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An approach of using ideal grading curve and coating paste thickness to evaluate the performances of concrete-(1) Theory and formulation

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Abstract. The performance of a concrete is significantly influenced by its mixture proportion and the coating thickness on aggregate surface. The concrete in this study is designed by estimating the blending ratio of aggregate using a densified mixture design algorithm (DMDA) based on an ideal grading curve and estimating the paste volume as the sum of the amount of paste needed to provide an assigned coating paste thickness. So as to obtain appropriate concrete amount, and thus can accurately estimate the property of concrete. Deduction of this mix design formula is simple and easy understanding, and meanwhile to obtain result is fast. This estimation model of mix design is expected to reward to industry and effectively upgrade concrete quality.

Keywords: densified mixture design algorithm (DMDA); ideal grading curve; coating paste thickness.

1. Introduction

Concrete is a composite material, but, due to interface bonding defects among various materials and associated imperfections, the mechanical properties are difficult to estimate according to mathematical formulae of composite material (Hsu and Hsu 2002). In the past, the mix proportion was obtained by laboratory tests and trial batching, each property of the concrete was estimated by empirical equations and test results, and the mix proportion was seldom designed by theory from a grading curve. Literature reviews from past studies (Hsu and Hsu 2002, Chang *et al*. 2001, Zhan 1988, Kessler 1944, Kwan *et al*. 1999, Mora *et al*. 1998, Mora *et al*. 2000). Present a theoretical approach to arranging aggregate to fit in an ideal grading curve, establishing the optimum blended ratio of aggregates and other solid particles in concrete, so as to reach the maximum packing density. The effect of fly ash/slag on the property of reactive powder mortar designed by using Fuller grading curve and error function, that mainly focused on Fuller grading curve to theoretically design blended ratio of all solid materials of a reactive powder mortar (RPM), also known as reactive powder concrete (RPC) (Chan and Chu 2004). By computing the total surface area of aggregates and particles in concrete and then assigning a coating paste thickness, based on nominal thickness of coating paste prescribed, one can determine the relationship between paste amount and

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content of coarse and fine aggregates. A complete and precise formula to estimate the optimum coating thickness so as to limit the paste amount can ensure sufficient coating paste and dense aggregate structure. In future various properties of concrete can be obtained and expected based on the paste amount. This approach is expected to solve the problem that current concrete mix proportion can only be determined by elaborated experimental work and operator's experience, to build up a set of fast and efficient mixture design algorithm to secure long term performance of concrete as required. RPC is also known as ultra-high-performance concrete (UHPC). That is characterizing by excellent physical properties, particularly high compressive (200-800 MPa) and flexure strength (40-50 MPa) (Matte *et al.* 2000).

The aggregate occupies about 60~70% of concrete volume and was previously regarded as filler, but is now known to be the main component conferring strength. The particle properties, size and gradation of aggregate will influence the packing density of a concrete mixture. A simple sieve analysis is generally employed to describe particle distribution, and the blended ratio of aggregate has a great impact on the workability and properties of concrete. To make a comprehensive description of the gradation of a blended aggregate, proper characteristic values are needed to describe the composition of each single particle. Normally, fineness modulus (FM), specific surface area (S_m) and nominal maximum particle diameter ($D_{\max}$) are taken as concrete characteristic values. The packing of individual aggregate can be expressed as an ideal gradation curve from many grading curves obtained from experimental work and error tests, may be expressed in a diagram, such as sieve analysis curve, or denoted in functions, such as a Fuller curve and Bolomey parabola curve. Consequently, calling such composite material a "concrete" is somewhat of a misnomer and it more closely approximates a mortar. However, RPM is unique in attempting to optimize the entire grain size distribution of the composite matrix in order to reach maximum compaction (Chan and Chu 2004, Lee *et al.* 2005). When aggregate packing reaches the maximum bulk density, such a mixture can be deemed as most closely packed mix (Chang *et al.* 2001). And the total surface area of aggregate can be determined based on the obtained proportion of aggregate, which facilitates calculation of concrete coating paste by assigning a thickness and assuming a round aggregate. In Taiwan, the Hwang's research group tried to achieve the maximum dry loose density by simply packing all solid particles including coarse aggregates, sand and fly ash, to reduce the quantity of lubricating paste but keep desired workability for HPC (Hwang 2003, Hwang and Chen 2002) as well as RPM. As the category of material is less than three, the blend ratio (α, β) (Hwang and Chen 2002, Lee and Hwang 2002) of solid materials can be easily obtained by experimental work, otherwise the packing seems difficult especially as the material size finer than 1 μm. Therefore, it is necessary to adopt numerical approach to obtain proper packing order of all granular materials. Mixing water is done primarily to increase cement hydration and desired concrete workability. Cement content affects the cost of concrete and CO_2 emission, and the quantity and quality of cement hydration influences concrete stability and durability, so minimizing the paste amount is essential.

The chemistry of the carbonation process of concrete is conceptually straightforward. It is a through-solution process, where usually CO_2 from the atmosphere diffuses into the concrete. This diffusion results in a lowering of the pH, destabilizing all the cement hydration products (C-S-H, AFt, AFm, CH). These hydration products become unstable at different pH-values (Claus and Maria 2007).

The employment of alternative fuels (i.e., waste materials), which have exploitable energy, is in use for many years in the cement industry and it presents one way of conserving natural energy sources. Through this, thanks to the characteristics of clinker baking, waste materials are disposed of in a way that is safer for the environment. With that kind of approach, CO_2 emission, which would have been released by combustion at, e.g. an incinerating plant, is being reduced (Popović *et al.* 2002, 2003). Three sources of CO_2 emission, which are the result of $CaCO_3$ decarburizations from raw material, the reaction of fuel combustion during the process of baking in rotary furnace and equivalent of electric energy consumption, are present in cement production (Popović 2003).

