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Theoretical impact of Kelvin's theory for vibration of double walled carbon nanotubes

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Abstract. In this article, free vibration of double-walled carbon nanotubes (DWNT) based on nonlocal Kelvin's model have been investigated. For this purpose, a nonlocal Kelvin's model is established to observe the small scale effect. The wave propagation is employed to frame the governing equations as eigenvalue system. The influence of nonlocal parameter subjected to different end supports has been overtly examined. The new set of inner and outer tubes radii investigated in detail against aspect ratio. The influence of boundary conditions via nonlocal parameter is shown graphically. Due to small scale effect fundamental frequency ratio decreases as length to diameter ratio increases. Small scale effect becomes negligible on all end supports for the higher values of aspect ratio. With the smaller inner tube radius double-walled CNT behaves more sensitive towards nonlocal parameter. The results generated furnish the evidence regarding applicability of nonlocal model and also verified by earlier published literature.

Keywords: free vibration; nonlocal material; double-walled CNTs; Kelvin's model; WPA

1. Introduction

With a vast area of potential innovation, however CNTs demands more understanding to investigate its mechanical properties. Carbon nanotubes (CNTs) is discovered by Iijima (1991), that may be used in a variety of fields like material reinforcement, aerospace, medicine, defense and microelectronic devices (Sosa *et al*. 2014, Soldano 2015, Fakhrabadi *et al*. 2015, Bouadi *et al*. 2018). The rapid development of nano science and nano technology is phenomenal as echoed with an increase of its application in scientific research. Owing the striking mechanical properties through the cylindrical mechanism, the carbon nanotubes hold purposeful role in conveying fluid and gas. Free vibration analysis of CNTs have been influential aspect in dynamical science for the last one decade. Well known two main classes of models used to analyze the theoretical aspects of CNTs have been atomic model and other is continuum model. The classical molecular dynamics (MD) has shown to exceed those of other techniques such as tight-binding molecular dynamics and ab initio method included in class of atomic modeling (Iijima *et al*. 1996, Yakobson *et al*. 1997, Hernandez *et al*. 1998, Sanchez-Portal *et al*. 1999, Qian *et al*. 2002). Vibration characteristics are investigated using thin shell theory by Yakobson *et al*. (1996), beam theory by Wang *et al*. (2006) and nonlocal beam theory (Zemri *et al*. 2015, Youcef *et al*. 2018). An eminent study found in based upon ring theory by Vodenitcharova and Zhang (2003) whereas theories of continuum models developed by Li and Chou (2003) in literature. The main reason continuum mechanics (Yoon *et al*. 2003, Fu *et al*. 2006, Ansari *et al*. 2011) turned noticeable tool is its computational capability to generate results of large range system in nanometer range.

Therefore, scientific community now propose to apply nonlocal continuum models to investigate nano-structured materials (Sudak 2003, Wang *et al*. 2006a, b, Pradhan and Phadikar 2009, Ansari *et al*. 2010, Hao *et al*. 2010, Amara *et al*. 2010, Shen and Zhang 2010). Flügge shell theory takes promising place to generate remarkably accurate developments to examine the CNTs. Donnell (1996) and Flügge (1962) have been two substantial shell theories practiced extensively in study of static and dynamic characteristics of CNTs. Usuki and Yogo (2009) formed beam equations based on Flugge shell theory with the help of refined model. Further, Wang and Zhang (2007) examined the bending and torsional stiffness of single-walled CNT applying the Flügge shell equations. They presented three-dimensional model of single-walled CNT in their work with effect of thickness. Rouhi *et al.* (2015) executed the axial buckling of double-walled CNT subject to various layer-wise conditions by using Rayleigh-Ritz based upon nonlocal Flügge shell theory. Their study showed that the number of different layer-wise boundary conditions dominates the choice of values for nonlocal parameter. In another paper, Natsuki *et al*. (2006) carried

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out the vibration analysis of nested CNTs in elastic matrix. Flügge shell theory again had been engaged to establish administrative shell equations while proposed method was wave propagation. Natsuki *et al*. (2007) investigated single and double-walled CNTs filled with fluids by adopting wave propagation approach. Flügge shell theory was proposed to form governing equations of motion for CNTs. The first ever work presented on use of nonlocal elasticity was by Peddieson *et al*. (2003).

Sharma *et al*. (2019) studied the functionally graded material using sigmoid law distribution under hygrothermal effect. The Eigen frequencies are investigated in detail. Frequency spectra for aspect ratios have been depicted according to various edge conditions. Hussain and Naeem (2017) examined the frequencies of armchair tubes using Flügge's shell model. The effect of length and thickness-to-radius ratios against fundamental natural frequency with different indices of armchair tube was investigated. Kolahchi and Cheraghbak (2017) studied with the nonlocal dynamic buckling analysis of embedded microplates reinforced by single-walled carbon nanotubes (SWCNTs). The material properties of structure are assumed viscoelastic based on Kelvin-Voigt model. Agglomeration effects are considered based on Mori-Tanaka approach. The elastic medium is simulated by orthotropic visco-Pasternak medium. Hussain *et al*. (2017) demonstrated an overview of Donnell theory for the frequency characteristics of two types of SWCNTs. Fundamental frequencies with different parameters have been investigated with wave propagation approach. Kolahchi (2017) investigated the bending, buckling and buckling of embedded nano-sandwich plates based on refined zigzag theory (RZT), sinusoidal shear deformation theory (SSDT), first order shear deformation theory (FSDT) and classical plate theory (CPT). In order to present a realistic model, the material properties of system are assumed viscoelastic using Kelvin-Voigt model. Hussain and Naeem (2018a) used Donnell's shell model to calculate the dimensionless frequencies for two types of single-walled carbon nanotubes. The frequency influence was observed with different parameters. Bilouei *et al*. (2016) used as concrete the most usable material in construction industry it's been required to improve its quality. Nowadays, nanotechnology offers the possibility of great advances in construction. For the first time, the nonlinear buckling of straight concrete columns armed with single- walled carbon nanotubes (SWCNTs) resting on foundation is investigated in the present study. The column is modeled with Euler-Bernoulli beam theory. Fatahi-Vajari *et al*. (2019) studied the vibration of single-walled carbon nanotubes based on Galerkin's and homotopy method. This work analyses the nonlinear coupled axial-torsional vibration of single-walled carbon nanotubes (SWCNTs) based on numerical methods. Two-second order partial differential equations that govern the nonlinear coupled axial-torsional vibration for such nanotube are derived. Kolahchi *et al*. (2016a) concerned with thedynamic stability response of an embedded piezoelectric nanoplate made of polyvinylidene fluoride (PVDF). In order to present a realistic model, the material properties of nanoplate are assumed viscoelastic using Kelvin-Voigt model. The visco-nanoplate is surrounded by viscoelastic medium which is simulated by orthotropic visco-Pasternak foundation. The PVDF visco-nanoplate is subjected to an applied voltage in the thickness direction. Asghar *et al*. (2019) conducted the vibration of nonlocal effect for double-walled carbon nanotubes using wave propagation approach. Many material parameters are varied for the exact frequencies of many indices of double-walled carbon nanotubes. Arani and Kolahchi (2016) used a concrete material in construction industry it's been required to improve its quality. Nowadays, nanotechnology offers the possibility of great advances in construction. For the first time, the nonlinear buckling of straight concrete columns armed with single-walled carbon nanotubes (SWCNTs) resting on foundation is investigated in the present study. The column is modeled with EulerBernoulli and Timoshenko beam theories. The characteristics of the equivalent composite being determined using mixture rule. The foundation around the column is simulated with spring and shear layer. Ansari and Rouhi (2013) summarized the effect of small scale, geometrical parameter and layer-wise end conditions of double-walled CNT by adopting frequencies model (FSM). They depicted that the continuum model considering the nonlocal effect compels the short double-walled CNT more flexible. Furthermore, Rouhi *et al*. (2015) investigated the vibration analysis of the multi-walled CNT by developing nonlocal FSM and presented the frequency spectrum against layer wise boundary conditions.

