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Forward and backward whirling of a spinning nanotube nano-rotor assuming gyroscopic effects

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Abstract. This work examines the fundamental vibrational characteristics of a spinning CNT-based nano-rotor assuming a nonlocal elasticity Euler-Bernoulli beam theory. The rotary inertia, gyroscopic, and rotor mass unbalance effects are all taken into consideration in the beam model. Assuming a nonlocal theory, two coupled 6th-order partial differential equations governing the vibration of the rotating SWCNT are first derived. In order to acquire the natural frequencies and dynamic response of the nano-rotor system, the nonlinear equations of motion are numerically solved. The nano-rotor system frequency spectrum is shown to exhibit two distinct frequencies: one positive and one negative. The positive frequency is known as to represent the forward whirling mode, whereas the negative characterizes the backward mode. First, the results obtained within the framework of this numerical study are compared with few existing data (i.e., molecular dynamics) and showed an overall acceptable agreement. Then, a thorough and detailed parametric study is carried out to study the effect of several parameters on the nano-rotor frequencies such as: the nanotube radius, the input angular velocity and the small scale parameters. It is shown that the vibration characteristics of a spinning SWCNT are significantly influenced when these parameters are changed.

Keywords: single-walled carbon nanotube; nano-rotor; forward and backward whirling; nonlocal elasticity

1. Introduction

Carbon nanotubes (CNTs), first designed by Iijima (1991), are mostly categorized into two structural groups: the single-walled nanotube group (SWCNTs) and the multi-walled carbon nanotube group (MWCNTs). SWCNTs represent tiny cylinders made from a sheet of carbon atoms (Basirjafari *et al*. 2013a) defined as a layer of a graphite rolled-up into a continuous cylindrical shape of one carbon atom thickness (Natsuki *et al*. 2013). These nano-structures possess numerous unique and attractive properties: strong structures, extremely light weight, excellent heat and electricity conducting properties. Thanks to these distinctive properties, they are substantially used in numerous nano-electronics, nano-devices, and nano-composites based devices. Very frequently, these CNT based nano-devices are subjected to external loadings during operation and their vibrational characteristics and resonant properties are therefore of major concern. Consequently, establishing a trustful model considering the accurate vibrational properties of CNT based nano- structures are important factors for a successful final design. Therefore, research work in the CNT based nano-structures have seen a tremendous growth in the past few decades.

There have been several published studies investigating the vibration behavior of beam like nanostructures using continuum model (Rao 2000, Gupta *et al*. 2010). It was established that the CNT mechanical properties depend strongly on their micro and nanoscopic structure (Bouaouina *et al*. 2018). The classical-continuum mechanics (CCM) rules are somehow shown to lead to inaccurate results in the material domains where the typical micro and nano-structural dimensions are alike the structural one. At present, numerous modifications to the Classical continuum mechanics theory have been suggested to take the micro-structural structures into account: for instance, the generalized continuum mechanics theory (Anderson *et al*. 2005). One particular example~~,~~ was proposed by Eringen (1972a) in the name of nonlocal mechanic’s theory. This particular theory states that: “the stress tensor at a point is a function of strains at all points in the continuum” (Ebrahimi *et al*. 2019, Aydogdu and Arda 2016). This principle is a bit dissimilar from the CCM theory which specifies that: “the stress at a point is a function of strain at that particular point” (Basirjafari *et al*. 2013a). Then, and only recently, a considerable effort has been devoted to the problem of the vibration of these nano-structures considering the micro and nano size effects (Gupta and Batra 2008). In addition, only assuming nonlocal gradient elasticity theory showed to capture simply softening behavior of nanostructures like CNTs (Rao 2000).

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Therefore, in order to predict the two different behaviors simultaneously (hardening and softening) and combine both possible features to mimic the real dynamical behavior of the nano-structure, it is convenient to merge two theories: the strain and velocity gradient energies theories. As an example, Ouakad *et al*. (2018) formulated the strain and velocity gradient theory for the Euler-Bernoulli beam theory to examine the nonlinear free vibration problem of CNT based nano-resonator. They generalized the strain energy to account for the strain and its gradient as well. In addition, they also generalized the kinetic energy to incorporate velocity and its gradient.

Because of their wide number of applications, and being very flexible in the transverse vibration mode (Farajpour *et al*. 2018), CNTs show promising potential to be used as rotating structures such as in miniature nano-rotor base devices. In fact, nanostructures possibly undergoing rotation are systems with potential to be used in nano-motor based devices such as fullerene gears and CNT gears (Mirtalaie and Hajabasi 2017, Huang and Han 2016, De Clerck 2014, Gopalakrishnan and Narendar 2013, Cai *et al*. 2014). They may be used as rotating machinery in many applications such as ultrafast laser spinning microdevices (Král and Sadeghpour 2002), nanofluidic desalination devices (Tu *et al*. 2016, Khodabakhshi and Moosavi 2017), shaft of nanomotors (Torkaman-Asadi *et al*. 2015, Kröner 1963) and MEMS gyroscope sensors (Yang *et al*. 2011). Commonly, rotating machinery consists of shafts whose diameters may change with their respective longitudinal position, and bearings placed at various positions along the rotor span. Nanostructures undergoing rotation include nanoturbines, nanoscale molecular bearings, shaft and gear, and multiple gear systems. These nanostructure machines are expected to receive considerable attention in the near future. Researchers have thus reported the feasibility of nanoscale rotating structures such as molecular gears, fullerene gears, and carbon nanotubes gears. Nonetheless, all the above declared applications and usefulness of such design, the precise prediction of their dynamic behavior is therefore indispensable for their final implementation.

