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Software for application of Newton-Raphson method in estimation of strains in prestressed concrete girders

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Abstract. Structures suffer from damages in their lifetime due to time-dependant effects, such as fatigue, creep and shrinkage, which can be expressed by concrete strains. These processes could be seen in the context of strain estimation of pre-stressed structures in two phases by using numerical methods. Their aim is checking and validating existing code procedures in determination of deformations of pre-tensioned girders by solving a system of nonlinear equations with strains as unknown parameters. This paper presents an approach based on the Newton-Raphson method for obtaining the stresses and strains in middle span section of pre-stressed girders according the equilibrium state.

Keywords: pre-tensioning; strains; linear theory; Newton-Raphson method; nonlinear equations.

1. Introduction

The basic idea of prestressing is improving load bearing capacity of concrete girders by creation of permanent stresses in a subjected structural element before it is imposed to any kind of loads. These elements are widely used in industrial building, bridge constructing as precast structural parts. In general, compared to reinforced concrete, prestressing has a many advantages as for the same spans, dimensions in girder cross section of pre-stressed elements can be smaller, full cross section is effective, lower amounts of plain reinforcement steel are necessary to guarantee structural bearing capacity.

In prestressed concrete, the most commonly used method of applying prestressing force to concrete is by tensioning high-strength reinforcement commonly referred to as tendons, against the concrete prior to the application of imposed loads (Marshall and Robberts 2000). Two different approaches can be distinguished in this regard to a time of casting concrete: pre-tensioning and post-tensioning.

As a consequence load strains are appearing. Designers and scientists are interested in strain value in top and bottom edge (flange) and its keeping below its limit value. Equations for calculating stresses, depending on material characteristics, are included in standards (EN 1992-1-1 2004, DIN 1045-1 2008). The importance of strain calculation could be seen as prevention of crack appearance in designing phase.

According to theories (Marshall and Robberts 2000) strains in cross section and position of

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neutral axes can be derived from nonlinear equation system in equilibrium state, which requires to be solved by appropriate numerical methods. This paper presents an approach based on the Newton-Raphson method (Polyak 2006), which leads to exact solution although it is possible to start with a roughly approximate one. In addition to improve algorithms other Newton methods, shown in (Grosan and Abraham 2008), can be considered in future. For easier handling of equations and results, software has been programmed based on Newton-Raphson method, and testing was done by numerical examples considering two general cross section shapes.

Summarizing the above mentioned methods, we can emphasize one of many existing definitions for this concrete strengthening method. Prestressing has been defined as the intentional creation of permanent stresses in a structure or assembly for the purpose of improving its behaviour and strength under various conditions of load.

2. Materials and methods

2.1 Study area

Both pre-tensioning and post-tensioning systems have specific theoretical and practical advantages and disadvantages. These must be considered in conjunction with the particular technical requirements and the prevailing economic considerations for a specific job. It is useful to remember, in this regard, that although pre-tensioning is generally perceived as being limited to permanent pre-casting factories, it can be economically feasible for the contractor to set up a temporary pre-tensioning yard at, or close to, the site on projects where a large number of pre-tensioned elements are to be used. On the other hand, post-tensioned members, which are usually constructed and tensioned *in situ*, can be manufactured in a pre-casting plant and subsequently transported to site (pre-cast segmental post-tensioned bridges) (Thomsing 2002).

The basic principle of pre-stressing is illustrated in Fig. 1, assembled by three states of a simple supported beam in different load cases. The diagram shows a response of a beam to uniformly distributed load and prestressing force. These are superposed stresses along the cross section height.

In this case, the method of pre-tensioning of structural elements should be enlightened. The tendons, which are placed in a bed in the form of wires or strands between fixed supports, are stretched. The tendons are stretched in a vertical plane to provide compression of concrete. As a remainder, eccentric pre-stress upward and downward stress components show different signs. Concrete is casted around the tendons. At the end of each member cutting of tendons after desired strengthening of concrete occurs. Restrained from shortening, tendons compress the concrete and transfer forces to

