model and a model based on fuzzy set theory, both of which were developed for predicting the shear strength of RC beams with stirrups. Ashour *et al*. (2003) developed an empirical model to predict the shear strength of RC deep beams by using genetic programming and studied the effects of various parameters on the shear strength of RC deep beams using this model. Perez *et al*. (2010, 2012) developed a genetic programming algorithm for adjusting existing expressions and applied the algorithm to the shear design formulation for RC beams without stirrups given by Eurocode 2 (2004). Gandomi *et al*. (2013) used a genetic programming technique to develop a model for predicting the shear strength of RC deep beams.

In this study, an ANN model was developed for predicting the diagonal cracking strength of RC slender beams without stirrups. The model was compared with six design code equations and twelve equations proposed by various researchers. Once the model was verified, a parametric study was conducted to study the effects of various parameters on the diagonal cracking strength of RC slender beams without stirrups.

2. Diagonal cracking strength

The shear behavior of RC beams without stirrups is generally studied by considering a simply supported RC beam with a rectangular cross-section having tensile reinforcement, loaded by either two symmetrically located concentrated loads or a concentrated load at its mid-span. The distance between the support and the load is referred to as shear span. Experimental results show that the failure mode of a RC beam without stirrups is determined by the ratio of its shear span to its effective depth (Kani 1964).

RC slender beams ($a/d \geq 2.5$) without stirrups failing in shear undergo diagonal cracking prior to failure. A typical diagonal cracking formation in a simply supported RC slender beam with two symmetrically located concentrated loads is as follows. At early stages of loading, flexural cracks perpendicular to the beam axis occur. As the load is increased, flexural cracks within the shear span progress vertically towards the neutral axis, and then some of these cracks are inclined towards the load (perpendicular to the principal tensile stress axis) while progressing upwards. At a certain instant, one of these inclined cracks progresses suddenly downwards to the level of longitudinal reinforcement. The inclined crack extending from the level of longitudinal reinforcement to the compression zone is called diagonal crack. The formation of the diagonal crack leads to a stress redistribution, where the tensile stress in the longitudinal reinforcement increases significantly. With a further increase in the load, the beam fails suddenly with the diagonal crack extending into the compression zone. Experimental studies (Moody *et al.* 1954, Cossio and Siess 1960, Taylor 1960, Van den Berg 1962, Bresler and Scordelis 1963, Mathey and Watstein 1963, Taylor and Brewer 1963, Krefeld and Thurston 1966, Mattock 1969, Mphonde and Frantz 1984, Ahmad *et al.* 1986, Elzanaty *et al.* 1986, Xie *et al.* 1994, Kim *et al.* 1999, Shin *et al.* 1999, Pendyala and Mendis 2000, Cladera and Mari 2005, Shah and Ahmad 2007, Hamrat *et al.* 2010, Sneed and Ramirez 2010, Garip 2011, Slowik and Nowicki 2012, Slowik and Smarzewski 2012) have identified concrete compressive strength (f_c), longitudinal reinforcement ratio (ρ), shear span-to-depth ratio and effective depth as the parameters affecting the diagonal cracking strength.

The ratio of diagonal cracking strength to ultimate strength is variable, depending on beam size and other factors (Bazant and Kazemi 1991). The failure in a slender beam occurs immediately after diagonal cracking (Rebeiz 1999). It is to be noted that the observed values of diagonal cracking load are sensitive to the observers' judgment since it is not easy to define the diagonal cracking load due to the gradual development of inclined cracks (Bazant and Kim 1984, Elzanaty *et al.* 1986). Mphonde and Frantz (1984) defined the diagonal cracking strength as the load when the critical crack becomes inclined and crosses mid-depth of the beam.

In general, the current design practice relies on diagonal cracking. A minimum amount of stirrups is required in order to ensure that the post-cracking shear strength is greater than the diagonal cracking stress. In addition, the contributions of concrete and stirrups to the shear strength are assumed to be independent, where the contribution of concrete is defined in terms of the diagonal cracking strength. Several design code equations predicting the shear strength of RC beams without stirrups are given below. ACI 318 (2011) provides two empirical equations -a detailed one and a simplified one- as

$$v_c = 0.16\sqrt{f_c} + 17\rho\left(\frac{V_u d}{M_u}\right) \leq 0.29\sqrt{f_c}\,, \tag{1}$$

$$v_c = 0.17\sqrt{f_c}\,, \tag{2}$$

respectively, where V_u is the external factored shear load at the section considered, M_u is the corresponding bending moment and $V_u d/M_u$ shall not be greater than 1.0. CEB-FIP Model Code (2010) presents an equation for RC beams without stirrups having compressive concrete strength and yield strength of longitudinal reinforcement less than 64 MPa and 500 MPa, respectively, as

$$v_{Rd,c} = k_v\sqrt{f_c}\left(\frac{z}{d}\right), \tag{3}$$

where $k_v = 200/(1000 + 1.3z) \leq 0.15$ and z is the internal moment arm which can be taken as $0.9d$. The previous version, CEB-FIP Model Code (1993), defines the diagonal cracking strength as

$$v_{cr} = 0.15\left(\frac{3}{a/d}\right)^{1/3}\left(1 + \sqrt{\frac{200}{d}}\right)(100\rho f_c)^{1/3}, \tag{4}$$

where d is in mm. The equation of Eurocode 2 (2004) is

$$v_{rd,c} = 0.18k(100\rho f_c)^{1/3} \geq 0.035k^{3/2}\sqrt{f_c}\,, \tag{5}$$

where $k = 1 + \sqrt{200/d} \leq 2.0$ (d is in mm) and $\rho \leq 0.02$. The equation of TS 500 (2000) is similar to the simplified equation of ACI 318 (2011), such that

$$v_c = 0.2275\sqrt{f_c}\,. \tag{6}$$

A number of equations have been derived empirically and theoretically for predicting the shear strength of RC beams without stirrups by various researchers. The equations considered in this study are as follows. Zsutty (1971) derived an equation as

$$v_u = 2.2\left(f_c\rho\frac{d}{a}\right)^{1/3}\text{for }\frac{a}{d} \geq 2.5 \tag{7}$$

by using a multiple regression analysis. The equation obtained empirically by Okamura and Higai (1980) is

$$v_c = 0.2\frac{(100\rho f_c)^{1/3}}{d^{1/4}}\left(0.75 + \frac{1.40}{a/d}\right), (d\text{ is in m}). \tag{8}$$