One of the most significant activities today aims at a higher percentage of replacement of clinker in cement with secondary raw materials, with the possibility of improvement of cement characteristics and durability of concrete. With this kind of approach, the consumption of natural raw materials, thermal and electric energy, together with the reduction of CO_2 emission, are decreasing (Popović 2001, Vanderborght and Brodmann 2001, Horton 2001, Onuma 2000).

Concrete mixture design algorithm evolves according to different control conditions, mainly including ACI mixture algorithm (ACI Committee 318), and densified mixture design algorithm (DMDA). This study used the DMDA as the basic deduction formula, since an ideal grading curve is hard to achieve theoretical grading curve deducted in this study was based on H-G Kessler *et al.*'s "sphere model theory of analyzing arrangement structure of varied size spheres densely packed" (Kessler 1994).

2. Theoretical deduction and verification (Hsu *et al.* 2002, Chang *et al.* 2001, Zhan 1998, Kessler 1994, Kwan *et al.* 1999)

2.1 Symbol indexes

$a_{i,j}$: Percent retained on the j^{th} sieve of the i^{th} aggregate
D : Nominal maximum size of aggregate, mm
dw : weight of d size aggregate, g
dN : particle quantity of d size aggregate
$\Delta F_{i,j\to j+1}$: Percent retained on $j\to j+1$ sieve of the i^{th} aggregate
$f_{i,j\to j+1}$: Density of the i^{th} aggregate retained on $j\to j+1$ sieve (base of log$x$)
K_{SS} : Surface area constant of a sieve grade, 1/m
K_{SS_j} : Surface area constant of j sieve grade, 1/m
$Ksst$: Surface area per kg of blended aggregate mixture, m^2/kg
k_j : Theoretical percent retained on the j^{th} sieve by theoretical grading curve
M : Discrete amount
P : Content of aggregate smaller than size d particle
P_i : Volume percent of the i^{th} aggregate in blended aggregate mixture
P_i : Aggregate weight ratio of the i^{th} aggregate, %w
p_i : Aggregate volume ratio of the i^{th} aggregate, %v
U : Maximum unit weight, kg/m^3
R_{limit} : Allowable deviation limit
γ_i : Unit weight of the i^{th} aggregate, kg/m^3
S : Total surface area of unit volume concrete
t : Coating paste thickness, μm
Vp : Total volume of cement paste, m^3
Vv : Total volume of voids and pores, m^3

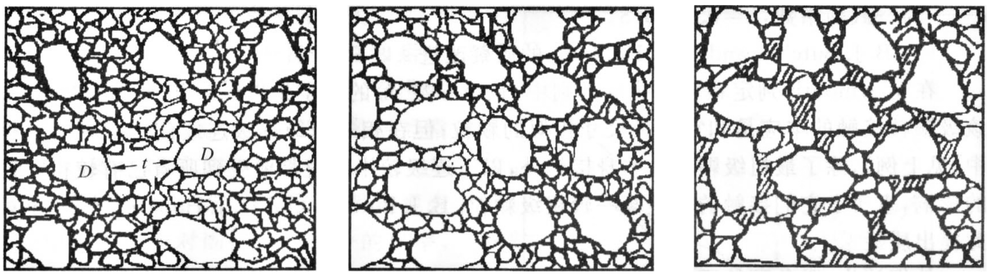
Va : Estimated volume of entrapped air, m^3
ξ : Slag ratio, slag/(cement + slag)

2.2 Theoretical estimation of aggregate blended ratio

In reality, as there are many types of aggregate, application of DMDA is time-consuming, so an effort to simplify mix design process is emphasized in this study by applying the simplest and most convenient Fuller's curve (as shown in Eq. (1) and Fig. 3) to make theoretical deduction.

$$P = \left(\frac{d}{D}\right)^h,\ h = \frac{1}{3} \sim \frac{1}{2} \tag{1}$$

As aggregate constitution close fits with Fuller's theoretical curve, its constitution will render more satisfactory compactness, to simplify the design process this study adopts the least square method to directly compute blended ratio of aggregate constitution. At first one need detailed aggregate information, suppose the concrete uses *n* type's aggregates, *m* grade particle sieve, use the



(a) $t > D_1$ (b) $t = D_1$ (c) $t < D_1$

Fig. 1 Particle disturbance status (Hsu *et al.* 2002)