Zamanian *et al*. (2017) considered the use of nanotechnology materials and applications in the construction industry. However, the nonlinear buckling of an embedded straight concrete columns reinforced with silicon dioxide (SiO_2) nanoparticles is investigated in the present study. The column is simulated mathematically with Euler-Bernoulli and Timoshenko beam models. Agglomeration effects and the characteristics of the equivalent composite are determined using Mori-Tanaka approach. The foundation around the column is simulated with spring and shear layer. Moreover, Benguediab *et al*. (2014) explored the mechanical buckling features of zigzag double-walled CNT. A comprehensive research presented by Brischotto (2015) to analyze the vibration characteristic of double-walled CNT by considering shell continuum model. The findings of article were evolved around effects of van der Waals interaction in terms of frequency ratio. Avcar (2015) examined the separate and combined effects of rotary inertia, shear deformation and material non-homogeneity (MNH) on the values of natural frequencies of the simply supported beam. MNH is characterized considering the parabolic variations of the Young's modulus and density along the thickness direction of the beam, while the value of Poisson's ratio is assumed to remain constant. Chemi *et al*. (2018) determined the nonlocal critical buckling loads of chiral double-walled carbon nanotubes embedded in an elastic medium, the nonlocal Timoshenko beam theory is implemented. The solution for the nonlocal critical buckling loads is obtained using governing equations of the nonlocal theory. The effect of the elastic medium, the buckling mode number, chirality, and aspect ratio on the nonlocal critical buckling loads of double-walled carbon nanotubes are studied and discussed. Xu *et al*. (2008) modeled the nested tubes of double-walled CNT as separate elastic beam. Their work revealed that double-walled CNT had no change for a particular invariable frequency subject to distinct edge conditions. Ke *et al*. (2009) investigated free nonlinear vibrations of double-walled CNT and applied differential quadrature technique to derive frequency equations. Kolahchi *et al*. (2017a, b) studied the dynamic buckling of sandwich nano plate (SNP) subjected to harmonic compressive load based on nonlocal elasticity theory. The material properties of each layer of SNP are supposed to be viscoelastic based on Kelvin-Voigt model. In order to mathematical modeling of SNP, a novel formulation, refined Zigzag theory (RZT) is developed. Furthermore, the surrounding elastic medium is simulated by visco-orthotropic Pasternak foundation model in which damping, normal and transverse shear loads are taken into account. Recently, Hussain and Naeem (2019a, b) performed the vibration of SWCNTs based on wave propagation approach and Galerkin's method.

In recent studies DWNT have been intensively attracted as that of single- walled CNT due to its effectively applicable thermal, mechanical and electronic features. Afterwards, Khosrozadeh and Hajabasi (2012) carried out vibration analysis of DWNT subject to nonlinear van der Waals forces. Aimed focus on values of nonlocal parameter, length of tube and surrounding elastic medium. Rouhi *et al*. (2013) adapted new numerical approach with nonlocal Donnell shell theory to inquire the small-scale effect on DWNT depending on boundary conditions. Motezaker and Eyvazian (2020) deals with the buckling and optimization of a nanocomposite beam. The agglomeration of nanoparticles was assumed by Mori-Tanaka model. The harmony search optimization algorithm is adaptively improved using two adjusted processes based on dynamic parameters. The governing equations were derived by Timoshenko beam model by energy method. The optimum conditions of the nanocomposite beam- based proposed AIHS are compared with several existing harmony search algorithms. Hu *et al*. (2008) reported a study on the transverse and torsion waves based on nonlocal shell model for single-walled and double-walled CNTs Gafour *et al*. (2020) focused the behavior of non-local shear deformation beam theory for the vibration of functionally graded (FG) nanobeams with porosities that may occur inside the functionally graded materials (FG) during their fabrication, using the nonlocal differential constitutive relations of Eringen. For this purpose, the developed theory accounts for the higher-order variation of transverse shear strain through the depth of the nanobeam. Kolahchi and Bidgoli (2016) presented a model for dynamic instability of embedded single-walled carbon nanotubes (SWCNTs). SWCNTs are modeled by the sinusoidal shear deformation beam theory (SSDBT). The modified couple stress theory (MCST) is considered in order to capture the size effects. The surrounding elastic medium is described by a visco-Pasternak foundation model, which accounts for normal, transverse shear, and damping loads. The motion equations are derived based on Hamilton's principle. Akgöz and Civalek (2015) developed a new non-classical sinusoidal plate model on the basis of modified strain gradient theory. This model takes into account the effects of shear deformation without any shear correction factors and also can capture the size effects due to additional material length scale parameters. Madani *et al*. (2016) presented vibration analysis of embedded functionally graded (FG)-carbon nanotubes (CNT) - reinforced piezoelectric cylindrical shell subjected to uniform and non-uniform temperature distributions. The structure is subjected to an applied voltage in thickness direction which operates in control of vibration behavior of system. Mehar *et al*. (2017a, b, c, d) studied the freqncy response of FG CNT and reinforced CNT using the simple deformation theory, finite element modeling and Mori-Tanaka scheme. They investigated a new frequency phenomenon with the combination of Lagrange strain, Green-Lagrange, for double curved and curved panel of FG and reinforced FG CNT. The charactrictics of sandwich and grades CNT Bwas found with labeling the temperarure environ. The thermoelastic frequency of single shaollow panel was determined using Mori-Tanake formaulation. The research of these authors has opened a new frequency spectrum for other material researchers. Kolahchi *et al*. (2016b) investigated the nonlinear dynamic stability analysis of embedded temperature-dependent viscoelastic plates reinforced by single-walled carbon nanotubes (SWCNTs). The equivalent material properties of nanocomposite are estimated based on the rule of mixture. For the carbon-nanotube reinforced composite (CNTRC) visco-plate, both cases of uniform distribution (UD) and functionally graded (FG) distribution patterns of SWCNT reinforcements are considered. The surrounding elastic medium is modeled by orthotropic temperature-dependent elastomeric medium. The visco-elastic properties of plate are assumed based on Kelvin-Voigt theory. Akgöz and Civalek (2011) proposed the higher-order continuum theories for the buckling analysis of single walled carbon nanotubes (SWCNT). Modified strain gradient elasticity and modified couple stress theories are proposed. The governing equations for buckling and related boundary conditions are obtained in conjunctions with the strain gradient elasticity and variational principle. Batou *et al*. (2019) studied the wave propagations in sigmoid functionally graded (S-FG) plates using new Higher Shear Deformation Theory (HSDT) based on two-dimensional (2D) elasticity theory. The current higher order theory has only four unknowns, which mean that few numbers of unknowns, compared with first shear deformations and others higher shear deformations theories and without needing shear corrector. Motezaker and Kolahchi (2017a) investigated the Seismic response of the concrete column covered by nanofiber reinforced polymer (NFRP) layer. The concrete column is studied in this paper. The column is modeled using sinusoidal shear deformation beam theory (SSDT). Mori-Tanaka model is used for obtaining the effective material properties of the NFRP layer considering agglomeration effects. Using the nonlinear strain-displacement relations, stress-strain relations and Hamilton's principle, the motion equations are derived. Mehar and Panda (2016a, b, 2018a) computed the vibration behavior, bending and dynamic response of FG reinforced