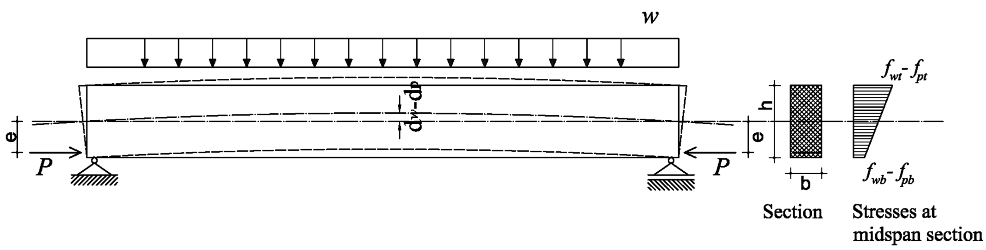
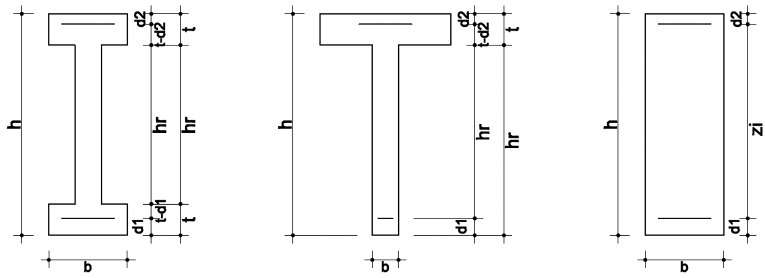


Fig. 1 Principle of pre-stressing on simple supported beam



(a) "*I*" cross section **(b) "*T*" cross section** **(c) Rectangular**

Fig. 2 The general cross section shapes

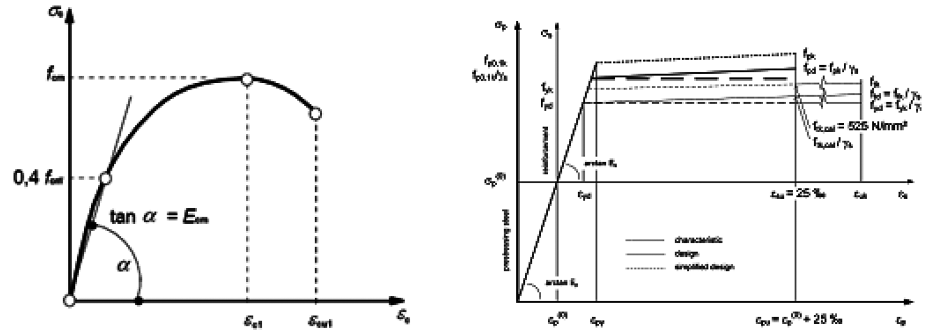
the element through the bond. This method can be used for industrial mass production. The casting happens in long lines, getting several finished elements at the same time. Thereby, it is important to ensure that the members are free to move along the pre-stressing bed. Otherwise, undesirable tensile stresses may be set up in them when releasing the ends of anchorages. This tensile stresses could lead to formation of cracks and finally to material damage or failure.

We aim to provide results of strains in a specified girder cross section under certain known load conditions. Therefore a straight pre-stressed simple supported beam has been discretized and analyzed. Due to the fact, that pre-stressed structures usually remain in a non-cracked state during the greater part of lifetime, we have considered linear elastic material behaviour in our study. The target profile of the investigated cross section, taken from (CEB/FIP 1993), equals a rectangular section-profile which is commonly used in bridge engineering. For the known values of ultimate outer loads, geometrical and material characteristics of concrete and steel, strains in several phases of loading can be determined. The concept is to program software as a kind of hand calculation tool, which can be used by engineers in designing pre-stressed structures as well as in confirmation of results, obtained by for example finite element calculations. Owing to some problems in programming the procedure of strain estimation, it has been necessary to make some simplification of the cross-section type. Therefore, one simplification of an "*I*"-cross-section is a "*T*"-beam (girder and half of deck slab and one footpath). Another simplification is from "*T*"-beam to simplest relevant case a rectangular cross section (Fig. 2).

2.1.1 Material properties

In any kind of prestressing, two basic materials can be used: concrete and high-strength steel. Primarily, the strength of concrete is founded on properties of ingredients of mixture, more precisely of water/cement ratio and strength of aggregates. In general, the strength of concrete develops with time. More precisely, it increases with age, whereby the rate at which it increases, is greatly affected by the curing conditions. Further, the composite material is exposed to phenomenon of time-dependent losses caused by creep, shrinkage and relaxation as well as physical deformations (deflections and fractures) and their properties demand complex numerical modelling (Yang *et al.* 2008). Thanks to the great compressive strength it is easily done to provide more effective cross-section with pre-stressing compared to plain reinforced situation.