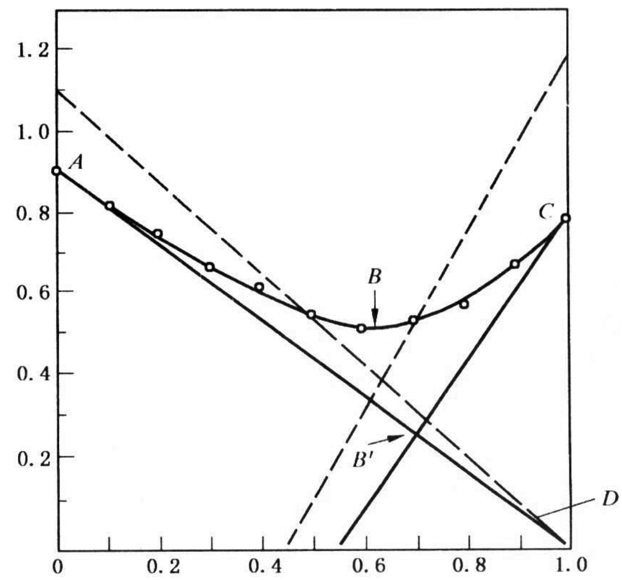


Fig. 2 Typical porosity

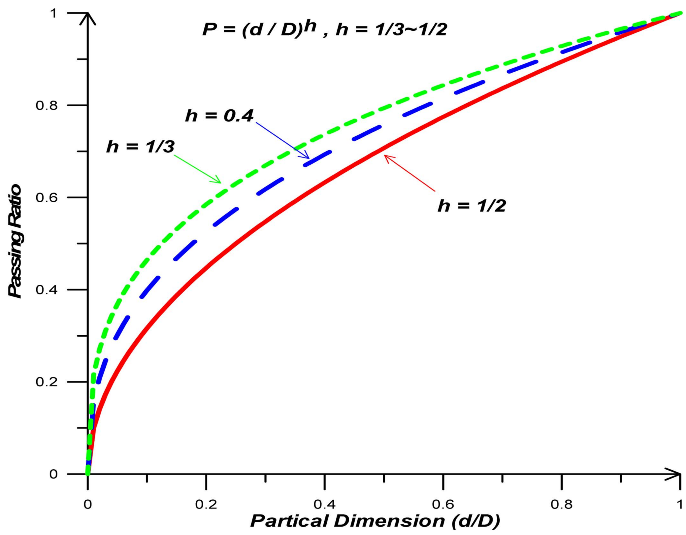


Fig. 3 Fuller's curve

least square method and discrete amount from theoretical curve (M) is expressed as Eq. (2)

$$M = R^2 = \sum_{j=1}^{m}\left(\sum_{i=1}^{n} P_i a_{i,j} - k_j\right)^2 = \sum_{j=1}^{m}\left(\sum_{i=1}^{n-1} P_i a_{i,j} + P_n a_{n,j} - k_j\right)^2 \tag{2}$$

From Eqs. (3) and (4) can be derived, substitute Eq. (4) to Eq. (2), and obtain Eq. (5)

$$\sum_{i=1}^{n} P_i = 100\% \tag{3}$$

$$P_n = 1 - \sum_{i=1}^{n-1} P_i \tag{4}$$

$$M = \sum_{j=1}^{m}\left(\sum_{i=1}^{n-1} P_i a_{i,j} + \left(1 - \sum_{i=1}^{n-1} P_i\right) a_{n,j} - k_j\right)^2 = \sum_{j=1}^{m}\left(\sum_{i=1}^{n-1} P_i (a_{i,j} - a_{n,j}) + a_{n,j} - k_j\right)^2 \tag{5}$$

Next, partial differentiate Eq. (5) by P_i, relationship of M increment (dM) and P_i can be obtained as Eq. (6)

$$dM = \sum_{i=1}^{n-1} \frac{\partial M}{\partial P_i} dP_i \tag{6}$$

Then obtain content of each aggregate P_i under minimum discrete state (Mmin), when M has extremism data set $dM = 0$ to satisfy Eq. (6) under each condition of dP_1, $dP_2 \cdots$, dP_n as in Eq. (7), there are totally n-1 conditional equations.

$$([a_{1,1} \quad a_{1,2} \quad \ldots \quad a_{1,m}] - [a_{n,1} \quad a_{n,2} \quad \ldots \quad a_{n,m}])$$

$$\times\left(\left(\begin{bmatrix} a_{1,1} & a_{2,1} & \cdots & a_{n-1,1} \\ a_{1,2} & a_{2,2} & \cdots & a_{n-1,2} \\ & & \vdots & \\ a_{1,m} & a_{2,m} & \cdots & a_{n-1,m} \end{bmatrix}-\begin{bmatrix} a_{n,1} & a_{n,1} & \cdots & a_{n,1} \\ a_{n,2} & a_{n,2} & \cdots & a_{n,2} \\ & & \vdots & \\ a_{n,m} & a_{n,m} & \cdots & a_{n,m} \end{bmatrix}\right)\begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_{n-1} \end{bmatrix}+\begin{bmatrix} a_{n,1} \\ a_{n,2} \\ \vdots \\ a_{n,m} \end{bmatrix}-\begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_m \end{bmatrix}\right)=0 \tag{7a}$$

$$([a_{2,1} \quad a_{2,2} \quad \cdots \quad a_{2,m}]-[a_{n,1} \quad a_{n,2} \quad \cdots \quad a_{n,m}])$$

$$\times\left(\left(\begin{bmatrix} a_{1,1} & a_{2,1} & \cdots & a_{n-1,1} \\ a_{1,2} & a_{2,2} & \cdots & a_{n-1,2} \\ & & \vdots & \\ a_{1,m} & a_{2,m} & \cdots & a_{n-1,m} \end{bmatrix}-\begin{bmatrix} a_{n,1} & a_{n,1} & \cdots & a_{n,1} \\ a_{n,2} & a_{n,2} & \cdots & a_{n,2} \\ & & \vdots & \\ a_{n,m} & a_{n,m} & \cdots & a_{n,m} \end{bmatrix}\right)\begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_{n-1} \end{bmatrix}+\begin{bmatrix} a_{n,1} \\ a_{n,2} \\ \vdots \\ a_{n,m} \end{bmatrix}-\begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_m \end{bmatrix}\right)=0 \tag{7b}$$

$$([a_{n-1,1} \quad a_{n-1,2} \quad \cdots \quad a_{n-1,m}]-[a_{n,1} \quad a_{n,2} \quad \cdots \quad a_{n,m}])$$

