Bending analysis of magneto-electro piezoelectric nanobeams system under hygro-thermal loading

Farzad Ebrahimi^{*1}, Mahsa Karimiasl¹ and Rajendran Selvamani²

¹ Department of Mechanical Engineering, Faculty of Engineering, Imam Khomeini International University, Qazvin, Iran ² Department of Mathematics, Karunya University, Coimbatore, TamilNadu, India

(Received May 16, 2019, Revised August 30, 2019, Accepted December 17, 2019)

Abstract. This paper investigated bending of magneto-electro-elastic (MEE) nanobeams under hygro-thermal loading embedded in Winkler-Pasternak foundation based on nonlocal elasticity theory. The governing equations of nonlocal nanobeams in the framework parabolic third order beam theory are obtained using Hamilton's principle and solved implementing an analytical solution. A parametric study is presented to examine the effect of the nonlocal parameter, hygro-thermal-loadings, magneto-electro-mechanical loadings and aspect ratio on the deflection characteristics of nanobeams. It is found that boundary conditions, nonlocal parameter and beam geometrical parameters have significant effects on dimensionless deflection of nanoscale beams.

Keywords: piezoelectric nanobeam; bending; hygro-thermal loading; nonlocal elasticity theory; magneto-electric

1. Introduction

The prime magneto-electro-elastic (MEE) used in 1970s, and MEE composite consisting of the piezoelectric and piezo magnetic phase was discovered in this year. Van den Boomgard et al. (1974) considered the MEE nanomaterials, (BiFeO₃, BiTiO₃-CoFe₂O₄, NiFe₂O₄-PZT) and their nanostructures and found the significant role in researches (Zheng et al. 2004, Martin et al. 2008, Wang et al. 2010, Prashanthi et al. 2012, Ke et al. 2014, Ramirez et al. 2006). For this reason to the major potential of nanostructure for amplification and for many applications, their mechanical behavior should be investigated and well identified before new designs can be proposed. The classical mechanic continuum theories demonstrated to predict the response of structures up to a minimum size, which they fail to provide accurate predictions. The nonlocal theories add a size parameter in the modeling of the continuum. In this paper studied models that developed according to the greatly used nonlocal elasticity theory of Eringen (1968, 1972, 1976, 2002, 2006). Akgoz and Civalek (2011) made buckling analysis of cantilever carbon nanotubes using the strain gradient elasticity and modified couple stress theories. Embedded in the nonlocal component relevance of Eringen, many articles were published in search of enlarging nonlocal beam models for nano structures. (Adda Bedia et al. 2019, Rakrak et al. 2016, Aydogdu et al. 2018). Bellifa et al. (2017) explained a nonlocal zeroth-order shear deformation theory for nonlinear postbuckling of nanobeams. Bouafia et al. (2017) developed a nonlocal quasi-3D theory for bending and free

*Corresponding author, Ph.D., Professor, E-mail: febrahimy@eng.ikiu.ac.ir flexural vibration behaviors of functionally graded nanobeams. Zemri et al. (2015) assessed refined nonlocal shear deformation theory beam theory and found the mechanical response of functionally graded nanoscale beam. Youcef et al. (2018) included the surface stress effects in nanoscale beams and discussed the dynamic behavior of the nanoscale beam. Ebrahimi and Daman (2017) provided an analytical investigation of the surface effects on nonlocal vibration behavior of nanosize curved beams. Peddieson et al. (2003) proposed nonlocal Euler-Bernoulli and Timoshenko beam theory, accepted by many studies to verify the bending by Wang et al. (Wang 2005, 2008, Wang et al. 2011). Elmerabet et al. (2017) investigated the buckling temperature of single-walled boron nitride nanotubes using a novel nonlocal beam model. Kheroubi et al. (2016) gave a new refined nonlocal beam theory accounting for effect of thickness stretching in nanoscale beams. During the years of research the smallsize agents in SWCNTs studied by Murmu and Pradhan (2009). Different papers have been studied the dynamic response of the mide-rise building, and composite structures under seismic events (Arabnejad Khanouki et al. 2010, Jalali et al. 2012, Khorami et al. 2017), also different types of loading scenarios as full-cyclic, half cyclic, reversed cyclic, and shack table have employed to evaluate structural behaviour of the specimens in full-scale or half-scale tests (Shariati et al. 2012, 2013, 2014, 2017). Self-consolidating concrete is a kind of concrete which fluidity and workability parameters have to be enhanced in it (Li et al. 2019, Shariati et al. 2019c, 2020a, b, c), hence using graphene sprayed on the surface of aggregated could improve the SCC fluidity properties. Furthermore, using graphene processed materials could be employed in a variety of construction applications, where the nano-scale properties of the graphene sheets or sprays are able to mitigate some micro-structural deficiencies as steel micro