Recently, a great effort is devoted to the vibration analysis of nanobeams and CNTS under rotation. Barooti *et al*. (2017) investigated the effects of critical speed on the vibrational behavior of spinning SWCNT using modified couple stress theory by considering the first-order shear deformation theory. They included the centrifugal, Coriolis and initial tension impacts in their formulation and utilized the differential quadrature method to reduce the order of the equation of motion. Ghafarian and Ariaei (2016) presented the free vibration analysis of a system containing a series of rotating nanobeams assuming the nonlocal elasticity theory. They conducted some parametric studies to examine the effect of small scale parameter, hub radius and rotational speed on the dynamic behavior of the rotating multiple-CNT system. They concluded that the transverse vibration of the considered structure may be significantly affected by the small scale parameter, elastic mediums and rotational speed of the base structure. Shojaeefard *et al*. (2019) analyzed the free vibration behavior of a rotating cylindrical nanoshell modeled by a FG piezomagnetic material. They derived the equation of motion by employing the Eringen's nonlocal elasticity. They solved the governing equations by applying the generalized differential quadrature method and associated boundary conditions. They demonstrated that the vibrational characteristics of the studied structure were influenced by key factors like the length scale, angular velocity, external amperage and viscoelastic media parameters. Narendar (2012) studied the flapwise bending vibration of a rotating short nanotubes by assuming the nanoscale effects. In his analysis, he took into account the effects of rotary inertia and shear deformation parameter and obtained the natural frequencies of nanotubes through differential quadrature method. The influence of the hub radii and nonlocal parameter on the vibrational frequencies of the system were investigated. Pradhan and Murmu (2010) developed a flapwise transverse vibration model for a rotating nanocantilever beam (as the blades of a nanoturbine) by considering the nonlocal theory of elasticity. They employed the differential quadrature method to discretize the governing equation and then obtained the normalized natural frequencies. They concluded that as the angular velocity of the base hub increases, the effect of small-scale parameter on the fundamental frequency is increased while it is decreased for higher modes of vibration. Ebrahimi and Shaghaghi (2015b) employed the nonlocal Euler–Bernoulli beam theory to analyze the transverse vibration behavior of a pre-stressed size-dependent rotating nanotube. They derived the governing equation of motion via Hamilton's principle and solved them using differential transform method. They performed several numerical studies to examine the effect of angular velocity, hub radius and preload stress on the vibration characteristics of rotating nanotubes. Kotwal *et al*. (2018) investigated the effect of nonlinear van der Pol damping on the planar to whirling transition of nanotubes by studying characteristic changes in resonance curves against changes in electrostatic actuation and nonlinear damping. A new regime was established that was not hitherto available in which no whirling dynamics exists no matter what values of the value of driving voltage. In addition, they were able to eliminate hysteretic jumps in both planar and nonplanar directions. According to the recent studies in the past decade, it is obvious that the most convenient/applicable small-scale model to deal with the mechanical behavior of nanotubes is the nonlocal theory of elasticity. This model is a popular methodology taking into account the size-dependent effects demonstrated by miniature structures, although, some other well-known size-dependent models are also valid and considered by many researchers (Akbas 2018, Barretta *et al*. 2019a, Demir and Civalek 2017a, b, Numanoğlu *et al*. 2018, Civalek and Demir 2016, Civalek 2008, Chemi *et al*. 2015, Civalek and Acar 2007, Akgöz and Civalek 2011). According to this supreme theory, the state of stress field at a specific point in an elastic domain is defined not only by the state of strain vector at that point, but also by the state of strain field at other points of the material. Hence, it is necessary to know different parameters over the entire domain to accurately capture the mechanical/physical properties of the nanoscale materials. As per this subtle hypothesis, the long-range interatomic interactions between different points are taken into consideration and therefore the obtained consequence depends on the size of the considered structure. Fortunately, the nonlocal theory grants us the feasible way to analyze the mechanical behavior of ultra-small structures without considering considerable amount of complicated equations (Ghavanloo *et al*. 2019).

As described earlier, most of the previously published research have not attempted to investigate the vibration characteristics of SWCNTs rotating about their own axis of symmetry. Most of the previous studies on CNT based nano-rotor dynamics have focused on the vibrations behavior of the nanobeams rotating on their fixed hub while neglecting the effects of lateral deformations with a suitable combination of the gyroscopic effects. These two factors, namely the geometric property and gyroscopic effect have crucial effects in rotor dynamic analyses. Therefore, a suitable combination of both effects is the motivation of our work: First, the free vibration of a rotating SWCNT is investigated assuming a nonlocal elasticity beam theory. The rotating carbon nanotube is assumed to be attached to supports and is undergoing rotation about its own axis of symmetry. The basic equation of motion for the structural vibration of rotating SWCNTs which considers small scale effect is therefore acquired. Finally, the resultant nonlinear equation is numerically solved to obtain the natural frequencies of the rotating SWCNTs while discovering the forward and backward whirling motions of the considered structure.

