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Investigation of microstructure and surface effects on vibrational characteristics of nanobeams based on nonlocal couple stress theory

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Abstract. The article brings the study of nonlocal, surface and the couple stress together to apparent the frequency retaliation of FG nanobeams (Functionally graded). For the examination of frequency retaliation, the article considers the accurate spot of neutral axis. This article aims to enhance the coherence of proposed model to accurately encapsulate the significant effects of the nonlocal stress field, size effects together with material length scale parameters. These considered parameters are assimilated through what are referred to as modified couple stress as well as nonlocal elasticity theories, which encompasses the stiffness-hardening and softening influence on the nanobeams frequency characteristics. Power-law distribution is followed by the functional gradation of the material across the beam width in the considered structure of the article. Following the well-known Hamilton's principle, fundamental basic equations alongside their correlated boundary conditions are solved analytically. Validation of the study is also done with published result. Distinct parameters (such as surface energy, slenderness ratio, as nonlocal material length scale and power-law exponent) influence is depicted graphically following the boundary conditions on non-dimensional FG nanobeams frequency.

Keywords: modified couple stress theory; surface effect; frequency response; FG nanobeams; nonlocal elasticity

1. Introduction

The Japanese scientists discovered Functionally Graded Materials (FGMs) in 1980's. These FGMs. The FGMs are heterogeneous composites. The FGMs have unique characteristics i.e., superior thermal and corrosive resistant properties, through which they are applicable at severe temperature. FGMs also having the material property distribution and that can be controlled with the help of volume fractions of the constituent's microstructures. FGMs have high application in science and in engineering such as nano-probes, as nano-electromechanical systems (NEMS), nano-actuators and nano sensors. From the design and manufacturing point of view, consideration of the size, effects of length together with the atomic forces within the equations' formulation, under mathematical science is very important.

A large amount of theories is available to incorporate size effects & brings the importance of size-dependency of microstructures. The nonlocal elasticity theory provided from Eringen (1972, 1983) and the modified couple stress theory by Yang *et al.* (2002) happen to be highly significant in order to examine microstructures. Mechanical retaliation of FG-Euler nanobeams examined by the researcher Eltaher *et al.* (2012, 2013) via finite element (FE) technique. Rahmani and Pedram (2014) presented an analytical technique to govern the constructional characteristics of FG materials nanobeams through nonlocal Timoshenko beam theory. In the assumption of the temperature-dependent material characteristics, the effect of external thermal environment with vibrational frequency characteristics as well as the buckling behaviour for FG nanobeams were probed by Ebrahimi and Salari (2015a, b), and stress-strain analysis related to material structures are studied by (Daneshmehr *et al.* 2015, Hadi *et al.* 2018a-b, Hosseini *et al.* 2016, 2019, Mazarei *et al.* 2016, Mohammadi *et al.* 2019, Nejad and Hadi 2016a, Nejad *et al.* 2014, 2016, 2017, 2018a-b, Shishesaz *et al.* 2017, Ebrahimi and Barati 2016a-d, Ghadiri *et al.* 2016, Chaht *et al.* 2015, Shafiei *et al.* 2016, Wang *et al.* 2018, Henderson *et al.* 2018, Tanaka 2018). Stability response examination of FG materials nanobeams was analyzed by Rahmani and Jandaghian (2015) following the nonlocal third-order shear deformation theory. The significant applications and the characteristics of Eringen's non-local elasticity theory were enhanced and have been provided by Nejad and Hadi (2016a) to study the bending retaliation of bi-directional functionally graded Euler–Bernoulli nano-beams. The model following the characteristics of differential of Eringen's nonlocal theory gives computational easiness, which is not equivalent to the

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integral model (Zhu and Li 2017a-c). Moreover, the few significant literatures have demonstrated that it is very important to take into account the size-dependent across the provided thickness from micro structures (Li *et al*. 2018, Deng *et al*. 2018, Tang *et al*. 2019). She *et al*. (2018a, b) exploited predicted the wave transference nature of FG porous materials nanobeams. A good amount of literature is available to study the nano and microstructures. Following the same field theory of vibrational frequency (She *et al*. 2018c), post buckling and buckling (She *et al*. 2017) and nonlinear bending characteristics (She *et al*. 2018d) of FG material thin porous nanotubes were also examined in the literature of the present article. She *et al*. (2019a) examined the snap-buckling nature of porous FG curved material nanobeam. Moreover, the group of researchers viz., She and co-researchers (2019) developed the evaluation to prove the nonlinear bending characteristics of FG porous materials curved nanobeam. Also, self-consolidating concrete has been turned to a favorable construction material due to its impressive properties, there are few existing studies on the inclusion of carbon nano-fibres or nano-sheets on the self-consolidating concrete mixtures. Furthermore, using nano-processed materials could be employed in a variety of construction applications, where the nano-scale properties of the nano sheets or sprays are able to mitigate some micro-structural deficiencies as steel micro crack and stain in cold-formed steel uprights or cover the flexural and compressive strength loss during cyclic and motonic loading scenarios. Furthermore, the nano polymers could be the reliable choice in steel-concrete composite systems as floor systems where the shear connectors have to interact with a ductile concrete, also in higher temperature where the composite beams experience a large amount of strength loss. Even novel metaheuristic artificial techniques could be employed for the most influential parameters on the performance of the CFRPs.

Free vibrational analysis of FG material microbeams (Ke *et al*. 2012, Asghari *et al*. 2010, 2011), axially graded microbeams (Akgöz and Civalek 2013), geometrically imperfect FG microbeams (Dehrouyeh-Semnani *et al*. 2016) are highly significant. Moreover, the static nature of FG material microbeams (Şimşek *et al*. 2013, Şimşek and Reddy 2013, Al-Basyouni *et al*. 2015) have also emphasized on the number of advantages through the modified couple stress theory. The effects that elastic foundation has in regard to the stability of the FG material's thin microbeams to thermal environment, was assessed by Akgöz and Civalek (2014). Relying on the variational differential quadrature technique, the nonlinear mechanical retaliation of third-order FG material microbeams was proposed by Ansari *et al*. (2016). Ebrahimi and Barati (2016a-m) utilized the nonlocal strain gradient theory for the sole purpose of examining nanobeams and nanoplates. Some of the most renowned authors and researchers worked on neglecting the shear deformation effect and other effects too (Kheroubi *et al*. 2016, Tounsi *et al*. 2013, Youcef *et al*. 2015, Zenkour 2016). Incorporating the modified couple stress theory, Khorshidi and Shariati (2015) explored that by considering the material length scale parameter enhance the vibrational natural frequencies of FG material nanobeam. A group of researchers Baghani *et al*. (2016) analyzed the nonlinear vibration retaliation of FG material tapered-nanobeams via analytical couple-stress technique. The post-buckling behavior of FG material nanobeams was considered by Khorshidi *et al*. (2016). Attia and Mahmoud (2016) provided the surface influence studied in microstructures and produced a nonlocal couple stress (NL-CS) theory.

