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Modal analysis of viscoelastic nanorods under an axially harmonic load

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Abstract. Axially damped forced vibration responses of viscoelastic nanorods are investigated within the frame of the modal analysis. The nonlocal elasticity theory is used in the constitutive relation of the nanorod with the Kelvin-Voigt viscoelastic model. In the forced vibration problem, a cantilever nanorod subjected to a harmonic load at the free end of the nanorod is considered in the numerical examples. By using the modal technique, the modal expressions of the viscoelastic nanorods are presented and solved exactly in the nonlocal elasticity theory. In the numerical results, the effects of the nonlocal parameter, damping coefficient, geometry and dynamic load parameters on the dynamic responses of the viscoelastic nanobem are presented and discussed. In addition, the difference between the nonlocal theory and classical theory is investigated for the damped forced vibration problem.

Keywords: nanorods; nonlocal elasticity theory; damped forced vibration; modal analysis

1. Introduction

With the development of science and technology, the applications of nano engineering have found many areas. The using of nanostructures is increasing in the engineering applications such as electro-mechanical devices, actuators, atomic microscopes from day to day.

In the mechanical solutions of the nano structures are major problems. The nonlocal continuum theories are preferred in the approximate solution of nano structures because of low computational cost. The nonlocal continuum theories consist of size effect in contrast with classical continuum theory. The main nonlocal elasticity theories are the couple stress theory, strain gradient theory, Eringen's nonlocal elasticity theory.

In the literature, the studies about forced vibration of nanorods are as follows; Adhikari *et al*. (2014) investigated dynamic analysis of nano rods resting on elastic medium by using finite element method within frequency domain. Akgöz and Civalek (2014) analyzed longitudinal vibration of microbars based on strain gradient elasticity theory based on strain gradient elasticity theory. Akbaş (2016a) investigated forced vibration responses of a viscoelastic nanobeam embedded elastic medium by using finite element method. Eltaher *et al*. (2016) investigated effects of the thermal load on the stability of nanobeams by using Nonlocal theory. Li *et al*. (2016) investigated longitudinal free vibration of the nanorods based on nonlocal strain gradient theory. Akbaş (2017d) examined forced vibration responses of functionally graded nanobeams. Xu *et al*. (2017) investigated vibration results of the nanorods with different boundary conditions based on nonlocal strain gradient theory. Hadji *et al*. (2017) investigated free vibration of carbon nanotubes reinforced composite beams embedded elastic foundation with stretching effect. Akbaş (2016b, 2017a, b, 2018c) analyzed vibration and static analysis of cracked nano structures. Ebrahimi and Salari (2018) examined effects of the temperature on the vibration and buckling behavior of functionally graded nanobeams based on nonlocal elasticity theory. Ghayesh (2018) presented an investigation about nonlinear vibrations of axially functionally graded microbeams by using couple stress theory and Galerkin method. Jena and Chakraverty (2018) investigated nonlocal vibration of nanobeams by using differential transform method. Akbaş (2018a, b, 2019a, b) analyzed the effects of cracks on the forced vibration responses of nano beams and rods. Kumar (2018) investigated vibration analysis of carbon nanotubes with considering Van der waals force by using differential transform method. Wu *et al*. (2018) examined nonlinear vibration of carbon nanotubes with Eringen's nonlocal elasticity theory Timoshenko beam theory. Pavlović *et al*. (2019) investigated dynamic stability of nanobeams with nonlocal strain gradient theory. Martin (2019) examined fractional dynamic analysis of viscoleastic nanobeams with nonlocal elasticity theory.