2. Mathematical modelling

Lately, the major concern for researchers in the dynamics of rotating devices is to accurately predict their respective critical speeds in order to avoid a possible resonant state. In rotor dynamics, the revolving speeds that results into resonance responses are explicitly named *critical speeds*. The vibration of a nano-rotor is maximum in the neighborhood of these critical speeds. In the below, we derive the equations governing the vibrational behavior of a rotating single walled carbon nanotube (SWCNT). SWCNT can be modeled as an elastic cylinder (the rotor) within the context of a distributed parameter beam model. It is represented as a simple cylindrical shaped beam having transverse displacements assumed perpendicular to its plane of rotation.

The below model is established within the framework the nonlocal elasticity Euler-Bernoulli (EB) beam theory where the effects of rotary inertia and gyroscopic forces are considered, however and for simplicity, shear deformations are not considered in the below problem formulation. Since CNT are seen as slender beam, the EB beam theory works fine. Fig. 1 shows a simplified sketch of an elastic SWCNT based nano-rotor with a circular cylindrical shape. The length of this presumed uniform nano-rotor is *L*. The CNT will be assumed as a cylindrical beam shape of radius R, and length *L*. It also has a Young's modulus *E* and a mass density ρ.

In Fig. 1, the rectangular coordinate system (*x y z*) represents the fixed reference in the 3D space. The *z-axis* represents the nano-rotor centerline. The respective transverse motions in the *x* and *y* directions at any position along the CNT centerline are denoted by $u(z,t)$ and $v(z,t)$, respectively. Therefore, the displacement fields at any cross-section of the CNT can be expressed as

$$u_x = u(z,t),\quad u_y = v(z,t),\quad u_z = x\frac{\partial v(z,t)}{\partial z} + y\frac{\partial v(z,t)}{\partial z} \tag{1}$$

In the same figure, the slope inclination angle of the tangent to the CNT based nano-rotor displacement curve is denoted through two angles ($\theta_x(z,t)$ and $\theta_y(z,t)$) which both represent its respective projections along planes (*y-z*) and (*x-z*) respectively, which both can be expressed as

$$\begin{cases} \theta_x = \dfrac{\partial u(z,t)}{\partial z} \\ \theta_y = \dfrac{\partial v(z,t)}{\partial z} \end{cases} \tag{2}$$

Next, within the framework of the Euler-Bernoulli Beam model, the bending reaction moments acting on planes (*y-z*) and (*x-z*), correspondingly M_{yz} and M_{xz} in Fig. 1, can be both expressed as follows

$$\begin{cases} M_{yz} = EI\dfrac{\partial \theta_x(z,t)}{\partial z} \\ M_{xz} = EI\dfrac{\partial \theta_y(z,t)}{\partial z} \end{cases} \tag{3}$$

The relationship between the stress state with the shear forces F_x, F_y and the bending moments M_{yz} and M_{xz} in a beam can be expressed as follows

$$F_x = \int_0^L \sigma_{xz} dA\,, \qquad F_y = \int_0^L \sigma_{yz} dA \tag{4}$$

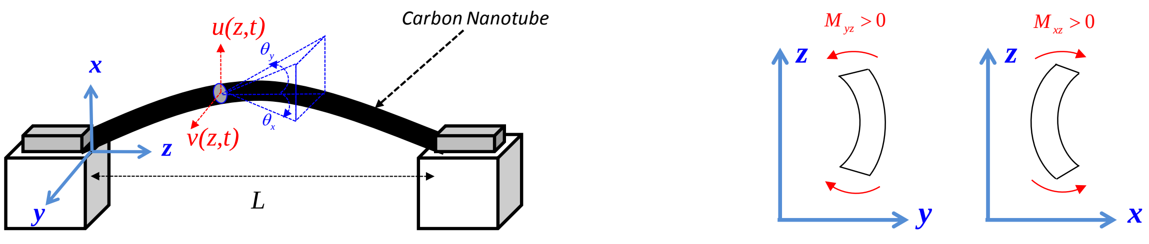


Fig. 1 Schematic drawings of the SWCNT based nano-rotor

$$M_{xz}=\int_0^L x\sigma_{xx}dA\,,\qquad M_{yz}=\int_0^L y\sigma_{xx}dA \tag{5}$$

