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Investigation on hygro-thermal vibration of P-FG and symmetric S-FG nanobeam using integral Timoshenko beam theory

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Abstract. In the current research, the free vibrational behavior of the FG nano-beams integrated in the hygro-thermal environment and reposed on the elastic foundation is investigated using a novel integral Timoshenko beam theory (ITBT). The current model has only three variables unknown and requires the introduction of the shear correction factor because her uniformed variation of the shear stress through the thickness. The effective properties of the nano-beam vary according to power-law and symmetric sigmoid distributions. Three models of the hygro-thermal loading are employed. The effect of the small scale effect is considered by using the nonlocal theory of Eringen. The equations of motion of the present model are determined and resolved via Hamilton principle and Navier method, respectively. Several numerical results are presented thereafter to illustrate the accuracy and efficiency of the actual integral Timoshenko beam theory. The effects of the various parameters influencing the vibrational responses of the P-FG and SS-FG nano-beam are also examined and discussed in detail.

Keywords: FG nano-beam; vibrational behavior; Integral Timoshenko beam theory; hygro-thermal effect

1. Introduction

Nanostructures are small-scale mechanical models that are widely used in recent years by many researchers (Shodja *et al.* 2012, Sedighi 2014, Eltaher *et al.* 2016, 2018b, 2019a, b, c, Ebrahimi and Barati 2016a, Sedighi and Bozorgmehri 2016, Ebrahimi and Barati 2017a, Sedighi and Sheikhanzadeh 2017, Romano *et al.* 2017, Khanik 2018, Hamidi *et al.* 2018, Bensaid *et al.* 2018, Faleh *et al.* 2018, Bensattalah *et al.* 2018, 2019, Akbas 2018, Belmahi *et al.* 2018 and 2019, Aria *et al.* 2019, Mohamed *et al.* 2019, Barati and Shahverdi 2019, Hussain and Naeem 2019, Aria and Friswell 2019, Forsat *et al.* 2020), because of these models of interesting structures, several research investigations have been carried out on the study of the behaviors of these nanostructures made from a novel class of materials such as functionally graded materials (FGM structures) which the material properties vary gradually and continually through a given direction. For example, Rezaiee-Pajand *et al.* (2018) investigated on static analysis of FG non-prismatic sandwich-beams. The large deformation of FG visco-hyperelastic structures is examined by Pascon (2018). The analytical solution for vibrational response of FG nanobeam is developed by Ebrahimi and Daman (2017).

The static and dynamic analysis of the porous FG nanobeam is studied by Eltaher *et al.* (2018b) using the FEM method (finite element method). Ebrahimi and Barati (2016b) examined the effect of the external load on the vibrational parameter of the nonlocal FG beam. These last years, several scientists have examined the influence of the thermal and hygro thermal environment on the behavior of FG nanostructures. Ebrahimi and Salari (2015) examined the thermal stability and vibrational behavior of FG nanobeam using nonlocal Timoshenko beam theory. Barati and Shahverdi (2016) analyzed the thermal vibration of the FG nanoplate under various non-uniform thermal loads. Sobhy (2017) used the HSDT to examine the buckling and Hygro-thermo-mechanical vibration of E-FG nanoplate. Ebrahimi and Heidari (2018) examined the effect of the humid-thermal environment on the vibrational characteristics of FG nanoplate using (DQM) method. Recently, several investigations that focuses on the effect of hygro-thermal environment are published as (Shahsavari *et al.* 2018, Hajmohammad *et al.* 2018, Hosseini and Kolahchi 2018, Akbas 2019a).

In this research work, the hygro-thermal vibrational behavior of the simply supported P-FG and symmetric S-FG nanobeams seated on Winkler-Pasternak elastic

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foundation is investigated using the nonlocal elasticity and novel integral Timoshenko beam theory. The developed model needs to assure the zero shear stresses at free surface of the beam. The analytical solution of the vibrational behavior is determined via Hamilton principle and Navier method. The accuracy of the current model is verified by comparing the obtained results with those found in the literature.

2. Theoretical formulations

2.1 Models of the FG nanobeam

In the present investigation, Consider a simply supported FG nanobeam with dimensions ($lenght$ "a", $width$ "b" and $thickness$ "h") reposed on the elastic foundation type Winkler Pasternak (as shown in the Fig. 1). Two types of the simply supported FG nanobeam are employed namely Power law FG nanobeam and Symmetric sigmoid FG nanobeam. The effective properties of the FG nanobeam of the both types are given as

$$P(z) = P_c V_c + P_m V_m \tag{1}$$

With $V_c + V_m = 1$.

Where P_i and V_i are the properties and volume fraction of the material with $(i = c, m)$.

2.1.1 Power law FG nanobeam (P-FG nanobeam)

In the first model of the properties of the FG nanobeam vary according to the power-law volume fraction (Fig. 2), the function of the ceramic volume fraction can be written as

$$V_c = \left(\frac{2z+h}{2h}\right)^p \quad \text{with} \quad p \geq 0 \tag{2}$$

Where E, G, ρ, α, and β are Young's modulus, shear modulus, mass density, thermal expansion and moisture expansion coefficient, respectively.

2.1.2 Symmetric sigmoid FG nanobeam (SS-FG nanobeam)

The second type of the volume fraction (SS-FG) varies symmetrically with respect to the mean axis by using two power law volume fractions (see Fig. 3). The symmetric sigmoid volume fraction is expressed as

$$V_m(z) = \begin{cases} \left(\dfrac{2z+h}{h}\right)^P & \text{for} \quad -\dfrac{h}{2} \leq z \leq 0 \\ \left(\dfrac{-2z+h}{h}\right)^P & \text{for} \quad 0 \leq z \leq \dfrac{h}{2} \end{cases} \tag{3}$$

To obtain the effective properties of the P-FG and SS-FG nanobeams such as $(E(z), G(z), \rho(z), \alpha(z), \beta(z))$ just replace the volume fraction in the corresponding model into Eq. (1).

For studying the behavior of the FG- nanobeam under thermal loading precisely, the temperature was taken depend on the material properties. The thermo-elastic Material properties "P" in function of the temperature "$T(k)$" can be given in the nonlinear form as (Ebrahimi and Salari 2015)

$$P(T) = P_0(P_{-1}T^{-1} + 1 + P_1T^1 + P_2T^2 + P_3T^3) \tag{4}$$

Where P and T are the material property and the environmental temperature, respectively. P_i indicates the temperature-dependent coefficients of the $SUS304$ (Metal) and Si_3N_4 (Ceramic) as mentioned in Table 1.