Bazant and Sun (1987) proposed an equation based on non-linear fractures mechanics as

$$v_u = 0.54\sqrt[3]{\rho}\left(\sqrt{f_c} + 249\sqrt{\frac{\rho}{(a/d)^5}}\right)\left(\frac{1+\sqrt{5.08/d_a}}{\sqrt{1+d/(25d_a)}}\right), \tag{9}$$

where d_a is the maximum aggregate size in mm. Kim and Park (1996) presented an equation based on basic shear transfer mechanisms, a modified version of Bazant's size effect law and experimental data as

$$v_u = 3.5{f_c}^{\alpha/3}\rho^{3/8}\left(0.4 + \frac{d}{a}\right)\left(\frac{1}{\sqrt{1+0.008d}} + 0.18\right), \tag{10}$$

where $\alpha = 2 - (a/d)/3$ for $1.0 \leq a/d < 3.0$, $\alpha = 1$ for $a/d \geq 3.0$ and d is in mm. The equation of Collins and Kuchma (1999) resulting from an enhancement of the modified compression field theory is

$$v_c = \frac{245}{1275+\left(\frac{25S_X}{d_a+16}\right)}\sqrt{f_c},\ S_x \approx 0.9d, (d \text{ and } d_a \text{ are in mm}). \tag{11}$$

Rebeiz (1999) employed a multiple regression analysis to derive an equation as

$$v_c = 0.4 + \sqrt{f_c\rho\frac{d}{a}}(2.7 - 0.4A_d), \tag{12}$$

where $A_d = a/d$ for $a/d < 2.5$ and $A_d = 2.5$ for $a/d \geq 2.5$. Khuntia and Stojadinovic (2001) derived an equation based on basic principles of mechanics and experimental data as

$$v_c = 0.54\sqrt[3]{\rho\left(f_c\frac{V_c d}{M_u}\right)^{0.5}},\ \frac{M_u}{V_c d} = \frac{a}{d} - 1\,. \tag{13}$$

Zararis and Papadakis (2001) presented an equation based on a theory assuming that diagonal tension failure results from a type of splitting of concrete occurring in a certain region of the shear span as

$$v_u = \left(1.2 - 0.2\frac{a}{d}d\right)\frac{c}{d}f_{ct}, \tag{14}$$

where c is the neutral axis depth, $f_{ct} = 0.30(f_c)^{2/3}$ is the splitting tensile strength of concrete, $(c/d)^2 + 600(\rho/f_c)(c/d) - 600(\rho/f_c) = 0$ and $(1.2 - 0.2(a/d)d) \geq 0.65$ (d is in meters). Arslan (2012) proposed an equation based on the principal shear strength carried in the compression zone as

$$v_c = 0.2{f_c}^{2/3}\left(\frac{c}{d}\right)\left(1 + 0.032{f_c}^{1/6}\right)\left(\frac{4}{a/d}\right)^{0.15}\left(\frac{400}{d}\right)^{0.25}, \tag{15}$$

where $a/d \geq 2.5$ and $(c/d)^2 + 600(\rho/f_c)(c/d) - 600(\rho/f_c) = 0$. In Eqs. (1)-(15), f_c is in MPa, and v_c (or v_{cr}) and v_u are the diagonal cracking strength and the ultimate shear strength of RC beams without stirrups, respectively.

Perez *et al*. (2012) applied a genetic programming algorithm to adjust the shear design formulation for RC beams without stirrups given by Eurocode 2 (2004). Three of the adjusted equations labeled as 7A1, 8H1 and 8I1 are considered in this study. These are given as

$$v_u = 0.81994081\left(1 + \frac{\left|\frac{14.2569}{\frac{4}{10\rho}-8} - f_c\right|}{d}\right)^{\frac{d}{8\left(f_c - \frac{5000\rho}{\frac{d}{16}-3} + 50\right)}}(100\rho)^{0.37058625}{f_c}^{-\frac{1}{\frac{5f_c}{7} - \frac{d}{252} + \frac{3}{20}}}, \tag{16}$$

$$v_u = 0.114\left(1 + \left(\frac{1600}{d}\right)^{0.42}\right)(100\rho)^{0.37}{f_c}^{\frac{1}{3}}\left(\frac{Vd}{M}\right)^{0.21}, \tag{17}$$

$$v_u = 1.75(100\rho)^{0.4} f_c^{\frac{2}{7}} \left(\frac{V}{M}\right)^{\frac{1}{4}}, \tag{18}$$

respectively.

3. Artificial neural networks

ANNs were inspired by the way in which a human brain organizes and operates its structural constituents, known as biological neurons (Haykin 1998). Accordingly, the fundamental processing unit of an ANN is referred to as neuron, which receives input from neighboring neuron(s), processes data and sends output to a neighboring neuron. A typical neuron receiving multiple input elements is shown in Fig. 1, where S is the number of input elements, p_i, $i = 1, \dots, S$, is the i-th input element, w_i, $w_{1,i} = 1, \dots, S$, is the weight of i-th input element, b is the bias that can be viewed as a weight of a constant input of 1, n is the net input, $f(.)$ is the transfer function, which must be differentiable, and q is the output. The output of a typical neuron with the most common net input function that is the summation of weighted inputs with the bias is calculated as

$$q = f(n) = f\left(\sum_{i=1}^{N} w_{1,i} p_i + b\right). \tag{19}$$

Neurons can be grouped into layers. The most common type of ANNs used in engineering applications is multi-layer feed forward network, which consists of an input layer, one or more hidden layers and an output layer. The input layer is responsible for receiving input elements from outside of the network and conveys this information to a hidden layer. The hidden layer processes the data received from the input layer, and passes the processed data to either another hidden layer or the output layer, which produces the final output. Multi-layer feedforward networks are adaptive data driven systems developed in two stages: training and testing. In the training stage, input data with known output is provided to the network. The main objective is to tune weights

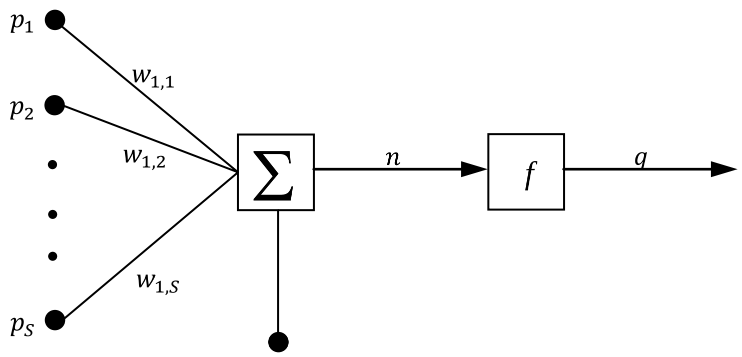


Fig. 1 Neuron with multiple input elements

and biases in such a way that the trained network is capable of producing reliable predictions over the data which is never presented to the network. The most common learning algorithm used for training ANNs is error back-propagation algorithm. Once the network is trained, its performance is evaluated in the testing stage.