The best way to explain any material characteristic is "σ-ε" diagram of stress-strain relation (Fig. 3). Such a diagram can be gained by laboratory experiments or alternatively taken from structural codes (EN 1992-1-1 2004, DIN 1045-1 2008, CEB/FIP 1993). In the first domain, straight lines



(a) (b)

Fig. 3 (a) "σ-ε" diagram of concrete after and (b) "σ-ε" diagram of steel after (CEN/EN 1992-1-1 2004)

(including both compression and tension), of the diagram (Fig. 3(a)) represent linear elastic behaviour of concrete or in other words linear dependency between stresses and strains connected by Young's modulus of elasticity. Until reaching a certain limiting strength (f_{ck}) in the end of domain two, we have presence and growth of small fractures, cracks, which are increasing with time. The third domain only exists in compression, describes the material behaviour after fracture. For concrete in tension, material is expected to fail after tensile strength (f_{ct}) is reached. The aim of our research is also to show and explain behaviour of girder in this field whether it is compression or tension due to a combination of pre-stressing force, outer loads and elastic losses.

Fig. 3(b). illustrates the same diagram but this time for composite partner steel. Comparing the stresses and strains for high-strength and plain reinforcement steel it is obvious to see the difference between them. In the case of pre-stressing steel there is an initial strain caused by impressing the prestressing force. Pushing over a limit state value, steel behaves different from concrete. By entering into this phase steel shows plastic characteristics and strains are increasing accompanied by low increases of stresses until failure of the material. Again, all required values for calculations are defined and fixed in codes as shown in DIN or Eurocodes.

As mentioned before, there are two aspects for estimation and element behaviour: linear and nonlinear theory. In these analyses linear theory takes the greatest place in further considerations. Also the supporting fact which allowed this aspect is certain reliability of construction assured with safety factors (Paik *et al*. 2009).

2.2 Linear theory and basic assumptions

The first fact, which is chosen in this study, is that many existing constructions are in linear elastic field, especially pre-stressed girders. Due to pre-tensioning, huge compressive forces are impressed in concrete cross section. However, it is true there are small strains in concrete thanks to great compressive strength of concrete. These strains are matching with concept of linear theory. Because of small strains there are no cracking in material. Nevertheless, many nonlinear problems can be approximated by linear formulation and solved with linear programming algorithms (Fereig 1994).

The basic assumptions for this theory and task are following: concrete and steel are isotropic homogenous elastic materials which are denoted with a straight line in "σ-ε" diagram or in other

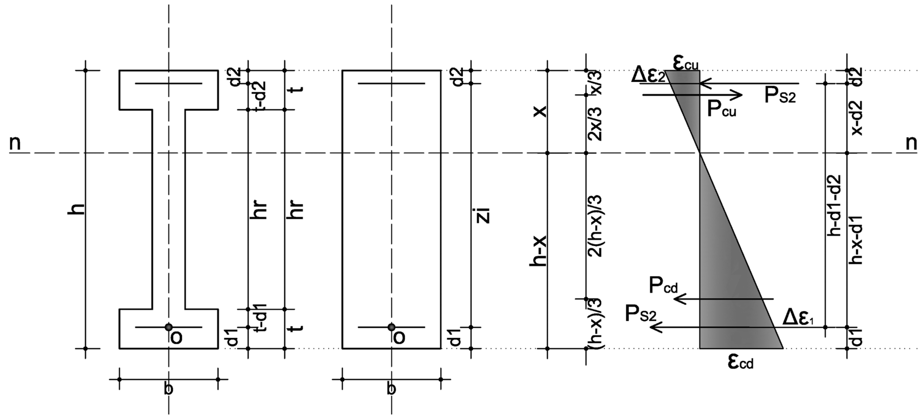


Fig. 4 Diagram of strains in general cross sections

words by Hooke's law: $\sigma = E \cdot \varepsilon$ leading to $\varepsilon = \sigma/E$; compatibility condition ($\Delta\varepsilon_{p,l} = \varepsilon_{cp,p}$), which means that strains in one fiber, regardless of the material they are connected to, are equal (perfect bond assumption); Bernoulli's hypothesis; assumption of linear progression of strains in cross section; mean value of prestressing force is decreased by certain losses.

2.2.1 Pre-tensioning state

The matter of consideration is a middle field cross section of a simply supported beam. Strains can be derived from the equilibrium state. There is the possibility to develop a system of equations from which we should obtain good results. The task also includes giving a graphical representation of the results, calculated by the developed algorithm, similar to the Fig. 4.

Fig. 4 illustrates all taken assumptions, which can be used for two layers of tendons in general. Due to the fact of eccentric pre-stress, the presence of tension in upper flange is found where the other remains in compression.