crack and stain in cold-formed steel uprights (Shah et al. 2015, 2016a, b, c, Shariati et al. 2018), or cover the flexural and compressive strength loss during cyclic and motonic loading scenarios. Timoshenko beam theory nonlocal elasticity investigated in their study. So, based on an elastic medium the stability response of SWCNT are described. Winkler and Pasternak parameter, aspect ratio of the SWCNT and nonlocal parameter were studied. Akgoz and Civalek (2012) investigated the size effects on static response of single-walled carbon nanotubes based on strain gradient elasticity. Ehyaei and Daman (2017) analyzed the free vibration analysis of double walled carbon nanotubes embedded in an elastic medium with initial imperfection. Thin-walled structures have demonstrated a sustainable strength against the axial compressive loads (Davoodnabi et al. 2019, Shariati et al. 2019a) and low flexural capacity, thermal stability and hysteresis response especially when their section was perforated or partially opened (Hosseinpour et al. 2018, Paknahad et al. 2018, Shariat et al. 2018). Yang et al. (2010) studied Nonlinear free vibration of SWCNTs based on strains Eringen's nonlocal elasticity theory. Vibration and buckling of piezoelectric and piezomagnetic nanobeams based on third order beam model were verified by Ebrahimi and Barati (2016a-e). The vibration, buckling and bending, free of Timoshenko nanobeams based on a meshless method investigated by Roque et al. (2011). Larbi Chaht et al. (2015) analyzed the bending and buckling of functionally graded material sizedependent nanoscale beams including the thickness stretching effect. Ahouel et al. (2016) discussed the mechanical behavior of functionally graded trigonometric shear deformable nanobeams including neutral surface position concept with the size. Akgoz and Civalek (2017) discussed the effects of thermal and shear deformation on vibration response of functionally graded thick composite microbeams. Ebrahimi et al. (2018) checked the vibration analysis thermally affected viscoelastic nanosensors subjected to linear varying loads. Ebrahimi and Habibi (2017) verified the low-velocity impact response of laminated FG-CNT reinforced composite plates in thermal environment. Attia et al. (2018) refined four variable plate theory for thermo elastic analysis of FGM plates resting on variable elastic foundations. Kadari et al. (2018) made the buckling analysis of orthotropic nanoscale plates resting on elastic foundations. Ahsi et al. (2017) made the analysis of mechanical and thermal buckling analysis of functionally graded plates using a four variable refined nth-order shear deformation theory. Khetir et al. (2017) developed a new nonlocal trigonometric shear deformation theory for thermal buckling analysis of embedded nanosize FG plates. Dastjerdi and Akgoz (2018) analyzed the latest static and dynamic behavior of macro and nano FGM plates using exact three-dimensional elasticity in thermal environment. Belkorissat et al. (2015) studied various vibration properties of functionally graded nano-plate using a new nonlocal refined four variable model Boutaleb et al. (2019) carried out the dynamic analysis of nano size functionally graded rectangular plates based on simple non local quasi 3D HSDT. Karami et al. (2018a) developed a size-dependent quasi-3D model for wave dispersion analysis of FG nanoplates. Bounouara et al. (2016) described a nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic foundation. Besseghier et al. (2017) considered the embedded nano size FG plates to analyze the free vibration using a new nonlocal trigonometric shear deformation theory. Mokhtar et al. (2018) developed a novel shear deformation theory for buckling analysis of single layer graphene sheet based on nonlocal elasticity theory. Sobhy (2013) studied the thermal buckling analysis of singlelayered graphene sheets lying on an elastic medium. Bouadi et al. (2018) analyzed a new nonlocal HSDT for the stability of single layer graphene sheet. Yazid et al. (2018) developed a novel nonlocal refined plate theory for stability response of orthotropic single-layer graphene sheet resting on elastic medium. Shahriar Dastjerdi et al. (2018) proposed a new approach for bending analysis of bilayer conical graphene panels considering nonlinear van der Waals force. Karami et al. (2017) studied the effects of triaxial magnetic field on the anisotropic nano plates. Later they (2018a) made variational approach for wave dispersion in anisotropic doubly-curved nano shells based on a new nonlocal strain gradient higher order shell theory. Cherif et al. (2018) used the differential transform method to analyze the vibration of nano beam including thermal effect. Kurtinaitiene et al. (2016) checked the effect of additives on the hydrothermal synthesis of manganese ferrite nanoparticles. Beldjelili et al. (2016) used the four variable trigonometric plate theory and analyzed the hygro-thermomechanical bending of S-FGM plates resting on variable foundations. Thermo-magneto-electro-elastic elastic analysis of a functionally graded nanobeam integrated with functionally graded piezomagnetic layers was studied by Arefi and Zenkour (2016). Mouffoki et al. (2017) analyzed the vibration of nonlocal advanced nanobeams in hygrothermal environment using a new two-unknown trigonometric shear deformation beam theory. Bousahla et al. (2016) showed the thermal stability of plates with functionally graded coefficient of thermal expansion. Simsek and Yurtcu (2013) presented three-unknown shear and normal deformations nonlocal beam theory for the bending analysis and researched bending and buckling of the FG based on the nonlocal Timoshenko and Euler-Bernoulli beam theory. They described that the power-law exponent has a wide influence on the responses of FG nanobeam. Chikh et al. (2017) made the thermal buckling analysis of cross ply laminated plates using a simplified HSDT. Bakhadda et al. (2018) carried out the dynamic and bending analysis of carbon nano tube reinforced composite plates with elastic foundation. Semmah et al. (2019) used non local FSDT for thermal buckling analysis of SWBNNT on Winkler foundation. Bending of Electro-mechanical sandwich nanoplate based on silica Aerogel foundation was examined by Ghorbanpour and Zamani (2017). They described the influence of parameters on nanostructure such as applied voltage, porosity index, foundation characteristics, parameter, plate aspect ratio, and thickness ratio on bending response of sandwich nanoplate. Sobhy (2013) investigated the buckling and free vibration of exponentially graded sandwich plates resting on elastic