In the open literature, the modal analysis of nanorods under dynamic loads has not been investigated in broadly. This is a blank for nonlocal nanorods. The main goal of this study is to investigate damped forced vibration of the nanorods with modal analysis and present the modal expressions of the nanorods. In this study, longitudinal damped forced vibration of a viscoelastic cantilever nanorod subjected to a harmonic load is presented by the nonlocal Elasticity theory. The considered problem is solved within the frame of the modal analysis. In the damped effect of the nanorod, the Kelvin-Voigt viscoelastic model is used within the nonlocal elasticity theory. The modal expressions of the problem are obtained and solved by analytically. The effects of the nonlocal parameter,

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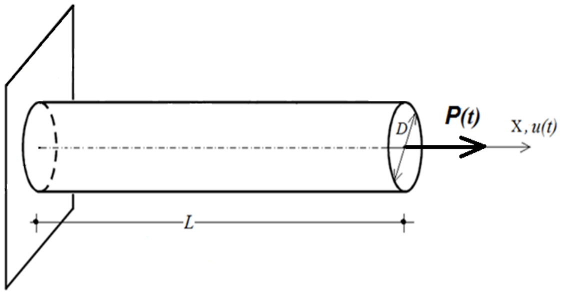


Fig. 1 A cantilever circular viscoelastic nanorod subjected to dynamically point load

damping coefficient, geometry and dynamic load parameters on the dynamic responses of the viscoelastic nanorod are investigated. Also, the difference between the nonlocal theory and classical theory is investigated within the dynamic responses of the nanorod.

2. Theory and formulation

Fig. 1 shows a cantilever viscoelastic nanorod with circular cross-section subjected to dynamically point force ($P(t)$). The load is subjected at the free endnjd d of nanorod. In Fig. 1, L and D indicate the length and the diameter of the nanarod, respectively.

Based on the nonlocal elasticity theory, constitutive equation of the problem is given (Eringen 1972, 1983)

$$\sigma_{xx} - \mu \frac{d^2\sigma_{xx}}{dx^2} = E\,\epsilon_{xx} \tag{1}$$

By adding the Kelvin-Voight viscoelastic model into the nonlocal constitutive relation in Eq. (1), the viscoelastic nonlocal constitutive equation is obtained as follows

$$\sigma_{xx} - \mu \frac{d^2\sigma_{xx}}{dx^2} = E\left(\epsilon_{xx} + \gamma \frac{d\epsilon_{xx}}{dt}\right) \tag{2}$$

where, σ_{xx} and ϵ_{xx} are nonlocal normal stress and strain, respectively. E, γ and μ are Young's modulus, viscous damping coefficient and nonlocal parameter, respectively. where $\mu = (e_0 a)^2$, e_0 indicates length scale parameter. t indicates the time. It is noted that, the Eq. (2) is induced to classical continuum theory when $\mu = 0$. By using equilibrium of forces in axially direction, the equation of motion is expressed as follows

$$\rho A \frac{\partial^2 u(x,t)}{\partial t^2} - EA\frac{\partial^2 u(x,t)}{\partial x^2} - EA\gamma \frac{\partial^2}{\partial x^2}\left(\frac{\partial u(x,t)}{\partial t}\right) - \rho A\mu \frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u(x,t)}{\partial t^2}\right) = f(x,t) \tag{3}$$

where, ρ and u are mass density and axial displacement function, respectively. $f(x,t)$ indicates the distributed axial load though the x direction.

For the free vibration solution, the Eq. (3) is reduced as following equation

$$\rho A \frac{\partial^2 u(x,t)}{\partial t^2} - EA\frac{\partial^2 u(x,t)}{\partial x^2} - \rho A\mu \frac{\partial^2}{\partial x^2}\left(\frac{\partial^2 u(x,t)}{\partial t^2}\right) = 0 \tag{4}$$

In the solution of the free vibration problem, the separation of variable technique is used in Eq. (4)

$$u_h(x,t) = U_h(x) e^{i\omega t} \tag{5}$$

where $U_h(x)$ is spatially function. ω is the natural frequency and i indicates imaginary number.