As far for the knowledge, this is a prior attempt for the examination of the response of the frequency of FG nanobeams together with both material length scale parameter and nonlocal stress field parameter. So, this works is the prior trial to develop a nonlocal couple stress beam model showing the surface energy influence. FGMs also having the material property distribution and that can be controlled with the help of volume fractions of the constituent's microstructures. FGMs have high application in science and in engineering such as nano-probes, as nano-electromechanical systems (NEMS), nano-actuators and nano sensors. From the design and manufacturing point of view, consideration of the size & length-scale effects and the atomic forces in the formulation of equations under mathematical science is very important. Time-consuming and costly experiments have always been a barrier in front of the new explorations; however, employing intelligence solutions are one of the practical ways to address these issues. Whereas, artificial intelligence techniques have performed on a variety of experimental studies and proved to be reliable not only in case of parameters estimation but also the prediction of crucial design characteristics. Different kind of algorithms has introduced which have their traits and advantages. Using the relevant algorithms in order to analytical assessment has been carried out on different types of studies. That being the case, performing the artificial intelligence algorithms is a potential method to avoid non-linearity and sophisticated analysis of the nanoscale problems.

2. Theory and formulation

2.1 Power-law FG nanobeam model

Fig. 1 represents the schematics of FG nanobeams. In the schematics, *L* and *h* represents beam length, thickness respectively, and L & h spans along the x- and z-axes. The FG nanobeams having the material gradation follow the power-law function. The density (ρ), surface elastic modulus E_s(z), Young's modulus E(z), and residual surface stress (τ_s) are as follows

$$E(z) = (E^+ - E^-)\left(\frac{z}{h} + \frac{1}{2}\right)^p + E^- \tag{1}$$

$$v(z) = (v^+ - v^-)\left(\frac{z}{h} + \frac{1}{2}\right)^p + v^- \tag{2}$$

$$\lambda_s(z) = (\lambda_s^+ - \lambda_s^-)\left(\frac{z}{h} + \frac{1}{2}\right)^p + \lambda_s^- \tag{3}$$

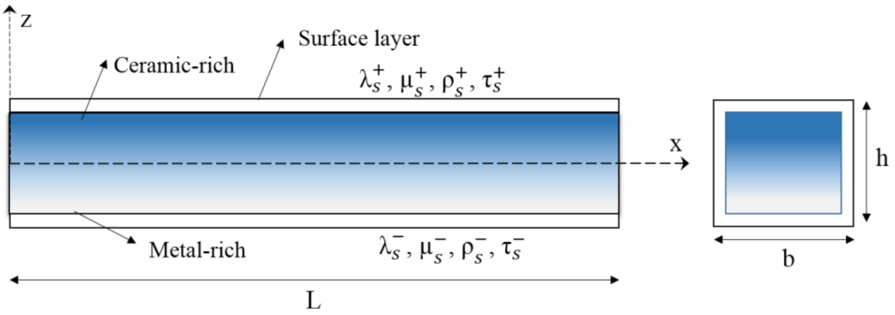


Fig. 1 Configuration and coordinates of FG nanobeam

Table 1 Material properties of FG nanobeam

Property	Si	Al
E(Gpa)	210×10^9	68.5×10^9
$\boldsymbol{\lambda_s}$	−4.448	6.842
$\boldsymbol{\mu_s}$	−2.774	−0.376
ρ	2331	3000
$\boldsymbol{\rho_s}$	3.17×10^{-7}	5.46×10^{-7}
$\boldsymbol{\tau_s}$	0.6048	0.9108
$\boldsymbol{v}$	0.24	0.35

$$\mu_s(z) = (\mu_s^+ - \mu_s^-)\left(\frac{z}{h} + \frac{1}{2}\right)^p + \mu_s^- \tag{4}$$

$$\rho(z) = (\rho_s^+ - \rho^-)\left(\frac{z}{h} + \frac{1}{2}\right)^p + \rho^- \tag{5}$$

$$\tau_s(z) = (\tau_s^+ - \tau_s^-)\left(\frac{z}{h} + \frac{1}{2}\right)^p + \tau_s^- \tag{6}$$

$$\rho_s(z) = (\rho_s^+ - \rho_s^-)\left(\frac{z}{h} + \frac{1}{2}\right)^p + \rho_s^- \tag{7}$$

Where power-law exponent is represented by p. FG nanobeams top surface having pure *Si*, on the other hand, bottom surface have pure *Al*. The '+' and '−' represents the material properties of top and bottom surface of the considered geometry. Table 1 carries the material properties of *Si* and *Al* materials. As a function of Lame's constant (λ and μ), the physical neutral axis of FG beams can be defined as follows

$$z_0 = \frac{\int_{-\frac{h}{2}}^{\frac{h}{2}} [\lambda(z) + 2\mu(z)] z dz}{\int_{-\frac{h}{2}}^{\frac{h}{2}} [\lambda(z) + 2\mu(z)]\, dz} \tag{8}$$

2.2 Kinematic relations

The displacement components of FG nanobeams are consider to follow theory (Euler–Bernoulli beam) as follows

$$\boldsymbol{u_x(x,z,t) = u(x,t) - (z - z_0)\frac{\partial w(x,t)}{\partial x}} \tag{9a}$$

$$\boldsymbol{u_z(x,z,t) = w(x,t)} \tag{9b}$$

where, u represents the axial and w indicates the transverse displacement components of the mid-surface. The nonzero normal strain are as follows

$$\begin{gathered} \varepsilon_{xx} = \varepsilon_{xx}^0 - (z - z_0)k^0, \\ \varepsilon_{xx}^0 = \frac{\partial u(x,t)}{\partial x}, k^0 = \frac{\partial^2 w(x,t)}{\partial x^2} \end{gathered} \tag{10}$$

where ε_{xx}^0 represents the extensional and k^0 bending strains.