foundations under several boundary conditions. Hamidi et al. (2015) explained a sinusoidal plate theory with 5unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates. Bouderba et al. (2016) analyzed the thermal stability of functionally graded sandwich plates using a simple shear deformation theory. Menasria et al. (2017) developed the new and simple HSDT for the thermal stability analysis of FG sandwich plates. Draoui et al. (2019) presented the Static and dynamic behavior of nanotubes-reinforced sandwich plates using FSDT. El-Haina et al. (2017) carried out a simple analytical approach for thermal buckling of thick functionally graded sandwich plates. There are different available techniques for data validations and predictions such as employing artificial neural networks (Safa et al. 2016, Sedghi et al. 2018, Shariati et al. 2019b), Finite element method, Finite strip method (Arabnejad Khanouki et al. 2011, 2016, Daie et al. 2011, Mohammadhassani et al. 2012, 2013a, b, 2014, Shariati et al. 2015). Finite element method which is generally carried out by FE programs as ABAQUS and ANSYS performed as a reliable technique for empirical data validation and response prediction.

Followed by previous work, bending characteristics of nanobeams under hygro-thermal loading is examined based on refined shear deformable theory. The analytical solution of the governing equations is solved by using third order beam theory via Hamilton principle and Eringen's nonlocal elasticity theory. The effect of several parameters such as hygro-thermal, magnet-electro loading, different boundary conditions and Winkler-Pasternak foundation on bending are investigated.

2. Theory and formulation

Based on parabolic third order beam theory, the displacement field at any point of the beam is supposed to be in the form

$$u_{x}(x,z) = u(x) + z\varphi(x) - \alpha z^{3} \left(\varphi + \frac{\partial w}{\partial x}\right), \qquad (1)$$
$$u_{z}(x,z) = w(x)$$

in which u and w are displacement components in the mid-plane along the coordinates x and z, respectively, while φ denotes the total bending rotation of the cross-section.

Considering strain-displacement relationships on the basis of parabolic beam theory, the non-zero strains can be stated as

$$\varepsilon_{xx} = \varepsilon_{xx}^{(0)} + z\varepsilon_{xx}^{(1)} + z^3\varepsilon_{xx}^{(3)}, \quad \gamma_{xz} = \gamma_{xz}^{(0)} + z^2\gamma_{xz}^{(2)}.$$
 (2)

Where

$$\varepsilon_{xx}^{(0)} = \frac{\partial u}{\partial x}, \quad \varepsilon_{xx}^{(1)} = \frac{\partial \psi}{\partial x}, \quad \varepsilon_{xx}^{(3)} = -\alpha \left(\frac{\partial \varphi}{\partial x} + \frac{\partial^2 w}{\partial x^2}\right),$$

$$\gamma_{xz}^{(0)} = \varphi + \frac{\partial w}{\partial x}, \qquad \gamma_{xz}^{(2)} = -\beta \left(\varphi + \frac{\partial w}{\partial x}\right),$$
(3)

and $\beta = 4/h^2$.

According to Maxwell's equation, the relation between electric field (Ex, Ez) and electric potential (ϕ) and magnet field (Qx, Qz) and magnet potential (ψ), can be obtained as: Ke *et al.* (2014)

$$E_{x} = -\phi_{,x} = \cos(\xi z) \frac{\partial \phi}{\partial x}$$

$$Q_{x} = -\psi_{,x} = \cos(\xi z) \frac{\partial \psi}{\partial x}$$
(4)

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$$E_z = -\phi_{,z} = \xi \sin(\xi z) - \frac{2v}{h},$$

$$Q_z = -\psi_{,z} = \xi \sin(\xi z) - \frac{2v}{h}$$
(5)

Where $\xi = \pi/h$. Also, V is the external electric applied to the nanobeam.