$$\times\left(\left(\begin{bmatrix} a_{1,1} & a_{2,1} & \cdots & a_{n-1,1} \\ a_{1,2} & a_{2,2} & \cdots & a_{n-1,2} \\ & & \vdots & \\ a_{1,m} & a_{2,m} & \cdots & a_{n-1,m} \end{bmatrix}-\begin{bmatrix} a_{n,1} & a_{n,1} & \cdots & a_{n,1} \\ a_{n,2} & a_{n,2} & \cdots & a_{n,2} \\ & & \vdots & \\ a_{n,m} & a_{n,m} & \cdots & a_{n,m} \end{bmatrix}\right)\begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_{n-1} \end{bmatrix}+\begin{bmatrix} a_{n,1} \\ a_{n,2} \\ \vdots \\ a_{n,m} \end{bmatrix}-\begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_m \end{bmatrix}\right)=0 \tag{7c}$$

Merge all conditional equations and simplify matrix Eqs. (8) to (9), using Eq. (10) to obtain optimum amount of each aggregate (dP_1, dP_2..., dP_{n-1}), as shown in formula Eq. (11).

$$[A]_{i,j}=\begin{bmatrix} a_{1,1} & a_{2,1} & \cdots & a_{n-1,1} \\ a_{1,2} & a_{2,2} & \cdots & a_{n-1,2} \\ & & \vdots & \\ a_{1,m} & a_{2,m} & & a_{n-1,m} \end{bmatrix},\ [A]_{n,j}=\begin{bmatrix} a_{n,1} \\ a_{n,2} \\ \vdots \\ a_{n,m} \end{bmatrix},\ [P]_{n-1}=\begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_{n-1} \end{bmatrix},\ [k]_m=\begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_m \end{bmatrix},\ [1_{n-1\times 1}]=\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \tag{8}$$

$$[([A]_{i,j}^T-[1_{n-1\times 1}][A]_{n,j}^T)([A]_{i,j}-[A]_{n,j}[1_{n-1\times 1}]^T)][P]_{n-1}=([A]_{i,j}^T-[1_{n-1\times 1}][A]_{n,j}^T)([k]_m-[A]_{n,j}) \tag{9}$$

$$\begin{aligned}[P]_{n-1}=&[([A]_{i,j}^T-[1_{n-1\times 1}][A]_{n,j}^T)([A]_{i,j}-[A]_{n,j}[1_{n-1\times 1}]^T)]^{-1} \\ &\times([A]_{i,j}^T-[1_{n-1\times 1}][A]_{n,j}^T)([k]_m-[A]_{n,j})\end{aligned} \tag{10}$$

Substitute P_1~P_n computed from Eqs. (10) and (4) to (2), and obtain M, divided by particle size grade m, so as to evaluate if the selected theoretical curve is applicable or not, as shown in Eq. (11)

$$\frac{M}{m}=\frac{1}{m}\sum_{j=1}^{m}\left(\sum_{i=1}^{n}P_i a_{i,j}-k_j\right)^2 \tag{11}$$

If it is less than or equal to R_{limit} (allowable deviation limit), then it indicates that the deviation falls in allowable range, the result is acceptable, alternatively, use different theoretical curves (theoretical curve) to calculate *kj*, and use Eq. (10) to recalculate P_1~P_n, and repeat check until appropriate grading curve obtained.

2.3 Estimation of concrete paste amount from coating paste thickness

Water/binder ratio is selected based on quality of coating paste of concrete, and determine paste amount needed in perspective of coating paste thickness (t) required to fill remaining space of packed aggregate and maintain appropriate flow property, as in Eq. (12).

$$V_p = V_v + S \cdot t \tag{12}$$

where V_p is paste amount needed, and S is the total surface area of aggregates.

This study suggests as concrete needs flowing behavior deem cement, fly ash, water-quenched slag, water and SP as the part of "paste", and use them to fill rest pores and voids of larger size aggregate and provide proper coating paste thickness, so that large particle size aggregate in concrete can "uniformly" flow without segregation, reaching the proper workability of "segregation resistance, high flowability, low pumping impedance".

2.3.1 Calculate void and porosity volume within densified packing aggregate, as in Eq. (13)

$$V_v = 1 - U_{\max} \cdot \left(\frac{p_1}{\gamma_1} + \frac{p_2}{\gamma_2}\right) \tag{13}$$

where p_1 is weight ratio of coarse aggregate in aggregate admixture, p_2 is weight ratio of fine aggregate in aggregate admixture, γ_1 is the unit weight of coarse aggregate, γ_2 is unit weight of fine aggregate, U_{max} is the maximum unit weight of aggregate.

2.3.2 Calculation of total surface area of coarse/fine aggregate in unit volume under densified packing condition

Mora *et al*. supposed spherical surface area have a linear relationship, so in this article the aggregate surface area was also estimated assuming spherical particles, so as to prevent complicated computation. Suppose logarithm function in terms of particle size of density function of the i^{th} aggregate between two adjacent sieves to be linear, one can obtain slope of density function of aggregate between two adjacent sieves (as in Eq. (14) and Fig. 2, from Eq. (14) deduce in series weight function of aggregate Eq. (15), aggregate particle function Eq. (16), total surface area of aggregate Eq. (17), finally integrate formula Eq. (17) and deduce the total surface area of particle size Eq. (18) (Hsu *et al*. 2002).