2.2 Integral Timoshenko's beam theory

Based on the Timoshenko beam theory and supposing that the total bending rotation equal $k_1 \int \theta(x,t)\, dx$. The current displacement field of the integral Timoshenko beam

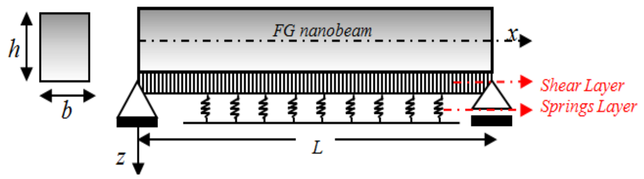

Fig. 1 Geometry of FG nanobeam resting on elastic foundation

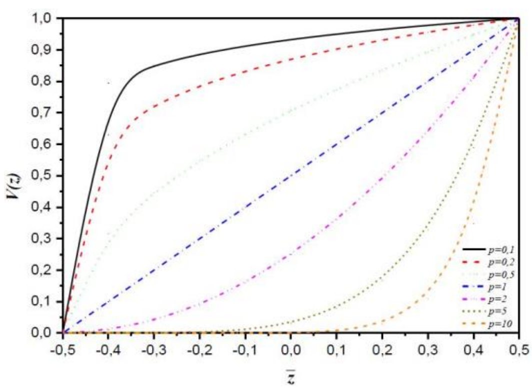

Fig. 2 Variation of ceramic volume fraction along the thickness of the P-FG nanobeam

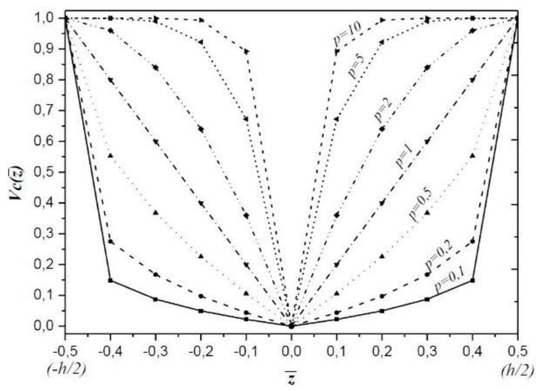

Fig. 3 Variation of ceramic volume fraction along the thickness of the Symmetric S-FG nanobeam

Table 1 Temperature-dependent material properties of FGM constituents (Ebrahimi and Salari 2015)

Material	Properties	P_0	P_{-1}	P_1	P_2	P_3
Si_3N_4	E (Pa)	3,4843e+11	0	-3,070e-04	2,160e-07	-8,946e-11
	α (K^{-1})	5,8723e-06	0	9,095e-04	0	0
	ρ (Kg/m^3)	2370	0	0	0	0
	v	0,24	0	0	0	0
SUS 304	E (Pa)	2,0104e+11	0	3,079e-04	-6,534e-07	0
	α (K^{-1})	1,233e-05	0	8,086e-04	0	0
	ρ (Kg/m^3)	8,17e+03	0	0	0	0
	v	3,262e-01	0	-2,002e-04	3,797e-07	0

theory can be expressed as

$$\begin{aligned} u(x,z,t) &= u_0(x,t) - zk_1 \int \theta(x,t)\,dx \\ w(x,z,t) &= w_0(x,t) \end{aligned} \tag{5}$$

Where $u_0(x,t), w_0(x,t)$ and $\theta(x,t)$ are unknowns' displacement. "$k_1 = \lambda^2$" with λ presented in Eq. (7).

The integral term appears in the Eq. (5) can be resolve via Navier method and can be expressed as

$$\int \theta dx = A' \frac{\partial \theta}{\partial x} \tag{6}$$

Where the coefficient is adopted according to the present solution (Navier Method) and can be obtained as

$$A' = -\frac{1}{\lambda^2} \quad \text{with} \quad \lambda = m\pi/a \tag{7}$$

The non-zero linear strains of the present integral Timoshenko beam theory are obtained as follow

$$\begin{aligned} \varepsilon_x &= \varepsilon_x^0 + zk_1 A' \eta_x \\ \gamma_{xz} &= \gamma_{xz}^0 + k_1 A' \beta_{xz} \end{aligned} \tag{8}$$

With

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x},\ \ \eta_x = -\frac{\partial^2 \theta}{\partial x^2},\ \ \gamma_{xz}^0 = \frac{\partial w_0}{\partial x},\ \ \beta_{xz} = -\frac{\partial \theta}{\partial x} \tag{9}$$

2.3 Hamilton's principle (HP)

In the actual investigation, the Three equations of motion of the FG beam are determined via Hamilton principle, which stipulate that the motion of FG nanobeam during the time $t \in [0,t]$. The analytical form of the Hamilton principle (HP) can be expressed as (Eltaher *et al.* 2018a, Yüksela and Akbaş 2018)

$$0 = \int_0^t \delta(U + V - K)dt \tag{10}$$

Where δU, δV and δK are the variations of the strain energy, work performed by external forces and kinetic energy of the FG-beam.

The formulation of the strain energy variation "δU" can be expressed as

$$\begin{aligned} \delta U &= \int_0^L \int_{-h/2}^{h/2} (\sigma_x \delta\varepsilon_x + \tau_{xz}\delta\gamma_{xz}) dxdz \\ &= \int_0^L (N_x \delta\varepsilon_x^0 + M_x k_1 A' \eta_x + Q_{xz}\gamma_{xz}) dx \end{aligned} \tag{11}$$

With

$$\begin{Bmatrix} N_x \\ M_x \end{Bmatrix} = \int_{-h/2}^{h/2} \sigma_x \begin{pmatrix} 1 \\ z \end{pmatrix} dz\,; \quad Q_{xz} = \int_{-h/2}^{h/2} \tau_{xz} dz \tag{12}$$

Where "N_x, M_x and Q_{xz}" are the stress resultants.

The variations of the work performed by applied forces (Hygro-thermal load and elastic foundation) can the following mathematical form

$$\delta V = \int_0^L \begin{bmatrix} (N^T + N^H)\left(\dfrac{\partial w_0}{\partial x}\dfrac{\partial \delta w_0}{\partial x}\right) \\ -K_w w_0 \delta w_0 + K_s \left(\dfrac{\partial w_0}{\partial x}\dfrac{\partial \delta w_0}{\partial x}\right) \end{bmatrix} dx \tag{13}$$

Where "K_w, K_s and N^T, N^H" are Winkler, Pasternak coefficients and the applied forces due to temperature and moisture change, respectively. The "N^H and N^T" can be given as

$$N^H = \int_{-h/2}^{h/2} E(z)\beta(z,T)(C - C_0)dz \tag{14a}$$

$$N^T = \int_{-h/2}^{h/2} E(z)\alpha(z,T)(T - T_0)dz \tag{14b}$$

Where "T_0 and C_0" are the moisture concentrations and reference temperature, respectively.

The variation of kinetic energy is expressed as

$$\delta K = \int_0^L \int_{-h/2}^{h/2} [\dot{u}\delta\dot{u} + \dot{w}\delta\dot{w}]\rho(z) dxdz \tag{15}$$

By replacing the displacement field of Eq. (5) in Eq. (15), we obtain

$$\delta K = \int_0^L [I_0(\dot{u}_0\delta\dot{u}_0 + \dot{w}_0\delta\dot{w}_0) \\ -I_1\left(k_1A^{'}\dot{u}_0\frac{\partial\delta\dot{\theta}}{\partial x} + k_1A^{'}\delta\dot{u}_0\frac{\partial\dot{\theta}}{\partial x}\right) \\ +I_2(k_1A^{'})^2\frac{\partial\dot{\theta}}{\partial x}\frac{\partial\delta\dot{\theta}}{\partial x}\Bigg] \tag{16}$$

with

$$\dot{u}_0 = \frac{\partial u_0}{\partial t}, \quad \dot{w}_0 = \frac{\partial w_0}{\partial t} \quad \text{and} \quad \dot{\theta} = \frac{\partial\theta}{\partial t} \tag{17}$$

and

$$(I_0, I_1, I_2) = \int_{-h/2}^{h/2}(1, z, z^2)\rho(z)dz \tag{18}$$

where "I_0, I_1 and I_2" are mass inertias. "$\rho(z)$" is the mass density.