Through extended Hamilton's principle, the governing equations can be derived as follows

$$\int_0^t \delta(\Pi_S - \Pi_W) \mathrm{d}t = 0 \tag{6}$$

where Π_S is the total strain energy, Π_W is the work done by external applied forces. The first variation of strain energy Π_S can be calculated as

$$\delta\Pi_{S} = \int \sigma_{ij} \delta\varepsilon_{ij} d\nu = \int \sigma_{x} \delta\varepsilon_{x} + \sigma_{xz} \delta\gamma_{xz}$$
(7)

Substituting Eqs. (1)-(2) into Eq. (6) yields

$$\begin{aligned} &= \int_{0}^{a} (N\varepsilon_{xx}^{(0)} + M\delta\varepsilon_{xx}^{(1)} + P\delta\varepsilon_{xx}^{(3)} + Q\delta\gamma_{xz}^{(0)} + R\delta\gamma_{xz}^{(2)})dx \\ &+ \int_{0}^{a} \int_{-h/2}^{h/2} \left[-D_{x}\cos(\xi z) \frac{\partial\delta\phi}{\partial x} + D_{z}\xi\sin(\xi z) \,\delta\phi \right] \\ &- B_{x}\cos(\xi z) \frac{\partial\delta\psi}{\partial x} + B_{z}\xi\sin(\xi z) \,\delta\phi\psi dz \end{aligned}$$
(8)

in which N, M and Q are the axial force, bending moment and shear force resultant respectively. Relations between the stress resultants and stress component used in Eq. (9) are defined as

$$\{N, M, P\} = \int_{-h/2}^{h/2} \sigma_{xx} \{1, z, z^3\} dz,$$

$$\{Q, R\} = \int_{-h/2}^{h/2} \sigma_{xz} \{1, z^2\} dz.$$
(9)

Variation of the work done due to external forces, $\delta \Pi_W$, can be written in the form

$$\delta\Pi_{W} = \int_{0}^{L} \left[\left(N_{x}^{0} \frac{\partial w}{\partial x} \frac{\partial}{\partial x} + \alpha P \frac{\partial^{2}}{\partial x^{2}} - k_{W} + k_{P} \frac{\partial^{2}}{\partial x^{2}} \right) \delta w \right] \\ -N \delta\varepsilon_{xx}^{(0)} - \bar{M} \frac{\partial \delta \psi}{\partial x} - \bar{Q} \delta \gamma_{xz}^{(0)} dx,$$
(10)

in which $\overline{M} = M - \alpha P$, $\overline{Q} = Q - \beta R$, k_w , k_p are linear, shear coefficients of medium, N^E , N^B , N^T , N^H electric

and magnet, thermal, hygro-thermal loading respectively.

$$N_{x}^{0} = N^{E} + N^{B} - N^{T} - N^{H},$$

$$N^{E} = -\int_{-h/2}^{h/2} e_{31} \frac{2V}{h} dz$$
(11)

$$N^{B} = -\int_{-h/2}^{h/2} e_{31} \frac{2\Omega}{h} dz$$
 (12)

$$N^{T} = \int_{-h/2-h_{0}}^{\frac{h}{2}-h_{0}} (\alpha \,\Delta T) dz \tag{13}$$

$$N^{H} = b \int_{-h/2-h_{0}}^{\frac{h}{2}-h_{0}} (\beta \,\Delta H) dz \tag{14}$$

Inserting Eqs. (10) and (8) in Eq. (6) and integrating by parts, and gathering the coefficients of δu , δw , $\delta \psi$ and $\delta \phi$, the following governing equations are obtained

$$\frac{\partial N_x}{\partial x} = 0 \tag{15}$$

$$\frac{\partial^2 \overline{M}}{\partial x^2} - \overline{Q} = 0, \tag{16}$$

$$\frac{\partial \bar{Q}}{\partial x} - (N^E + N^B + N^T + N^H) \frac{\partial^2 w}{\partial x^2} + \alpha \frac{\partial^2 P}{\partial x^2} - k_w w + k_P \frac{\partial^2 w}{\partial x^2} = 0$$
(17)

$$\int_{-h/2}^{h/2} \left[\cos(\xi z) \frac{\partial D_x}{\partial x} + \xi \sin(\xi z) D_z \right]$$

$$+ B_x \cos(\xi z) \frac{\partial \delta \psi}{\partial x} + B_z \xi \sin(\xi z) \delta \psi dz = 0$$
(18)

2.3 Nonlocal elasticity theory

The nonlocal theory can be extended for the piezoelectric nanobeams as

$$\sigma_{ij-}(ea)^2 \nabla^2 \sigma_{ij} = \left[c_{ijkl} \varepsilon_{kl} - e_{mij} E_m - q_{nij} H_n \right] \quad (19)$$

$$D_{ij-}(ea)^2 \nabla^2 D_{ij} = [e_{ikl} \varepsilon_{kl} + k_{im} E_m + d_{in} H_n]$$
(20)

$$B_{i-}(ea)^2 \nabla^2 B_i = [q_{ikl} \varepsilon_{kl} + d_{im} E_m + \varkappa_{in} H_n]$$
(21)

$$p_{i-}(a)^2 \nabla^2 p_i = \left[\varepsilon_0 \chi_{ij} E_j + e_{ikl} \varepsilon_{kl} \right]$$
(22)