CNT using shear deformation theory and finite element method. For the sake of generality, the mathematical model was presented with the mixture of Green Lagrange method. The convergence of these methodologies has been checked for the variety of results. The composite paltes with differenct greded was investigated with isotropic and core phase. Motezaker and Kolahchi (2017b) presented the dynamic analysis of a concrete pipes armed with Silica (SiO_2) nanoparticles subjected to earthquake load. The structure is modeled with first order shear deformation theory (FSDT) of cylindrical shells. Mori-Tanaka approach is applied for obtaining the equivalent material properties of the structure considering agglomeration effects. Prominent computational competence and accuracy makes nonlocal models an attractive choice for further advancements in field. The nonlocal elasticity introduced by Eringen (1983, 2002) becomes a turning point as small scale effect was inculcated into fundamental equations as simply material parameter. Kolahchi *et al*. (2017a, b) focused with general wave propagation in a piezoelectric sandwich plate. The core is consisted of several viscoelastic nanocomposite layers subjected to magnetic field and is integrated with viscoelastic piezoelectric layers subjected to electric field. The piezoelectric layers play the role of actuator and sensor at the top and bottom of the core, respectively. Mehar and Panda (2018b) investigated the curved shell and CNT vibration with thermal environment using higher order deformation theory. This CNT was mixed with different configurations of the layers. The results have been verified with the earlier investigations. Motezaker *et al*. (2020) presented the present research post-buckling of a cut out plate reinforced through carbon nanotubes (CNTs) resting on an elastic foundation. Mehar *et al*. (2018a, b, c) evaluated the frequency behavior of nanolpate structure using FEM including the nonlocal theory of elasticity. Computer generated results are created by using the software first time roubustly to check the vibration of nanoplate. The efficiency was checked by comparing the results of available data. Material characteristics of CNTs are hypothesized to be altered within thickness orientation which is calculated according to Mori-Tanaka model. For modeling the system mathematically, first order shear deformation theory (FSDT) is applied and using energy procedure, the governing equations can be derived. Ebrahimi and Mahmoodi (2018) presented the static analysis of SWCNTs and vibration of CNTs using Eringen's beam theory. The bending moment and function of strain were performed with different boundary conditions. Motezaker *et al*. (2020) analysis the vibration, buckling and bending of annular nanoplate integrated with piezoelectric layers at the top and bottom surfaces. The higher order nonlocal theory for size effect and Gurtin-Murdochtheory for surface effects are utilized. The governing equations are derived based on the layer-wise (LW) theory and Hamilton's principle. The differential cubature method (DCM) as a new numerical procedure is utilized to solve the motion equations for obtaining the frequency, buckling load and deflection.

Many material researchers calculated the frequency of nano structur using different techniques, for example, Timoshenko beam model (Zidour *et al*. 2014), SiO2 nanoparticles (Zarei *et al*. 2017, Amnieh *et al*. 2018, Jassas *et al*. 2019), layerwise theory (Hajmohammad *et al*. 2018a, 2019), Flugge shell theory (Zidour *et al*. 2014), Grey Wolf algorithm (Kolahchi *et al*. 2020), reinforced polymer layer (Hajmohammad *et al*. 2018b), agglomerated CNTs (agglomerated CNTs), zigzag theory (Kolahchi *et al*. 2017a, b), viscoelastic cylindrical shell (Hosseini and Kolahchi 2018, Hajmohammad *et al*. 2018c), deformation theory (Mehar *et al*. 2016), nonlocal elasticity theory (Mehar *et al*. 2018a, b, c), multiscale modeling approach (Mehar and Panda 2019, Mehar *et al*. 2019, Das *et al*. 2013).

Vibration analysis of armchair double-walled CNTs are rarely done in recent past. A limited number of researchers performed analysis first time to investigate the vibration of double-walled CNTs (Wang *et al*. 2006a, b, Natsuki *et al*. 2007, Shen and Zhang 2010, Ansari and Rouhi 2012, Ansari and Arash 2013). So far as reviewed from the literature, vibration response of armchair double-walled CNT using wave propagation approach based on nonlocal kelvin model has not been investigated/assumed. Many material researchers calculated the frequency of CNTs using different techniques, for example, structural mechanics approach (Li and Chou 2003, Tahouneh 2017, Moradi-Dastjerdi and Payganeh 2017, Shafiei and Setoodeh 2017), shear deformation theory (Arefi *et al*. 2018, Lei and Zhang 2018), nonlocal continuum models (Sudak 2003, Wang *et al*. 2006a ,b, Pradhan and Phadikar 2009, Ansari *et al*. 2010, Hao *et al*. 2010, Amara *et al*. 2010, Shen and Zhang 2010, She *et al*. 2019), stress and strain theory (Karami *et al*. 2018), shell theory (Yakobson *et al*. 1996), beam theory (Wang *et al*. 2006a, b), atomic modeling (Iijima *et al*. 1996, Yakobson *et al*. 1997, Hernandez *et al*. 1998, Sanchez-Portal *et al*. 1999, Qian *et al*. 2002), Rayleigh-Ritz (Ansari and Rouhi 2012), Galerkin method (Do *et al*. 2019) and axially loaded double beam system (Xiaobin *et al*. 2014). Moreover, the existing novel theoretical model contributes inventive computational outputs for the vibration of CNTs as compare to prior models presented (Iijima *et al*. 1996, Qian *et al*. 2002, Peddison *et al*. 2003, Sudak 2003, Natsuki *et al*. 2006, Shen and Zhang 2010, Ansari and Rouhi 2012, Avcar 2019, Ehyaie and Daman 2017, Hanjayah and Khadem 2015, Mercan and Civalek 2016, Rakrak *et al*. 2016, Tounsi *et al*. 2013). Another group of material researcher used novel approached to investigate the features of nanostructures (Bensattalah *et al*. 2018, Sedighi and Sheikhanzadeh 2017, Ghodrati *et al*. 2018, Salah *et al*. 2019, Batou *et al*. 2019, Behera and Kumari 2018, Safa *et al*. 2019, Sahouane *et al*. 2019, Lal *et al*. 2017, Zouatnia and Hadji 2019, Narwariya *et al*. 2018). Recently, Hussain and Naeem (2019a, b, c, d, 2020a) and performed the vibration of SWCNTs based on wave propagation approach and Galerkin's method. They investigated many physical parameters for the rotating and non-rotating vibrations of armchair, zigzag and chiral indices. Moreover, the mass density effect of single walled carbon nanotubes with in-plane rigidity has been calculated for zigzag and chiral indices.