2.2.1 The modified couple stress theory

Based on modified couple stress theory model, U represents the strain energy of an elastic material-covering Ω region, which corresponds to strain and curvature tensors as

$$U = \frac{1}{2}\int_{\Omega} (\sigma_{ij}\varepsilon_{ij} + m_{ij}\chi_{ij})dV, (i.j = 1.2.3) \tag{11}$$

where σ represents the Cauchy stress tensor, ε represent the classical strain tensor, m denotes deviatoric part of the couple stress tensor and χ symbolizes the symmetric curvature tensor. The strain and curvature tensors are stated in the equation form as

$$\varepsilon_{ij} = \frac{1}{2}(u_{i.j} + u_{j.i}) \tag{12a}$$

$$\chi_{ij} = \frac{1}{2}(\theta_{i.j} + \theta_{j.i}) \tag{12b}$$

where $u_{i,j}$ represents the components of the displacement vector and $\theta_{i,j}$ represents the components of the rotation vectors are

$$\theta_i = \frac{1}{2}e_{ijk}u_{k.j} \tag{13}$$

where e_{ijk} symbolizes the permutation symbol.

The engrained combination in the equation format can be expressed as

$$\sigma_{ij} = \lambda(z)\epsilon_{kk}\delta_{ij} + 2\mu(z)\epsilon_{ij} \tag{14a}$$

$$m_{ij} = 2\mu(z)[l(z)]^2 X_{ij} \tag{14b}$$

where δ_{ij} symbolizes the Kroenke delta, l represent the length of the material scale parameter which governs the influence of couple stress.

Láme's constants related to the considered study can be defined by

$$\lambda(z) = \frac{E(z)\nu(z)}{[1+\nu(z)][1-2\nu(z)]} \tag{15a}$$

$$\mu(z) = \frac{E(z)}{2[1+\nu(z)]} \tag{15b}$$

2.2.2 Surface elasticity theory

For a FGM beam in the absence of any residual stresses in the bulk because of surface stress, the prominent stress–strain combination in the equation format is expressed as

$$\sigma_{\mathrm{xx}} = (\lambda + 2\mu)\varepsilon_{xx} + \nu\sigma_{zz} \tag{16}$$

Now, the Gurtin–Murdoch elasticity theory is used, where the nanobeams surface is two-dimensional thin membrane in connection with the beneath the bulk material. Therefore, the surface stress which is non-zero is given as

$$\begin{aligned}\sigma^s_{\alpha\beta} = \tau_s\delta_{\alpha\beta} + (\tau_s + \lambda_s)\varepsilon_{\gamma\gamma}\delta_{\alpha\beta} \\ +2(\mu_s - \tau_s)\varepsilon_{\alpha\beta} + \tau_s u^s_{\alpha,\beta}\end{aligned} \tag{17}$$

$$\sigma^s_{\alpha z} = \tau_s u^s_{z,\alpha} \tag{18}$$

Non-zero components of nanobeams for surface stress is

$$\sigma^s_{xx} = \tau_s + (\lambda_s + 2\mu_s)\varepsilon_{xx} \tag{19}$$

$$\sigma^s_{xz} = \tau_s\frac{\partial w}{\partial x} \tag{20}$$

The stresses at the nanobeam surface thin membrane must follow the equilibrium relations

$$\begin{aligned}&(\tau_{\beta i,\beta})^+ - (\sigma_{iz})^+ = \rho^+(\frac{\partial^2 u_i}{\partial t^2})^+ \\ &\text{at } \; z = +h/2\end{aligned} \tag{21}$$

$$\begin{aligned}&(\tau_{\beta i,\beta})^- - (\sigma_{iz})^- = \rho^-(\frac{\partial^2 u_i}{\partial t^2})^- \\ &\text{at } \; z = -h/2\end{aligned} \tag{22}$$

Where which $(\tau_{\beta i})^+$ and $(\tau_{\beta i})^-$ symbolizes the surface stresses; $(\sigma_{iz})^+$ and $(\sigma_{iz})^-$ symbolizes the bulk stresses. Introducing Eqs. (9), (17) and (18) into Eqs. (21) and (22) yields

$$(\sigma_{zz})^+ = (\tau_s^+)(\frac{\partial^2 w}{\partial x^2}) - \rho_s^+(\frac{\partial^2 w}{\partial t^2}) \tag{23}$$

$$(\sigma_{zz})^- = -(\tau_s^-)(\frac{\partial^2 w}{\partial x^2}) + \rho_s^-(\frac{\partial^2 w}{\partial t^2}) \tag{24}$$

In this study σ_{zz} is considered to be in the following form

$$\sigma_{zz} = \frac{z}{h}[(\sigma_{zz})^+ - (\sigma_{zz})^-] + \frac{1}{2}[(\sigma_{zz})^+ + (\sigma_{zz})^-] \tag{25}$$

Introducing Eqs. (23) and (24) into (25) which leads to

$$\begin{aligned}\sigma_{zz} = \frac{z}{h}[-(\rho_s^+ + \rho_s^-)\frac{\partial^2 w}{\partial t^2} + (\tau_s^+ + \tau_s^-)\frac{\partial^2 w}{\partial x^2}] \\ + \frac{1}{2}[-(\rho_s^+ - \rho_s^-)\frac{\partial^2 w}{\partial t^2} + (\tau_s^+ \\ - \tau_s^-)\frac{\partial^2 w}{\partial x^2}]\end{aligned} \tag{26}$$

2.2.3 The nonlocal constitutive relations

Following Eringen nonlocal elasticity model, stress state at a fixed point at interior of a body is a function of strains at all distinct points in the nearest regions. The equivalent differential form of the nonlocal native equation can be expressed by

$$(1 - (e_0 a)\nabla^2)\sigma_{kl} = t_{kl} \tag{27}$$

where ∇^2 represents Laplacian operator. Scale length is represented by the $e_0 a$, which consider the small-scale effect on nano-structures response. At last, the inherent equations relations of nonlocal nanobeams is given as (Attia and Mahmoud 2016)

$$\sigma_{xx} - \beta^2\frac{\partial^2 \sigma_{xx}}{\partial x^2} = [\lambda(z) + 2\mu(z)]\varepsilon_{xx} \tag{28}$$

$$m_{xy} - \beta^2\frac{\partial^2 m_{xy}}{\partial x^2} = 2\mu(z)l^2\chi_{xy} \tag{29}$$

$$\sigma^s_{xx} - \beta^2\frac{\partial^2 \sigma^s_{xx}}{\partial x^2} = \tau_s + (\lambda_s(z) + 2\mu_s(z))\varepsilon_{xx} \tag{30}$$

where $\beta = (e_0 a)^2$.