Substituting the Eqs. (11)-(15) and Eq. (15) into Eq. (10), integrating by parts the results equation, and separating the terms of displacement "δu_0, δw_0 and $\delta\theta$". The equations of motion of the FG beam expressed by resultants stresses "N_x, M_x and Q_{xz}" are obtained as

$$\delta u_0: \quad \frac{\partial N_x}{\partial x} = I_0\ddot{u}_0 + I_1k_1A^{'}\frac{\partial\ddot{\theta}}{\partial x} \tag{19a}$$

$$\delta w_0: \quad -\frac{\partial Q_{xz}}{\partial x} - (N^T + N^H)\frac{\partial^2 w_0}{\partial x^2} \\ -K_w w_0 - K_s\frac{\partial^2 w_0}{\partial x^2} = -I_0\ddot{w}_0 \tag{19b}$$

$$\delta\theta: \quad -k_1A^{'\frac{\partial^2 M_x}{\partial x^2}} + k_1A^{'\frac{\partial Q_{xz}}{\partial x}} \\ = I_1k_1A^{'}\frac{\partial\ddot{u}_0}{\partial x} + I_2{k_1}^2A^{'2}\frac{\partial^2\ddot{\theta}}{\partial x^2} \tag{19c}$$

2.4 Nonlocal elasticity

The nonlocal theory of (Eringen 1972, 1983) is employed herein to derive the non-local equations of motion, which take into account the small scale effect. Thus, the normal and shear stresses "σ and τ" of nonlocal theory For FG nanobeam can be obtained as

$$(1 - \mu\nabla^2)\begin{pmatrix}\sigma_{xx}\\ \tau_{xz}\end{pmatrix} = \begin{bmatrix}Q_{11} & 0\\ 0 & Q_{44}\end{bmatrix}\begin{pmatrix}\varepsilon_{xx}\\ \gamma_{xz}\end{pmatrix} \tag{2}$$

Where $\mu = (e_0a)^2$ and Q_{ii} are the small scale effect and stiffnesscoefficients, and can be defined as

$$Q_{11} = E(z), \quad Q_{44} = G(z) \tag{21}$$

Substituting Eq. (12) into Eq. (20). The resultants forces and moment "N_x, M_x and Q_{xz}" can be obtained in the nonlocal form as follow

$$(1 - \mu\nabla^2)\begin{pmatrix}N_x\\ M_x\end{pmatrix} = \begin{bmatrix}A_{11} & B_{11}\\ -k_1A^{'}B_{11} & -k_1A^{'}D_{11}\end{bmatrix}\begin{pmatrix}{\varepsilon_x}^0\\ \eta_x\end{pmatrix} \tag{22}$$

With

$$(1 - \mu\nabla^2)Q_{xz} = A_{44}^s\gamma_{xz} \tag{23}$$

Where the stiffness components A_{11}, B_{11}, D_{11} and A_{55} are defined as

$$\{A_{11}, B_{11}, D_{11}\} = \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{11}(z)(1, z, z^2)dz, \\ A_{44}^s = F_c^s\int_{-h/2}^{h/2} Q_{44}(z)dz \tag{24}$$

With "F_c^s" is the shear correction factor.

To obtain the Equations of motion as function of displacement terms "δu_0, δw_0 and $\delta\theta$", just we replace the Eq. (22) in (19). the equations of motion become

$$A_{11}\frac{\partial^2 u_0}{\partial x^2} - k_1A'B_{11}\frac{\partial^3\theta_0}{\partial x^3} = I_0\ddot{u}_0 + I_1k_1A'\frac{\partial^3\ddot{\theta}_0}{\partial x^3} \tag{25a}$$

$$-A_{55}\left(\frac{\partial^2 w_0}{\partial x^2} - k\,{}_1A'\frac{\partial^2\theta_0}{\partial x^2}\right) - (N^T + N^H)\left(\frac{\partial^2 w_0}{\partial x^2}\right) \\ -k_w w_0 - k_s\left(\frac{\partial^2 w_0}{\partial x^2}\right) = -I_0\ddot{w}_0 \tag{25b}$$

$$-k_1A^{'B_{11}\frac{\partial^3 u_0}{\partial x^3}} - k_1A^{'D_{11}\frac{\partial^4\theta_0}{\partial x^4}} \\ +A_{44}^s k_1A'\left(\frac{\partial^2 w_0}{\partial x^2} - k_1A'\frac{\partial^2\theta_0}{\partial x^2}\right) \\ = I_1k_1A'\frac{\partial\ddot{u}_0}{\partial x} + I_2(k_1A')^2\frac{\partial^2\ddot{\theta}_0}{\partial x^2} \tag{25c}$$

3. Analytical solution

To solve analytically the above equations of motion for studying the vibrational behavior of the simply supported FG nanobeam, it is better to use the Navier method which the term of displacement is assumed as follows (Ebrahimi and Salari 2015)

$$\begin{Bmatrix}u_0\\ w_0\\ \theta\end{Bmatrix} = \sum_{m=1}^{\infty}\begin{Bmatrix}u_m\,cos(\lambda x)\,e^{i\omega t}\\ w_m\,sin(\lambda x)\,e^{i\omega t}\\ \theta_m\,sin(\lambda x)\,e^{i\omega t}\end{Bmatrix} \tag{26}$$

Where the terms "u_m, w_m and θ_m" are arbitrary parameters to be found, "ω" is the eigenfrequency correspond to m-th eigenmode

Substituting the analytical solution (Navier method) of Eq. (26) in equations of motion of Eq. (25). We obtain the following matrix system

$$\left(\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix} - \xi\omega\begin{bmatrix}m_{11} & m_{12} & m_{13}\\ m_{21} & m_{22} & m_{23}\\ m_{31} & m_{32} & m_{33}\end{bmatrix}\right)\begin{Bmatrix}u_m\\ w_m\\ \theta_m\end{Bmatrix} \\ = \begin{Bmatrix}0\\ 0\\ 0\end{Bmatrix} \tag{27}$$

Where

$$
\begin{aligned}
&a_{11} = -A_{11}\lambda^2; && a_{12} = 0 \\
&a_{13} = -B_{11}k_1 A'\lambda^3; && a_{21} = a_{12} \\
&a_{22} = -A^s{}_{55}\lambda^2 + (N^T + N^H)\lambda^2 && \\
&a_{23} = -A^s{}_{55}k_1 A'\lambda^2; && a_{31} = B_{11}\lambda^3 \\
&a_{32} = A^s{}_{55}k_1 A'\lambda^2 + D_{11}k_1 A'\lambda^4 && \\
&a_{33} = -A^s{}_{55}\lambda^2 && \\
&m_{11} = -I_0; && m_{12} = 0 \\
&m_{13} = I_1 k_1 A'; && m_{21} = m_{12} \\
&m_{22} = -I_0; && m_{23} = 0 \\
&m_{31} = -I_1\lambda; && m_{32} = m_{23} \\
&m_{33} = I_2 k_1 A'\lambda^2; && \xi = 1 + \mu\lambda^2
\end{aligned}
\tag{28}
$$

4. External loads types (hygro-thermal loads)

4.1 Uniform model

In the first model the moisture and temperature raise uniformly .The moisture from “C_0” to a final value “C” with “$\Delta C = C - C_0$”. Also, the temperature from “T_0” to final value “T” with $\Delta T = T - T_0$.