In recent years, a plenty of research works have focused on the different aspects and models of size-dependent models to appropriately capture the nanoscale behavior of micro/nanostructures (Pinnola *et al*. 2020, Barretta *et al*. 2019b, 2020, Zhu and Li 2017a, 2019, Romano *et al*. 2017, Barretta and de Sciarra 2019). Among them and in accordance with the nonlocal elasticity theory, and through considering the nonlocal effects of higher-order strain gradients $\varepsilon_{ij,k}$, in which the index k symbolizes the differentiation with respect to the parameter x_k, according to the extended Eringen's model (Eringen 1972), the nonlocal stress tensor σ at any point of a continuum medium can be expressed as

$$\sigma_{ij}=\int_V K\left(\left|x-x^{'}\right|,e_0a\right)t(x^{'})dV \tag{6}$$

where $t(x)$ is the classical macroscopic stress tensor at any point x and the kernel function $K\left(\left|x-x^{'}\right|,e_0a\right)$ represents the nonlocal modulus, and e_0a represents the material constant that depends on the medium lattice spacing and wavelength lengths. The macroscopic Hookean solid stress tensor $t(x)$ can be related to the strain tensor ε using the following generalized form

$$t(x)=C(x):\varepsilon(x) \tag{7}$$

where C represents the 4^{th}-order elasticity tensor and ":" denotes the tensor product operator. Eqs. (6) and (7) represent the weighted average of the contributions of the strain field of all points in the body to the stress field at a point. The above integral constitutive relations make the elasticity problems difficult to solve. However, it is possible to represent the integral constitutive relations in an equivalent differential form as follows

$$t=\sigma-(ea)^2\nabla\sigma; \tag{8}$$

where e and a are the material constant and the internal characteristic lengths, respectively. The resultant stress components can be expressed in terms of the scale parameter and the nonlocal constitutive relations can be consequently written as follows, using Eqs. (7) and (8)

$$\begin{cases}E\varepsilon_{xz}=\sigma_{xz}-(ea)^2\dfrac{\partial^2\sigma_{xz}}{\partial z^2};\\ E\varepsilon_{yz}=\sigma_{yz}-(ea)^2\dfrac{\partial^2\sigma_{yz}}{\partial z^2};\end{cases} \tag{9}$$

Using Eq. (1), the strains of the rotating CNT can be obtained as follow

$$\begin{gathered}\varepsilon_{xz}=\frac{\partial u(z,t)}{\partial z};\quad \varepsilon_{yz}=\frac{\partial v(z,t)}{\partial z};\\ \varepsilon_{zz}=x\frac{\partial^2u(z,t)}{\partial z^2}+y\frac{\partial^2 v(z,t)}{\partial z^2};\end{gathered} \tag{10}$$

Next, substituting Eq. (9) into Eq. (4) with using Eq. (10) yields the following nonlocal shear forces for the CNT based nano-rotor

$$\begin{cases}EA\dfrac{\partial u(z,t)}{\partial z}=F_x-(ea)^2\dfrac{\partial^2F_x}{\partial z^2};\\ EA\dfrac{\partial v(z,t)}{\partial z}=F_y-(ea)^2\dfrac{\partial^2F_y}{\partial z^2};\end{cases} \tag{11}$$

Similarly, the nonlocal shear moments are obtained with substituting Eq. (9) into Eq. (5) along with using Eq. (10) as

$$\begin{cases}EI\dfrac{\partial^2 u(z,t)}{\partial z^2}=M_{xz}-(ea)^2\dfrac{\partial^2M_{xz}}{\partial z^2};\\ EI\dfrac{\partial v^2(z,t)}{\partial z^2}=M_{yz}-(ea)^2\dfrac{\partial^2M_{yz}}{\partial z^2};\end{cases} \tag{12}$$

To finally get the equations of motion, an infinitesimal component with thickness "*dz*" is considered as illustrated in the following Fig. 2. In the same figure, deviations in the angular momentums about the principal axes of the moment of inertia are shown in addition to the resultant shearing force $F(z,t)$ and bending moment $M(z,t)$. The polar moment of inertia dI_p and the diametric moment of inertia dI_d of this carbon nanotube element can both be expressed as

$$\begin{cases}dI_p=\rho AdA\dfrac{R^2}{2}\\ dI_d=\rho AdA\dfrac{R^2}{4}\end{cases} \tag{13}$$

Assuming that the nano-rotor is undergoing whirling motions and the resultant disposition of the CNT infinitesimal element, as shown in the 3D schematic of Fig. 1, changes from (θ_x,θ_y) to $(\theta_x+d\theta_x,\theta_y+d\theta_y)$ during an infinitesimal time *dt*, the resultant equations of motion for the CNT based nano-rotor can be obtained using the considered Free-Body Diagrams (FBDs) of Fig. 2. The equations of motion are obtained from the relationships among momentum and angular momentum changes, and shearing forces and moments, in addition to considering the Newton's Second Law as follows