3. Governing equations

After applying the extended Hamilton's principle, fundamental equation is obtained as follows

$$\int_0^t \delta(U - T + V)dt = 0 \tag{31}$$

Here U, $T$and V represents the strain energy, kinetic energy & external forces work, respectively. The strain energy is given as

$$\begin{aligned}\delta U = \int_v (\sigma_{ij}\delta\varepsilon_{ij})dV + \int_{S^+}(\sigma^s_{ij}\delta\varepsilon_{ij})dS^+ \\ + \int_{S^-}(\sigma^s_{ij}\delta\varepsilon_{ij})dS^-\end{aligned} \tag{32}$$

Introducing the Eq. (10) into Eq. (32) gives

$$\delta U=\int_0^L[(N+N^s)(\delta\varepsilon_{xx}^0)-(M+M^s)(\delta k^0)]dx \quad (33)$$

where N represents the axial force moment and M denotes the bending moment. The variation of kinetic energy is given as

$$\delta T=\int_0^L\left[I_0(\dot u\delta\dot u+\dot w\delta\dot w)-I_1(\dot u\frac{\partial\delta\dot w}{\partial x}+\delta\dot u\frac{\partial\dot w}{\partial x})+I_2\frac{\partial\dot w}{\partial x}\frac{\partial\delta\dot w}{\partial x}\right]dx \quad (34)$$

Where (I_0,I_1,I_2) are the mass moment of inertias, defined as follows

$$(I_0,I_1,I_2)=b\int_{-h/2}^{+h/2}\rho(z)(1,(z-z_0),(z-z_0)^2)\,dz \quad (35)$$

Introducing the Eqs. (32)-(34) in Eq. (31) and the coefficients of $\delta u,\delta w$ equating to zero

$$\frac{\partial(N+N^s)}{\partial x}=I_0\ddot u-I_1\frac{\partial\ddot w}{\partial x} \quad (36)$$

$$\frac{\partial^2(M+M^s)}{\partial x^2}=I_0\ddot w+I_1\frac{\partial\ddot u}{\partial x}-I_2\frac{\partial^2\ddot w}{\partial x^2} \quad (37)$$

Solving the Eqs. (28)-(30) and which yields

$$N-\beta\frac{\partial^2N}{\partial x^2}=A_{11}\frac{\partial u}{\partial x}-B_{11}\frac{\partial^2w}{\partial x^2}+\frac{A_{44}}{2}\left[(\tau_s^+-\tau_s^-)\frac{\partial^2w}{\partial x^2}-(\rho_s^+-\rho_s^-)\frac{\partial^2w}{\partial t^2}\right]+\frac{B_{44}}{h}[(\tau_s^++\tau_s^-)\frac{\partial^2w}{\partial x^2}-(\rho_s^++\rho_s^-)\frac{\partial^2w}{\partial t^2}] \quad (38)$$

$$N^s-\beta\frac{\partial^2N^s}{\partial x^2}=[b(\lambda_s^++2\mu_s^++\lambda_s^-+2\mu_s^-)+2A_{11}^s]\frac{\partial u}{\partial x}-[\frac{bh}{2}(\lambda_s^++2\mu_s^+-\lambda_s^--2\mu_s^-)+2B_{11}^s]\frac{\partial^2w}{\partial x^2} \quad (39)$$

$$M-\beta\frac{\partial^2M}{\partial x^2}=B_{11}\frac{\partial u}{\partial x}-D_{11}\frac{\partial^2w}{\partial x^2}+\frac{B_{44}}{2}[(\tau_s^+-\tau_s^-)\frac{\partial^2w}{\partial x^2}-(\rho_s^+-\rho_s^-)\frac{\partial^2w}{\partial t^2}]+\frac{D_{44}}{h}[(\tau_s^++\tau_s^-)\frac{\partial^2w}{\partial x^2}-(\rho_s^++\rho_s^-)\frac{\partial^2w}{\partial t^2}] \quad (40)$$

$$M^s-\beta\frac{\partial^2M^s}{\partial x^2}=\left[\frac{bh}{2}(\lambda_s^++2\mu_s^+-\lambda_s^--2\mu_s^-)+2B_{11}^s\right]\frac{\partial u}{\partial x}-[4(\lambda_s^++2\mu_s^++\lambda_s^-+2\mu_s^-)+2D_{11}^s]\frac{\partial^2w}{\partial x^2} \quad (41)$$

$$Y_1-\beta^2\frac{\partial^2Y_1}{\partial x^2}=-A_{13}l^2\frac{\partial^2w}{\partial x^2} \quad (42)$$

where the cross-sectional rigidities of the considered structure are given as follows

$$(A_{11},B_{11},D_{11})=b\int_{-h/2}^{h/2}[\lambda(z)+2\mu(z)](1,(z-z_0),(z-z_0)^2)\,dz \quad (43)$$

$$(A_{11}^s,B_{11}^s,D_{11}^s)=\int_{-h/2}^{h/2}[\lambda_s(z)+2\mu_s(z)](1,(z-z_0),(z-z_0)^2)\,dz \quad (44)$$

$$(A_{44},B_{44},D_{44})=b\int_{-h/2}^{h/2}\frac{v(z)}{1-v(z)}(1,(z-z_0),(z-z_0)^2)\,dz \quad (45)$$

Solving Eqs. (38)-(42), required equations of motion related to FG nanobeam carrying displacements are as

$$A_{11}\frac{\partial^2u}{\partial x^2}-B_{11}\frac{\partial^3w}{\partial x^3}+\frac{A_{44}}{2}[(\tau_s^+-\tau_s^-)\frac{\partial^3w}{\partial x^3}-(\rho_s^+-\rho_s^-)\frac{\partial^3w}{\partial x\partial t^2}]+\frac{B_{44}}{h}[(\tau_s^++\tau_s^-)\frac{\partial^3w}{\partial x^3}-(\rho_s^++\rho_s^-)\frac{\partial^3w}{\partial x\partial t^2}]+[b(\lambda_s^++2\mu_s^++\lambda_s^-+2\mu_s^-)+2A_{11}^s]\frac{\partial^2u}{\partial x^2}-[\frac{bh}{2}(\lambda_s^++2\mu_s^+-\lambda_s^--2\mu_s^-)+2B_{11}^s]\frac{\partial^3w}{\partial x^3}-I_0\frac{\partial^2u}{\partial t^2}+I_1\frac{\partial^3w}{\partial x\partial t^2}+\beta(I_0\frac{\partial^4u}{\partial x^2\partial t^2}-I_1\frac{\partial^5w}{\partial x^3\partial t^2})=0 \quad (46)$$