The boundary conditions of the cantilever (clamped-free) the nanorod are presented as follows

$$u_h(0,t) = 0, \quad \frac{\mathrm{d}u_h(L,t)}{\mathrm{dx}} = 0 \tag{6}$$

Substituting Eq. (5) into Eq. (4) gives following equations of motion

$$\left(\frac{d^2 U_h(x)}{dx^2} + \beta^2 U_h(x)\right) e^{i\omega t} = 0 \tag{7}$$

where

$$\beta^2 = \frac{\rho\omega^2}{E - \rho\,\mu\,\omega^2} \tag{8}$$

With implementing the boundary conditions, the solution of the Eq. (7) gives the following frequency equation and mode function

$$cos\beta L = 0 \tag{9a}$$

$$\beta_k L = (k - 0.5)\pi, \quad k = 1,2,3 \ldots \tag{9b}$$

$$U_k(x) = sin\left(\frac{(k - 0.5)\pi x}{L}\right) \tag{9c}$$

where $U_k(x)$ is the mode shape functions of the problem. By substitute Eq. (8) into Eq. (9b) gives following equations of frequency

$$\omega_k = \sqrt{\frac{E}{\rho}} \frac{(\mathrm{k} - 0.5)\pi}{\sqrt{\mathrm{L}^2 + \mu\,(\mathrm{k} - 0.5)^2\,\pi^2}}, \quad k = 1,2,3 \ldots \tag{10}$$

In the solution of the damped forced vibration problem, the modal analysis technique is used. In the solution of Eq. (3) in the modal analysis, the solution function $u(x,t)$ is defined by mode superposition method in the modal space as follows

$$u(x,t) = \sum_{\mathrm{k}} U_k(x) T_k(t) \tag{11}$$

where $T_k(t)$ is modal coordinate functions. Substituting Eq. (11) into Eq. (3) gives following equation

$$\rho A \sum_k U_k \frac{\partial^2 T_k(t)}{\partial t^2} - EA \sum_k \frac{\partial^2 U_k(x)}{\partial x^2} T_k(t) \tag{12}$$

$$-EA\gamma\sum_k \frac{\partial^2 U_k(x)}{\partial x^2}\frac{\partial T_k(t)}{\partial t} -\rho A\mu\sum_k \frac{\partial^2 U_k(x)}{\partial x^2}\frac{\partial^2 T_k(t)}{\partial x^2} = f(x,t) \tag{12}$$

With using the orthogonality properties of modal coordinate functions, the following modal equations can be obtained

$$\widehat{M}_{\mathrm{k}}\frac{d^2T_k(t)}{dt^2}+\hat{C}_{\mathrm{k}}\frac{\mathrm{d}T_k(t)}{\mathrm{d}t}+\widehat{K}_{\mathrm{k}}T_k(t)=\hat{f}_k(t) \tag{13}$$

where $\widehat{M}_k$ is the modal mass, $\hat{C}_k$ is the modal damping, $\widehat{K}_k$ is the modal rigidity and $\hat{f}_k$ is the modal load. The detail of these expressions are given as follows

$$\widehat{M}_{\mathrm{k}} = \int_0^{\mathrm{L}} \rho A\left(sin\left(\frac{(k-0.5)\pi x}{L}\right)^2 + \mu\left(cos\left(\frac{(k-0.5)\pi x}{L}\right)\frac{(k-0.5)\pi x}{L}\right)^2\right)dX \tag{14}$$

$$\hat{C}_{\mathrm{k}} = \int_0^{\mathrm{L}} EA\gamma\left(cos\left(\frac{(k-0.5)\pi x}{L}\right)\frac{(k-0.5)\pi x}{L}\right)^2 dX \tag{15}$$

$$\widehat{K}_{\mathrm{k}} = \int_0^{\mathrm{L}} EA\left(cos\left(\frac{(k-0.5)\pi x}{L}\right)\frac{(k-0.5)\pi x}{L}\right)^2 dX \tag{16}$$

$$\hat{f}_k(t) = \int_0^L f(x,t)U_k(x)\,dX \tag{17a}$$

$$f(x,t) = \begin{Bmatrix} P(t) & x=L \\ 0 & other \end{Bmatrix} \tag{17b}$$

After integration process of the Eqs. (14)-(17), the modal expressions can be obtained as follows