The foremost intension of this paper to investigate vibrations characteristics of armchair double-walled CNT by means of nonlocal elasticity shell model. The nonlocal shell model is established by inferring the nonlocal elasticity equations into Kelvin's theory, which is our particular motivation. The suggested method to investigate the solution of fundamental eigen relations is wave propagation, which is a well-known and efficient technique to develop the fundamental frequency equations. It is keenly seen from the literature, no evidence is found concerning current model where such problem has been studied so it gave impetus to conduct present work. The specific influence of four different end supports based on nonlocal FSM such as clamped-clamped (FSM-CC), clamped-simply supported (FSM-CS), simply supported-simply supported (FSM-SS) and clamped-free (FSM-CF) with assorted values of nonlocal parameter and distinguish inner tube radii is examined in detail.

2. Formation of model

Basically, nanotubes are two types depending upon the layers. They are single-walled carbon nanotube and multi-walled carbon nanotube. SWCNTs can be prepared by rolling a single graphene layer and DWCNTs obtained by rolling a twice the graphene layer with hexagonal cells to form the structure of cylindrical fullerene. They are generated by rolling of the graphene sheet. Carbon nanotube sheets include hexagonal cells that are ideally cut to produce carbon atoms of the tube. These cylindrical structures have many fascinating and valuable properties which have several potential applications in different fields. When a graphene sheet is rolled with its hexagonal cells, the structure can be conceptualized as DWCNT and its circumference and quantum properties depend upon the chirality and diameter described as a pair of (*n*, *m*). The indices pair occur during the rolling of tube Fig. 1 shows the schema of the pair indices as (*m*, *n*) which occurs on rolling of the tube and this pair of in indices formed as armchair, if $m = n$, respectively.

The dynamic equilibrium equations for DWNT are

$$\begin{cases} \dfrac{\partial N_\alpha}{\partial \alpha} + \dfrac{\partial S_\beta}{\partial \beta} + \kappa = \rho h R \dfrac{\partial^2 u}{\partial t^2} \\ \dfrac{\partial N_\beta}{\partial \beta} + \dfrac{\partial S_\alpha}{\partial \alpha} + Q_\beta = \rho h R \dfrac{\partial^2 v}{\partial t^2} \\ \dfrac{\partial Q_\alpha}{\partial \alpha} + \dfrac{\partial Q_\beta}{\partial \beta} + N_\beta + p = \rho h R \dfrac{\partial^2 w}{\partial t^2} \end{cases} \tag{1}$$

$$\begin{cases} \dfrac{\partial M_{\alpha\beta}}{\partial \alpha} + \dfrac{\partial M_\beta}{\partial \beta} - RQ_\beta = 0 \\ \dfrac{\partial M_{\beta\alpha}}{\partial \beta} + \dfrac{\partial M_\alpha}{\partial \alpha} - RQ_\alpha = 0 \end{cases} \tag{2}$$

where ρ is the mass density.

Where p denotes the exerted pressure on i tube through van der Waals (vdW) interaction forces. The proposed vdW model accounts the effects of interlayer interactions between the tubes of DWNT.

$$p = w_i \sum_{j=1}^{2} c_{ij} - \sum_{j=1}^{2} c_{ij}\, w_j \qquad (i = 1,2) \tag{3}$$

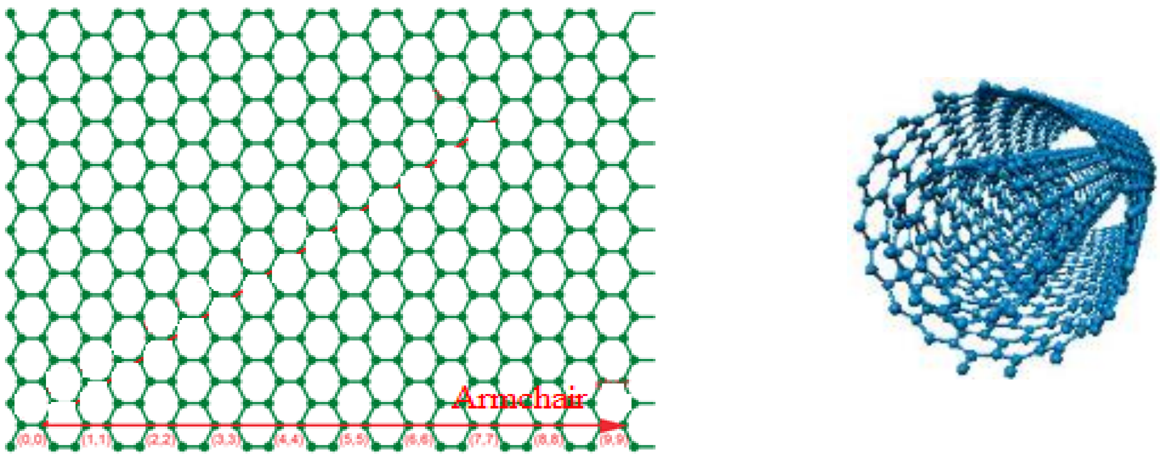
c_{ij} is vdW coefficient, depicting the pressure increment contributing from ith to jth tube.

$$c_{ij} = \left[\frac{1001\pi\varepsilon\sigma^{12}}{3a^4} {E_{ij}}^{13} - \frac{1120\pi\varepsilon\sigma^6}{9a^4} {E_{ij}}^{7}\right] R_j \tag{4}$$

Here C-C bond length is given by $a = 1.42\dot{A}$, depth of potential by ε, σ as parameter concluded by equilibrium distance, R_j as radius of j^{th} tube and ${E_{ij}}^m$ be as elliptic integral which is given as

$${E_{ij}}^m = (R_j + R_i)^{-m} \int_0^{\frac{\pi}{2}} \frac{d\theta}{(1 - K_{ij} \cos^2\theta)^{\frac{m}{2}}} \tag{5}$$

being m as integer and coefficient K_{ij} is defined by



(a) (b)

Fig. 1 Hexagonally description of armchair DWCNTs on the (a) graphene sheet; (b) Rolled DWCNTs

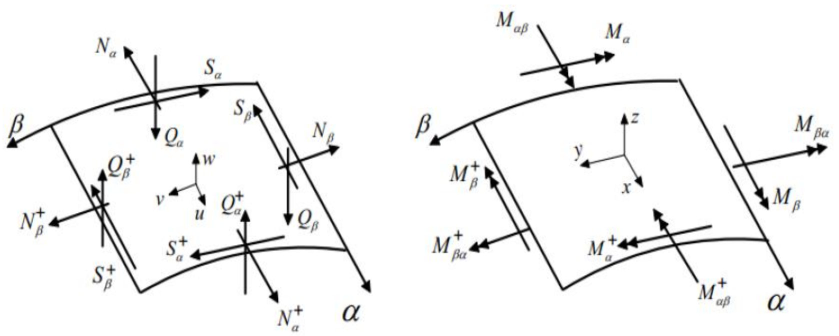


Fig. 2 Stress and moments components of the middle surface of CNTs

$$K_{ij} = \frac{4R_jR_i}{(R_j + R_i)^2} \tag{6}$$

The resultants (N, S, Q) are derived from above set of integral equations using the stress components.