The system of equations describing the whole problem is assembled of the following three equations. The first one is taken from the proportion

$$\varepsilon_{cd} = \varepsilon_{cu} \cdot \frac{h-x}{x} \tag{1}$$

Similar equation is obtained in (Eurviriyanukul and Askes 2010, Moesly *et al.* 2007) for ultimate limit state (ULS) of concrete.

The equilibrium state in middle span section, precisely defined by the sum of horizontal forces as well as the sum of outer and inner moments which must be zero, leads to

$$P_{cu} - P_{cd} - (P_{p1} - \Delta P_{p1}) - (P_{p2} - \Delta P_{p2}) = 0 \tag{2}$$

$$P_{cu} \cdot \left(h - d_1 - \frac{x}{3}\right) - P_{cd} \cdot \left(\frac{h-x}{3} - d_1\right) - (P_{p1} - \Delta P_{p1}) \cdot z_{s1} - (P_{p2} - \Delta P_{p2}) \cdot z_{s2} = 0 \tag{3}$$

After input of geometrical and material characteristic values we obtain Eq. (2) and Eq. (3) for rectangular section in this form

$$\varepsilon_{cu}\cdot\left(\frac{x\cdot b}{2}+A_{s2}\cdot\frac{x-d_2}{x}\cdot n\right)-\varepsilon_{cd}\cdot\left(\frac{(h-x)\cdot b}{2}-A_{s1}\cdot\frac{h-x-d_1}{h-x}\cdot n\right)-(\varepsilon_{s2}\cdot A_{s2}+\varepsilon_{s1}\cdot A_{s1})\cdot n \tag{4}$$

$$\begin{gathered}\varepsilon_{cu}\cdot\left(\frac{x\cdot b}{2}\cdot\left(h-\frac{d_1-x}{3}\right)+A_{s2}\cdot z_{s2}\cdot\frac{x-d_2}{x}\cdot n\right)-\\ \varepsilon_{cd}\cdot\left(\frac{(h-x)^2\cdot b}{6}-\frac{b\cdot(h-x)\cdot d_1}{2}+A_{s1}\cdot z_{s1}\cdot\frac{h-x-d_1}{h-x}\cdot n\right)-(\varepsilon_{s2}\cdot A_{s2}\cdot z_{s2}-\varepsilon_{s1}\cdot A_{s1}\cdot z_{s1})\cdot n\end{gathered} \tag{5}$$

The system of equations has been expressed by following unknown parameters: x (position of neutral axes X), ε_{cu} (concrete strain upward) and ε_{cd} (concrete strain downward).

There is a difference which appears in values or equations for concrete forces for different section shapes (P_{cu}, P_{cd}). Forces for rectangular shape are expressed as

$$\begin{gathered}P_{cu}=\varepsilon_{cu}\cdot E_c\cdot\frac{x\cdot b}{2}\\ P_{cd}=-\varepsilon_{cd}\cdot E_c\cdot\frac{(h-x)\cdot b}{2}\end{gathered} \tag{6}$$

In other case it is more complex due to change of width along the girder height

$$\begin{gathered}P_{cu}=\varepsilon_{cu}\cdot E_c\cdot\left[\frac{b\cdot t\cdot\left(1+\frac{x-t}{x}\right)}{2}+b_r\cdot\frac{(x-t)^2}{2\cdot x}\right]\\ P_{cd}=-\varepsilon_{cd}\cdot E_c\cdot\left(b\cdot t\cdot\frac{h-2\cdot t}{2}+b_r\cdot\frac{(h-x-t)^2}{2\cdot(h-x)}\right)\end{gathered} \tag{7}$$

This mean that Eq. (1) is common for any section profile we choose for our girder, and the other two must be modified regard to geometrical characteristics and section area.