$$\widehat{M}_{\mathrm{k}} = \frac{\rho AL}{2}+\frac{\rho\, A\,\mu\,\pi^2}{8L}(2k-1)^2 \tag{18}$$

$$\widehat{K}_{\mathrm{k}} = \frac{E\,A\,\pi^2}{8L}(2k-1)^2 \tag{19}$$

$$\hat{C}_{\mathrm{k}} = \frac{E\,A\,\gamma\,\pi^2}{8L}(2k-1)^2 \tag{20}$$

$$\hat{f}_k(t) = P(t)\,sin((k-0.5)\pi) \tag{21}$$

After the simplifying expression (13), the following modal equation for *k*th mod is obtained as follows

$$\frac{d^2T_k(t)}{dt^2}+2\,\xi_k\,\omega_k\frac{\mathrm{d}T_k(t)}{\mathrm{d}t}+\omega^2{}_k\,T_k(t)=\frac{\hat{f}_k(t)}{\widehat{M}_{\mathrm{k}}} \tag{22}$$

where ξ_k is modal damping ratio which expressed as follows

$$\xi_k = \frac{\hat{C}_{\mathrm{k}}}{2\,\omega_k\,\widehat{M}_{\mathrm{k}}} \tag{23}$$

The external dynamically load (P(t)) is considered a harmonic function as follows

$$P(t) = P_0\,cos\,(\Omega t) \tag{24}$$

where P_0 and Ω are the amplitude and frequency of load, respectively. Substituting Eq. (24) into Eq. (22) gives following equation

$$\frac{d^2T_k(t)}{dt^2}+2\,\xi_k\,\omega_k\frac{\mathrm{d}T_k(t)}{\mathrm{d}t}+\omega^2{}_k\,T_k(t) = \frac{P_0\,sin((k-0.5)\pi)cos\,(\Omega t)}{\widehat{M}_{\mathrm{k}}} \tag{25}$$

The solution form of the differential Eq. (25) is given as follows

$$T_k(t) = Q_k\,cos\,(\Omega t-\emptyset_k) \tag{26}$$

where Q_k and $\emptyset_k$ indicate the modal amplitude and phase angle for *k*th mod, respectively. Substituting Eq. (26) into Eq. (25) and solving the differential equation, the expressions of Q_k and $\emptyset_k$ are obtained as follows

$$Q_k = \frac{P_0\,sin((k-0.5)\pi)}{\widehat{K}_k}\frac{1}{\sqrt{(1-\beta_k^2)^2+(2\,\xi_k\,\beta_k)^2}} \tag{27}$$

$$\emptyset_k = \mathrm{arctg}\left(\frac{2\,\xi_k\,\beta_k}{1-\beta_k^2}\right) \tag{28}$$

where β_k is the frequency ratio

$$\beta_k = \frac{\Omega}{\omega_k} \tag{29}$$

For *k*th mod, the static displacement function (q_k^{st}) can expressed as follows

$$q_k^{st} = \frac{P_0\,sin((k-0.5)\pi)}{\widehat{K}_k} \tag{30}$$

The modal magnification ratio (θ) for *k*th mod is expressed as follows

$$\theta(\beta_k) = \frac{Q_k}{q_k^{st}} = \frac{1}{\sqrt{(1-\beta_k^2)^2+(2\,\xi_k\,\beta_k)^2}} \tag{31}$$

The modal magnification ratio (θ) depends on the frequency ratio (β_k) and the modal damping ratio. The dimensionless quantities are expressed as follows

$$\eta=\frac{e_0a}{D},\quad \bar{\Omega}=\sqrt{\frac{\rho\,D^2}{E}}\Omega,\quad \lambda=\frac{L}{D},\quad \bar{U}=\frac{U_p}{L} \tag{32}$$

where η and $\bar{\Omega}$ indicate the dimensionless nonlocal

Table 1 Comparison study: Dimensionless longitudinal frequencies of the clamped-free nanorod