3.1 Experimental data

The performance of an ANN depends strongly on the data provided to the network in the training stage. The database needs to be sufficiently large, accurate and evenly distributed so that the network can extract the hidden relationships between the parameters involved. A database of diagonal cracking strength was compiled by scanning experimental studies on RC slender beams without stirrups (Moody *et al*. 1954, Cossio and Siess 1960, Taylor 1960, Van den Berg 1962, Bresler and Scordelis 1963, Mathey and Watstein 1963, Taylor and Brewer 1963, Krefeld and Thurston 1966, Mattock 1969, Mphonde and Frantz 1984, Ahmad *et al*. 1986, Elzanaty *et al*. 1986, Xie *et al*. 1994, Kim *et al*. 1999, Shin *et al*. 1999, Pendyala and Mendis 2000, Cladera and Mari 2005, Shah and Ahmad 2007, Hamrat *et al*. 2010, Sneed and Ramirez 2010, Garip 2011, Slowik and Nowicki 2012, Slowik and Smarzewski 2012). Thirteen beams were excluded from the database since they cause large gaps in the distributions of effective depth, concrete compressive strength and longitudinal reinforcement ratio. A total of 271 simply supported beams loaded by either two symmetrically located concentrated loads or a concentrated load at mid-span were included in the database. The ranges of parameters stored in the database are 12.2≤ f_c ≤87 (MPa), 0.33%≤ ρ ≤5.03%, 2.50≤ a/d ≤8.52, 133≤ d ≤530 (mm) and 0.41≤ v_{cr} ≤2.43 (MPa).

3.2 ANN model

A multi-layer feedforward network was developed using MATLAB Neural Network Toolbox. The network consists of an input layer of four neurons receiving concrete compressive strength, longitudinal reinforcement ratio, shear span-to-depth ratio and effective depth as input parameters, a hidden layer of five neurons and an output layer of a single neuron delivering an estimate of diagonal cracking strength. The network topology is shown in Fig. 2 schematically.

All of the neurons in the network use the summation of weighted inputs with the bias as the net input function. The transfer functions of hidden and output layers are log-sigmoid and linear transfer functions, respectively (Fig. 3). The input parameters were normalized before they were presented to the network to prevent the log-sigmoid function from becoming saturated which slows down the network training. The diagonal cracking strength of each beam was also normalized so that the network output fell into the normalized range and then it was converted into the corresponding diagonal cracking strength.

A common issue with training ANNs is overfitting, which results in a very small error on the training set but a large error on a set of new data. In other words, the network memorizes the training data but fails to generalize to new data. MATLAB Neural Network Toolbox offers two methods for improving generalization: Bayesian regularization and early stopping. The details of Bayesian regularization can be found elsewhere (Foresee and Hagan 1997). Early stopping technique was used for training the ANN model. It requires the experimental database to be divided into three subsets: training, validation and test sets. The training set is used to optimize the network performance by tuning the network weights and biases according to the Levenberg-Marquardt back-propagation algorithm (Hagan *et al.* 1996). The performance function is the mean

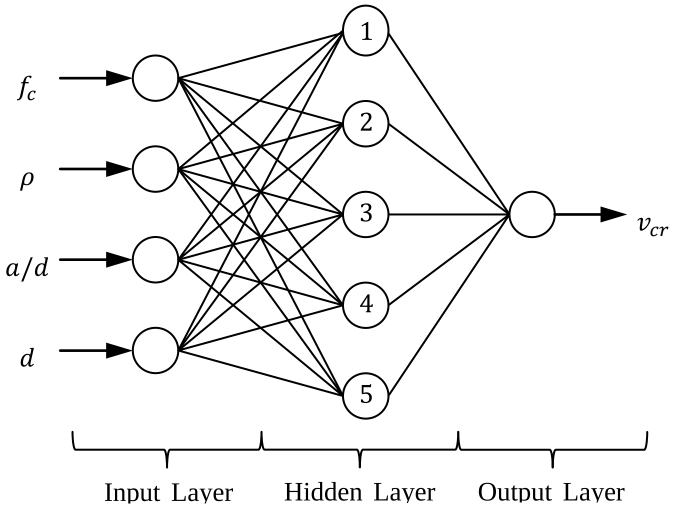
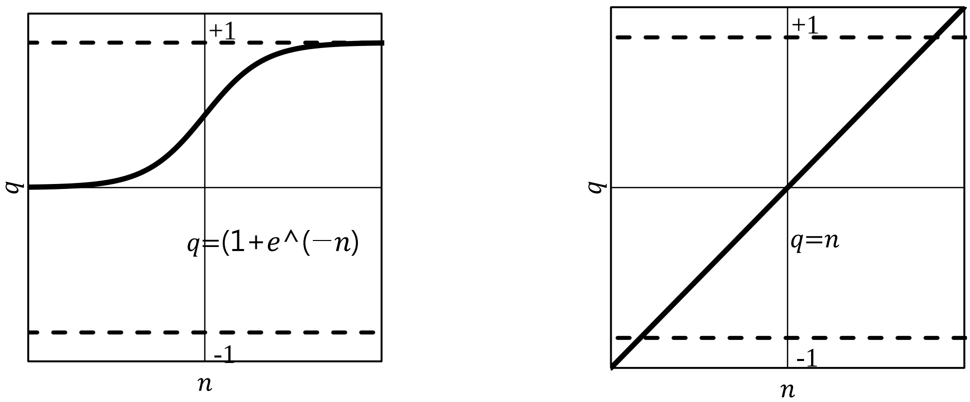


Fig. 2 The architecture of the developed ANN model



(a) Log-sigmoid transfer function (b) Linear transfer function

Fig. 3Transfer functions

squared error (MSE) between the network outputs q_i and the corresponding experimental values t_i, that is,

$$F = \frac{1}{N}\sum_{\mathrm{i}=1}^{N}(t_i - q_i)^2, \tag{20}$$

where N is the number of beams in the set. The validation set is used to prevent the network from overfitting the data. The errors on the training and validation sets are monitored simultaneously and the training process is stopped when the validation set error starts to increase. The test set is not involved in the training process. It is used for comparing various ANN models. Accordingly, the database was divided into training, validation and test sets having 217, 27 and 27 beams, respectively, where the beams were distributed randomly.