$$B_{11}\frac{\partial^3u}{\partial x^3}-(D_{11}+l^2A_{13})\frac{\partial^4w}{\partial x^4}+\frac{B_{44}}{2}[(\tau_s^+-\tau_s^-)\frac{\partial^4w}{\partial x^4}-(\rho_s^+-\rho_s^-)\frac{\partial^4w}{\partial x^2\partial t^2}]+\frac{D_{44}}{h}[(\tau_s^++\tau_s^-)\frac{\partial^4w}{\partial x^4}-(\rho_s^++\rho_s^-)\frac{\partial^4w}{\partial x^2\partial t^2}]+[\frac{bh}{2}(\lambda_s^++2\mu_s^+-\lambda_s^--2\mu_s^-)+2B_{11}^s]\frac{\partial^3u}{\partial x^3}-[\frac{bh^2}{4}(\lambda_s^++2\mu_s^++\lambda_s^-+2\mu_s^-)+2D_{11}^s]\frac{\partial^4w}{\partial x^4}-I_0\frac{\partial^2w}{\partial t^2}-I_1\frac{\partial^3u}{\partial x\partial t^2}+I_2\frac{\partial^4w}{\partial x^2\partial t^2}+\beta(+I_0\frac{\partial^4w}{\partial x^2\partial t^2}+I_1\frac{\partial^5u}{\partial x^3\partial t^2}-I_2\frac{\partial^6w}{\partial x^4\partial t^2})=0 \quad (47)$$

4. Solution method

Analytical solution is utilize to obtain the fundamental expressions of nonlocal couple stress nanobeams of FGM with distinct boundary edges. To please the distinct boundary conditions, following solution for displacement variables is used

$$u(x,t)=\sum_{n=1}^{\infty}U_n\frac{\partial X_m(x)}{\partial x}e^{i\omega_nt} \quad (48)$$

$$w(x,t)=\sum_{n=1}^{\infty}W_nX_m\,(x)e^{i\omega_nt} \quad (49)$$

where (U_n,W_n) represents the unknown Fourier coefficients. Introducing the Eqs. (48)-(49) in Eqs. (46)-(47) respectively, yields

$$\{[K]+[M]\omega^2\}\begin{Bmatrix}U_n\\W_n\end{Bmatrix}=0 \tag{50}$$

where [K] represents the stiffness matrix and [M] represents the mass matrixes for nanobeams of FG material, respectively.

$$\begin{aligned}
k_{1,1} &= \big(A_{11}+2A_{11}^s+b(\lambda_s^+ +2\mu_s^+ +\lambda_s^- +2\mu_s^-)\big)\alpha_3,\\
k_{1,2} &= \big(B_{11}+2B_{11}^s+0.5bh(\lambda_s^+ +2\mu_s^+ -\lambda_s^- -2\mu_s^-)\big)\alpha_9,\\
k_{2,1} &= -\big(B_{11}+2B_{11}^s+0.5bh(\lambda_s^+ +2\mu_s^+ -\lambda_s^- -2\mu_s^-)\big)\alpha_3\\
&\quad +\left[0.5A_{44}(\tau_s^+ -\tau_s^-)+\frac{B_{44}}{h}(\tau_s^+ +\tau_s^-)\right]\alpha_3,\\
k_{2,2} &= +(-D_{11}-l^2A_{13}-2D_{11}^s\\
&\quad -\frac{bh^2}{4}(\lambda_s^+ +2\mu_s^+ +\lambda_s^- +2\mu_s^-))\alpha_9\\
&\quad +\left[0.5B_{44}(\tau_s^+ -\tau_s^-)+\frac{D_{44}}{h}(\tau_s^+ +\tau_s^-)\right]\alpha_9,\\
m_{1,1} &= (a_1-\beta a_3)I_0,\\
m_{1,2} &= (a_7-\beta a_9)I_1,\\
m_{2,1} &= -(a_1-\mu a_3)I_1\\
&\quad +\left[0.5A_{44}(\rho_s^+ -\rho_s^-)+\frac{B_{44}}{h}(\rho_s^+ +\rho_s^-)\right]\alpha_1\\
m_{2,2} &= (a_5-\mu a_7)I_0-(a_7-\mu a_9)I_2+[0.5B_{44}(\rho_s^+ -\rho_s^-)\\
&\quad +\frac{D_{44}}{h}(\rho_s^+ +\rho_s^-)]\alpha_7,
\end{aligned}$$

where

$$\begin{aligned}
\alpha_1 &= \int_0^L X_m' X_m' dx, \alpha_3 = \int_0^L X_m''' X_m'\, dx\\
\alpha_5 &= \int_0^L X_m X_m dx, \alpha_7 = \int_0^L X_m'' X_m\, dx, \alpha_9\\
&= \int_0^L X_m'''' X_m\, dx
\end{aligned} \tag{51}$$

The function X_m for different boundary conditions is defined by

S-S
$$X_m(x)=\sin(\lambda_n x)\qquad \lambda_n=\frac{n\pi}{L} \tag{52}$$

C-C
$$\begin{aligned}
X_m(x) &= \sin(\lambda_n x)-\sinh(\lambda_n x)\\
&\quad -\xi_m(\cos(\lambda_n x)-\cosh(\lambda_n x))\\
\xi_m &= \frac{\sin(\lambda_n x)-\sinh(\lambda_n x)}{\cos(\lambda_n x)-\cosh(\lambda_n x)}\\
\lambda_1 &= 4.730, \qquad \lambda_2 = 7.853,\\
\lambda_3 &= 10.996, \qquad \lambda_4 = 14.137,\\
\lambda_{n\geq 5} &= \frac{(n+0.5)\pi}{L}
\end{aligned} \tag{53}$$

C-S
$$\begin{aligned}
X_m(x) &= \sin(\lambda_n x)-\sinh(\lambda_n x)\\
&\quad -\xi_m(\cos(\lambda_n x)-\cosh(\lambda_n x))\\
\xi_m &= \frac{\sin(\lambda_n x)+\sinh(\lambda_n x)}{\cos(\lambda_n x)+\cosh(\lambda_n x)}\\
\lambda_1 &= 3.927, \qquad \lambda_2 = 7.069,\\
\lambda_3 &= 10.210, \qquad \lambda_4 = 13.352,\\
\lambda_{n\geq 5} &= \frac{(n+0.25)\pi}{L}
\end{aligned} \tag{54}$$