4.2 Linear model

In the second model, the moisture and temperature raise linearly across the thickness of the FG nanobeam as Kiani and Eslami (2013)

$$T = T_m + \Delta T\left(\frac{z}{h} + \frac{1}{2}\right), \tag{29a}$$

$$C = C_m + \Delta C\left(\frac{z}{h} + \frac{1}{2}\right) \tag{29b}$$

Where $(\Delta T, \Delta C)$ are defines as

$$\Delta T = T - T_0 \quad \text{and} \quad \Delta C = C - C_0 \tag{30}$$

4.3 Sinusoidal model

In the Third model, the moisture and temperature rise are supposed to vary according to sinusoidal function as (Na and Kim 2004, Ebrahimi and Barati 2016b)

$$T = T_m + \Delta T\left(1 - cos\left(\frac{\pi}{2}\right)\left(\frac{z}{h} + \frac{1}{2}\right)\right), \tag{31a}$$

$$C = C_m + \Delta C\left(1 - cos\left(\frac{\pi}{2}\right)\left(\frac{z}{h} + \frac{1}{2}\right)\right) \tag{31b}$$

With $\Delta T = T - T_0$ and $\Delta C = C - C_0$.

5. Numerical results and discussions

In this work, the free vibrational behavior of the P-FG and symmetric S-FG nanobeam is investigated using an integral nonlocal shear deformation beam theory. The beam is supposed seated on elastic foundation type (Winkler-Pasternak). In the first section, several comparisons are provided to valid the current model and the second part is dedicated to the parametric studies to determine the different factors influencing the fundamental frequency of simply supported FG nanobeam reposed on elastic foundation.

5.1 Comparison and validation

To compare the current results obtained using an integral nonlocal shear deformation theory with those obtained by the others theory existing in the literature, the nondimensional fundamental frequencies and foundation parameters are presented in the following adimensional form

$$\widehat{\omega} = \omega L^2\sqrt{\frac{\rho_c A}{E_c I}}, \quad K_w = k_w\frac{L^4}{E_c I}, \quad K_s = k_s\frac{L^2}{E_c I} \tag{32}$$

Table 2 present a comparison of the nondimensional fundamental frequency “$\widehat{\omega}$” of the simply supported P-FG nanobeam under linear thermal temperature rise with ($L/h = 20$; $F_c^s = 5/6$ and $K_w = K_s = 0$). From the table, it can be seen that the current results are in good agreement with those given by Timoshenko beam theory “TBT” developed by Ebrahimi and Salari (2015) and the Classical beam theory “CBT” overestimates slightly the fundamental frequency “$\widehat{\omega}$” because of the neglect of shear deformation effect and this is insured for all values of the power law index “p” and scale effect “μ”.

The comparison of the adimensional fundamental frequency “$\widehat{\omega}$” of the P-FG nanobeam without elastic foundation ($K_w = K_s = 0$) under the three type proposed of the hygro-thermal loading (uniform, linear and sinusoidal) are presented in the Tables 3-5, respectively. Form the results shown in the tables, it is confirmed again that the current model gives almost the same values of the adimensional fundamental frequency “$\widehat{\omega}$” with the “TBT” model published by Ebrahimi and Salari (2015) and the

Table 2 Comparison of the nondimensional fundamental frequency “$\widehat{\omega}$” of the simply supported P-FG nanobeam under linear temperature rise without elastic foundation with various gradient indexes ($L = 20h$)

μ	$p = 0$			$p = 0.2$			$p = 1$			$p = 5$		
	CBT	TBT	Present	CBT	TBT	Present	CBT	TBT	Present	CBT	TBT	Present
0	9,1796	9,1475	9,1454	7,3681	7,342	7,3423	5,374	5,3537	5,3545	4,3059	4,2875	4,2878
1	8,691	8,6601	8,6601	6,967	6,9419	6,9422	5,0676	5,048	5,0488	4,0496	4,0317	4,0320
2	8,2608	8,231	8,2310	6,6135	6,5892	6,5895	4,7967	4,7777	4,7785	3,8223	3,8049	3,8052
3	7,8777	7,8488	7,8488	6,2983	6,2747	6,2750	4,5545	4,536	4,5367	3,6185	3,6015	3,6018
4	7,5334	7,5053	7,5053	6,0145	5,9916	5,9918	4,3357	4,3177	4,3184	3,4338	3,1472	3,4175

Table 3 Variation of the fundamental nondimensional frequencies "$\hat{\omega}$" of the simply supported P-FG nanobeam under uniform hygro-thermal loading (UH-TL) for various beam theories ($L = 20\ h$ and $K_w = K_s = 0$)

μ	Beam theory	$(\Delta T, \Delta C) = (0,0)$			$(\Delta T, \Delta C) = (20,1)$			$(\Delta T, \Delta C) = (40,2)$		
		$p = 0.2$	$p = 1$	$p = 5$	$p = 0.2$	$p = 1$	$p = 5$	$p = 0.2$	$p = 1$	$p = 5$
0	CBT	7,9923	5,9506	4,8629	7,4706	5,2423	4,0328	6,9006	4,4134	2,9692
	TBT	7,9683	5,9324	4,8466	7,4449	5,2216	4,0132	6,8728	4,3887	2,9423
	present	7.9687	5.9333	4.847	7.4452	5.2224	4.0134	6.8731	4.3894	2.9425
1	CBT	7,6249	5,677	4,6393	7,0759	4,9289	3,7594	6,471	4,0354	2,5845
	TBT	7,602	5,6597	4,6238	7,0512	4,9089	3,7402	6,4439	4,0108	2,5564
	Present	7.6023	5.6606	4.6242	7.0515	4.9097	3.7405	6.4442	4.0115	2.5566
2	CBT	7,3039	5,438	4,444	6,7285	4,6511	3,5148	6,0889	3,6902	2,2125
	TBT	7,2819	5,4214	4,4292	6,7047	4,6316	3,496	6,0625	3,6655	2,1824
	present	7.2823	5.4223	4.4295	6.705	4.6323	3.4962	6.0628	3.6661	2.1826
3	CBT	7,0203	5,2269	4,2714	6,4193	4,4018	3,2933	5,7452	3,37	1,8393
	TBT	6,9992	5,2109	4,2572	6,3963	4,3828	3,2747	6,7193	3,345	1,8057
	present	6.9995	5.2118	4.2575	6.3966	4.3835	3.2749	5.7196	3.3456	1.8058
4	CBT	6,7673	5,0385	4,1175	6,1415	4,176	3,0904	5,4328	3,0686	1,4438
	TBT	6,747	5,0231	4,1038	6,1191	4,1574	3,0721	5,4074	3,0431	1,4039
	present	6.7473	5.024	4.1041	6.1194	4.1581	3.0723	5.4077	3.0436	1.404

Table 4 Variation of the fundamental nondimensional frequencies "$\hat{\omega}$" of the simply supported P-FG nanobeam under linear hygro-thermal loading (LH-TL) for various beam theories ($L = 20\ h$ and $K_w = K_s = 0$)