4. Results and discussion

The mean squared errors on the training, validation and test sets monitored during the training process are plotted in Fig. 4. The optimum solution was obtained at the sixty-first epoch after which the validation set error failed to decrease for five successive epochs. The mean squared errors on the training, validation and test sets are 0.023, 0.013 and 0.029, respectively, at the sixty-first epoch. A possible sign of overfitting is a significant increase in the test error before the error on the validation set increases. No such behavior is observed in Fig. 4. ANN models resulting in much smaller errors were developed, however they were rejected due to overfitting issues. It is not always possible to capture overfitting from the performance curves, but it can be detected through a parametric study. A network which seems to be well trained at first may produce weird relationships, e.g. largely oscillating curves, in the parametric study.

The ANN model outputs ($v_{cr,\mathrm{ANN}}$) against the experimental results ($v_{cr,exp}$) for the training, validation and test sets, and the whole database are plotted in Fig. 5. The correlation coefficients (R) are 0.907, 0.910, 0.898 and 0.906, respectively. The statistics of the ratio of the ANN model outputs to the experimental values, $v_{cr,\mathrm{ANN}}/v_{cr,exp}$ are given in Table 1. The mean, standard deviation (SD) and coefficient of variation (COV) of $v_{cr,\mathrm{ANN}}/v_{cr,exp}$ for the whole database are 1.02, 0.14 and 0.14, respectively. A good agreement between the experimental data and the ANN model outputs is observed through Fig. 5 and Table 1.

Table 1 Statistics of $v_{cr,ANN}/v_{cr,exp}$

Set	Min.	Max.	Mean	SD	COV
Training	0.74	1.85	1.02	0.15	0.14
Validation	0.83	1.38	1.01	0.13	0.13
Test	0.78	1.41	1.01	0.13	0.13
All	0.74	1.85	1.02	0.14	0.14

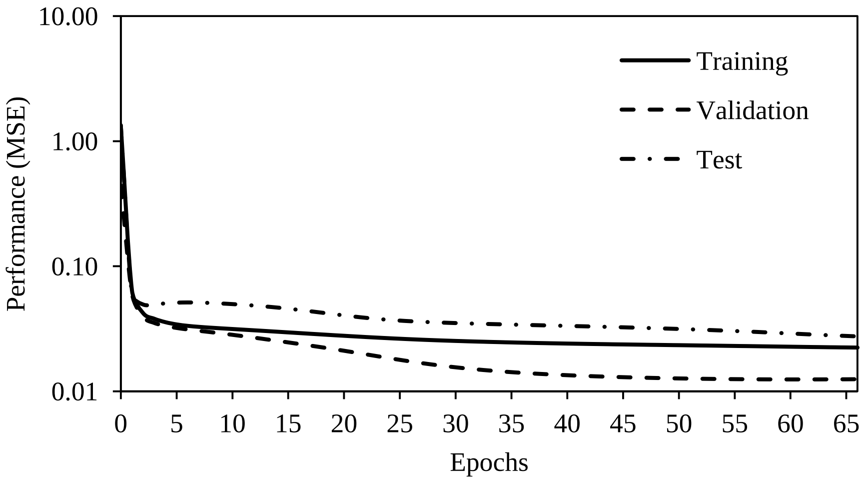
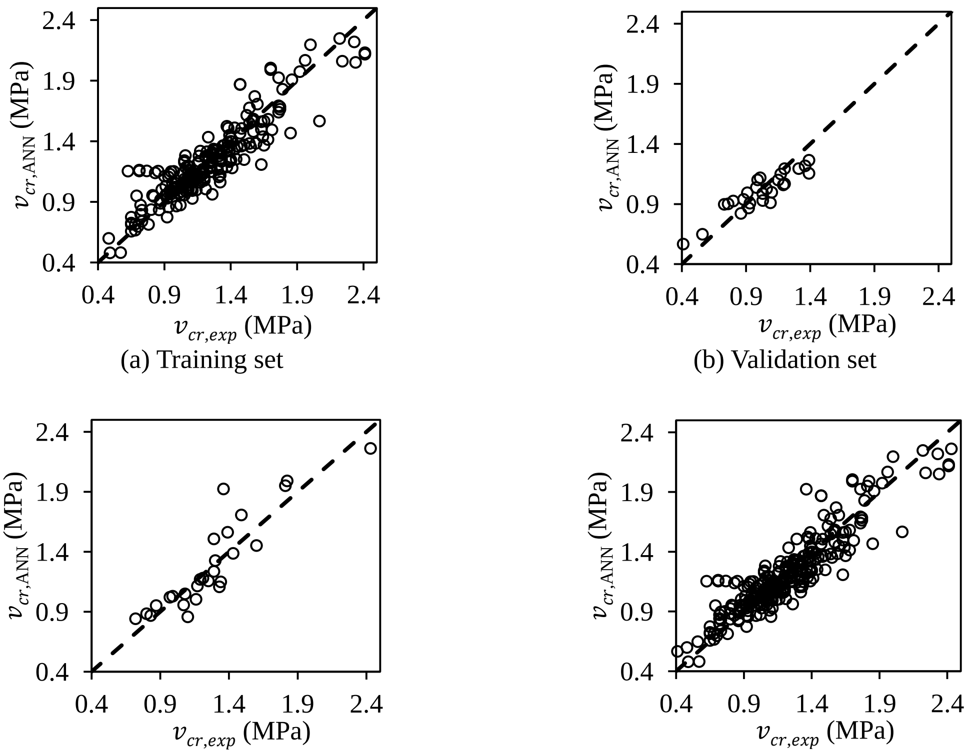