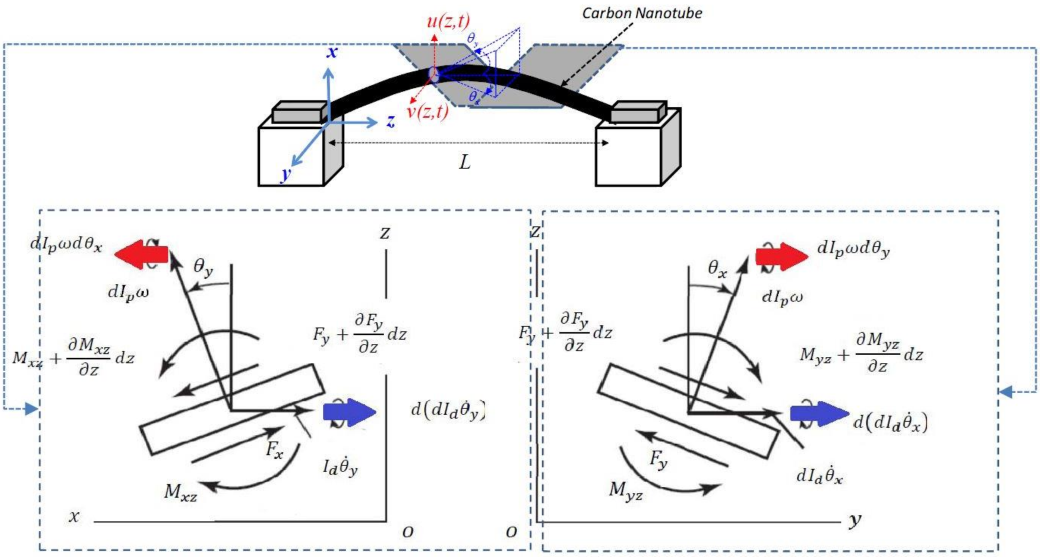
$$\left(\rho A\frac{\partial^2u}{\partial t^2}\right)dz=-F_x+\left(F_x+\frac{\partial F_x}{\partial z}dz\right)-c\frac{\partial u}{\partial t}dz \tag{14}$$

$$\left(\rho A\frac{\partial^2v}{\partial t^2}\right)dz=-F_y+\left(F_y+\frac{\partial F_y}{\partial z}dz\right)-c\frac{\partial v}{\partial t}dz \tag{15}$$

$$\begin{aligned}&\frac{d}{dt}\left(dI_d\dot{\theta}_x+dI_p\omega d\left(d\theta_y\right)\right)\\ &=-M_{xz}+\left(M_{xz}+\frac{\partial M_{xz}}{\partial z}dz\right)+F_xdz\end{aligned} \tag{16}$$

$$\begin{aligned}&\frac{d}{dt}\left(dI_d\dot{\theta}_y+dI_p\omega d(d\theta_x)\right)\\ &=-M_{yz}+\left(M_{yz}+\frac{\partial M_{yz}}{\partial z}dz\right)+F_ydz\end{aligned} \tag{17}$$

where c is the viscous damping coefficient per unit length of the nano-rotor, F_x and F_y are the components of the



(a) (b)

Fig. 2 Free body diagrams of the CNT infinitesimal element of length "*dz*" in (a) the *x*-*z* plane

shearing force in the *x*- and *y*-directions, respectively, and $F_x dz$ and $F_y dz$ both symbolize the moments due to the shearing forces in planes (*x*-*z*) and (*y*-*z*), respectively.

Next, substituting Eq. (2) into Eqs. (16) and (17) and after further simplifications, the following equations are obtained

$$\rho A\frac{\partial^2 u}{\partial t^2}+c\frac{\partial u}{\partial t}=\frac{\partial F_x}{\partial z} \tag{18}$$

$$\rho A\frac{\partial^2 v}{\partial t^2}+c\frac{\partial v}{\partial t}dz=\frac{\partial F_y}{\partial z} \tag{19}$$

$$\frac{dI_d}{dz}\frac{\partial^3 u}{\partial z\partial t^2}+\frac{dI_p}{dz}\omega\frac{\partial^2 v}{\partial z\partial t}=\frac{\partial M_{xz}}{\partial z}+F_x \tag{20}$$

$$\frac{dI_d}{dz}\frac{\partial^3 v}{\partial z\partial t^2}-\frac{dI_p}{dz}\omega\frac{\partial^2 u}{\partial z\partial t}=\frac{\partial M_{yz}}{\partial z}+F_y \tag{21}$$

In the above moment equations (Eqs. (20) and (21)), the left hand side terms are repressing the rotary inertia as well as the gyroscopic moment effects. By differentiating once these latter equations with respect to *z* and substituting Eqs. (18) and (19) in order to eliminate F_x and F_y, the following equations are therefore obtained

$$\begin{aligned}&\frac{dI_d}{dz}\frac{\partial^4 u}{\partial z^2\partial t^2}+\frac{dI_p}{dz}\omega\frac{\partial^3 v}{\partial z^2\partial t}\\&=\frac{\partial^2 M_{xz}}{\partial z^2}+\rho A\frac{\partial^2 u}{\partial t^2}+c\frac{\partial u}{\partial t}\end{aligned} \tag{22}$$

$$\begin{aligned}&\frac{dI_d}{dz}\frac{\partial^4 v}{\partial z^2\partial t^2}-\frac{dI_p}{dz}\omega\frac{\partial^3 u}{\partial z^2\partial t}\\&=\frac{\partial^2 M_{yz}}{\partial z^2}+\rho A\frac{\partial^2 v}{\partial t^2}+c\frac{\partial v}{\partial t}\end{aligned} \tag{23}$$