μ	Beam theory	$(\Delta T, \Delta C) = (0,0)$			$(\Delta T, \Delta C) = (20,1)$			$(\Delta T, \Delta C) = (40,2)$		
		$p = 0.2$	$p = 1$	$p = 5$	$p = 0.2$	$p = 1$	$p = 5$	$p = 0.2$	$p = 1$	$p = 5$
0	CBT	7,9053	5,868	4,7844	7,688	5,5817	4,4153	7,46	5,2763	4,0085
	TBT	7,881	5,8496	4,7679	7,6631	5,5623	4,3974	7,4343	5,2557	3,9887
	present	7.8814	5.8505	4.7683	7.6634	5.5632	4.3977	7.4346	5.2565	3.989
1	CBT	7,5336	5,5904	4,557	7,3051	5,2885	4,167	7,0644	4,9645	3,7326
	TBT	7,5105	5,5728	4,5413	7,2812	5,2698	4,1498	7,0397	4,9445	3,7132
	Present	7.5108	5.5737	4.5416	7.2815	5.2707	4.1501	7.04	4.9453	3.7135
2	CBT	7,2086	5,3476	4,358	6,9691	5,0306	3,9478	6,7162	4,6882	3,4854
	TBT	7,1864	5,3307	4,3429	6,9461	5,0126	3,9311	6,6923	4,6688	3,4663
	present	7.1867	5.3316	4.3432	6.9464	5.0134	3.9314	6.6927	4.6695	3.4666
3	CBT	6,9211	5,1327	4,1819	6,6711	4,8011	3,7519	6,4063	4,4405	3,2612
	TBT	6,8997	5,1164	4,1674	6,6489	4,7836	3,7357	6,3831	4,4216	3,2424
	present	6.9	5.1173	4.1677	6.6492	4.7844	3.7359	6.3834	4.4223	3.2426
4	CBT	6,6644	4,9408	4,0246	6,4042	4,5949	3,5752	6,1277	4,2163	3,0556
	TBT	6,6437	4,9251	4,0105	6,3827	4,578	3,5593	6,1052	4,1978	3,037
	present	6.644	4.9259	4.0108	6.383	4.5788	3.5596	6.1055	4.1985	3.0372

"CBT" model give the biggest values of the frequency "$\hat{\omega}$" because of the omission of the shear deformation effect. It can be also seen from the tables that the fundamental frequency "$\hat{\omega}$" is in inverse relation with the power index "p" for the various values of the scale effect "μ" and all type of the hygro-thermal loading (uniform, linear and sinusoidal loads) with $(\Delta T, \Delta C) = (0,0)$, $(20,1)$ and $(40,2)$. It can be concluded that the increase in the values of the hygro-thermal load $(\Delta T, \Delta C)$ lead to decrease the values of the frequency "$\hat{\omega}$".

Table 5 Variation of the fundamental nondimensional frequencies "$\widehat{\omega}$" of the simply supported P-FG nanobeam under sinusoidal hygro-thermal loading (SH-TL) for various beam theories ($L = 20\ h$ and $K_w = K_s = 0$)

μ	Beam theory	$(\Delta T, \Delta C) = (0, 0)$			$(\Delta T, \Delta C) = (20, 1)$			$(\Delta T, \Delta C) = (40, 2)$		
		$p = 0.2$	$p = 1$	$p = 5$	$p = 0.2$	$p = 1$	$p = 5$	$p = 0.2$	$p = 1$	$p = 5$
0	CBT	7,9053	5,868	4,7844	7,7592	5,6833	4,5349	7,607	5,4897	4,2683
	TBT	7,881	5,8496	4,7679	7,7344	5,6642	4,5174	7,5818	5,4699	4,2497
	present	7,881	5,851	4,768	7.7348	5.6651	4.5178	7,582	5,471	4,25
1	CBT	7,5336	5,5904	4,557	7,3799	5,3956	4,2936	7,2194	5,1907	4,0102
	TBT	5,5105	5,5728	4,5413	7,3562	5,3773	4,2768	7,1953	5,1717	3,9922
	Present	7,511	5,574	4,542	7.3565	5.3782	4.2771	7.1955	5.1725	3.9925
2	CBT	7,2086	5,3476	4,358	7,0475	5,143	4,0811	6,8791	4,9272	3,7812
	TBT	7,1864	5,3307	4,3429	7,0248	5,1254	4,0649	6,8558	4,9087	3,7637
	present	7,187	5,332	4,343	7.0251	5.1263	4.0652	6.8561	4.9095	3.7640
3	CBT	6,9211	5,1327	4,1819	6,753	4,9188	3,892	6,5768	4,6921	3,5756
	TBT	6,8997	5,1164	4,1674	6,731	4,9018	3,8763	6,5543	4,6742	3,5585
	present	6,9	5,117	4,168	6.7313	4.9026	3.8766	6.5546	4.6750	3.5588
4	CBT	6,6644	4,9408	4,0246	6,4894	4,7178	3,7219	6,3058	4,4806	3,3892
	TBT	6,6437	4,9251	4,0105	6,4682	4,7013	3,7067	6,284	4,4631	3,3724
	present	6,644	4,926	4,011	6.4685	4.7021	3.7070	6.2842	4.4639	3.3727

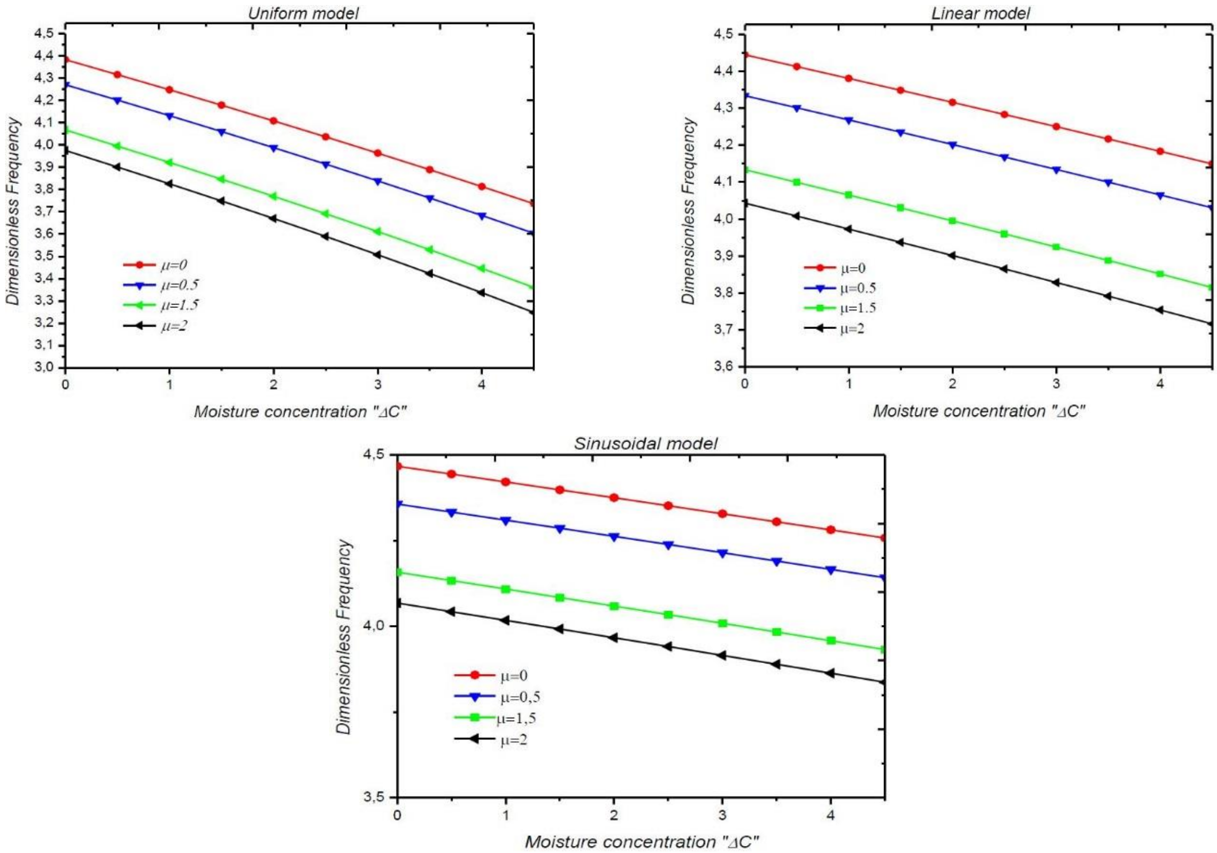

Fig. 4 Effect of moisture "ΔC" and nonlocal parameter "μ" on the dimensionless frequency "$\widehat{\omega}$" of the Symmetric S-FG nanobeam under various hygro-thermal loadings "$p = 0.1,\ L = 10\ h$ and $\Delta T = 40K$"