5. Numerical results and discussions

This section represents the FG nanobeams frequency response characteristics of L= 10 nm length which contains the nonlocal couple stress elasticity. Furthermore, the influence of distinct consider parameters on natural frequencies is obtained in the present article, where the important considered parameters are surface elasticity, power-law exponent, material length scale parameter, nonlocality parameter, slenderness ratio and boundary conditions. The non-dimensional natural frequencies obtained in this analysis can be represented as follows

$$\bar{\omega}=\frac{\omega L^2}{h}\sqrt{\frac{\rho_c}{E_c}} \tag{55}$$

Following the considered distinct boundary conditions viz., C-S, S-S and C-C, the natural frequency that are dimensionless of FG nanobeams are examined and comparison is done relying on the nonlocal couple stress

Table 2 Comparison of the dimensionless frequency for nonlocal FG nanobeams (L/h = 20)

B.C.	β	p = 0.1		p = 0.5		p = 1	
		CBT (Eltaher *et al*. 2012)	Present	CBT (Eltaher *et al*. 2012)	Present	CBT (Eltaher *et al*. 2012)	Present
S-S	0	9.2129	9.18873	7.8061	7.73775	7.0904	6.9885
	1	8.7879	8.76631	7.4458	7.38203	6.7631	6.66723
	2	8.4166	8.39725	7.1312	7.07125	6.4774	6.38655
	3	8.0887	8.07120	6.8533	6.79669	6.2251	6.13857
C-C	0	20.8529	20.8240	17.5613	17.5355	15.8612	15.8375
	1	19.6741	19.6438	16.5686	16.5413	14.9645	14.9396
	2	18.6707	18.6439	15.7235	15.6991	14.2013	14.1789
	3	17.8037	17.7827	14.9934	14.9737	13.5419	13.5237

Table 3 Dimensionless frequency of FG nanobeams with S-S boundary conditions for various elasticity theories (l = 0.5 h)

	β	NL-SE			NL-CS			NL-CS-SE		
		p = 0.2	p = 1	p = 5	p = 0.2	p = 1	p = 5	p = 0.2	p = 1	p = 5
L/h = 10	0	2.67343	2.35267	2.0961	4.02805	3.35256	2.77671	3.91118	3.31535	2.76324
	1	2.55041	2.24439	1.99964	3.84287	3.19844	2.64906	3.73121	3.16277	2.63607
	2	2.44295	2.14981	1.91537	3.68109	3.06379	2.53754	3.57399	3.02948	2.52498
	3	2.34801	2.06625	1.84093	3.53816	2.94483	2.43901	3.43511	2.91174	2.42685
	4	2.26335	1.99174	1.77454	3.41068	2.83873	2.35113	3.31125	2.80674	2.33933
L/h = 20	0	2.41584	2.18294	1.94774	4.04068	3.36367	2.78563	3.7459	3.2018	2.65563
	1	2.30473	2.08254	1.85816	3.85493	3.20904	2.65757	3.57362	3.05454	2.53349
	2	2.20767	1.99482	1.7799	3.69264	3.07394	2.54569	3.42312	2.92589	2.42678
	3	2.12191	1.91734	1.71076	3.54926	2.95459	2.44685	3.29015	2.81223	2.33252
	4	2.04544	1.84823	1.6491	3.42138	2.84813	2.35869	3.17157	2.71087	2.24845

Table 4 Dimensionless frequency of FG nanobeams with C-S boundary conditions for various elasticity theories (l = 0.5 h)

	β	NL-SE			NL-CS			NL-CS-SE		
		p = 0.2	p = 1	p = 5	p = 0.2	p = 1	p = 5	p = 0.2	p = 1	p = 5
L/h = 10	0	4.17471	3.67391	3.27332	6.28948	5.23456	4.33554	6.10754	5.17722	4.31513
	1	3.95146	3.4774	3.09829	5.95347	4.9548	4.10388	5.78092	4.90031	4.08439
	2	3.76057	3.3094	2.94863	5.66614	4.71558	3.90578	5.50165	4.66356	3.8871
	3	3.59492	3.1636	2.81875	5.41676	4.50796	3.73385	5.25931	4.45811	3.71589
	4	3.44939	3.03552	2.70466	5.19765	4.32556	3.5828	5.0464	4.27762	3.56548
L/h = 20	0	3.77421	3.4104	3.04294	6.3125	5.25479	4.35179	5.85214	5.00217	4.14887
	1	3.57353	3.22906	2.88115	5.97703	4.97551	4.12052	5.54097	4.73619	3.92827
	2	3.4018	3.07388	2.74269	5.68994	4.7365	3.92259	5.27469	4.50858	3.73949
	3	3.25266	2.93912	2.62244	5.44059	4.52892	3.75069	5.04344	4.31092	3.57555
	4	3.12156	2.82066	2.51675	5.2214	4.34644	3.59957	4.84016	4.13717	3.43143

Table 5 Dimensionless frequency of FG nanobeams with C-C boundary conditions for various elasticity theories (l = 0.5 h)

	β	NL-SE			NL-CS			NL-CS-SE		
		p = 0.2	p = 1	p = 5	p = 0.2	p = 1	p = 5	p = 0.2	p = 1	p = 5
L/h = 10	0	6.01006	5.28912	4.71246	9.12162	7.59152	6.28777	8.79262	7.45337	6.21232
	1	5.66594	4.98627	4.44281	8.59626	7.15375	5.92543	8.28917	7.02659	5.85685
	2	5.37488	4.73012	4.21472	8.15233	6.78391	5.61928	7.86335	6.66562	5.55615
	3	5.1245	4.50978	4.01849	7.77076	6.46607	5.35615	7.49706	6.35512	5.29747
	4	4.90615	4.31761	3.84735	7.4382	6.18908	5.12683	7.17761	6.08432	5.07186
L/h = 20	0	5.43454	4.91071	4.38159	9.15726	7.62286	6.31294	8.42658	7.20273	5.97404
	1	5.12839	4.63411	4.13479	8.63829	7.19072	5.95512	7.95187	6.79703	5.63754
	2	4.86876	4.39953	3.92549	8.19861	6.82462	5.65197	7.54929	6.45296	5.35217
	3	4.64495	4.19732	3.74505	7.81988	6.50928	5.39086	7.20226	6.15636	5.10616
	4	4.44941	4.02064	3.58742	7.48921	6.23397	5.16288	6.89907	5.89723	4.89123