$$(1-(e_0a)^2\nabla^2)\begin{bmatrix} N_\alpha, S_\alpha, \\ M_\alpha, M_{\alpha\beta} \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \sigma_\alpha, \tau_{\alpha\beta}, \\ z\,\sigma_\alpha, z\,\tau_{\alpha\beta} \end{bmatrix}\left(1+\frac{z}{R}\right)dz \tag{7}$$

$$(1-(e_0a)^2\nabla^2)\begin{bmatrix} N_\beta, S_\beta, \\ M_\beta, M_{\beta\alpha} \end{bmatrix} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \begin{bmatrix} \sigma_\beta, \tau_{\beta\alpha}, \\ z\,\sigma_\beta, z\,\tau_{\beta\alpha} \end{bmatrix} dz \tag{8}$$

$$(1-(e_0a)^2\nabla^2)(Q_\alpha, Q_\beta) = \int_{-\frac{h}{2}}^{\frac{h}{2}} [\tau_{\alpha z}, \tau_{\beta z}]\,dz \tag{9}$$

where h is thickness of the shell. Above equations result in

$$N_\alpha - (e_oa)^2\nabla^2 N_\alpha = \frac{K}{R}\left[\frac{\partial u}{\partial\alpha} + \mu_1\left(\frac{\partial v}{\partial\beta} + w\right) - c^2\frac{\partial^2 w}{\partial\alpha^2}\right] \tag{10}$$

$$N_\beta - (e_oa)^2\nabla^2 N_\beta = \frac{Kk_1}{R}\left[\frac{\partial v}{\partial\beta} + \mu_2\frac{\partial u}{\partial\alpha} + w + c^2\left(\frac{\partial^2 w}{\partial\beta^2} + w\right)\right] \tag{11}$$

$$S_\alpha - (e_oa)^2\nabla^2 S_\alpha = \frac{Kk_2}{R}\left[\frac{\partial u}{\partial\beta} + \frac{\partial v}{\partial\alpha} - c^2\left(\frac{\partial^2 w}{\partial\alpha\partial\beta} - \frac{\partial v}{\partial\alpha}\right)\right] \tag{12}$$

$$S_\beta - (e_oa)^2\nabla^2 S_\beta = \frac{Kk_2}{R}\left[\frac{\partial u}{\partial\beta} + \frac{\partial v}{\partial\alpha} + c^2\left(\frac{\partial^2 w}{\partial\alpha\partial\beta} + \frac{\partial v}{\partial\alpha}\right)\right] \tag{13}$$

$$M_\alpha - (e_oa)^2\nabla^2 M_\alpha = -Kc^2\left[\frac{\partial u}{\partial\alpha} + \mu_1\frac{\partial v}{\partial\beta} - \left(\frac{\partial^2 w}{\partial\alpha^2} + \mu_1\frac{\partial^2 w}{\partial\beta^2}\right)\right] \tag{14}$$

$$M_\beta - (e_oa)^2\nabla^2 M_\beta = Kk_1c^2\left(\frac{\partial^2 w}{\partial\beta^2} + w + \mu_2\frac{\partial^2 w}{\partial\alpha^2}\right) \tag{15}$$

$$M_{\alpha\beta} - (e_oa)^2\nabla^2 M_{\alpha\beta} = 2Kk_2c^2\left(\frac{\partial v}{\partial\alpha} - \frac{\partial^2 w}{\partial\alpha\partial\beta}\right) \tag{16}$$

$$M_{\beta\alpha} - (e_oa)^2\nabla^2 M_{\beta\alpha} = Kk_2c^2\left(\frac{\partial u}{\partial\beta} - \frac{\partial v}{\partial\alpha} + 2\frac{\partial^2 w}{\partial\alpha\partial\beta}\right) \tag{17}$$

$$Q_\alpha - (e_oa)^2\nabla^2 Q_\alpha = \frac{Kc^2}{R}\begin{bmatrix} \frac{\partial^2 u}{\partial\alpha^2} - k_2\frac{\partial^2 u}{\partial\beta^2} + (k_2+\mu_1)\frac{\partial^2 v}{\partial\alpha\partial\beta} - \\ \frac{\partial^3 w}{\partial\alpha^3} - (2k_2+\mu_1)\frac{\partial^3 w}{\partial\alpha\partial\beta^2} \end{bmatrix} \tag{18}$$

$$Q_\beta - (e_oa)^2\nabla^2 Q_\beta = \frac{Kk_1c^2}{R}\begin{bmatrix} 2\frac{k_2}{k_1}\frac{\partial^2 v}{\partial\alpha^2} - \frac{\partial^3 w}{\partial\beta^3} - \\ \frac{\partial w}{\partial\beta} - \left(2\frac{k_2}{k_1} + \mu_2\right)\frac{\partial^3 w}{\partial\alpha^2\partial\beta} \end{bmatrix} \tag{19}$$

where $K = E_1h/(1-\mu_1\mu_2)$, $k_1 = E_2/E_1$, $k_2 = G(1-\mu_1\mu_2)/E_1$, $c^2 = h_o^3/(12R^2h)$.

Using Kelvin model and Eqs. (1) and (2), we get Kelvin-like nonlocal orthotropic elastic shell model.

The obtained model is as follow

$$\left[\frac{\partial^2}{\partial\alpha^2} + k_2(1+c^2)\frac{\partial^2}{\partial\beta^2}\right]u + \left[(\mu_1+k_2)\frac{\partial^2}{\partial\alpha\partial\beta}\right]v + \left[6 + \frac{\partial}{\partial\alpha} + c^2\left(k_2\frac{\partial^3}{\partial\alpha\partial\beta^2} - \frac{\partial^3}{\partial\alpha^3}\right)\right]w = \frac{\rho hR^2[1-(e_oa)^2\nabla^2]}{K}\frac{\partial^2 u}{\partial t^2} \tag{20}$$

$$\left[(\mu_1+k_2)\frac{\partial^2}{\partial\alpha\partial\beta}\right]u + \left[k_2(1+3c^2)\frac{\partial^2}{\partial\alpha^2} + k_1\frac{\partial^2}{\partial\beta^2}\right]v + \left[k_1\frac{\partial}{\partial\beta} - c^2(\mu_1+3k_2)\frac{\partial^3}{\partial\alpha^2\partial\beta}\right]w = \frac{\rho hR^2[1-(e_oa)^2\nabla^2]}{K}\frac{\partial^2 v}{\partial t^2} \tag{21}$$

$$\left[\mu_1\frac{\partial}{\partial\alpha} - c^2\left(\frac{\partial^3}{\partial\alpha^3} - k_2\frac{\partial^3}{\partial\alpha\partial\beta^2}\right)\right]u + \left[k_1\frac{\partial}{\partial\beta} - c^2(\mu_1+3k_2)\frac{\partial^3}{\partial\alpha^2\partial\beta}\right]v \tag{22}$$

$$+\left[\left(1+\frac{1}{c^2}\right)k_1+\frac{\partial^4}{\partial\alpha^4}+k_1\frac{\partial^4}{\partial\beta^4}\right.$$
$$\left.+2k_1\frac{\partial^2}{\partial\beta^2}+(2\mu_1+4k_2)\frac{\partial^4}{\partial\alpha^2\partial\beta^2}\right]c^2w$$
$$+\frac{R^2}{K}(1-(e_0a)^2\nabla^2)\left[Ew+\eta\frac{\partial w}{\partial t}\right] \quad (22)$$
$$\left[w_i\sum_{j=1}^{2}c_{ij}-\sum_{j=1}^{2}c_{ij}\,w_j\right]=-\frac{\rho hR^2[1-(e_oa)^2\nabla^2]}{K}\frac{\partial^2 w}{\partial t^2}$$

where $K=\frac{E_1h}{1-\mu_1\mu_2}$, medium has stiffness E, and the viscosity of the medium is η and the nonlocal parameter is $\Im=(e_oa)^2$.