Also, χ_{ij} is the relative dielectric susceptibility. Also, e_0a is nonlocal parameter which is introduced to describe the size-dependency of nanostructures. Where ∇^2 is the Laplacian operator. The stress relations can be expressed by

$$(1 - \mu \nabla^2) \sigma_{xx} = [C_{11} \varepsilon_{xx} - e_{31} E_z - q_{31} H_z]$$
(23)

$$(1 - \mu \nabla^2) \sigma_{xz} = [C_{55} \gamma_{xz} - e_{15} E_x - q_{15} H_x]$$
(24)

$$(1 - \mu \nabla^2) D_x = [e_{15} \gamma_{xz} + k_{11} E_x + d_{11} H_x]$$
(25)

$$(1 - \mu \nabla^2) D_z = [e_{31} \varepsilon_{xx} + k_{33} E_z + d_{33} H_z]$$
(26)

$$(1 - \mu \nabla^2) B_x = [q_{15} \gamma_{xz} + d_{11} E_x + \varkappa_{11} H_x]$$
(27)

$$(1 - \mu \nabla^2) B_z = [q_{31} \varepsilon_{xx} + d_{33} E_z + \varkappa_{33} H_x]$$
(28)

Integrating Eq. (21) over the cross-section area of nanobeam provides the following nonlocal relations for a refined beam model as: Also, normal forces and moments due to electrical field can be defined by

$$N - \mu \frac{\partial^2 N}{\partial x^2} = A_{xx} \frac{\partial u}{\partial x} + (B_{xx} - \alpha E_{xx}) \frac{\partial \varphi}{\partial x} - \alpha E_{xx} \frac{\partial^2 w}{\partial x^2} + A_{31} \phi + G_{31} \psi - N^E - N^B + N^T + N^H$$
⁽²⁹⁾

$$M - \mu \frac{\partial^2 M}{\partial x^2} = B_{xx} \frac{\partial u}{\partial x} + (D_{xx} - \alpha F_{xx}) \frac{\partial \varphi}{\partial x} - \alpha F_{xx} \frac{\partial^2 w}{\partial x^2} + B_{31} \phi + Q_{31} \psi$$
(30)

$$P - \mu \frac{\partial^2 P}{\partial x^2} = E_{xx} \frac{\partial u}{\partial x} + (F_{xx} - \alpha H_{xx}) \frac{\partial \varphi}{\partial x} - \alpha H_{xx} \frac{\partial^2 w}{\partial x^2} + E_{31} \phi + J_{31} \psi$$
(31)

$$Q - \mu \frac{\partial^2 Q}{\partial x^2} = (A_{xz} - \beta D_{xz}) \left(\frac{\partial w}{\partial x} + \varphi\right) - F_{11} \frac{\partial \phi}{\partial x} \quad (32)$$

$$R - \mu \frac{\partial^2 R}{\partial x^2} = (D_{xz} - \beta F_{xz}) \left(\frac{\partial w}{\partial x} + \varphi\right) - F_{33} \frac{\partial \phi}{\partial x}$$
(33)

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \left(D_x - \mu \frac{\partial^2 D_x}{\partial x^2} \right) \cos(\xi z) \, \mathrm{d}z$$

$$= (F_{11} - \beta F_{33}) \left(\frac{\partial w}{\partial x} + \varphi \right) + F_{11} \frac{\partial \phi}{\partial x}$$
(34)

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \left(D_z - \mu \frac{\partial^2 D_z}{\partial x^2} \right) \xi \sin(\xi z) dz$$

$$= A_{31} \frac{\partial u}{\partial x} + (B_{31} - \alpha E_{31}) \frac{\partial \psi}{\partial x} - \alpha E_{31} \frac{\partial^2 w}{\partial x^2} - F_{33} \phi$$

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \left(B_x - \mu \frac{\partial^2 B_x}{\partial x^2} \right) \cos(\xi z) dz$$

$$= (\varkappa_{11} - \beta \varkappa_{33}) \left(\frac{\partial w}{\partial x} + \varphi \right) + \varkappa_{11} \frac{\partial \psi}{\partial x}$$
(35)
(36)

$$\int_{-\frac{h}{2}}^{\frac{h}{2}} \left(B_z - \mu \frac{\partial^2 B_z}{\partial x^2} \right) \xi \sin(\xi z) dz$$

$$= G_{31} \frac{\partial u}{\partial x} + (Q_{31} - \alpha J_{31}) \frac{\partial \psi}{\partial x} - \alpha J_{31} \frac{\partial^2 w}{\partial x^2} - \varkappa_{33} \psi$$
(37)

in which

$$\{A_{11}, B_{11}, D_{11}, E_{xx}, F_{xx}, H_{xx}\} = \int_{-h/2}^{h/2} c_{11}\{1, z, z^2, z^3, z^4, z^6\} dz,$$
(38)