$$f_{i,j\to j+1} = \frac{\Delta F_{i,j\to j+1}}{\log x_{i,j+1} - \log x_{i,j}} \tag{14}$$

$$dw = U_i \cdot f_{i,j\to j+1} \cdot d(\log x) = U_i \cdot \frac{\Delta F_{i,j\to j+1}}{\log x_{j+1} - \log x_j} \cdot d(\log x) \tag{15}$$

$$dN = \frac{dw}{\gamma_i \cdot \frac{\pi x^3}{6}} = \frac{6U_i}{\gamma_i \cdot \pi} \cdot \frac{\Delta F_{i,j\to j+1}}{\log x_{i,j+1} - \log x_{i,j}} \cdot \frac{d(\log x)}{x^3} \tag{16}$$

$$dS = \pi x^2 dN = \frac{6U_i}{\gamma_i} \cdot \frac{\Delta F_{i,j\to j+1}}{\log x_{i,j+1} - \log x_{i,j}} \cdot \frac{d(\log x)}{x} \tag{17}$$

$$S_{i,j\to j+1} = \frac{U_i \cdot \Delta F_{i,j\to j+1}}{\gamma_i} \cdot \frac{6}{\ln 10(\log x_{i,j+1} - \log x_{i,j})} \int_{x_{i,j}}^{x_{i,j+1}} \frac{1}{x^2} dx \tag{18}$$

Except that Ui, γ_i and $\Delta F_{i,j\to j+1}$ are obtained from experiment, others do not change under "fixed standard sieve" condition, thus can be regarded as constants. Hence let it be Kss_j (constant surface area of specific sieve), and because all aggregates are separated by the same sieve, so this constant is often denoted by subscript of particle grade j, simplified to Eq. (19).

$$S_{i,j\to j+1} = \frac{U_i \cdot \Delta F_{i,j\to j+1} \cdot Kss_j}{\gamma_i} \tag{19}$$

Where

$$Kss_j(1/m) = Kss_{i,j\to j+1}(1/m) = \frac{6}{(\ln x_{j+1} - \ln x_j)} \left(\frac{1}{x_j} - \frac{1}{x_{j+1}} \right) \tag{20}$$

when $\Delta F_{i,j\to j+1}$ in Eq. (19) applies to one single material, it is the corresponding percent retained on sieve of i material, aggregate in this study are blended coarse and fine aggregates, thus the surface area is calculated as Eq. (21).

$$S = \sum S_i = \sum \frac{U_i \cdot \Delta F_{i,j\to j+1} \cdot Kss_j}{\gamma_i} \tag{21}$$

Then use previously deducted p_1, p_2 (%w), and measure unit weight of aggregate mixture as U (kg/m^3), then content of coarse and fine aggregates can be achieved as $U_k = U \cdot p_k$, substitute into Eq. (19) to get Eq. (22)

$$S = U \sum_{i=1}^{2} \sum_{j=1}^{10} \frac{p_i \cdot \Delta F_{i,j\to j+1} \cdot Kss_j}{\gamma_i} = U \cdot \sum_{j=1}^{10} \left[\left(\frac{p_1}{\gamma_1} a_{1,j} + \frac{p_2}{\gamma_2} a_{2,j} \right) Kss_j \right] = U \cdot Ksst \tag{22}$$

Where this can be further simplified to Eq. (23), indicating surface area of per kg weight of blended aggregate mixture, unit m^2/kg

$$Ksst = \sum_{j=1}^{10} \left[\left(\frac{p_1}{\gamma_1} a_{1,j} + \frac{p_2}{\gamma_2} a_{2,j} \right) Kss_j \right] \tag{23}$$

2.3.3 Calculate the content of each constitutional material of concrete

To solve the content of seven materials, w_{stone}, w_{sand}, w_{flyash}, w_{cement}, w_{slag}, w_{water} and w_{sp}, seven conditional equations are needed.

(1) at first, use Eqs. (13) and (19) to calculate Vv and S, then one can compute paste content, Vp, as Eq. (24). Vv can be regarded as constant value as coating thickness is small enough, substitute Eq. (22) into Eq. (24) to get Eq. (25)

$$V_p = V_v + S \cdot t \tag{24}$$

$$V_p = V_v + U \cdot Ksst \cdot t = V_v + (w_{stone} + w_{sand}) \cdot Ksst \cdot t \tag{25}$$

It is known that "cement paste" is composed of "cement, fly ash, quenched slag, water and *SP*", transpose to get Eq. (26)

$$Ksst\cdot t\cdot w_{stone}+Ksst\cdot t\cdot w_{sand}-\frac{1}{\gamma_{flyash}}w_{flyash}-\frac{1}{\gamma_{cement}}w_{cement}-\frac{1}{\gamma_{slag}}w_{slag}-\frac{1}{\gamma_{water}}\cdot w_{water}=-V_v \quad (26)$$

(2) From concrete subtract "cement paste", V_p, and estimate air content *Va*, usually 2%, that is coarse aggregate volume, fine aggregate volume, transpose to get Eq. (27)

$$\left(Ksst\cdot t+\frac{1}{\gamma_{stone}}\right)w_{stone}+\left(Ksst\cdot t+\frac{1}{\gamma_{sand}}\right)w_{sand}=1-V_a-V_v \quad (27)$$

(3) From experiment get weight ratio of coarse aggregate, fine aggregate and fly ash, that is

$$\frac{w_{stone}}{p_1}=\frac{w_{sand}}{p_2}=\frac{w_{flyash}}{p_3} \quad (28)$$

Therefore revise Eq. (27) into Eqs. (29) and (30)

$$w_{stone}-\frac{p_1}{p_2}w_{sand}=0 \quad (29)$$

$$w_{stone}-\frac{p_1}{p_3}w_{flyash}=0 \quad (30)$$

(4) Choose water/binder ratio (*w/b*=water/(cement + fly ash + slag)) according to cement paste quality from existing information, get Eq. (31)

$$\frac{w}{b}w_{flyash}+\frac{w}{b}w_{cement}+\frac{w}{b}w_{slag}-w_{water}=0 \quad (31)$$