$\frac{e_0 a}{L}$		$\bar{\omega}_1$	$\bar{\omega}_2$	$\bar{\omega}_3$	$\bar{\omega}_4$
0	Xu *et al*. (2017)	1.57080	4.71239	7.85398	10.99557
	Li *et al*. (2016)	1.57080	4.71239	7.85398	10.99557
	Present	1.57080	4.71239	7.85398	10.99557
0.1	Xu *et al*. (2017)	1.55177	4.26279	6.17668	7.39805
	Li *et al*. (2016)	1.55177	4.26279	6.17668	7.39805
	Present	1.55177	4.26279	6.17668	7.39805
0.2	Xu *et al*. (2017)	1.49858	3.42933	4.21782	4.55152
	Li *et al*. (2016)	1.49858	3.42933	4.21782	4.55152
	Present	1.49858	3.42933	4.21782	4.55152

parameter and the dimensionless the frequency of the dynamic load, respectively. λ is the aspect ratio and $\bar{U}$ is dimensionless the longitudinal displacement. When $\eta = 0$, the problem is induced to classical continuum theory.

3. Numerical results

In numerical examples, effects of the damping coefficient, dimensionless nonlocal parameter and dimensionless the frequency of the dynamic load on the forced vibration responses of the viscoelastic nanorod are presented. The material of the nanorod is considered as epoxy (E = 1,44 GPa, $\rho = 1600\ \text{kg/m}^3$). The diameter of the nanorod is taken as D = 1 nm. The amplitude of dynamic load is taken as $P_0 = 1$ nN. The length of the nanorod is selected according to the aspect ratio (λ).

In Table 1, dimensionless longitudinal frequencies of the clamped-free nanorod are presented for different values of nonlocal parameters on the purpose of the accuracy of the present study. In the comparison study of the Table 1, the results of the Xu *et al*. (2017) and Li *et al*. (2016) are compared with the results of this study. It is seen from table 1, the results of this study are in good agreement with the results of the Xu *et al*. (2017) and Li *et al*. (2016).

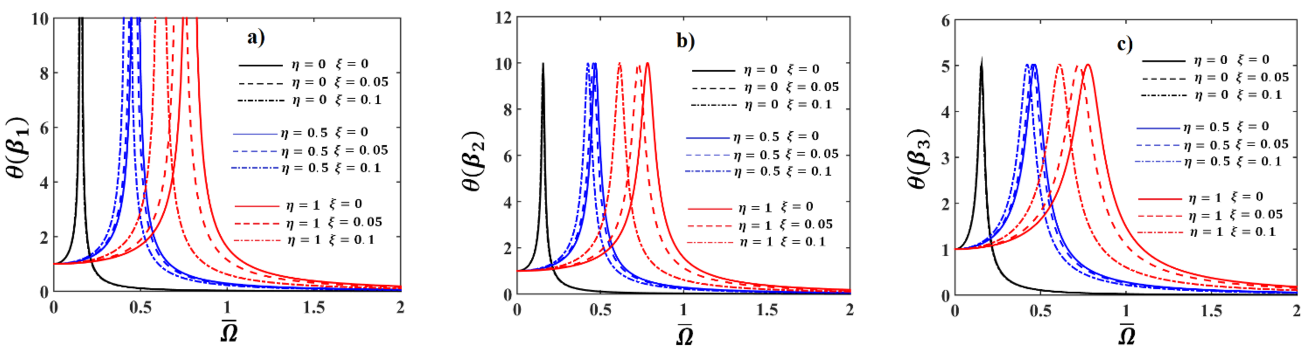
Fig. 2 shows the relationship between the modal magnification ratio (θ) and the dimensionless frequency of the dynamic load ($\bar{\Omega}$) is presented in first three modes for different values of damping ratio (ξ) and the dimensionless nonlocal parameter (η) for $\lambda = 10$.