Over the past several years vibration of nanostructures of various configurations and boundary conditions have been extensively studied (Hussain *et al*. 2018a, b, c, 2019a, b, 2020a, b, c, d, e, f, g, Hussain and Naeem 2018b, 2020b, Asghar *et al*. 2020, Taj *et al*. 2020a, b, c). Here, we will discuss wave solutions for DWNTs.

The solutions of system of Eqs. (20)-(22) for axisymmetric waves is given by (Wang and Gao 2016)

$$\begin{cases} u(\alpha,t)=Ae^{ik\left(\alpha-\frac{vt}{R}\right)} \\ v(\alpha,t)=Be^{ik\left(\alpha-\frac{vt}{R}\right)} \\ w(\alpha,t)=Ce^{ik\left(\alpha-\frac{vt}{R}\right)} \end{cases} \quad (23)$$

where U, V and W are the amplitudes of waves along the direction of x, y and z respectively, the dimensionless wave vector in the longitudinal direction is $k=\frac{\pi mR}{L}$, in longitudinal direction m is the half axial wave number and ν is the wave phase velocity.

Substituting Eq. (23) in system of Eqs. (20)-(22) and simplifying, in matrix form, we get the following system

$$[M^{(1)}(k,\nu)]_{3\times3}\begin{bmatrix}A\\B\\C\end{bmatrix}=[0\quad 0\quad 0]^T \quad (24)$$

3. Results and discussion

In this section, the significance of boundary conditions on the vibration behavior of DWNT is investigated employing wave propagation approach. The versatility and accuracy of proposed method is seen by numerous studies (Natsuki *et al*. 2006, 2007) to determine natural frequencies in shell and CNTs. This study specifically scrutinizes the small scale effect in the vibration analysis of DWNT. The numerical values of Young modulus, Poisson's ratio, thickness¬ and density are $E=1\,\text{TPa}$, $\nu=0.3, h=0.34$ nm and $\rho=2.3\,\text{g/cm}^3$ reported (Ansari and Arash 2013). Moreover, distinguished values of inner tube radius together with nonlocal parameter signify the present non-local shell-based model to analyze frequency spectra. CNT is well known structure in shapes of (i) armchair; (ii) chiral; and (iii) zigzag, here the vibration analysis is carried out of armchair CNT subjected to four conditions FSM-CC, FSM-CS, FSM-SS and FSM-CF. For the convergence rate of

Table 1 Comparison of MD results (Hu *et al*. 2012) with existing result

	f (THz)							
	(5, 5)		(10, 10)		(15, 15)		(20, 20)	
	C-C	C-F	C-C	C-F	C-C	C-F	C-C	C-F
Hu *et al*. (2012)	2.12	1.07	2.05	1.07	1.99	1.02	1.83	1.01
Present	2.12	1.07	2.05	1.07	1.99	1.02	1.83	1.01

Table 2 Non dimensional results comparison with present results

Method	e_oa				
	0	1	2	3	4
Reddy (2007)	9.8696	8.983	8.2426	7.6149	47.0761
Aydogdu (2009)	9.8696	9.6319	9.4055	9.1894	8.983
Elather *et al*. (2013)	9.86973	8.98312	8.24267	7.61499	7.07614
Karami *et al*. (2019)	9.80601	8.92692	8.19176	7.56846	7.03246
Present	9.80601	8.92692	8.19176	7.56846	7.03246

CNT, the non-dimensional frequency parameters enumerated in the current work, i.e., using FSM, are happened to be in a good consistency along with the so-called exact results furnished by Hu *et al*. (2012), those were established by working out with the deformation theory provided in Table 1. Table 1 displays the fundamental frequencies of CNTs with two certain end conditions with the same length of 4.12 nm. The boundary conditions have a significant effect on the fundamental frequency. In the table cantilevered end support is showed by C-F whereas clamped-clamped by C-C for comparison. The preliminary focus of the investigation was on the precision of the proposed technique with existing model, whose results are summarized in Table 2 in nondimensional form for an S-S condition, while varying the nonlocal parameter e_oa. Based on a comparative evaluation between our predictions and those obtained by Reddy (2007), Aydogdu (2009), Eltaher *et al*. (2013) and Karami *et al*. (2019) a very good match was observed, which confirms the accuracy of the proposed formulation for similar problems.

Fig. 3 exhibits the variation of fundamental eigen frequencies against values of nonlocal parameter that changes within a limit from 0 to 2. Three distinct aspect ratio (length to radius) $L/R_1=5, 10, 20$ are discussed subject to four boundary conditions FSM-CC, FSM-CS, FSM-SS and FSM-CF. The radius of inner tube is considered here as $R_1=0.35\,nm$ with all above mentioned numerical estimates of physical parameters incorporating also with vdW interaction between two tubes of double-walled CNT. The graph in figure shows that with a decrease in values of nonlocal parameter, frequency corresponding to each boundary condition tends to decrease. For smaller values of e_oa there is slight variation in frequencies of FSM-CC, FSM-CS, FSM-SS and FSM-CF respectively at the same time for lower aspect ratio the observation remains alike. Two main findings depicted by