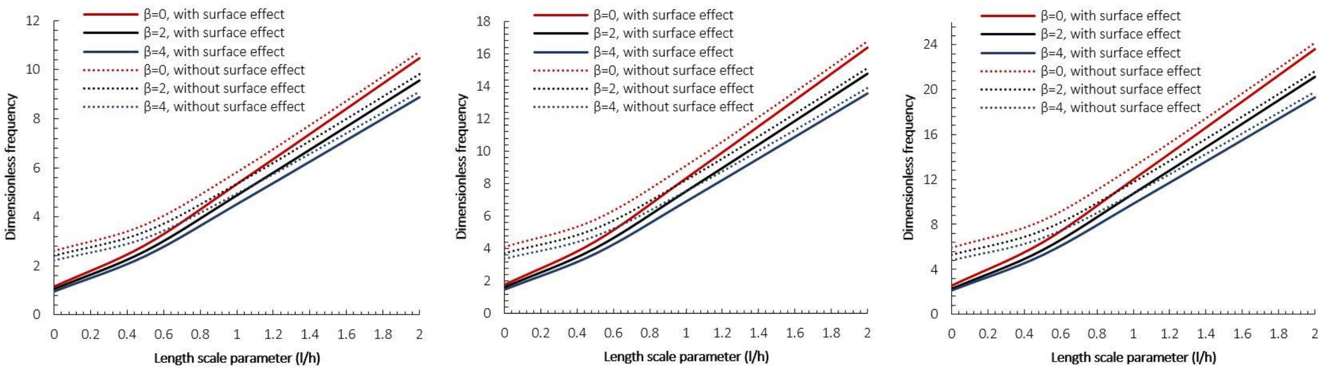
elasticity (NL-CS), nonlocal surface elasticity (NL-SE) and nonlocal couple stress based surface elasticity (NL-CS-SE). Moreover, the effect of gradient index (p) and nonlocality parameter (β) is also find. The outcomes tabulated in Tables from 3 to 5 affirms that for a selected measure of gradient index and slenderness ratio a significant influence of NL-CS on natural frequency over NL-CS-SE and NL-SE theories can be witnessed. It is notice that fact, NL-CS theory carries a length scale parameter which shows that the influence of stiffness-hardening of FG nanobeams.

However, there is only the consideration of nonlocal stress field parameter and deterioration of length scale parameter; the natural frequencies were underestimated in the previous works. Furthermore, while ignoring the boundary conditions, it is found that with the help of nonlocal parameter (β) in the examination have a detrimental influence on frequencies because of stiffness-softening influence. Therefore, FG nanobeams natural frequencies are evaluated by the assumption of modified couple stress theory overestimated.

The influence of the length scale parameter (l/h) on the non-dimensional frequency of NL-CS FG nanobeams with varied nonlocal parameters (β) is shown in Fig. 2. Moreover, the influence of surface effect is also been emphasize in the study. It is noticed that in Fig. 2 at constant nonlocal parameter value with all distinct boundary conditions, as the value of l/h ratio increases, then there is great impact on natural frequency, as a result natural frequency starts increases significantly. So, at large l/h ratio, the influence of nonlocal parameter is remarkable. This result governs that the FG nanobeams turns into the high rigid form with the increment of length scale parameter, then the softening influence of nonlocal parameter is noticeable. Therefore, the presence of both the parameters, which are discussed above is highly important for the accurate examination of the FG nanobeams. Moreover, high frequency is observed in the absence of surface influence. In the view of S-S and C-S boundary conditions, a high measure of frequency is noticed for C-C boundary condition because of Fg nanobeams have higher rigidity with addition of number of strong supports.

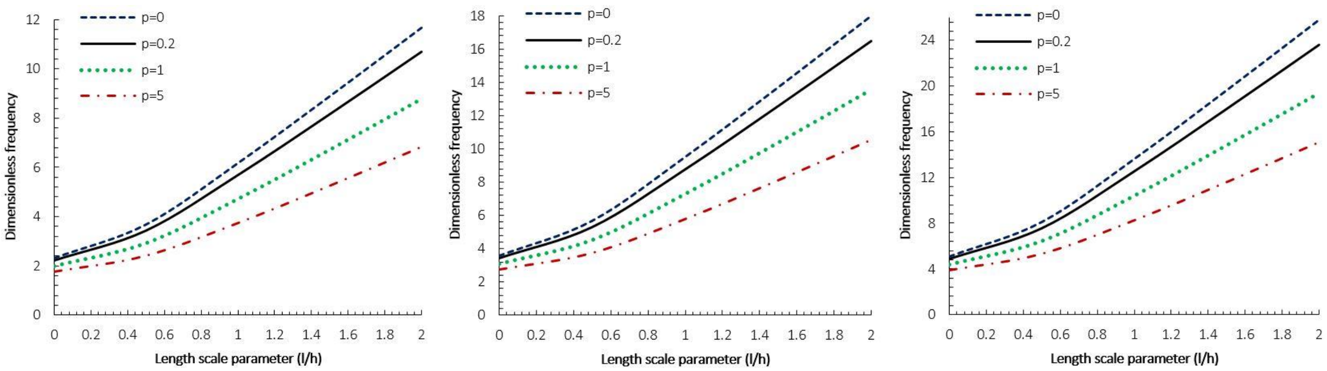
In view of NL-CS-SE FG nanobeam, the non-dimensional frequency variation versus length scale parameter is examined in the assumption of distinct gradient indices (p). Fig. 3, it is easily noticed that for a chosen measure of material length scale parameter, a higher measure of frequency is observed for the mow measure of gradient index. Now, as outcomes, it is noticed that in the increment of the gradient index, the percentage of the metal gets increases and which also diminished the FG nanobeams rigidity. Afterwards, it is found that the effect of gradient index is remarkably noticeable at larger material length scale parameter. Therefore, as conclusion it is observed that, a less influence of gradient index on frequency vibrations prevails when the examination is done while ignoring the couple stress effect (l/h = 0). As the couple stress influence is considered ($l/h \neq 0$), the remarkable influence of the gradient index (p) becomes remarkable.