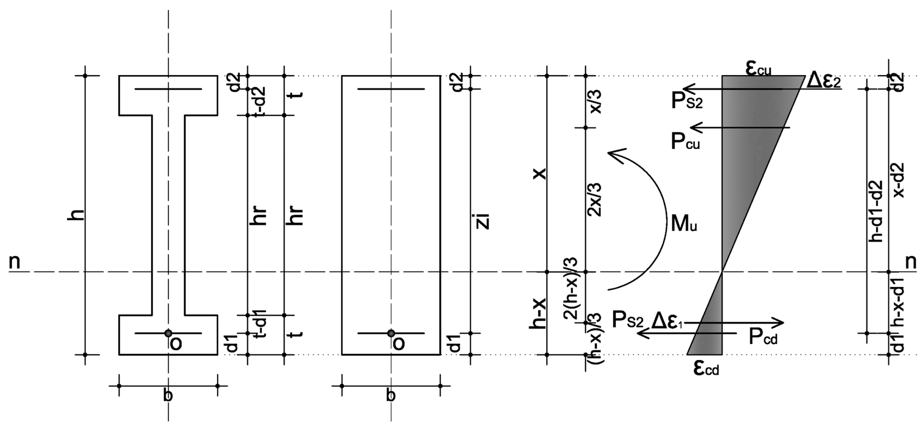


Fig. 5 Diagram of strains in general cross sections ULS

2.2.2 Ultimate load state

The second case of consideration and analysis was an additional loading step (Fig. 5). It has been included in this calculation and therefore in the equations there is summarized bending moment due to live load and self weight. The whole solution procedure has been repeated. When we wanted to fulfill main state of pre-stressing we had the expectation to get an opposite stress picture. This case is close to the aim of pre-stressing, avoiding possible tensile forces in downward zone.

Equations are similar to the former case just with one additional extension. Taken from the proportion, the first equation of the system is the same as Eq. (1), but the equilibrium state in middle span section includes two equations different from those described in Eq. (2) and Eq. (3).

Therefore, sum of horizontal forces appears in the following shape

$$-P_{cu}+P_{cd}-(P_{p1}-\Delta P_{p1})-(P_{p2}-\Delta P_{p2})=0 \tag{8}$$

Sum of moments has been extended for certain value of ultimate bending moment and as consequence a slight different equation been given

$$-P_{cu}\cdot\left(h-d_1-\frac{x}{3}\right)+P_{cd}\cdot\left(\frac{h-x}{3}-d_1\right)-(P_{p1}-\Delta P_{p1})\cdot z_{s1}-(P_{p2}-\Delta P_{p2})\cdot z_{s2}=M_u \tag{9}$$

It is important to note that point O (Fig. 5), according to which bending moments are regarded to is chosen freely. There were two handy options to choose point O, in tendon layer level or in center of gravity. Here, we been decided to pick the first option. Regardless which point we take, the equilibrium must be fulfilled. In shown equations, elastic material properties have been considered. Ahn *et al.* (2010) presented a similar method for solving equation system for plastic material.

2.3 Software solution and features

The proposed software is written for approximate solving of a system of equations

$$\begin{aligned} F(\varepsilon_{cd},\ \varepsilon_{cu},\ x) &= 0 \\ G(\varepsilon_{cd},\ \varepsilon_{cu},\ x) &= 0 \\ H(\varepsilon_{cd},\ \varepsilon_{cu},\ x) &= 0 \end{aligned} \tag{10}$$

where F, G and H are corresponding to Eqs. (1-5), respectively. They are continuous differentiable functions, using Newton-Raphson method (Stoer and Bulirsch 2002, Faires and Burden 2002)

$$\begin{aligned} \varepsilon_{cd\,n+1} &= \varepsilon_{cd\,n}-\Delta\varepsilon_{cd\,n} \\ \varepsilon_{cu\,n+1} &= \varepsilon_{cd\,n}-\Delta\varepsilon_{cu\,n} \qquad (n=0,1,2,\ldots) \\ x_{n+1} &= x_n-\Delta x_n \end{aligned} \tag{11}$$

starting with approximate values of $\varepsilon_{cd}=1.0$, $\varepsilon_{cu}=1.0$ and $x_0=1.0$ where

$$
\begin{aligned}
\Delta\varepsilon_{cd\,n} &= \frac{1}{J(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n)}\begin{vmatrix} F(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) & F'_{\varepsilon_{cu}}(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) & F'_x(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) \\ G(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) & G'_{\varepsilon_{cu}}(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) & G'_x(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) \\ H(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) & H'_{\varepsilon_{cu}}(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) & H'_x(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) \end{vmatrix} \\
\Delta\varepsilon_{cu\,n} &= \frac{1}{J(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n)}\begin{vmatrix} F'_{\varepsilon_{cd}}(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) & F(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) & F'_x(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) \\ G'_{\varepsilon_{cd}}(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) & G(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) & G'_x(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) \\ H'_{\varepsilon_{cd}}(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) & H(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) & H'_x(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) \end{vmatrix} \\
\Delta x_n &= \frac{1}{J(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n)}\begin{vmatrix} F'_{\varepsilon_{cd}}(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) & F'_{\varepsilon_{cu}}(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) & F(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) \\ G'_{\varepsilon_{cd}}(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) & G'_{\varepsilon_{cu}}(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) & G(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) \\ H'_{\varepsilon_{cd}}(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) & H'_{\varepsilon_{cu}}(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) & H(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) \end{vmatrix} \\
J(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) &= \begin{vmatrix} F'_{\varepsilon_{cd}}(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) & F'_{\varepsilon_{cu}}(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) & F'_x(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) \\ G'_{\varepsilon_{cd}}(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) & G'_{\varepsilon_{cu}}(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) & G'_x(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) \\ H'_{\varepsilon_{cd}}(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) & H'_{\varepsilon_{cu}}(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) & H'_x(\varepsilon_{cd\,n},\ \varepsilon_{cu\,n},\ x_n) \end{vmatrix} \neq 0
\end{aligned} \tag{12}
$$