Substituting Eq. (12) into Eqs. (22) and (23), the following resulting equations of motion (EOMs) are obtained

$$\begin{aligned}&\frac{dI_d}{dz}\frac{\partial^4 u}{\partial z^2\partial t^2}+\frac{dI_p}{dz}\omega\frac{\partial^3 v}{\partial z^2\partial t}=(ea)^2\left(\frac{dI_d}{dz}\frac{\partial^6 u}{\partial z^4\partial t^2}\right.\\&\left.+\frac{dI_p}{dz}\omega\frac{\partial^5 v}{\partial z^4\partial t}-\rho A\frac{\partial^4 u}{\partial z^2\partial t_2}-c\frac{\partial^3 u}{\partial z^2\partial t}\right)\\&+E\frac{dI_d}{dz}\frac{\partial^4 u}{\partial z^4}+\rho A\frac{\partial^2 u}{\partial t^2}+c\frac{\partial u}{\partial t}\end{aligned} \tag{24}$$

$$\begin{aligned}&\frac{dI_d}{dz}\frac{\partial^4 v}{\partial z^2\partial t^2}-\frac{dI_p}{dz}\omega\frac{\partial^3 u}{\partial z^2\partial t}=(ea)^2\left(\frac{dI_d}{dz}\frac{\partial^6 v}{\partial z^4\partial t^2}\right.\\&\left.+\frac{dI_p}{dz}\omega\frac{\partial^5 u}{\partial z^4\partial t}-\rho A\frac{\partial^4 v}{\partial z^2\partial t_2}-c\frac{\partial^3 v}{\partial z^2\partial t}\right)\\&+E\frac{dI_d}{dz}\frac{\partial^4 v}{\partial z^4}+\rho A\frac{\partial^2 v}{\partial t^2}+c\frac{\partial v}{\partial t}\end{aligned} \tag{25}$$

From Eq. (13), we can express the component $\frac{dI_d}{dz}$ and $\frac{dI_p}{dz}$ considering the hollow SWCNT under consideration for this work as follows

$$\begin{cases}\bar{I}_d=\dfrac{dI_d}{dz}=\dfrac{\pi}{4}(R_0^4-R_i^4),\\[2mm]\bar{I}_p=\dfrac{dI_p}{dz}=\dfrac{\pi}{2}(R_0^4-R_i^4),\end{cases} \tag{26}$$

Finally, and since the problem seem to be showing a symmetric like behavior, a complex variable $r = u + \mathrm{i}v$ is introduced and the above EOMs cab be further simplified as follows

$$\begin{aligned}&\bar{I}_d\frac{\partial^4 r}{\partial z^2\partial t^2}-i\bar{I}_p\omega\frac{\partial^3 r}{\partial z^2\partial t}=(ea)^2\left(\bar{I}_d\frac{\partial^6 r}{\partial z^4\partial t^2}\right.\\&\left.-i\bar{I}_p\omega\frac{\partial^5 r}{\partial z^4\partial t}-\rho A\frac{\partial^4 r}{\partial z^2\partial t^2}-c\frac{\partial^3 r}{\partial z^2\partial t}\right)\\&+E\bar{I}_d\frac{\partial^4 r}{\partial z^4}+\rho A\frac{\partial^2 r}{\partial t^2}+c\frac{\partial r}{\partial t}\end{aligned} \tag{27}$$

The above equation of motion, Eq. (27), represents the basic equation for the vibration of a SWCMT based nano-rotor considering small scale effect, and both gyroscopic and rotary inertia effects. The respective boundary conditions would depend on the considered supports for the SWCNT based nano-rotor and can be summarized as follows

$$\begin{cases} r(z=0,t)=r(z=L,t) \\ =\dfrac{\partial r}{\partial z}(z=0,t)=\dfrac{\partial r}{\partial z}(z=L,t)=0 \\ \rightarrow \text{ for fixed-fixed boundary conditions} \\ r(z=0,t)=r(z=L,t) \\ =\dfrac{\partial^2 r}{\partial z^2}(z=0,t)=\dfrac{\partial^2 r}{\partial z^2}(z=L,t)=0 \\ \rightarrow \text{ for simply-sopported boundary condition} \\ \dfrac{\partial r}{\partial z}(z=0,t)=\dfrac{\partial r}{\partial z}(z=L,t) \\ =\dfrac{\partial^2 r}{\partial z^2}(z=0,t)=\dfrac{\partial^2 r}{\partial z^2}(z=L,t)=0 \\ \rightarrow \text{ for free-free boundary condition} \end{cases} \quad (28)$$

It is worth mentioning that for the subsequent analysis, the simply-supported case will only be considered as a case study.

3. Eigenvalue problem formulation

In order to compute the frequencies variation of the SWCNT based nano-rotor, the free vibration solution of Eq. (27) assuming simply supported beam boundary conditions can be expressed accordingly as follows

$$r(z,t)=r\,sin\left(\frac{n\pi}{L}z\right)e^{i\omega_n t}; \quad (29)$$

where r and ω_n symbolize both the amplitude of vibration and its respective angular natural frequency of the rotating nano-rotor vibration, and n its respective mode of vibration number. Then, substituting Eq. (29) into Eq. (27) yields

$$\begin{aligned} &\bar{I}_d\omega_n^2\left(\frac{n\pi}{L}\right)^2-\bar{I}_p\omega_n\left(\frac{n\pi}{L}\right)^2\omega=-\rho A\omega_n^2+ic\omega_n+ \\ &+(ea)^2\left(-\bar{I}_d\left(\frac{n\pi}{L}\right)^4\omega_n^2+\bar{I}_p\omega_n\left(\frac{n\pi}{L}\right)^4\omega\right. \\ &\left.-\rho A\omega_n^2\left(\frac{n\pi}{L}\right)^2+ic\omega_n\left(\frac{n\pi}{L}\right)^2\right)+E\bar{I}_d\left(\frac{n\pi}{L}\right)^4; \end{aligned} \quad (30)$$