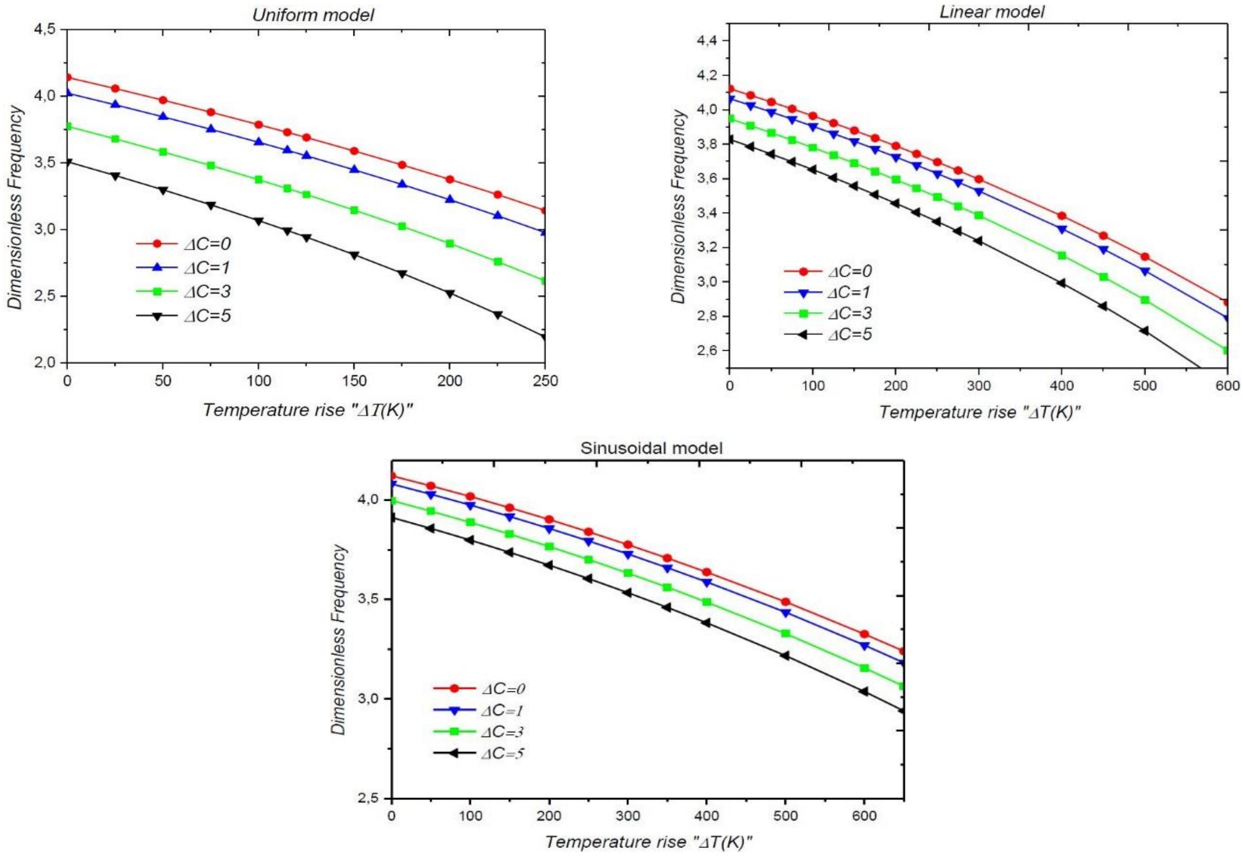

Fig. 5 Effect of moisture concentration “ΔC” on the dimensionless frequency “$\widehat{\omega}$” of the Symmetric S-FG nanobeam with respect to various temperature rises “ΔT” with “$p = 0.1$ and $L = 10\ h$”

5.2 Parametric studies

5.2.1 Symmetric S-FG nanobeam without elastic foundation

In this part present the analysis of the dynamic behavior of the symmetric S-FG nanobeam under hygro-thermal loads (HTL) without elastic foundation ($K_w = K_s = 0$) with “$p = 0.1$ and $L = 10\ h$”.

The Fig. 4 presents the variation of the nondimensional frequencies “$\widehat{\omega}$” of the simply supported symmetric S-FG nanobeam under uniform (UHTL), linear (LHTL) and sinusoidal (SHTL) hygro-thermal loads versus the moisture concentration “ΔC” and small scale effect “μ” with “$\Delta T = 40K$”. From the plotted graphs, it can be observed that the dimensionless frequencies “$\widehat{\omega}$” decrease with increasing of both moisture concentration “ΔC” and small scale effect “μ”.

The effect of the temperature rises “ΔT” and moisture “ΔC” on the dimensionless frequency “$\widehat{\omega}$” of the Symmetric S-FG nanobeam under various hygro-thermal

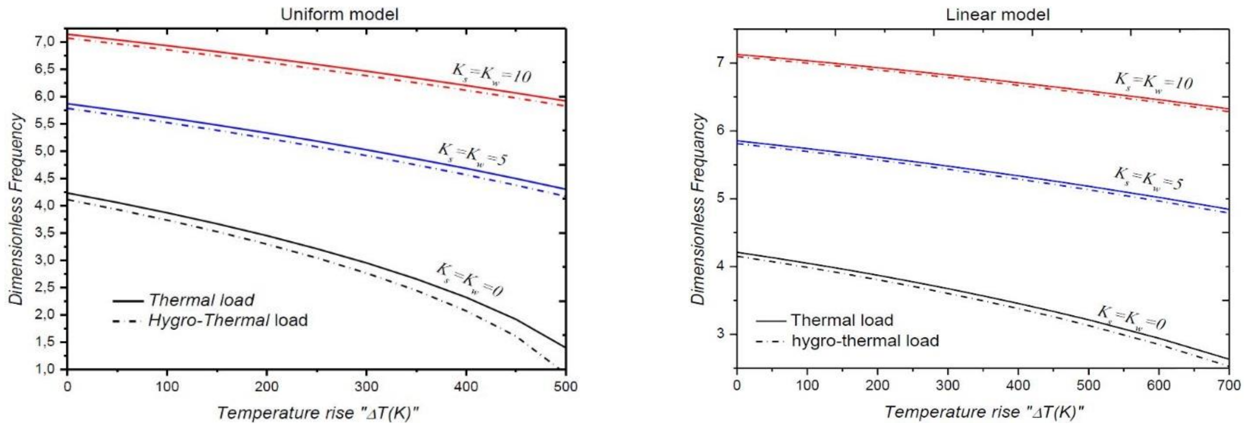

Fig. 6 Influence of elastic foundation on the dimensionless frequency “$\widehat{\omega}$” of the symmetric S-FG nanobeam versus the temperature change “ΔT” for thermal, “$\Delta C = 0$” and hygro-thermal, “$\Delta C = 1$” environments with “$p = 0.1,\ L = 10\ h$ and $\mu = 1.5\ nm$”