Fig. 4 Performance of the developed ANN model



(a) Training set

(b) Validation set

(c) Test set

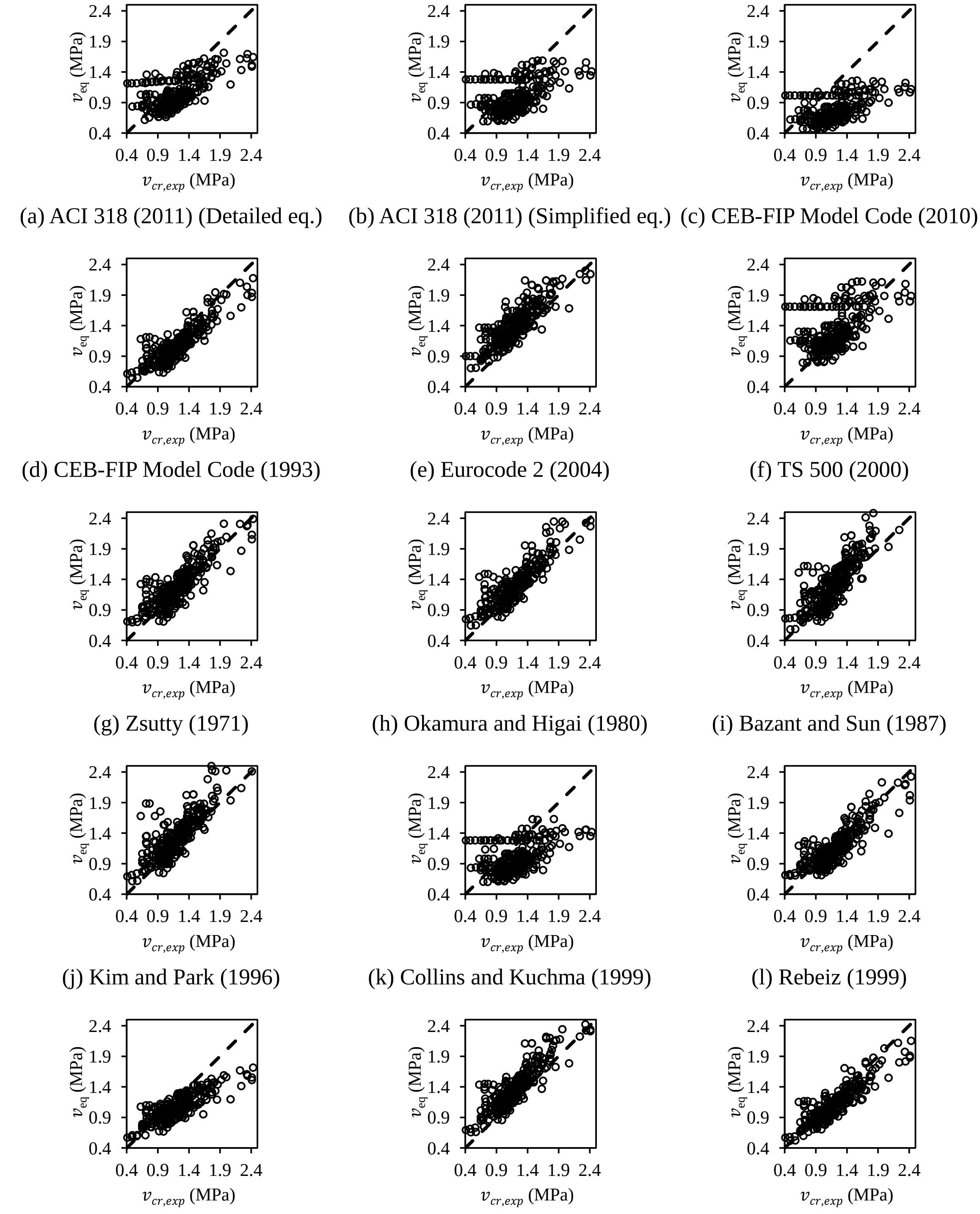
(d) Whole database

Fig. 5 $v_{cr,\mathrm{ANN}}$ vs. $v_{cr,exp}$

4.1 Comparison with the existing equations

Six design code equations (ACI 318 2011, CEB-FIP Model Code 2010, CEB-FIP Model Code 1993, Eurocode 2 2004, TS 500 2000) and twelve equations proposed by various researchers (Zsutty 1971, Okamura and Higai 1980, Bazant and Sun 1987, Kim and Park 1996, Collins and Kuchma 1999, Rebeiz 1999, Khuntia and Stojadinovic 2001, Zararis and Papadakis 2001, Arslan 2012, Perez *et al*. 2012) were applied to the database to compare the performance of the ANN model with the performances of those equations. The predicted values (v_{eq}) from Eqs. (1)-(15) and Eqs. (16)-(18) against the experimental results are plotted in Figs. 6 and 7, respectively. Table 2 presents the statistics of the ratios of the predicted values from Eqs. (1)-(18) to the experimental values, $v_{eq}/v_{cr,exp}$.

The equations of ACI 318 (2011), CEB-FIP Model Code (2010) and CEB-FIP Model Code (1993) underestimate the diagonal cracking strength of most of the beams in the database. On the other hand, the equations of Eurocode 2 (2004) and TS 500 (2000) estimate the diagonal cracking strength greater than the experimental values for most of the considered beams. The correlation coefficients between the experimental values and the predictions by the detailed and simplified equations of ACI 318 (2011), and the equations of CEB-FIP Model Code (2010), CEB-FIP Model



(a) ACI 318 (2011) (Detailed eq.) (b) ACI 318 (2011) (Simplified eq.) (c) CEB-FIP Model Code (2010)

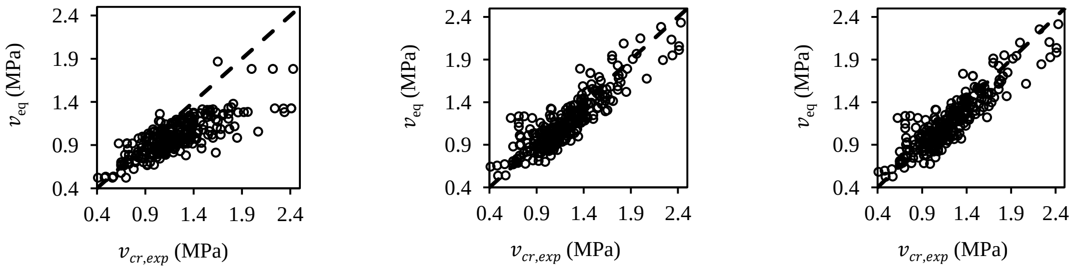
(d) CEB-FIP Model Code (1993) (e) Eurocode 2 (2004) (f) TS 500 (2000)