(5) Assign weight ratio of quenched slag replace cement (ξ = slag/(cement + slag)), get Eq. (32)

$$\xi\cdot w_{cement}+(\xi-1)w_{slag}=0 \quad (32)$$

(6) Convert 6 conditional Eqs. (26)-(32) into matrix Eq. (33)

$$\begin{bmatrix} Ksst\cdot t & Ksst\cdot t & -\frac{1}{\gamma_{flyash}} & -\frac{1}{\gamma_{cement}} & -\frac{1}{\gamma_{slag}} & -\frac{1}{\gamma_{water}} \\ Ksst\cdot t+\frac{1}{\gamma_{stone}} & Ksst\cdot t+\frac{1}{\gamma_{sand}} & 0 & 0 & 0 & 0 \\ 1 & -\frac{p_1}{p_2} & 0 & 0 & 0 & 0 \\ 1 & 0 & -\frac{p_1}{p_3} & 0 & 0 & 0 \\ 0 & 0 & \frac{w}{b} & \frac{w}{b} & \frac{w}{b} & -1 \\ 0 & 0 & 0 & \xi & \xi-1 & 0 \end{bmatrix}\begin{bmatrix} w_{stone} \\ w_{sand} \\ w_{flyash} \\ w_{cement} \\ w_{slag} \\ w_{water} \end{bmatrix}=\begin{bmatrix} -V_v \\ 1-V_a-V_v \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (33)$$

Solve Eq. (33) to obtain content of each constituent as in Eq. (34)

$$\begin{bmatrix} w_{stone} \\ w_{sand} \\ w_{flyash} \\ w_{cement} \\ w_{slag} \\ w_{water} \end{bmatrix} = \begin{bmatrix} Kss t\cdot t & Kss t\cdot t & -\frac{1}{\gamma_{flyash}} & -\frac{1}{\gamma_{cement}} & -\frac{1}{\gamma_{slag}} & -\frac{1}{\gamma_{water}} \\ Kss t\cdot t+\frac{1}{\gamma_{stone}} & Kss t\cdot t+\frac{1}{\gamma_{sand}} & 0 & 0 & 0 & 0 \\ 1 & -\frac{p_1}{p_2} & 0 & 0 & 0 & 0 \\ 1 & 0 & -\frac{p_1}{p_3} & 0 & 0 & 0 \\ 0 & 0 & \frac{w}{b} & \frac{w}{b} & \frac{w}{b} & -1 \\ 0 & 0 & 0 & \xi & \xi-1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -V_v \\ 1-V_a-V_v \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (34)$$

(7) Densified concrete coupled with sufficient *SP*, approximates more to high performance concrete (HPC) with high flowability. The SP is mainly used to control the workability under fixed water and *SP* content for the purpose of volume stability and durability. This study chose HPC1000, a kind of naphthenic-formaldehyde based SP, and the recommended dosage is in Eq. (35) (Hwang 2009).

$$SP(\%) = 3.8329+2.7156\times 10^{-2}\cdot W_{water}-1.8038\times 10^{-4}\cdot W_{water}^{2} \quad (35)$$

3. Example of mixture calculation by Fuller's grading curve

3.1 Blended ratio of aggregate

3.1.1 Standard calculation procedure

1. Define aggregate data

Take three basic constituents in concrete, coarse aggregate, fine aggregate and fly ash as examples, designated as No. 1, No. 2 and No. 3 aggregate, respectively; particle grading is as per standard sieve system, in ascending order, (1) finer than #100, (2) #100, (3) #50, …, (10) 3/8" is No. 1, No. 2, No.3, …, 10 grade, respectively. Get sieve analysis data of coarse and fine aggregates; since fly ash particle size is normally finer than #100, thus directly set $a_{3,1} = 1$, $a_{3,2\sim10} = 0$, and specific gravity of coarse aggregate, fine aggregate, fly ash is γ_1, γ_2, γ_3, respectively. It should be noted that when specific gravity of each grade of aggregate is congruent ($\gamma_1 = \gamma_2 = \gamma_3 = \ldots = \gamma_n$), the weight ratio is the same as volume ratio, as shown in Table 2.

2. Use Fuller's curve to compute ideal percent retained on specific sieve of blended mixture, separately.

set $h = 0.5$, $D = 2/3" = 37.5$ mm, substitute in Eq. (1) to get ideal percent retained on specific sieve, e.g. percent retained on 3/4" sieve (19.0 mm) is $(\sqrt{37.5}-\sqrt{19.0})/\sqrt{37.5} = 28.82\%$. The ideal percent retained on specific sieve is shown in Table 2.