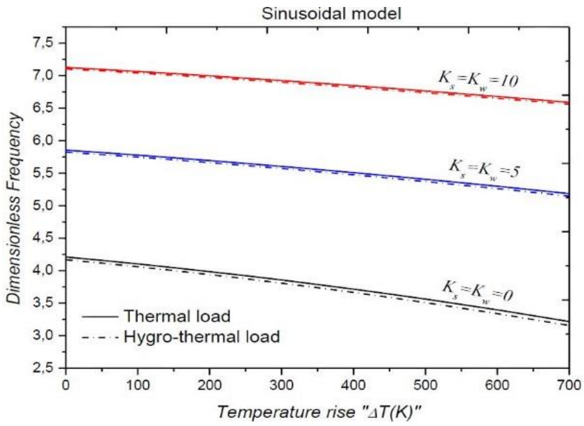

Fig. 6 Continued

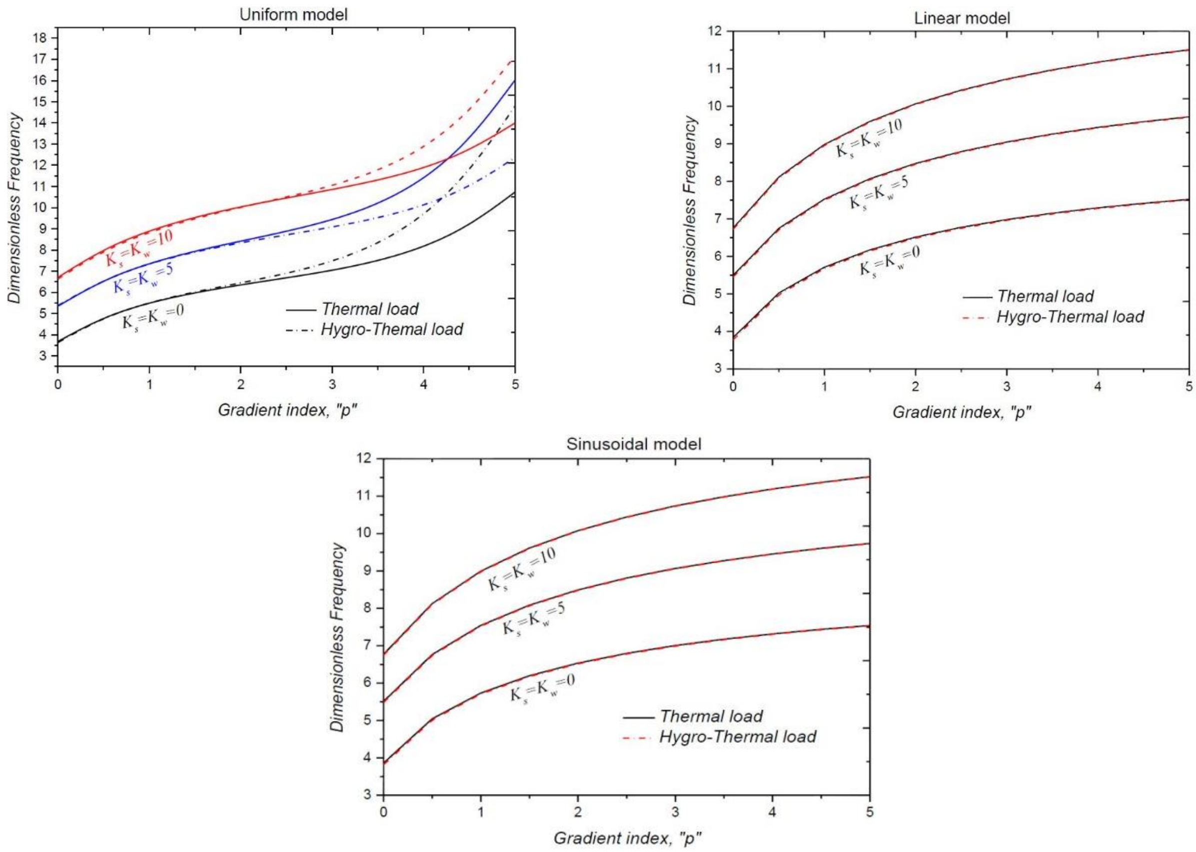

Fig. 7 Influence of power index "p" on the dimensionless frequency "$\widehat{\omega}$" of the symmetric S-FG nanobeam for thermal, "$\Delta C = 0$" and hygro-thermal, "$\Delta C = 1$" environments with "$L = 10\ h, \Delta T = 40\ K$ and $\mu = 1.5\ nm$"

loadings are drawn in Fig. 5. It is remarkable from the obtained results that the increasing in the temperature "ΔT" and moisture "ΔC" leads to the decrease in the values of the frequency "$\widehat{\omega}$" and this is valid to the three types of the hygro-thermal loads (UHTL, LHTL and SHTL). The lower values of the dimensionless frequencies "$\widehat{\omega}$" are obtained by UHTL model (uniform).

5.2.2 Symmetric S-FG nanobeam on elastic foundation

The Fig. 6 illustrate the variation of the values of the dimensionless frequency "$\widehat{\omega}$" of the symmetric S-FG nanobeam under uniform, linear and sinusoidal thermal and hygro-thermal loads versus the elastic foundation parameters "K_w and K_s" and the temperature rise "ΔT" with "$p = 0.1, L = 10\ h$ and $\mu = 1.5\ nm$". It can be seen from the figures that the dimensionless frequency "$\widehat{\omega}$" is in inverse relation with the elastic foundation parameters and temperature "ΔT". The frequencies of the FG nanobeam under only thermal load "$\Delta C = 0$" gives higher values of frequency relative to the FG nanobeam under hygro-thermal load with "$\Delta C = 1$".

The variation of the values of the dimensionless frequency "$\widehat{\omega}$" of the symmetric S-FG nanobeam under

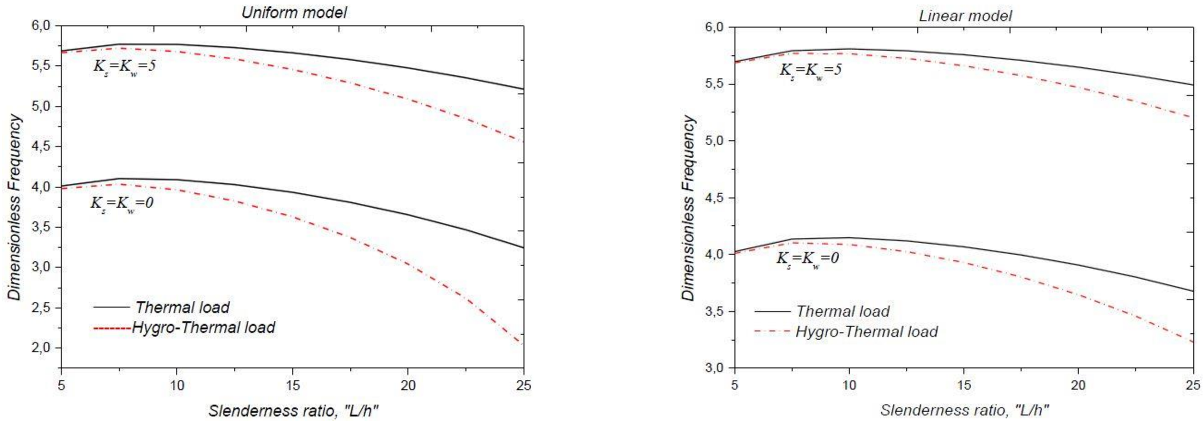

Fig. 8 Effect of slenderness ratio *“*L/h*”* on the dimensionless frequency *“*$\widehat{\omega}$*”* of the symmetric S-FG nanobeam under uniform and linear moisture rises *“*$p = 0.1, L = 10\ h, \Delta T = 40\ K$ and $\mu = 1.5\ nm$*”*