It is seen from in the Fig. 2, increasing dimensionless nonlocal parameter (η) yields to increase the resonance frequency. The resonance phenomenon can be observed in the vertical asymptote regions in Fig. 2. With increase of the dimensionless nonlocal parameter, the damping effect increases considerably in all modes. The difference among the results of the damping ratios increases while the dimensionless nonlocal parameter increases. In the higher value of the dimensionless nonlocal parameter, the damping effect becomes even more important on the forced vibration responses of the nanorods.

Another result of the Fig. 2 that effects of the damping ratio on the magnification ratio are highest level in higher value of modes. With increase of the number of modes, the damping effects increases significantly. Also, the difference among the results of the damping ratios increase significantly with increase of modes.

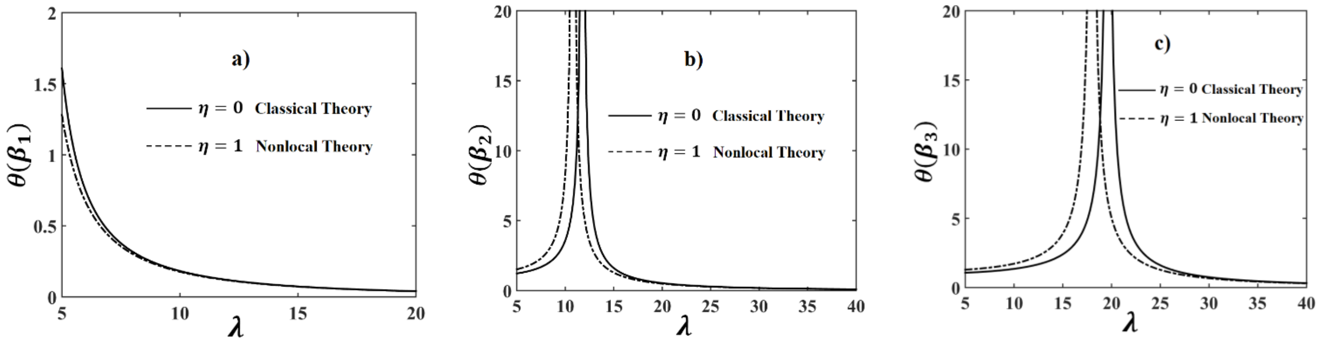
In Fig. 3, effects of the aspect ratio (λ) on the forced vibration responses of the nanorod are investigated. For this purpose, the modal magnification ratios are calculated for different the dimensionless nonlocal parameters in first three modes for $\bar{\Omega} = 0.4$ and $\xi = 0.01$. Also, the difference between the nonlocal theory and classical continuum theory is studied. The dimensionless nonlocal parameters are selected as $\eta = 0$ and $\eta = 1$. It is stated before that the results of classical theory can be obtained when $\eta = 0$. So, the results of the $\eta = 0$ represents the classical theory. Whereas, the results of the $\eta = 1$ represents the nonlocal theory in Fig. 3.

As seen from Fig. 3 that increasing the aspect ratio (λ) yields to decrease the difference between results of the nonlocal theory and classical theory significantly. In higher values of the aspect ratios, the results of the two theories coincide completely. It shows that the classical continuum theory can be used instead of nonlocal theory for higher values of the aspect ratio. Another result of Fig. 3 that with increase of the number of modes, the difference between results of the nonlocal theory and classical theory increases.