3. Build the computation matrix, as in Eq. (7), here $m = 10$, $n = 3$.

4. Substitute each matrix into Eq. (10), one can obtain P_1 and P_2, then use Eq. (4) to obtain P_3.

Table 1 Concrete mix design algorithm

Mix method	Principle	Shortcoming	Recommended improvement
ACI mix	• Formulated by ACI • Simple table checking method was induced from experiment statistics	• Fresh concrete, curing property is hard to meet special requirement of high flowability, high strength • Obsolete design, limited concrete performance	• Can still be used in concrete for general requirement, yet has to be improved for special requirement
Surface area theory mix	• In 1988, first of the time lubrication paste thickness was tried to determine concrete paste amount $Vp = Vv + St$	• Not good workability	• Due to common availability of pozzolan material, high performance reducer, workability may be greatly improved
Densified mix	• To secure strength and durability, set up the most close packing of aggregate $Vp = Vn$, $n \geq 1$	• Slower flowability • Higher cost	• $Vp = Vn$ is an empirical rule, hard to explain concrete property and mechanism of various mix ratio • Use formulae to simply obtain most close packing, avoid much waste of resources by repeated tests

Table 2 Aggregate numbering, sieve analysis and ideal sieve intercepting percent

Screen size	Less than #100	#100	#50	#30	#16	#8	#4	$\frac{3}{8}''$	$\frac{1}{2}''$	$\frac{3}{4}''$
Sieve size (mm)	<0.15	0.15	0.30	0.60	1.18	2.36	4.75	9.5	12.5	19.0
Serial number of material	1	2	3	4	5	6	7	8	9	10
#1 coarse aggregate	$a_{1,1}$	$a_{1,2}$	$a_{1,3}$	$a_{1,4}$	$a_{1,5}$	$a_{1,6}$	$a_{1,7}$	$a_{1,8}$	$a_{1,9}$	$a_{1,10}$
#2 fine aggregate	$a_{2,1}$	$a_{2,2}$	$a_{2,3}$	$a_{2,4}$	$a_{2,5}$	$a_{2,6}$	$a_{2,7}$	$a_{2,8}$	$a_{2,9}$	$a_{2,10}$
#3 fly ash	$a_{3,1}$	0	0	0	0	0	0	0	0	0
Percent retained of ideal sieve	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}

5. Deviation check: substitute result to Eq. (11) to evaluate its properness, if deviation is too large, then use other ideal curves (such as gap grading) to reevaluate.

3.1.2 Tips

The blended aggregate ratio computed in this study varied with experimental result, primarily due to only few types of aggregates have limited the accuracy for approaching ideal curve. If aggregate types used are greater than a critical value and after assign weight ratio to the significant aggregate, a better result will be achieved. One may use different ideal curves to obtain corresponding mix proportion, and choose some of them to precede dry ridding test, obtain the maximum unit weight to determine the appropriate of grading. This approach may not be the absolute true value of blended aggregate, but it greatly shortens testing cycle and meets construction requirement.

3.2 Evaluating optimum coating paste thickness

3.2.1 Computation procedure

1. Set up aggregate grading data

Set coarse aggregate as No. 1 aggregate, sand as No. 2 aggregate; #100~#50 sieve as No. 1 grade, #50~#30 as the No. 2 grade, #30~#16 as No. 3 grade, #16~#8 as No. 4 grade, #8~#4 as No. 5 grade, #4~3/8" as No. 6 grade, 3/8~1/2" as No. 7 grade, 1/2"~3/4" as No. 8 grade, and pro rata, set up data of percent retained on specific sieve of each aggregate $a_{i,j}$(where, i denotes aggregate number, j denotes grade). Verified by experimental result of literature (Tsai 1997), percent retained on specific sieve of each aggregate after sieve analysis are shown in Table 3.

2. Specific surface area of mixed aggregate (*Ksst*)

From definition of Eq. (19), one can deduce Eq. (1) to compute Kss_j of each grade as shown in Table 4; result of EC-32 optional aggregate dry ridding test: (1) $\alpha = w_{flyash}/(w_{sand} + w_{flyash}) = 0.14$ (2) $\beta = (w_{sand} + w_{flyash})/(w_{stone} + w_{flyash}) = 0.6$ and (3) $U_{\max} = 2056.88\ \text{kg/m}^3$. From (1) and (2), deduce inversely specific weight ratio of each aggregate

$$p_{stone} : p_{sand} : p_{flyash} = (1-\beta) : (1-\alpha)\cdot\beta : \alpha\cdot\beta = 0.4 : 0.516 : 0.084 \tag{36}$$

As (1) coarse aggregate is No. 1 aggregate, (2) sand is No. 2 aggregate and associating the above result, obtain $p_1 = 0.4$ and $p_2 = 0.516$, substitute in Eq. (20) to estimate *Ksst*, and obtain 1.99437 m^2/kg; or, in this blended ratio, total surface area of a unit weight (1 kg) blended aggregate is 1.99437

Table 3 Sieve intercepting percent of EC-32 aggregate

Sieve	#100	#50	#30	#16	#8	#4	$\frac{3''}{8}$	$\frac{1''}{2}$
No material	1	2	3	4	5	6	7	8
	$a_{1,1}$	$a_{1,2}$	$a_{1,3}$	$a_{1,4}$	$a_{1,5}$	$a_{1,6}$	$a_{1,7}$	$a_{1,8}$
No 1 (coarse aggregate)	0	0	0	0.0121	0.0407	0.554	0.372	0.0212
Material No 2	$a_{2,1}$	$a_{2,2}$	$a_{2,3}$	$a_{2,4}$	$a_{2,5}$	$a_{2,6}$	$a_{2,7}$	$a_{2,8}$
(fine aggregate)	0.0967	0.2511	0.2882	0.2502	0.0878	0.0086	0	0