(g) Zsutty (1971) (h) Okamura and Higai (1980) (i) Bazant and Sun (1987)

(j) Kim and Park (1996) (k) Collins and Kuchma (1999) (l) Rebeiz (1999)

(m) Khuntia and Stojadinovic (2001) (n) Zararis and Papadakis (2001) (o) Arslan (2012)

Fig. 6 v_{eq} from Eqs. (1)-(15) vs. $v_{cr,exp}$



(a) Perez *et al.* (2012) (7A1) (b) Perez *et al.* (2012) (8H1) (c) Perez *et al.* (2012) (8I1)

Fig. 7 v_{eq} from Eqs. (16)-(18) vs. $v_{cr,exp}$

Table 2 Statistics of $v_{eq}/v_{cr,exp}$

Equation	Min.	Max.	Mean	SD	COV
Eq. (1) (ACI 318 2011)	0.56	2.96	0.91	0.29	0.32
Eq. (2) (ACI 318 2011)	0.48	3.12	0.87	0.33	0.38
Eq. (3) (CEB-FIP Model Code 2010)	0.38	2.48	0.68	0.25	0.37
Eq. (4) (CEB-FIP Model Code 1993)	0.64	1.89	0.92	0.17	0.18
Eq. (5) (Eurocode 2 2004)	0.81	2.20	1.21	0.21	0.17
Eq. (6) (TS 500 2000)	0.65	4.17	1.17	0.44	0.38
Eq. (7) (Zsutty 1971)	0.72	2.12	1.08	0.21	0.19
Eq. (8) (Okamura and Higai 1980)	0.79	2.31	1.11	0.20	0.18
Eq. (9) (Bazant and Sun 1987)	0.72	2.42	1.14	0.23	0.20
Eq. (10) (Kim and Park 1996)	0.75	2.69	1.16	0.25	0.22
Eq. (11) (Collins and Kuchma 1999)	0.48	3.12	0.87	0.32	0.37
Eq. (12) (Rebeiz 1999)	0.67	1.91	0.99	0.19	0.19
Eq. (13) (Khuntia and Stojadinovic 2001)	0.57	1.72	0.91	0.16	0.18
Eq. (14) (Zararis and Papadakis 2001)	0.84	2.30	1.17	0.19	0.16
Eq. (15) (Arslan 2012)	0.71	1.84	0.94	0.15	0.16
Eq. (16) (Perez *et al.* 2012)	0.50	2.41	0.85	0.18	0.21
Eq. (17) (Perez *et al.* 2012)	0.71	1.94	0.99	0.16	0.17
Eq. (18) (Perez *et al.* 2012)	0.68	1.94	0.98	0.16	0.16

Table 3 Correlation coefficients and errors for the ANN model and the considered equations

Model/Equation	R	MSE	NMSE	MAPE
The ANN model	0.906	0.022	0.179	0.096
Eq. (1) (ACI 318 2011)	0.643	0.101	0.812	0.236
Eq. (2) (ACI 318 2011)	0.502	0.145	1.168	0.277
Eq. (3) (CEB-FIP Model Code 2010)	0.507	0.282	2.269	0.375
Eq. (4) (CEB-FIP Model Code 1993)	0.873	0.044	0.357	0.150
Eq. (5) (Eurocode 2 2004)	0.860	0.078	0.629	0.218
Eq. (6) (TS 500 2000)	0.502	0.132	1.060	0.263
Eq. (7) (Zsutty 1971)	0.849	0.041	0.331	0.148
Eq. (8) (Okamura and Higai 1980)	0.875	0.044	0.357	0.151

Table 3 Continued

Eq. (9) (Bazant and Sun 1987)	0.864	0.086	0.688	0.191
Eq. (10) (Kim and Park 1996)	0.836	0.099	0.793	0.193
Eq. (11) (Collins and Kuchma 1999)	0.504	0.147	1.182	0.276
Eq. (12) (Rebeiz 1999)	0.852	0.036	0.291	0.137
Eq. (13) (Khuntia and Stojadinovic 2001)	0.839	0.065	0.522	0.152
Eq. (14) (Zararis and Papadakis 2001)	0.889	0.063	0.503	0.186
Eq. (15) (Arslan 2012)	0.891	0.035	0.278	0.124
Eq. (16) (Perez *et al.* 2012)	0.674	0.113	0.908	0.188
Eq. (17) (Perez *et al.* 2012)	0.888	0.028	0.225	0.116
Eq. (18) (Perez *et al.* 2012)	0.892	0.027	0.220	0.112

Code (1993), Eurocode 2 (2004) and TS 500 (2000) are 0.643, 0.502, 0.507, 0.873, 0.860 and 0.502, respectively. Although the predictions by the equations of ACI 318 (2011) and CEB-FIP Model Code (2010) are not well correlated with the experimental results, they are generally on the safe side.

The predictions calculated from the equations proposed by Zsutty (1971), Bazant and Sun (1987), Kim and Park (1996) and Zararis and Papadakis (2001) for the beams in the database are mostly greater than the experimental values. This is expected since those equations were derived for predicting the ultimate shear strength of RC beams without stirrups. The correlation coefficients between the experimental results and the predictions by the equations of Zsutty (1971), Bazant and Sun (1987), Kim and Park (1996) and Zararis and Papadakis (2001) are 0.849, 0.864, 0.836 and 0.889, respectively. The equations proposed by Okamura and Higai (1980), Collins and Kuchma (1999), Rebeiz (1999), Khuntia and Stojadinovic (2001) and Arslan (2012) predict the diagonal cracking strength of RC beams without stirrups. The equation of Okamura and Higai (1980) overestimates the diagonal cracking strength of most of the considered beams. The correlation coefficient between the experimental values and the predictions by the equation of Okamura and Higai (1980) is 0.875. The predictions calculated from the equation of Collins and Kuchma (1999) for the considered beams generally remain on the safe side, but the correlation between the predictions and the experimental values are not good, where the correlation coefficient is 0.504. The predictions obtained through the equations of Rebeiz (1999), Khuntia and Stojadinovic (2001) and Arslan (2012) are mostly less than the experimental values and have a satisfactory correlation with the experimental values, where the correlation coefficients are 0.852, 0.839 and 0.891, respectively.