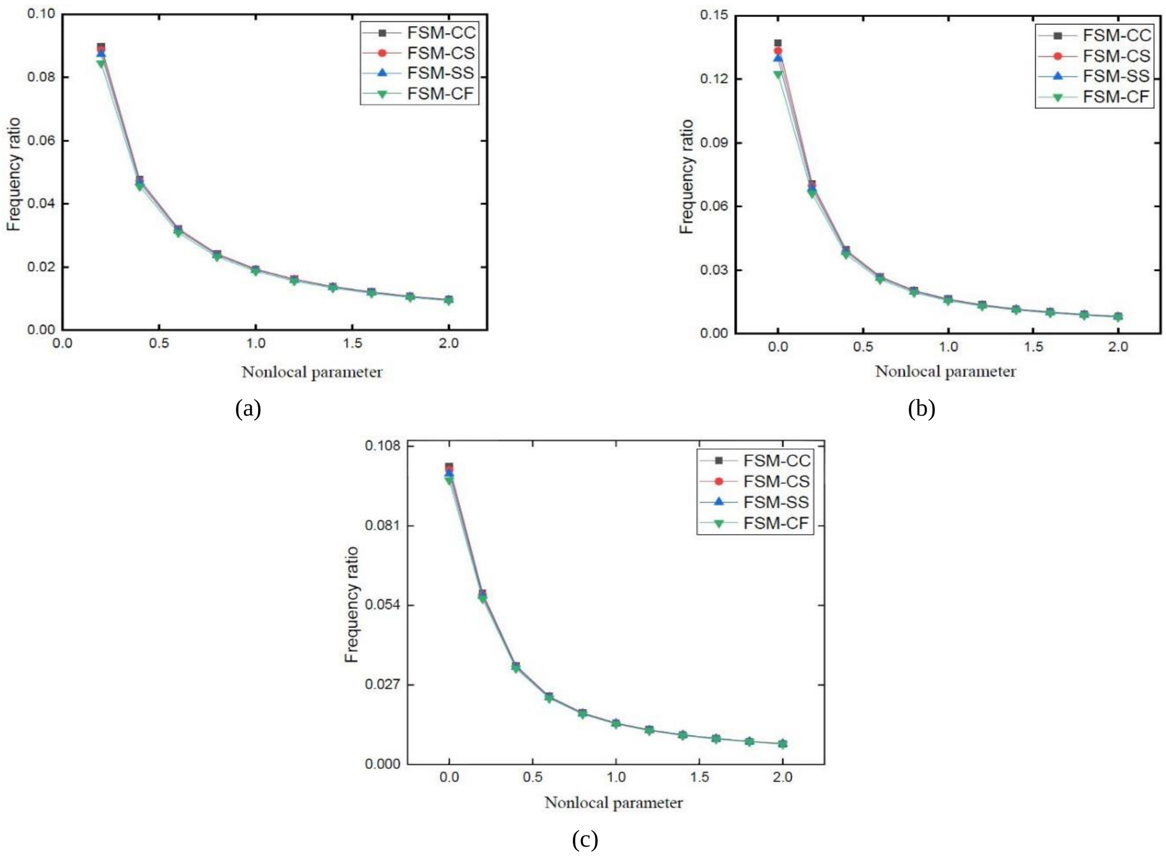


Fig. 3 Frequency with respect to nonlocal parameter $e_o a$ for aspect ratio $L/R_1 =$ 5, 10, 20

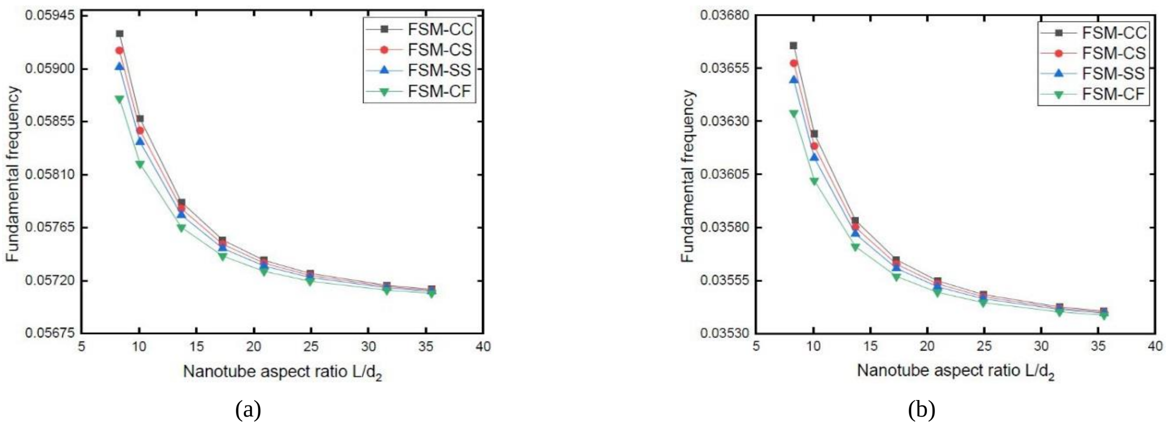


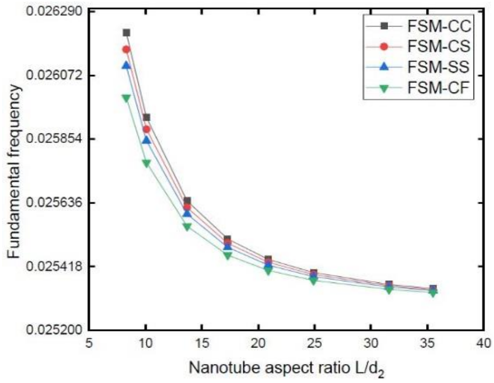
Fig. 4 Influence of distinct boundary condition against numerous values of $e_o a$ = 0.2, 0.35, 0.5 of armchair (7, 7) (a-c) and armchair (9, 9) (d-f) double-walled CNTs with $R_1 = 0.35\ nm$

graph are, calculated frequencies coincide for all boundary condition and continue to decrease with a rise in aspect ratio. The rooted nonlocal elasticity model also produces more significant results for minimal radius of tubes.

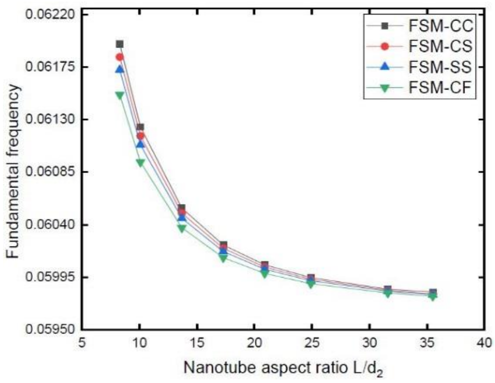
The graphs in Fig. 4 included the fundamental frequencies of armchair (7, 7) and (9, 9) showing diversity with the $e_o a = 0.2, 0.35$ and 0.5. The all depicted frequencies in graphs are facing length to diameter ratio. It is noticed that there is uniform increase in frequencies of arm chair corresponding to all four conditions FSM-CC, FSM-CS, FSM-SS and FSM-CF. Corresponding to $e_o a =$ 0.2, the clamped-clamped (FSM-CC) condition of armchair (7, 7) and (9, 9) obtained frequencies 0.054, 0.0595 and 0.0619 respectively.

It is obviously seen there is an increasing trend and which remains unchanged for all boundary conditions as well as other two values of nonlocal parameter possess the identical behavior. Moreover, the more accretion in the nonlocal parameter, the lower the fundamental frequencies are observed.

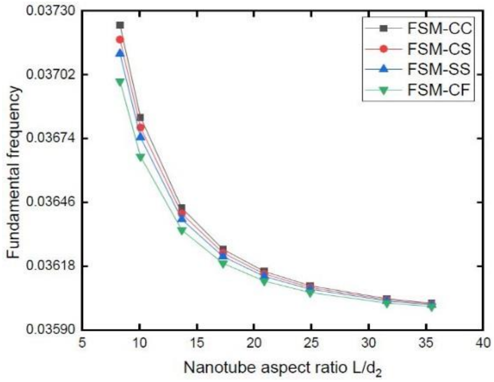
Figs. 5 and 6 shows the influence of boundary conditions for armchair (7, 7) and (9, 9) respectively considering the $R_1 = 1.5\ nm$. The frequency decreases versus length to diameter ratio affirms the nonlocal effect.