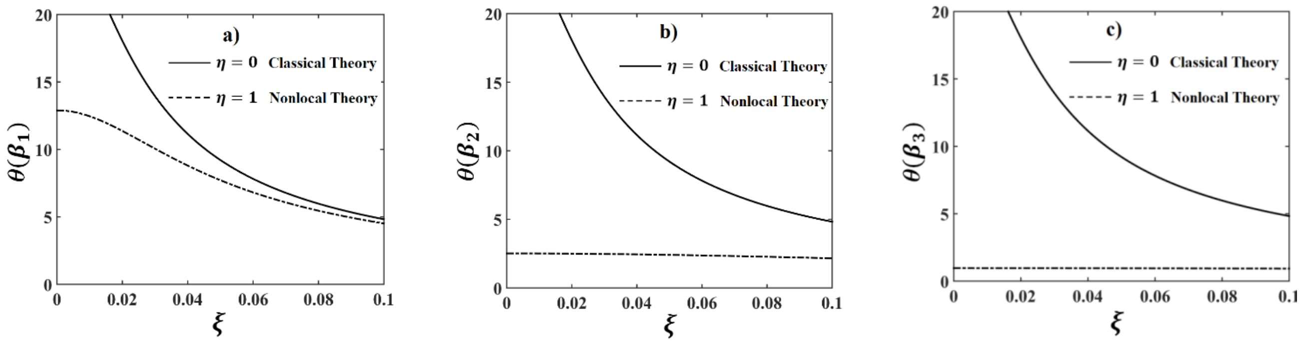
(a) For first vibration mod (b) For second vibration mod (c) For third vibration mod

Fig. 2 The modal magnification ratio (θ) versus the dimensionless frequency of the dynamic load ($\bar{\Omega}$) rising different values of damping ratio (ξ) and the dimensionless nonlocal parameter (η)



(a) For first vibration mod (b) For second vibration mod (c) For third vibration mod

Fig. 3 The relationship between modal magnification ratio and aspect ratio (λ) for nonlocal and classical theories



(a) For first vibration mod for $\bar{\Omega} = 0.2$ (b) For second vibration mod for $\bar{\Omega} = 0.6$ (c) For third vibration mod for $\bar{\Omega} = 01$

Fig. 4 The relationship between modal magnification ratio and damping ratio (ξ) for nonlocal and classical theories

Although, this difference completely closes after values of $\lambda = 12$ in first mode, the difference closes after values of $\lambda = 30$ in third mode. So, the mode numbers have important role on the difference between results of the nonlocal theory and classical theory.

Fig. 4 displays the relationship between the modal magnification ratio (θ) and the damping ratio (ξ) in first three modes for different values of dimensionless nonlocal parameters for $\eta = 0$ and $\eta = 1$. In these figures, $\eta = 0$ represents the classical theory. In addition, the difference between results of the nonlocal theory and classical continuum theory for different the damping ratios can be observed in Fig. 4.

It is observed from Fig. 4 that the increasing in the damping ratio causes a decreasing in magnification ratio and the difference between results of the nonlocal theory and classical continuum theory significantly. Also, the damping effect is more effect in the first vibration mode in contrast with the second and third modes. In first mode, the difference between results of the nonlocal theory and classical continuum theory is very small in smaller values of damping ratio. However, this difference is not small in smaller values of damping ratio. This difference closes in higher values of damping ratio with increase of mode number. It shows that the damping ratio play important role on forced vibration responses of nonlocal rods. Also, the damping effect change with different modes, significiantly.

4. Conclusions

Damped forced vibration results of viscoelastic nanorods under axially harmonic load are studied based on the nonlocal elasticity theory. The Kelvin-Voight viscoelastic model is used within the nonlocal elasticity theory. The modal analysis technique is used in the solution of the dynamic problem and modal expressions are developed for the nonlocal nanorods. In the numerical examples, effects of the damping coefficient, dimensionless nonlocal parameter and dimensionless the frequency of the dynamic load on the forced vibration responses of the viscoelastic nanorod are presented and discussed. In the obtained the results, the major findings are as follows:

- By increasing the nonlocal parameter and the number of modes, effects of damping on the nanorod increases significantly.
- By increasing the aspect ratios, the results of the nonlocal and classical theory coincide completely.
- The damping ratio is very effective in the nonlocal results and the difference between results of the nonlocal and classical theories.
- By increasing the mode numbers, the effects of the nonlocal parameters and damping ratio on the dynamic responses of the nanorods change considerably.
- The classical continuum theory can be used instead of nonlocal theory for higher values of the aspect ratios.

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