$$\{A_{xz}, D_{xz}, F_{xz}\} = \int_{-h/2}^{h/2} c_{55}\{1, z^2, z^4\} dz.$$
(39)

$$\{A_{31}, B_{31}, E_{31}\} = \int_{-h/2}^{h/2} e_{31}\{1, z, z^3\} \xi \sin(\xi z) \, \mathrm{d}z \qquad (40)$$

$$(F_{11}, F_{33}) = \int_{-h/2}^{h/2} \{s_{11} \cos^2(\xi z), s_{33}\xi^2 \sin^2(\xi z)\} dz \quad (41)$$

$$(G_{31}, Q_{31}J_{31}) = \int_{-h/2}^{h/2} q_{31}\xi \sin(\xi z) \{1, z, z^3\}_z dz \qquad (42)$$

$$(\varkappa_{11},\varkappa_{33}) = \int_{-h/2}^{h/2} \{\varkappa_{11}\cos^2(\xi z),\varkappa_{33}\xi^2\sin^2(\xi z)\}dz \quad (43)$$

$$\frac{\partial N}{\partial x} = A_{xx} \frac{\partial u}{\partial x} + K_{xx} \frac{\partial \phi}{\partial x} - \alpha E_{xx} \frac{\partial^2 w}{\partial x^2} + A_{31} \phi + G_{31} \psi$$
(44)

The governing equations of nonlocal strain gradient nanoplate under electrical field in terms of the displacement can be derived by substituting Eqs. (36)-(42), into Eqs. (15)-(16) as follows

$$\frac{\partial^{2}\overline{M}}{\partial x^{2}} - \overline{Q} = (B_{xx} - \alpha E_{xx}) \frac{\partial^{2}u}{\partial x^{2}} + (D_{xx} - \alpha F_{xx}) \frac{x^{2}\psi}{\partial x^{2}}
-\alpha (F_{xx} - \alpha H_{xx}) \frac{\partial^{3}w}{\partial x^{3}} + k_{w} \frac{\partial w}{\partial x} - k_{P} \frac{\partial^{3}w}{\partial x^{3}}
+ (B_{31} - \alpha E_{31}) \frac{\partial \phi}{\partial x} + (Q_{31} - \alpha J_{31}) \frac{\partial \psi}{\partial x}
-\mu[(-N^{E} - N^{B} + N^{T} + N^{H}) \frac{\partial^{3}w}{\partial x^{3}} - \alpha \frac{\partial^{3}P}{\partial x^{3}}
+ k_{w} \frac{\partial w}{\partial x} - k_{P} \frac{\partial^{2}w}{\partial x^{2}}] - (A_{xz} - \beta D_{xz}) \left(\frac{\partial w}{\partial x} + \varphi\right)
- (F_{11} - \beta F_{33}) \frac{\partial \phi}{\partial x}$$
(45)

$$\frac{\overline{\partial \overline{Q}}}{\partial x} = (A_{xz} - \beta D_{xz}) \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial \varphi}{\partial x} \right) - (F_{11} - \beta F_{33}) \frac{\partial^2 \phi}{\partial x^2}
+ \mu \left[(N^E + N^B - N^T - N^H) \frac{\partial^3 w}{\partial x^3} - \alpha \frac{\partial^3 P}{\partial x^3} + k_w \frac{\partial w}{\partial x} - k_P \frac{\partial^3 w}{\partial x^3} \right]$$
(46)

$$(F_{11} - \beta F_{33}) \left(\frac{\partial w}{\partial x} + \varphi \right) + F_{11} \frac{\partial \phi}{\partial x} + A_{31} \frac{\partial u}{\partial x} + (B_{31} - \alpha E_{31}) \frac{\partial \psi}{\partial x} - \alpha E_{31} \frac{\partial^2 w}{\partial x^2} - F_{33} \phi = 0$$

$$(47)$$

$$(\varkappa_{11} - \beta \varkappa_{33}) \left(\frac{\partial w}{\partial x} + \varphi \right) + \varkappa_{11} \frac{\partial \psi}{\partial x} + G_{31} \frac{\partial u}{\partial x} + (Q_{31} - \alpha J_{31}) \frac{\partial \psi}{\partial x} - \alpha J_{31} \frac{\partial^2 w}{\partial x^2} - \varkappa_{33} \psi$$

$$(48)$$

3. Solution procedure

To satisfy above-mentioned boundary conditions, the displacement quantities are presented in the following form

$$u = \sum_{n=1}^{\infty} U_n \frac{\partial X_n(x)}{\partial x} e^{i\omega_n t}$$
(49)

$$\varphi = \sum_{n=1}^{\infty} \varphi_n X_n(x) e^{i\omega_n t}$$
(50)

$$w = \sum_{n=1}^{\infty} W_n X_n(x) e^{i\omega_n t}$$
(51)

$$\emptyset = \sum_{n=1}^{\infty} \emptyset_n X_n(x) e^{i\omega_n t}$$
(52)

$$\psi = \sum_{n=1}^{\infty} \psi_n X_n(x) e^{i\omega_n t}$$
(53)