The FG nanobeams frequency variation is observed in presence of distinct elasticity theories and it is compared against with slenderness ratio in Fig. 4. In the examination



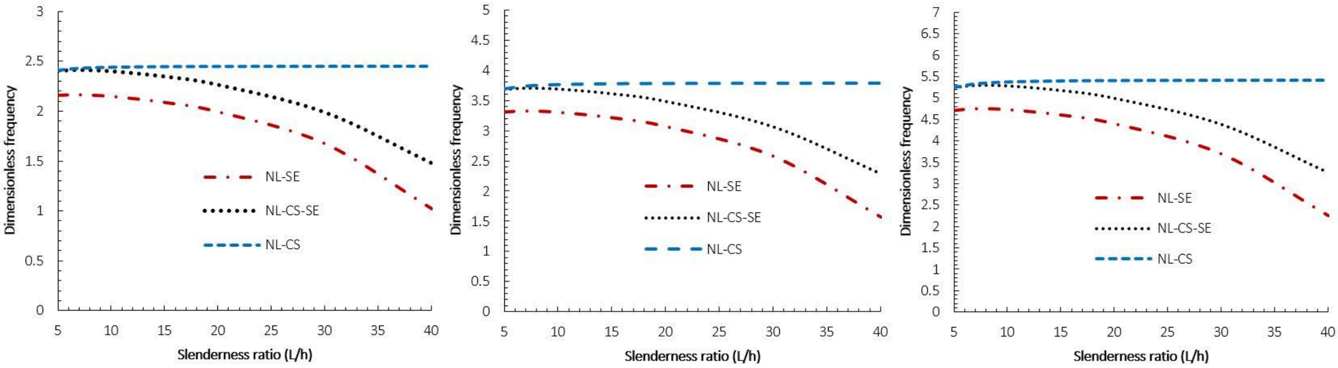
(a) S-S (b) C-S (c) C-C

Fig. 2 Variation of dimensionless frequency of NL-CS FG nanobeam versus length scale parameter for various nonlocal parameters (L/h = 40, p = 0.5)



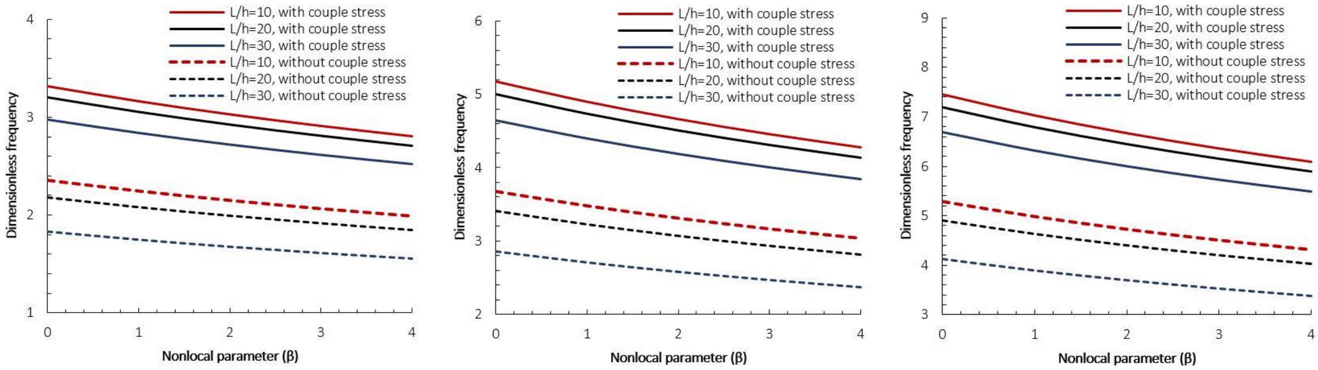
(a) S-S (b) C-S (c) C-C

Fig. 3 Variation of dimensionless frequency of NL-CS-SE FG nanobeam versus length scale parameter for various gradient indices (L/h = 20, β = 2 nm^2)



(a) S-S (b) C-S (c) C-C

Fig. 4 Variation of dimensionless frequency of FG nanobeam versus slenderness ratio for NL-SE, NL-CS and NL-CS-SE theories (p = 1, l = 0.25 h, β = 2 nm^2)



(a) S-S (b) C-S (c) C-C

Fig. 5 Variation of dimensionless frequency of NL-SE FG nanobeam versus nonlocal parameter for various slenderness ratios (l/h = 0.5, p = 1)

of the study, the measure of the gradient index is selected as p = 1. Furthermore, for NL-SE theory, the measure of the nonlocal parameter is fixed (β = 2 nm^2). In the considered structure the examination of the frequency vibrations is done while following the nonlocal CS and nonlocal NL-CS-SE theories, at the assumption of l = 0.25h and β = 2 nm^2. As an outcomes, it governs that from the three theories NL-CS theory shows a negligible influence of slenderness ratio on non-dimensional frequency vibrations while the if FG-nanobeam follows the NL-CS-SE and NL-SE, it is more effected by slenderness ratio. Furthermore, as the measure of the slenderness ratio gets increment, the non-dimensional frequency gets decrement.

Couple stress effect influence on the non-dimensional frequency of NL-SE FG nanobeam together with nonlocal parameter and slenderness ratio id delineated in Fig. 5. The outcomes from the figure is that, β has the higher value which shows the detrimental influence on vibrational frequency of the FG nanobeams. Moreover, it is shows that if FG nanobeams have less slenderness ratio which creates a higher frequency. Furthermore, the outcome of the classical beam model, for NL-SE FG nanobeams along with nonlocal parameter, the natural frequencies variation pattern is same, in absence slenderness ratio. In view of achieved FG nanobeam natural frequency in absence of the couple stress effect, a large measure value of frequency is witnessed after the addition of influence of couple stress in examination.

6. Conclusions

This research article represents the more exact forecasting of FG nanobeams natural frequency carry out creates a nonlocal couple stress theory relying on surface elasticity where dual scale parameters are launched to capture size effects. The basic fundamental equations are achieved as per modified couple stress theory to represents the effect of local rational degree of freedom. Such influence is carried out in the structure considered by Eringen following nonlocal elasticity theory, which carries long-range and nonlocal interactions between the particles. Variable material properties are described by power-law model. Numerical outcomes show the following:

(1) Incorporation of nonlocal parameter conduct to underneath frequencies after diminishing the FG nanobeams bending stiffness.
(2) Incorporation of the couple stress influence creates the FG nanobeams stiffer, and which cause the monotonic supplement in the vibration frequencies.
(3) In the company of elasticity theories, the frequency outcomes of NL-SE and NL-CS theories are lowest

and highest respectively. NL-CS-SE theory forecast the natural frequencies which is all the time is between NL-SE and NL-CS theories.

(4) The FG nanobeams frequency influenced by surface elasticity and which rely on the measure of slenderness ratio (L/h). As outcomes, it is achieved that the FG nanobeams having larger slenderness ratios are further influenced by surface effect.

(5) Frequencies, which are dimensionless, are decreasing continuously by the increment in the index value of power-law. Influence of gradient index on the resultant frequencies is highly remarkable at excessive material length scale parameter.

(6) NL-CS-SE clamped-clamped FG nanobeam produces excessive frequencies in comparison to simply-supported at a constant length scale and nonlocal parameters.

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