Partial derivates are to be obtained numerically. The iterations interrupt when the conditions

$$|\varepsilon_{cd\,n+1}-\varepsilon_{cd\,n}|\leq\varepsilon, |\varepsilon_{cu\,n+1}-\varepsilon_{cu\,n}|\leq\varepsilon \text{ and } |x_{n+1}-x_n|\leq\varepsilon \tag{13}$$

are fulfilled. ε is accuracy given in advance, such as $\varepsilon = 10^{-10}$.

2.3.1 Input and output data

The input data are: modulus of elasticity of concrete (E_c), modulus of elasticity of steel (E_s), cross section area of one strand (A_s), number of strands in downer zone (k_1), number of strands in upper zone (k_2), nominal pre-tensioning stress in layer 1 (σ_{spl}), nominal pre-tensioning stress in layer 2 (σ_{sp2}), girder height (h), girder width (b), tendon distance from downer edge (d_1) and tendon distance from upper edge (d_2).

The output data are: concrete strain downward (ε_{cd}), concrete strain upward (ε_{cu}) and position of neutral axes X (x), which are calculated using Newton-Raphson method, prestressing force (Fp), prestressing bending moment (Mp), stress in top ($\sigma_{c,u}$) and stress in bottom ($\sigma_{c,d}$) flange for prestressing phase, strain of top flange ($\varepsilon_{cu,t}$) and strain of bottom flange ($\varepsilon_{cd,t}$).

2.3.2 Object-oriented concept

This software is based on object-oriented programming (OOP) and is written in *C#*. Therefore, the software design allows adding further functionalities and a code is reusable and extendible.

Because of the problems which can occur during the execution of an application, we used the concept of exception handling. This concept is used to provide a custom message to inform users which component is not correctly filled in.

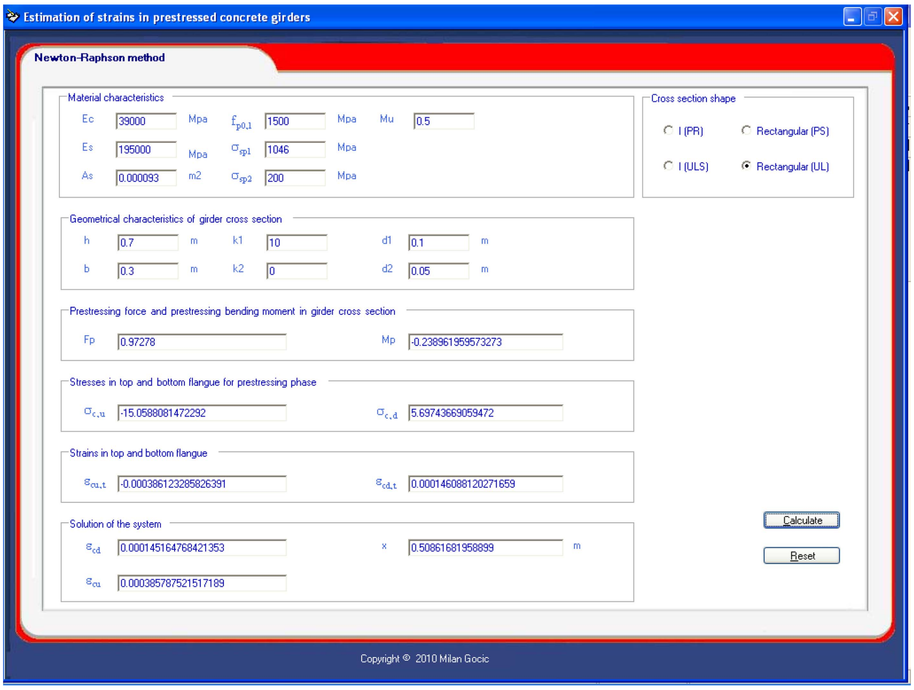


Fig. 6 Screen printout of the application for estimation of strains in prestressed concrete girders

2.3.3 Graphical user interface

A screen printout of the application is shown in Fig. 6. The window of the interface consists of two major parts. The top window includes information on material characteristics and geometrical characteristics of girder cross section. The bottom window contains estimated values of concrete strain downward, concrete strain upward, position of neutral axes X, prestressing force, prestressing bending moment, stress in top and stress in bottom flangue for prestressing phase, strain of top flange and strain of bottom flange.