The above equation represents the SWCNT based nano-rotor frequency equation. With neglecting the eccentricity and damping effect, solving numerically the above frequency equation would result into two distinct roots both functions including the scale parameter ea and the nano-rotor input rotating speed ω. It is also worth mentioning that these solutions are the natural frequencies for the rotating SWCNT incorporating both the gyroscopic and small scale effects. The main advantage of this simple frequency equation is that it explicitly shows the dependency of the natural frequencies of the nano-rotor with its respective geometrical and mechanical properties. The two resultant frequencies in the frequency spectrum are either positive root demonstrating the frequency of a forward whirling mode, while the negative root characterizes that of a backward whirling mode, respectively.

4. Results and analysis

4.1 Validation of the present analysis

As a case study, a SWCNT whose geometrical and mechanical properties are summarized in Table 1 is considered. As to certify the validity of the above obtained eigenvalue problem equation, we propose to compare our numerical results with the below case study in Table 1.

Table 2 shows the frequencies of a SWCNTs for several values of the CNT aspect ratios based on some published theoretical and experimental data in the literature. In the table, the first column shows the aspect ratios of the nanotube and the next columns are summarizing the numerical data obtained from the Classical Continuum Mechanics (CCM) theory and the Molecular Dynamics (MD) simulations as compared to the results of the present study. It can be observed from the table that the fundamental frequency predictions of different non-rotating SWCNTs are showing satisfactory agreement with the available MD simulations and that the small scale effects shouldn't be neglected as compared to the CCM theory.

Table 1 The geometrical and physical properties of the considered SWCNT in this investigation

Parameter	Respective value
SWCNT Young's modulus (Basirjafari *et al.* 2013b)	E = 1 TPa
SWCNT mass density (Basirjafari *et al.* 2013b)	ρ = 2300 kg/m^3
Carbon-Carbon bond scale parameter (Gupta and Batra 2008)	ea = 0.142 nm
The speed of light (vacuum condition) (Anderson *et al.* 2005)	$C \approx 3\times10^8$ m/s
Frequency of vibration (Anderson *et al.* 2005)	$f=\frac{\omega_n}{2\pi C}$ (Hz)

Table 2 The fundamental natural frequencies (in *THz*) for several SWCNTs aspect ratio as acquired from the nonlocal theory (this study), the CCM, and Molecular Dynamics (MD) simulations, for $\omega = 0$

SWCNT aspect ratio	Results of Eq. (30)	Classical continuum mechanics ($\boldsymbol{ea = 0\ nm}$)	Molecular dynamics Eringen (1983), Nahvi and Boroojeni (2013), Hayat *et al.* (2017)
10.1	0.3629	0.364	0.3618
20.9	0.0753	0.0853	0.0724
31.6	0.0361	0.0373	0.0358
39.1	0.0252	0.0244	0.0259

4.2 Whirling Motion Analysis

In this section, the forward and backward frequencies obtained in the previous section by the present method are compared in graphic form through Figs. 3 to 6 for SWCNT and considering simply-supported boundary conditions.

Fig. 3 displays the variation of the fundamental frequency of spinning CNT, governed by Eq. (30), versus the small scale parameter *ea*. It is visibly acknowledged from this variation graph that the fundamental frequency of the system is split into two branches namely forward and backward modes as a consequence of the gyroscopic effect. Moreover, it should be pointed out that the rotational and whirling motions have the same directions in forward whirl while the whirl is opposed to the rotational speed in backward whirl. According to the demonstrated results of Fig. 3, it can be deduced that as the small scale parameter increases, the absolute values of the forward and backward speeds are increased accordingly. Moreover, the backward whirling speed is more sensitive to the variation of nanoscale parameter in comparison with the forward mode as shown in the same figure.

Fig. 4 shows the variation of the first natural frequency of spinning nano-rotor as a function of CNT length L. One can observe that the absolute value of both forward and backward speeds both increase with the CNT effective length. In contrast, it is also shown that this increase trend is completely different as compared to the previous case. As can be observed, the slope of the solid black line corresponding to the forward mode increases with the nanotube length whereas the same slope for the dashed blue line corresponding to the backward mode is decaying as the nanotube length increases.

Because of the non-symmetric characteristic of the gyroscopic effect, computing the whirling speeds variation with the input spinning frequency is of great significance in applications involving rotating CNTs. According to the results of Fig. 5, it is demonstrated that as the input angular velocity increases, the gyroscopic hardening effect (contributing as a negative kinetic energy) tends to increase the forward whirl speed. Contrariwise, the gyroscopic softening effect (contributing as a positive kinetic energy) results in decreasing the absolute value of the corresponding backward whirl. Another interesting consequence can be noticeably observed from the results of Fig. 5 is the non-trivial frequency separation among the forward and backward modes respective frequencies when the angular velocity is around zero. In fact, this is due to the asymmetric properties of the gyroscopic effects as demonstrated by Eq. (30).