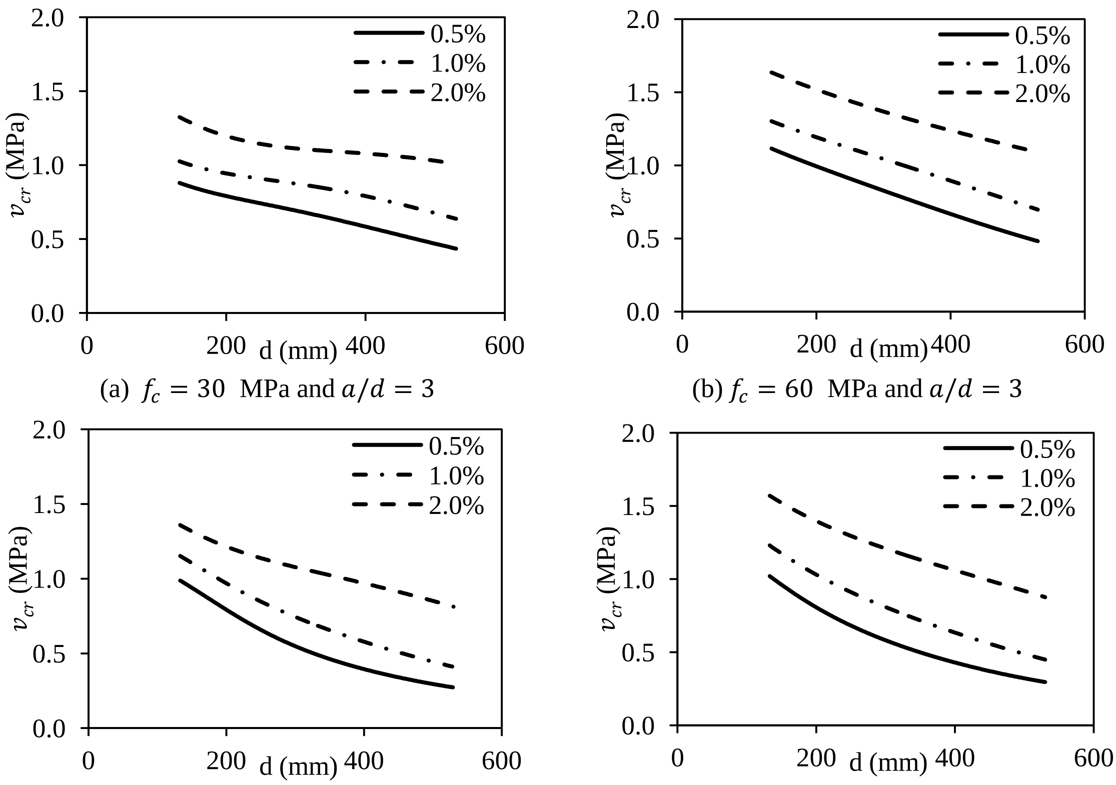
The correlation coefficients between the experimental values and the predictions by the equations of Perez *et al.* (2012) given by Eqs. (16)-(18) are 0.674, 0.888 and 0.892, respectively. Although the predictions by Eq. (16) are poorly correlated with the experimental results, it generally delivers conservative estimates. The predictions by Eqs. (17)-(18) have a better correlation with the experimental values compared to those by Eq. (16) do.

Among the considered equations, it is observed that Eq. (12) by Rebeiz (1999), Eq. (15) by Arslan (2012) and Eqs. (17)-(18) by Perez *et al.* (2012) are superior to the others in predicting the diagonal cracking strength of the beams in the database. In general, the ANN model generates outputs closer to the experimental values than Eqs. (1)-(18) do. Table 3 presents the correlation coefficients, the mean squared errors, the normalized mean squared errors (NMSE) and the mean absolute percentage errors (MAPE) for the ANN model and Eqs. (1)-(18). It can be observed through Table 3 that the ANN model has a better performance of predicting the diagonal cracking strength of the beams in the database than Eqs.(1)-(18) do.

4.2 Parametric study

A parametric study was conducted to study the effects of various parameters on the diagonal cracking strength of RC beams without stirrups using the ANN model. The ranges of parameters stored in the database were considered in the parametric study. Fig. 8 shows the change in the diagonal cracking strength with respect to the effective depth for f_c of 30 MPa and 60 MPa, a/d of 3 and 5, and ρ of 0.5%, 1.0% and 2.0%. Fig. 9 plots the change in the diagonal cracking strength against the concrete compressive strength for a/d of 3 and 5, and ρ of 0.5%, 1.0% and 2.0%, where the effective depth is 300 mm. Fig. 10 depicts the relationship between the diagonal cracking strength and the concrete compressive strength for a/d of 3 and 5, and d of 200 mm, 300 mm, 400 mm and 500 mm, where the longitudinal reinforcement ratio is 1.0%.

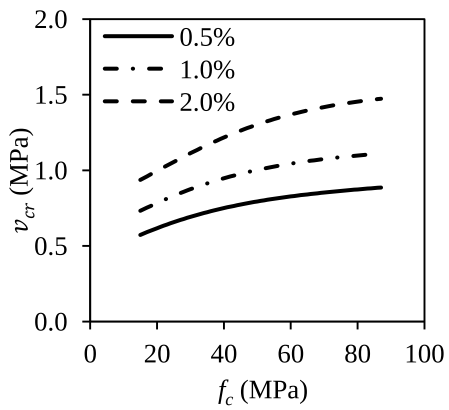
A significant size effect on the diagonal cracking strength is observed in Fig. 8. The decrease in the diagonal cracking strength ranges from 24% to 72% with the increase in the effective depth from 133 mm to 530 mm, depending on the concrete compressive strength, the longitudinal reinforcement ratio and the shear span-to-depth ratio. The reduction in the diagonal cracking strength with respect to the effective depth increases as the longitudinal reinforcement ratio decreases. For instance, in the case that the concrete compressive strength is 60 MPa and the shear



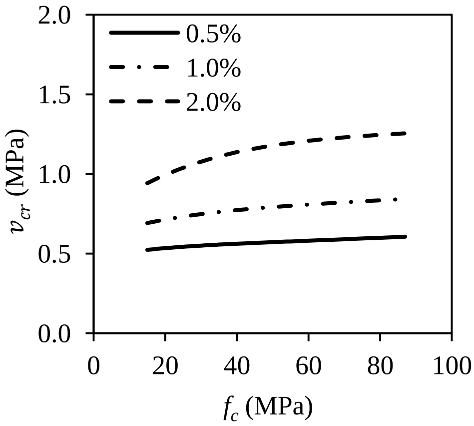
(a) $f_c = 30$ MPa and $a/d = 3$ (b) $f_c = 60$ MPa and $a/d = 3$