2.3.4 Availability and documentation

The complete source code is presented in a documented hypertext help file. The proposed software could be used as a component. Therefore, the application and documentation are available for free download from the website http://www.gaf.ni.ac.rs/mgocic/EstimationOfStrains.htm.

3. Results and discussion

Two cross-section types have been studied. The procedure has been solved for ideal cross section of both represented shapes. All introduced parameters in these calculations are represented in Table 1.

These are results for prestressing state without any type of outer load, but including elastic losses in tendons occurring during transfer of forces between steel and concrete. For using a linear theory it is supposed to retrieve discrepancies between the results not greater than 5%. If there is a problem, when expectation is not fulfilled, it might be that it has not been possible to distinguish

Table 1 Material and geometrical properties for rectangular and *I* cross sections

Rectangular cross section			*I* cross section		
Symbol	Value	Units	Symbol	Value	Units
E_c	39000	MPa	E_c	39000	MPa
E_s	195000	MPa	E_s	195000	MPa
f_{pk}	1770	MPa	f_{pk}	1770	MPa
M_g	200	kNm	M_g	300	kNm
M_l	300	kNm	M_l	460	kNm
h	0.70	m	h	0.85	m
b	0.30	m	b	0.30	m
			t	0.12	m
			b_r	0.10	m
d_1	0.10	m	d_1	0.065	m
d_2	0.04	m	d_2	0.04	m
k_1	10	pcs	k_1	13	pcs
k_2	0	pcs	k_2	2	pcs

Table 2 Values estimated introducing of forces and moments in a girder

Cross section	ε_{cu}	ε_{cd}	x (m)
Rectangular (*PS*)	-0.000123	-0.000374	0.173576
Rectangular (*UL*)	0.000386	0.000145	0.508617

clearly the centers of gross and effective cross section. In this research we made an attempt to skip this obstacle by defining that position with lever arms of section forces in equation of bending moments.

Table 2 represents values estimated using proposed software with introducing of all forces and moments in a girder.

Results were checked with basic equations of prestressed concrete *UL* state given in (Marshall and Robberts 2000, Marmo *et al.* 2011, Nilson *et al.* 2004, Gilbert and Mickleborough 2004), and compared with results of software analysis made in Inca 2.0. These results are consequence of simplifications, but more accurate results can be derived with application of some integral methods for better generation of stresses in cross sections (Zupan and Saje 2005).

4. Conclusions

Increasing of strains over limiting value in pre-tensioned girder might be a possible reason for fracture appearance and endangering of load bearing capacity of girder. Because of that the accuracy of calculating procedure and introducing adequate parameters is important for design. In this case and to improve computation accuracy software has been developed, which estimates strains in certain cross section concrete profile basing its procedure on numerical method known as Newton-Raphson.

If the equilibrium is fulfilled in all girder sections, the solution of system of nonlinear equation figuring strains as variables has to exist, and could be solved with recommended software. In the other hand, existence of solution means that equilibrium in observed middle span section is valid and achieved, taking into account elastic losses occurring in the transfer of pre-stress to the concrete as well as various load conditions.

For some estimation it is necessary to use simplification in a sense of generalization or searching for better model of cross section. For that reason it is used I and rectangular cross sections. More accurate results could be derived after finite element discretisation of prestressed girder (Ayoub and Filippou 2010).

With assumption of linear elastic behaviour of materials correct results as a solution of equation system can be obtained. The procedure can be based on nonlinear theory, which is a future concept of developing the introduced software.

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References

Ahn, J.H., Jung, C.Y. and Kim, S.H. (2010), "Evaluation on structural behaviors of prestressed composite beams using external prestressing member", *Struct. Eng. Mech.,* **34**(2), 247-275.

Ayoub, A. and Filippou, C.F. (2010), "Finite-element model for pretensioned prestressed concrete girders", *J. Struct. Eng.-ASCE,* **136**(4), 401-409.