The first three fundamental natural frequencies of the spinning nano-rotor as a function of CNT radius R, are displayed in Fig. 6. As shown in the figure, the second and third modes frequencies are shown to be the most sensitive to the radius of the SWCNT: these two frequencies increase considerably with increasing CNT. It is also inferred that

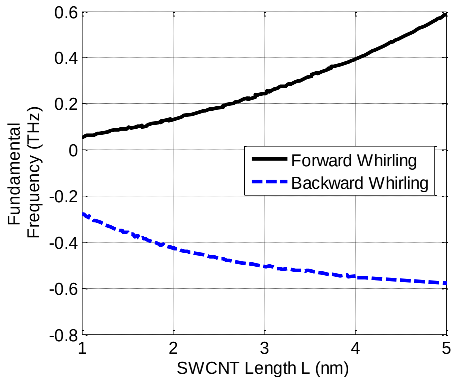


Fig. 4 Fundamental frequency of CNT based nanorotor vs. SWCNT length L

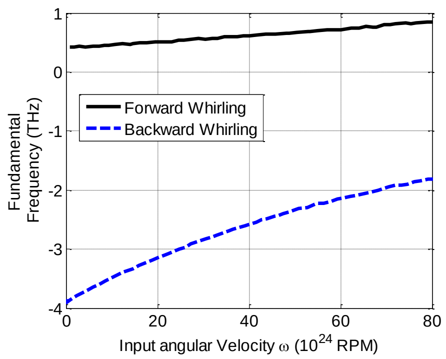


Fig. 5 Fundamental frequency of CNT based nanorotor vs. angular frequency ω

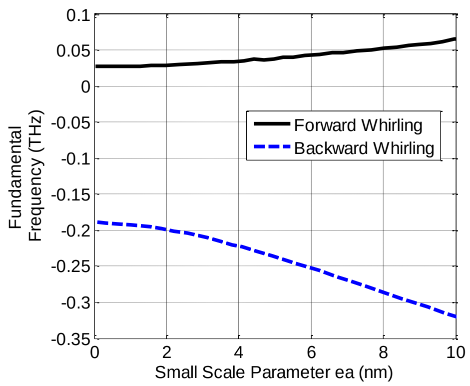


Fig. 3 Fundamental frequency of CNT based nanorotor vs. small scale parameter *ea*

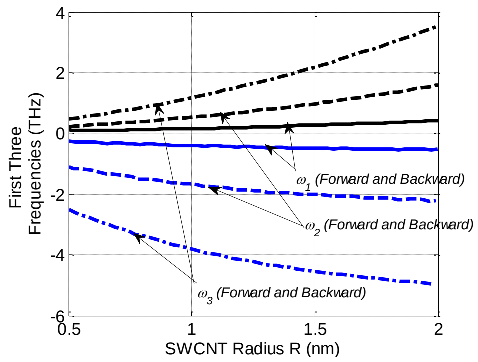


Fig. 6 The first three natural frequencies of CNT based nanorotor vs. SWCNT radius R

as the number of mode increases, the separation between the forward and backward speeds of the CNT absed nano-rotor are increasing considerably.

5. Conclusions

In the present investigation, a detailed numerical study to explore the forward and backward whirling motions of a spinning CNT-based nano-rotor while considering the size-dependency and gyroscopic effects is presented. The governing equations of motion were derived within the framework of the Eringen nonlocal elasticity theory and the eigenvalue problem was established accordingly to extract the natural frequencies of the system.

To demonstrate the accurateness of the simulated results in this study, the obtained frequencies were compared with those available in the literature, and excellent agreement has been demonstrated. Then, it was shown that the fundamental natural frequencies of the rotating SWCNT, consisting of a forward mode and a backward mode, are both influenced by the CNT basic geometric properties (such as the length and the radius) in addition to its material properties (such as Young's modulus, density and Poisson's ratio). In fact, due to the coupling in the CNT lateral motions in both the x and y directions, backward and forward nonlinear natural frequencies were discussed in this numerical study. Indeed, due to the assumed asymmetric properties of the whirling motions of the CNT based nano-rotor, an important frequency separation occurred between the forward and backward modes even for a zero input spinning angular velocity. These two modes of vibration were also shown to be significantly depending on the spinning CNT input rotating speed and the nonlocal scale parameter (mainly due to microstructural nature of the nano-system). The absolute values of the forward and backward critical speeds increase with the small scale parameter. In addition, the backward natural frequency was revealed to be more sensitive to the variation of nanoscale parameter as compared to the forward mode. Dissimilarly, the forward whirling speed was shown to be more sensitive, as compared to the backward mode, to the SWCNT length. Actually, the inclusion of the nonlocal effect increased the forward natural frequency and decreased the backward one. The nonlocal effect is considerably different and more pronounced in the higher vibration modes. In addition, it was shown that the forward natural frequencies decrease by the increase in the value of the radius-to length ratio of the CNT based nano-rotor. The opposite effect was realized for the backward mode. This outcome was shown to be more prominent in the higher modes of vibration.

Author contributions

All authors contributed to the study conception and design. Material preparation, data collection and analysis were performed by all authors. All authors read and approved the final manuscript.

Conflict of interest

The authors declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

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