Where $(u_n, \varphi, w, \phi, \psi)$ are the unknown coefficients and for different boundary conditions $(\alpha = m\pi/a, \beta = n\pi/b)$.

$$[K] \begin{cases} u_{n} \\ \varphi \\ w \\ \psi \\ \psi \end{cases} = \begin{cases} 0 \\ Q_{n}(1+\mu\frac{n^{2}\pi^{2}}{L^{2}}) \\ Q_{n}\left(1+\mu\frac{n^{2}\pi^{2}}{L^{2}}\right) \\ 0 \\ 0 \end{cases}$$
(54)

Where [K], [F] are the stiffness, loading matrixes for nanobeam, respectively.

$$\begin{aligned} k_{1,1} &= A_{xx}\alpha_1, \quad K_{1,2} &= K_{xx}\alpha_6, \quad K_{1,3} &= \alpha E_{xx}\alpha_2, \\ K_{1,4} &= A_{31}\alpha_3, \quad K_{1,5} &= G_{31}\alpha_3 \\ K_{2,1} &= (B_{xx} - \alpha E_{xx})\alpha_6, \\ k_{2,2} &= (D_{xx} - \alpha F_{xx})\alpha_6 + (A_{xx} - \beta D_{xZ})\alpha_5 \\ &- (F_{11} - \beta F_{33})\alpha_3, \end{aligned}$$
(55)
$$K_{2,3} &= \alpha (F_{xx} - \alpha H_{xx})\alpha_6 + k_w\alpha_3 - k_p\alpha_2 \\ &+ \mu (-N^B - N^E - N^T - N^H)\alpha_2 \\ &- k_p\alpha_6 + k_w\alpha_3 - (A_{xx} - \beta D_{xZ})\alpha_3 \\ K_{2,4} &= (B_{31} - \alpha E_{31})\alpha_3, \quad K_{2,5} = (Q_{31} - \alpha J_{31})\alpha_3 \end{aligned}$$

In which

$$\alpha_{1} = \int_{0}^{a} X'(x)X''(x) dx, \quad \alpha_{2} = \int_{0}^{a} X(x)X'''(x) dx,$$

$$\alpha_{7} = \int_{0}^{a} X(x)X'''(x) dx, \quad \alpha_{5} = \int_{0}^{a} X(x)X(x)dx,$$

$$\alpha_{3} = \int_{0}^{a} X(x)X'(x)dx, \quad \alpha_{11} = \int_{0}^{a} X'(x)X'''(x) dx,$$

$$\alpha_{6} = \int_{0}^{a} X(x)X''(x) dx$$
(56)

	Boundary conditions	The functions X_m
	At $x = 0$, a	$X_m(x)$
SS	$X_m(0) = X_m''(0) = 0$	$Sin(\alpha x)$
	$X_m(a) = X_m''(a) = 0$	
CC	$X_m(0) = X'_m(0) = 0$	$Sin^2(\alpha x)$
	$X_m(a) = X'_m(a) = 0$	

Table 1 The admissible functions $X_m(x)$ (Sobhy 2013)

Table 2 Material properties of BiTiO₃-CoFe₂O₄ composite materials Ramirez *et al.* (2006)

Properties	BiTiO ₃ -CoFe ₂ O ₄				
Elastic (GPa)	c11 = 226, c12 = 125, c13 = 124, c33 = 216, c44 = 44.2, c66 = 50.5				
$\begin{array}{c} Piezoelectric \\ /(C \ \cdot \ m^{-2}) \end{array}$	<i>e</i> 31 = -2.2, <i>e</i> 33 = 9.3, <i>e</i> 15 = 5.8				
Dielectric /(10^{-9} C · V ⁻¹ · m ⁻¹)	<i>k</i> 11 = 5.64, <i>k</i> 33 = 6.35				
Piezomagnetic /(N · A^{-1} · $m-1$)	q15 = 275, q31 = 290.1, q33 = 349.9				
$\begin{array}{c} Magnetoelectric \\ /(10^{-12}Ns \cdot V^{-1} \cdot C^{-1}) \end{array}$	<i>s</i> 11 = 5.367, <i>s</i> 33 = 2 737.5				
Magnetic $/(10^{-6} Ns^2 c^{-2}/2)$	$\kappa_{11} = -297, \ \kappa_{33} = 83.5$				
Mass density /(103 Kg/m ³)	$\rho = 5.55, \ \alpha = \beta = 5 \times 10^{-6}$				

The uniform load is supposed that leading to bending and is expressed by the following form

$$q_{dynamics} = \sum_{n=1}^{\infty} Q_n \sin\left[\frac{n\pi}{L}x\right] \sin \omega t$$
 (57)

$$Q_n = \frac{2}{L} \int_{x_0 - c}^{x_0 + c} \sin\left[\frac{n\pi}{L}x\right] q_x dx$$
(58)

in which Q_n are the Fourier coefficients and q(x) = q0 is the uniform load density and x0 is the centroid coordinate. Also, in the case of concentrated point load the following expression for the harmonic load intensity can be written

$$q(x) = p\delta(x - x_0)\sin\omega t \tag{59}$$

$$Q_n = \frac{2p}{L} \sin\left[\frac{n\pi}{L}x_0\right] \tag{60}$$

In which δ is the Dirac delta.