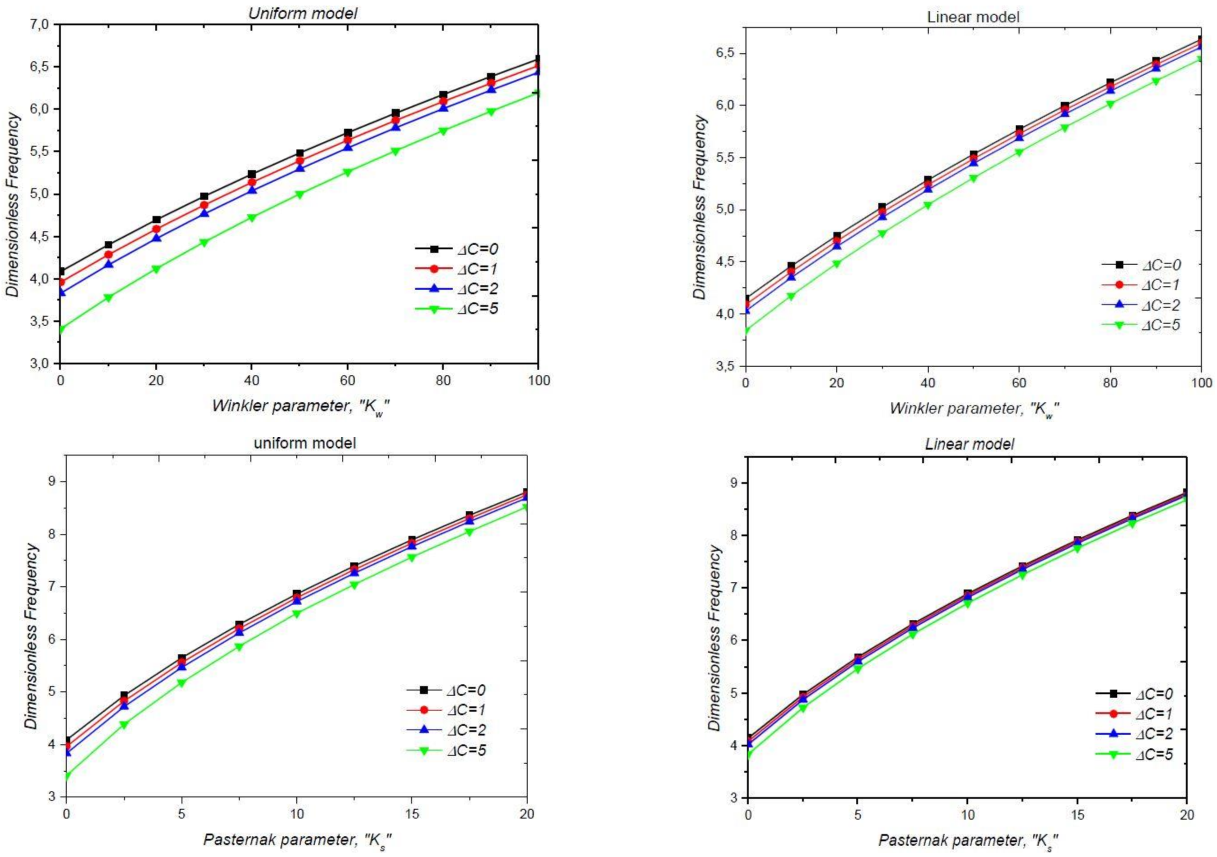

Fig. 9 Influence of the elastic foundation parameters *“*K_w, K_s*”* on the dimensionless frequency *“*$\widehat{\omega}$*”* of the FG nanobeam under uniform and linear moisture rises *“*$p = 0.1, L = 10\ h, \Delta T = 40\ K$ and $\mu = 1.5\ nm$*”*

thermal “$\Delta C = 0$” and hygro-thermal “$\Delta C = 1$” loads versus the power index “p” and elastic foundation parameter “K_w and K_s” is presented in Fig. 7. It can be noted from the plotted curves that the dimensionless frequency “$\widehat{\omega}$” is in direct correlation relation with the material index *“*p*”* and elastic foundation and this is valid for all type of distributions (uniform, linear and sinusoidal).

Effect of the geometry ratio “L/h” on the adimensional frequency “$\widehat{\omega}$” of the symmetric S-FG nanobeam under uniform and linear moisture rises “ΔC” with “$p = 0.1, L = 10\ h, \Delta T = 40\ K$ and $\mu = 1.5\ nm$*”* is illustrated in the plotted curves in the Fig. 8.

We can observe that the the increase of the slenderness ratio lead to the slight increase in the adimensional frequency “$\widehat{\omega}$” to a maximum value for “$L/h = 7.5$” than the frequency *“*$\widehat{\omega}$*”* decreases because the nanobeam becomes slender.

Fig. 9 shows the variations of the dimensionless frequency “$\widehat{\omega}$” of the symmetric S-FG nanobeam under uniform and linear moisture rises “ΔC” with ”$p = 0.1$, and $\mu = 1.5\ nm$” versus the effects of the Winkler and Pasternak parameter “K_w, K_s”. From the obtained results, it can be concluded that the presence of the elastic foundation leads to increase the nondimensional frequency "$\hat{\omega}$" because the symmetric S-FG becomes stiffer. It is also remarkable that the increasing in the value of the moisture rises decreases the values of the frequency "$\hat{\omega}$" and this is valid for the uniform and linear hygro-thermal loading.

6. Conclusions

In the current paper, the nonlocal integral Timoshenko beam theory is developed for the free vibration analysis of P-FG and symmetric S-FG nanobeam seated on Winkler-Pasternak foundation subjected to the thermal and hygro-thermal loading. The thermo-elastic materials propertiesare considered nonlinearly distributed. The analytical solution of the present investigation is obtained based on the Hamilton and Navier model. The several numerical comparisons and parametric studies has been presented and discussed in detail to show the validity of the present model and determining the various parameters influencing the fundamental frequency of the FG nanobeam.An improvement of the present formulation will be considered in the future work to consider other type of materials (Sharma *et al*. 2009, Kolahchi *et al*. 2016, Bozyigit and Yesilce 2016, Daouadji 2017, Lal *et al*. 2017, Salamat and Sedighi 2017, Kar *et al*. 2017, Panjehpour *et al*. 2018, Selmi and Bisharat 2018, Shahadat *et al*. 2018, Ayat *et al*. 2018, Li *et al*. 2018, Behera and Kumari 2018, Narwariya *et al*. 2018, Akbas 2019b, Katariya *et al*. 2019, Safa *et al*. 2019, Othman *et al*. 2019, Yüksela and Akbaş 2019, Abdou *et al*. 2019, Rajabi and Mohammadimehr 2019, Avcar 2019, Selmi 2019, Esmaeili and Beni 2019, Hadji *et al*. 2019, Sahouane *et al*. 2019).

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