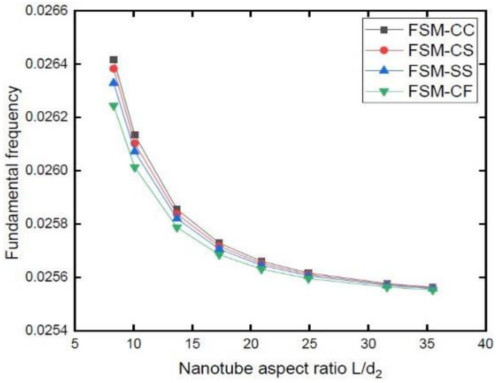
(c)



(d)

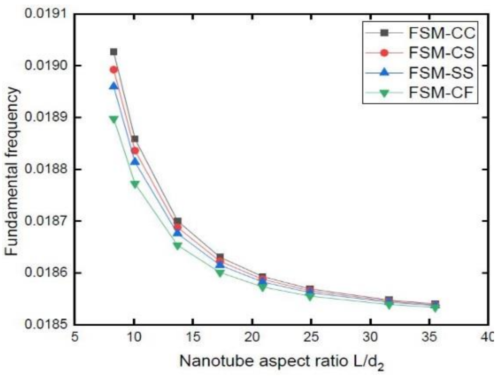


(e)

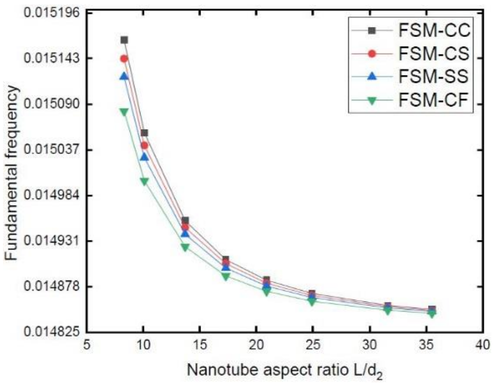


(f)

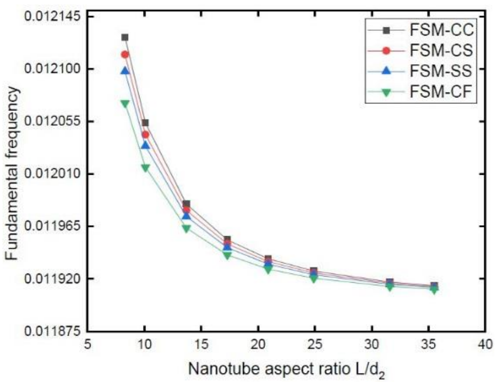
Fig. 4 Continued



(a)

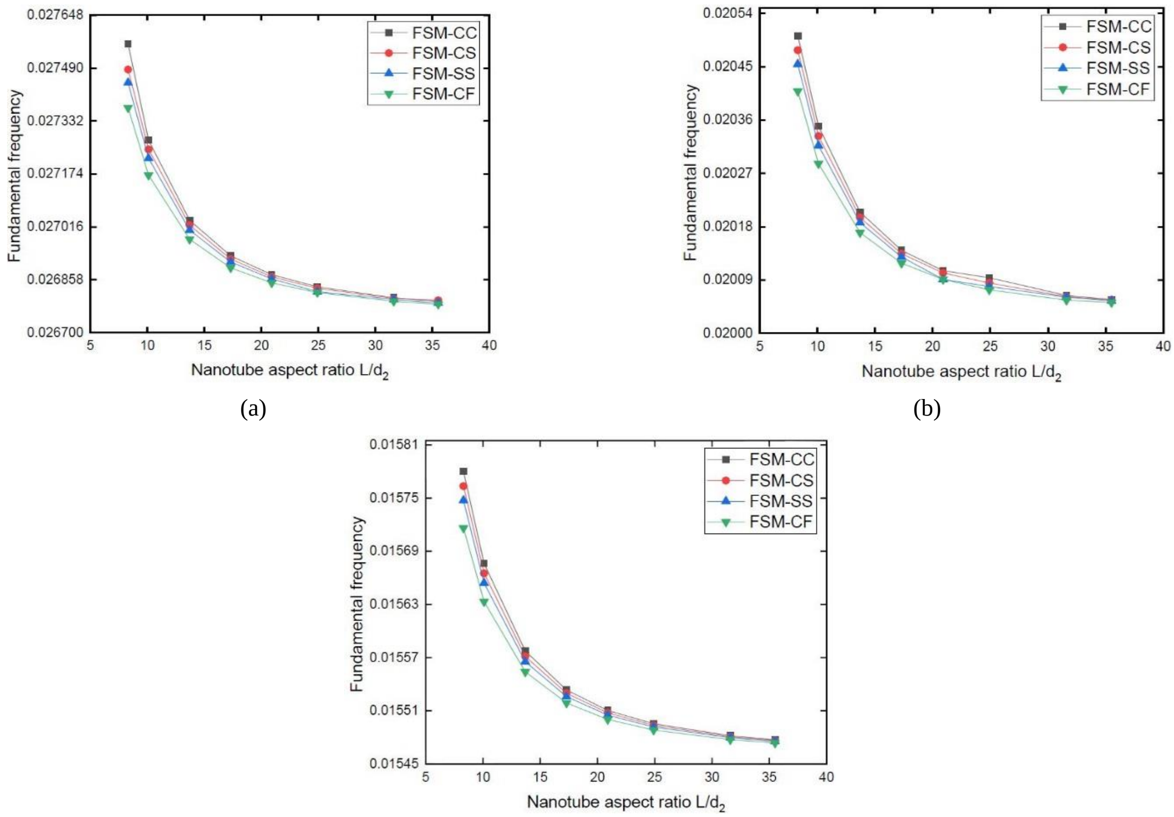


(b)



(c)

Fig. 5 Influence of distinct boundary conditions against numerous values of $e_o a$ = 0.2, 0.35, 0.5 of armchair (7, 7) double-walled CNTs with $R_1 = 1.5\ nm$



(a) (b)

(c)

Fig. 6 Influence of distinct boundary conditions against numerous values of $e_o a$ = 0.2, 0.35, 0.5 of armchair (9, 9) double-walled CNTs with $R_1 = 1.5\ nm$

Corresponding to armchair (7, 7) and (9, 9), there is seen drop in the frequencies as inflates the nonlocal parameter value. However, on increasing the indices of armchair, the frequency curves increases. The expanded values of length to diameter ratio exhibit the reality that nonlocal effect becomes negligible on boundary conditions. On the other hand, the frequency curve showed the difference in contrast of the boundary conditions becomes infinitesimal with an increase in inner tube radius. The gap presented in four boundary conditions is obvious in start of the curves as FSM-CF have the lowest frequency in comparison of FSM-SS, FSM-CS and FSM-CC.

4. Conclusions

The Kelvin's model based on nonlocal elasticity theory investigates the vibration characteristics of DWNT. Theoretical formation of the nonlocal model involves the van der Waals interactions between the tubes and impact of small-scale effect subjected to four boundary supports. The wave propagation approach is employed to determine eigen frequencies for armchair CNT. The fundamental frequencies scrutinized with assorted length to diameter ratios. The raised in value of nonlocal parameter reduces the corresponding fundamental frequency estimates. Due to small scale effect fundamental frequency ratio decreases as length to diameter ratio increases. Small scale effect becomes negligible on all end supports for the higher values of aspect ratio. With the smaller inner tube radius double-walled CNT behaves more sensitive towards nonlocal parameter. The present study can be appropriate to employ for analyzing the vibrations in double-walled CNTs with Galerkin and finite element methods.

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