4. Numerical results and discussions

Bending of piezoelectric nanobeam is analyzed in this section. The material properties are shown in Table 2 Ramirez *et al.* (2006). The validity of the present study is proved by the means of comparing the bending of this model with those of Ebrahimi and Mahesh (2019) for various nonlocal parameters as presented in Table 3. The length of nanobeam is considered to be L = 10 nm. Also, the dimensionless deflection is adopted as

Table 3 Comparison of dimensionless deflections of nanobeam for electric voltage and magnet potential

L/h	μ (nm ²)	$\psi = 0.001$		L/h	μ (nm ²)	$\phi = 0.001$	
		Arefi and Zenkour (2016)	Present			Arefi and Zenkour (2016)	Present
10	1	3.68	3.5781	10	0	3.68	3.59892
	2	3.71	3.6482		1	1.3333	3.66921
	3	3.77	3.7302		2	1.3645	3.74018
	4	3.84	3.79011		3	1.3958	3.80234
	5	3.94	3.8952		4	1.4270	3.92011



Fig. 1 Geometry of nanobeam resting on elastic foundation



Fig. 2 Effect of nonlocal parameters on dimensionless deflection for uniform load for various magnet potential parameter (L/h = 10, V = 0, $K_w = K_p$ = 20, ΔT = 20, ΔH = 1)

$$W = 100 \frac{C_{11}I}{q_0 L^4} \tag{61}$$

Fig. 2 is investigated the effect of nonlocal parameter versus various magnet potential, it is found that increasing value of magnet potential caused increase of dimensionless deflection so when nonlocal parameter arise the magnitude of deflection increase, and we understand from this subject that magnet potential has significant role under deflection. Dimensionless deflection nanobeam with respect to slenderness ratio and nonlocal scale parameter $\mu = 2$ presented in Fig 3. It is found; external electric voltage caused that increasing of bending of nanobeam and external voltage for negative values of nanobeam demonstrated a reducing effect for positive voltages. So the axial tensile and compressive forces produced in the nanobeams via the constructed positive and negative voltages, respectively. In addition, it is lightly observed that the dimensionless deflection is approximately independent of slenderness ratio for zero electric voltages (V = 0).

The variations of the dimensionless deflection of nanobeams versus the Winkler and Pasternak parameters for various electric voltages and nonlocal parameters at L/h = 10 and = are shown in Fig. 4, respectively. It is found from this figure that regardless of the sign and magnitude of electric voltage, the dimensionless deflection increase with the increase of Winkler and Pasternak parameters, So the increment in stiffens of the nanobeam. It must be mentioned that at a constant electric voltage the increase of dimensionless deflection with Pasternak parameter measurement with a higher rate than those of Winkler parameter.

The effect of the aspect ratio versus the dimensionless deflection of nanobeam for various values of Winkler foundations shown in Fig. 5. One can observe that the dimensionless static deflections increase from L/h = 10-20



Fig. 3 Effect of slenderness ratio dimensionless deflection for uniform load for various of electric voltage with voltage without elastic foundation and with elastic foundation (L/h = 10, $\Omega = 0$, $\Delta T = 20$, $\Delta H = 1$)



Fig. 4 Effect of the Pasternak foundation on dimensionless deflection for uniform load and various electric voltage $(L/h=10, K_w=K_p=20, \mu=2, \Omega=0, \Delta T=20, \Delta H=1)$



Fig. 5 Effect of aspect ratio on dimensionless deflection for uniform load and various Winkler foundation $(L/h = 10, K_p = 20, V = 0, \Omega = 0, \Delta T = 20, \Delta H = 1)$

after that the deflection decrees with increase of aspect ratio. It is found to the fact that an increase Winkler foundation yields increase magnitude in the stiffness of the nanobeam.

5. Conclusions

Deflection of magneto-electro-elastic (MEE) nanobeams under hygro-thermal loading studied in this article. The governing equations of nonlocal nanobeams based on higher order refined beam theory are obtained using Hamilton's principle and solved by analytical solution. Obtained numerical result shows that:

- The effect of the nonlocal parameter, hygro-thermal loading, magneto-electro-mechanical loadings and aspect ratio on the deflection characteristics of nanobeams is more significant.
- The effects of dimensionless deflection have important influence on slenderness.
- The dimensionless deflection is approximately independent of slenderness ratio for zero electric voltages.
- The external electric voltage caused that increasing of bending of nanobeam and external voltage for negative values of nanobeam demonstrated a reducing effect for positive voltages.
- Also, it is clear the fact that an increase Winkler foundation yields increase magnitude in the stiffness of the nanobeam.

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