Table 4 Computational results of EC-32 mix series *Ksst*

Sieve	No	Particle ratio		γ_1 = 2650 kg/m^3, p_1 = 0.4 γ_2 = 2650 kg/m^3, p_1 = 0.516			*Kss,j*	*Ksst*
		#1	#2				(1/m)	(m^2/kg)
(1)	(2)	(3)	(4)	(5)=(3) × p_1/γ_1	(6)=(4) × p_2/γ_2	(7)=(5)+(6)	(8)	(9)=(7)*(8)
#100	1	0	0.0967	0.0000*E*+00	1.8829*E*-05	1.8829*E*-05	28853.901	0.54329
#50	2	0	0.2511	0.0000*E*+00	4.8893*E*-05	4.8893*E*-05	14426.950	0.70538
#30	3	0	0.2882	0.0000*E*+00	5.6117*E*-05	5.6117*E*-05	7267.430	0.40783
#16	4	0.0121	0.2502	1.8264*E*-06	4.8718*E*-05	5.0545*E*-05	3667.869	0.18539
#8	5	0.0407	0.0878	6.1434*E*-06	1.7096*E*-05	2.3240*E*-05	1828.801	0.04250
#4	6	0.554	0.0086	8.3623*E*-05	1.6746*E*-06	8.5297*E*-05	911.176	0.07772
3"/8	7	0.372	0	5.6151*E*-05	0.0000*E*+00	5.6151*E*-05	552.327	0.03101
1"/2	8	0.0212	0	3.2000*E*-06	0.0000*E*+00	3.2000*E*-06	392.182	0.00125
Σ(9) = Ksst = 1.99437 m^2/kg								

m^2. Computation process is shown in Table 4.

3. Total surface area of aggregate (*S*)

Take EC-32 mix as example, total weight of blended aggregate is coarse aggregate 766 + fine aggregate 988 = 1754 kg/m^3.

$$S = Ksst \times w_{partical} = 1.99437\ m^2/kg \times 1754\ kg/m^3 = 3498.16\ m^2/m^3 \quad (37)$$

4. Cement paste coating volume (V_l)

It is known that mixed aggregate U_{max} = 2056.88 kg/m^3, coarse aggregate: 40% and fine aggregate: 51.6%; so sand and stone volume can be calculated as

$$(1)\ V_{stone} = V_1 = U \cdot p_1/\gamma_1 = 0.3105\ m^3/m^3 \quad (38)$$

$$(2)\ V_{sand} = V_2 = U \cdot p_2/\gamma_2 = 0.4005\ m^3/m^3 \quad (39)$$

Porosity and void volume is the rest volume

$$V_v = 1 - V_{stone} - V_{sand} = 0.2890\ m^3/m^3 \quad (40)$$

EC-32 mix and its property: (1) fly ash: 161 kg/m^3, Gs = 2.18, (2) cement: 318 kg/m^3, GS = 3.15, (3) quenched slag: 16.7 kg/m^3, Gs = 2.88, (4) water: 140 kg/m^3 and (5) SP: 18.4 kg/m^3, Gs = 1.18. Then get cement paste volume

$$Vp = \sum w_i/(Gsi \cdot \gamma_w) = 0.3390\ m^3/m^3 \quad (41)$$

Subtracting filling void volume V_v, it is coating paste amount

$$Vc = Vp - Vv = 0.339 - 0.289 = 0.050\ m^3/m^3 \quad (42)$$

5. Cement paste coating thickness (*t*)

Divide cement coating paste volume V_l by total surface area of mixed aggregate (*S*) and get paste thickness

$$t = Vc/S = \frac{0.050\ m^3/m^3}{3498.16\ m^2/m^3} = 14.3\ \mu m \quad (43)$$

6. Repeated calculating coating paste thickness of each mix

Repeating above computation procedure one can obtain coating paste thickness of different mix, as shown in Table 5.

Table 5 Summary of computational coating paste thickness of EC-32 mix series

n -	w/b -	n -	$Ksst$ m^2/kg	$w_{partical}$ kg/m^3	S m^2/m^3	V_p m^3/m^3	V_v m^3/m^3	V_c m^3/m^3	t μm	w/c -
(1)	(2)	(3)	(4)	(5)	(6) =(4)*(5)	(7)	(8)	(9) =(7)-(8)	(10) =(9)/(6)	(11)
1.2		1.20		1754.00	3498.16	0.339		0.050	14.3	0.473
1.4		1.40		1649.00	3288.74	0.379		0.090	27.3	0.439
1.6	0.32	1.60	1.9944	1543.00	3077.34	0.419	0.289	0.130	42.1	0.413
1.8		1.80		1438.00	2867.93	0.458		0.169	59.1	0.395
2.0		2.00		1331.00	2654.53	0.498		0.209	78.7	0.382

3.2.2 Tips

As DMDA shall meet the requirement of HPC (Chang *et al.* 2001, Hwang 2009), so all computational mixture proportion have to be verified, including $W/C = (w_{water} + w_{sp})/w_{cement} \geq 0.42$, to ensure cement is fully hydrated to avoid autogenously shrinkage and paste N shall be $1.2 \leq N \leq V$ $(w_{cement+slag+water+sp})/Vv \leq 1.6$, to prevent from excessive paste leading to segregation and bleeding, or insufficient paste leading to poor workability.

4. Conclusions

Based on densified mix structure, partial correction was made so that mixing design process becomes simpler and more rational, and coating paste thickness was replacing previous empirical paste amount coefficient "*n*", more fitting industry demand in reality.

Ideal grading curve was used to estimate aggregate constitution ratio, coupled with more kinds of aggregate, the confidence would be greatly increased till replacing conventional experiment method, resulted in much less test cycle and cost, and using the maximum unit weight as the selection criterion.

Advantage of ideal curve algorithm is simple and rapid computation, yet it has shortcoming, e.g. its result can only approximate true value, and subject to number of aggregate types, if it is lower than critical value, then the result will be non-representative. There are many available ideal curves, but various computational results have to be compared, if the shortcoming is improved, the application value of this method will be greatly elevated.

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