CEB/FIP (1993), *CEB-FIP Model code 1990: Design code*, Thomas Telford.

DIN 1045-1 (2008), *Tragwerke aus Beton, Stahlbeton und Spannbeton, Teil 1: Bemessung und Konstruktion*, Deutsches Institut für Normung, Beuth-Verlag, Berlin.

EN 1992-1-1 (2004), *Eurocode 2: Design of concrete structures, Part 1-1: General rules and rules for buildings*, Brussels.

Eurviriyanukul, S. and Askes, H. (2010), "The equilibration of configurational forces in the tendon layout optimisation of pre-stressed concrete beams", *Compos. Struct.*, **88**(23-24), 1412-1418.

Faires, D. and Burden, D. (2002), *Numerical methods*, third ed., Brooks Cole, England.

Fereig, S.M. (1994), "An application of linear programming to bridge design with standard prestressed girders", *Compos. Struct.*, **50**(4), 455-469.

Gilbert, R.I. and Mickleborough, N.C. (2004), *Design of prestressed concrete*, Spon Press.

Grosan, C. and Abraham, A. (2008), "A new approach for solving nonlinear equations systems", *IEEE T. Syst. Man. Cy. A*, **38**(3), 698-714.

Marmo, F., Serpieri, R. and Rosati, L. (2011), "Ultimate strength analysis of prestressed reinforced concrete sections under axial force and biaxial bending", *Compos. Struct.*, **89**(1-2), 91-108.

Marshall, V. and Robberts, J.M. (2000), *Prestressed concrete design and practice*, Concrete Society of Southern Africa, Prestressed Concrete Division, Midrand, South Africa.

Moesly, B., Bungey, J. and Hulse, R. (2007), *Reinforced concrete design to Eurocode 2*, sixth ed., Palgrave MacMillan, England.

Nilson, A.H., Darwin, D. and Dolan, C.W. (2004), *Design of concrete structures*, thirteenth ed., The McGraw-Hill Companies.

Paik, I., Hwang, E.S. and Shin, S. (2009), "Reliability analysis of concrete bridges designed with material and member resistance factors", *Comput. Concrete,* **6**(1), 59-78.

Polyak, B.T. (2006), "Newton-Kantorovich method and its global convergence", *J. Math. Sci.*, **133**(4), 1513-

1523.
Stoer, J. and Bulirsch, R. (2002), *Introduction to numerical analysis*, third ed., Series: Texts in Applied Mathematics, Vol. 12, Springer, Berlin.
Thomsing, M. (2002), *Spannbeton, Grundlagen, Berechnungsverfahren, Beispiele*, third edition, Teubner Verlag, Stuttgart.
Yang, X.S., Lees, J.M. and Morley, C.T. (2008), "Modelling crack propagation in structures: Comparison of numerical methods", *Commun. Numer. Meth. En.*, **24**(11), 1373-1392.
Zupan, D. and Saje, M. (2005), "Analytical integration of stress field and tangent material moduli over concrete cross-sections", *Compos. Struct.*, **83**(28-30), 2368-2380.

NB

Notation

The following symbols are used in this paper:

P_0 = initial prestressing force
ΔP_c = elastic losses due to concrete shortening
$\Delta P_\mu(x)$ = friction losses
ΔP_{sl} = losses due to slip (draw-in of wedges)
$\Delta P_t(t)$ = time-dependent losses (csr)
f_{ck} = material limiting strength
σ = material stress
σ_{ps1} = nominal stress in layer 1
σ_{ps2} = nominal stress in layer 2
ε = longitudinal strain
E = Young's modulus of elasticity
E_c = modulus of elasticity of concrete
E_s = modulus of elasticity of steel
h = girder height
b = girder width
t = thickness of flanges
b_r = width of rib
d_1 = tendon distance from downer edge
d_2 = tendon distance from upper edge
k_1 = number of strands in layer 1
k_2 = number of strands in layer 2
$\Delta\varepsilon_{p,l}$ = strain reduction due to elastic losses
x = position of neutral axes X
ε_{cu} = concrete strain upward
ε_{cd} = concrete strain downward
A_{s1} = area of pre-tensioning steel in layer 1
A_{s2} = area of pre-tensioning steel in layer 2
z_s = lever arm of cross section forces
J = Jacobian matrix
w = self weight

P = prestressing force
e = eccentricity
d_w, d_p = deflections due to loads w and P
f_{wt}, f_{pt} = stress in concrete top edge due to w, P
f_{wd}, f_{pd} = stress in concrete downer edge due to w, P