(c) $f_c = 30$ MPa and $a/d = 5$ (d) $f_c = 60$ MPa and $a/d = 5$

Fig. 8 Diagonal cracking strength vs. effective depth (ρ =0.5%, 1.0% and 2.0%)

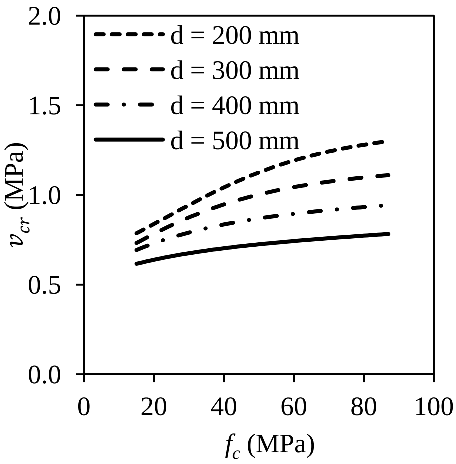


(a) $d = 300$mm and $a/d = 3$

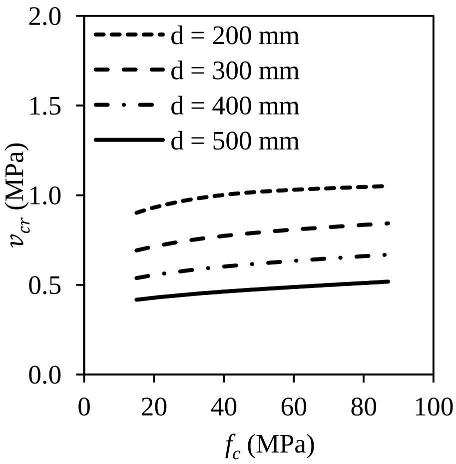


(b) $d = 300$ mm and $a/d = 5$

Fig. 9 Diagonal cracking strength vs. concrete compressive strength (ρ =0.5%, 1.0% and 2.0%)



(a) $a/d = 3$



(b) $a/d = 5$

Fig. 10 Diagonal cracking strength vs. concrete compressive strength (ρ =1.0%)

span-to-depth ratio is 3, the reduction in the diagonal cracking strength is 57%, 46% and 33% when the longitudinal reinforcement ratio is 0.5%, 1.0% and 2.0%, respectively. It should be noted that the shear design equations of ACI 318 (2011) (Eqs. 1 and 2) and TS 500 (2000) (Eq. 6) does not consider size effect on the diagonal cracking strength of RC beams.

It is observed in Figs. 9 and 10 that the diagonal cracking strength increases significantly with the concrete compressive strength. Fig. 9 presents that the increase in the diagonal cracking strength of a RC beam with an effective depth of 300 mm ranges from 52% to 57% and 16% to 33% when the shear span-to-depth ratio is 3 and 5, respectively, depending on the longitudinal reinforcement ratio, with the increase in the concrete compressive strength from 15 MPa to 87 MPa. Fig. 10 shows that the increase in the diagonal cracking strength due to an increase in the concrete compressive strength gets more pronounced as the effective depth gets smaller when the shear span-to-depth ratio is 3. In the case that the shear span-to-depth ratio is 5, the influence of the effective depth is less pronounced. For example, the increase in the diagonal cracking strength is 65% for an effective depth of 200 mm, while it is 27% for an effective depth of 500 mm when the shear span-to-depth ratio is 3.

It can be inferred from Figs. 8 and 9 that the diagonal cracking strength increases significantly with the longitudinal reinforcement ratio. For a beam with an effective depth of 300 mm and a shear span-to-depth ratio of 3, the increase in the diagonal cracking strength ranges from 61% to 66% depending on the concrete compressive strength.

As previously stated, the most important issue with developing an ANN model is whether the network has memorized the training examples or has succeeded in generalizing to new data. Since both the performance curves plotted in Fig. 4 and the resulting trends in the parametric study do not exhibit any sign of overfitting, the developed ANN model can be considered to be able to predict the diagonal cracking strength of an RC beam without stirrups with a reasonable error, provided that the parameters of the beam are within the ranges considered in training the ANN model.

5. Conclusions

The diagonal cracking strength of a RC beam is a critical parameter in the current practice of shear design since it is essential for determining the minimum amount of stirrups and the contribution of concrete to shear strength. The diagonal cracking strength of RC slender beams without stirrups is of interest to this research. A database of 271 beams compiled from the experimental studies available in the literature was used to develop an ANN model for predicting the diagonal cracking strength of RC slender beams without stirrups. The model consists of an input layer of four neurons accepting concrete compressive strength, longitudinal reinforcement ratio, shear span-to-depth ratio and effective depth as input parameters, a hidden layer of five neurons and an output layer of a single neuron delivering an estimate of diagonal cracking strength. The mean, standard deviation and COV of the estimates of diagonal cracking strength are 1.02, 0.14 and 0.14, respectively. The coefficient of correlation between the estimates and the experimental values are 0.906. ANN models generating outputs with much smaller errors were discarded due to overfitting issues, which can be detected through performance curves plotted in the training stage and/or incoherent trends arising in the parametric study. The developed ANN model outputs were compared with the predictions obtained through six shear design equations and twelve equations proposed by various researchers. The ANN model has a better performance over the compiled database than the considered equations do.

The effects of effective depth, concrete compressive strength and longitudinal reinforcement ratio on the diagonal cracking strength of RC slender beams without stirrups were examined through a parametric study using the ANN model. A significant size effect is observed that the diagonal cracking strength decreases as the effective depth increases. The reduction in the diagonal cracking strength with respect to the effective depth increases as the longitudinal reinforcement ratio gets smaller. It is also observed that the diagonal cracking strength increases significantly with the concrete compressive strength and the longitudinal reinforcement ratio. The effect of concrete compressive strength on the diagonal cracking strength gets more pronounced as the effective depth gets smaller. The performance curves obtained in the training stage and the resulting trends in the parametric study show that the ANN model is able to generalize to new data. Based on the results of the parametric study, it is recommended that the design codes should consider size effect in the shear design equations.

Even though the ANN model generates satisfactory outputs, it needs to be improved since the data used for developing the model is limited. For example, 75% of the beams in the database have concrete compressive strength less than 50 MPa. The ANN model will be updated as new data becomes available.

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