# Dynamic characteristics of multi-phase crystalline porous shells with using strain gradient elasticity

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**Abstract.** This paper studies forced vibrational behavior of porous nanocrystalline silicon nanoshells under radial dynamic loads using strain gradient theory (SGT). This type of material contains many pores inside it and also there are nano-size grains which define the material character. The formulation for nanocrystalline nanoshell is provided by first order shell theory and a numerical approach is used in order to solve nanoshell equations. SGT gives a scale factor related to stiffness hardening provided by nano-grains. For more accurate description of size effects due to nano-grains or nano-pore, their surface energy influences have been introduced. Surface energy of inclusion exhibit extraordinary influence on dynamic response of the nanoshell. Also, dynamic response of the nanoshell is affected by the scale of nano-grain and nano-pore.

Keywords: nanocrystalline material; forced vibration; porous nanoshell; strain gradient; Mori-Tanaka scheme

## 1. Introduction

Many elements of structures made of silicone have gained extraordinary applications as construction blocks of sensing devices or actuating systems. Silicone material may has a nano-crystalline nature due to the fact that it contains many nano-pores and nano-grains. The nano-pores and nano-grains act as inclusions in the material texture and are able to determine all material properties of silicone (Wang *et al.* 2003). So, elastic stiffness of silicone is affected by the nano-pores and nano-grains especially their amounts and sizes. Also, it must be stated that there is a zone between grains and pores which is called interface region.

For mathematical formulating of a nanocrystalline silicon nanoshell, classic mechanics is unable to characterize small scale affects. So, there are other theories which able to do this. For example, nonlocal theory or strain gradient theory have scale factors which makes them suitable to be applied in nanoshell modeling (Aydogdu 2009, Thai 2012, Ke et al. 2012, Eltaher et al. 2013, Barati 2017, Al-Maliki et al. 2019, Ahmed et al. 2019). The scale factor used by these theories can incorporate small scale influences by taking into account nano-scale interactions of atoms (Mohammadi et al. 2016, Zenkour and Abouelregal 2014, Ebrahimi and Barati 2016, 2017a, Barati and Shahverdi 2016, Bounouara et al. 2016, Besseghier et al. 2017, Mokhtar et al. 2018, Ebrahimi et al. 2018). The theories may have one or more scale factors based on the nature of small scale structure, however a one parameter strain gradient theory is used in the present study taking into account non-uniform strain field. The one parameter strain gradient theory is adopted due to the fact that nano-grains may induce stiffness hardening influence to the nanoshell, so there is a need to a mechanism to characterize this influence. This theory is also used by other authors to capture mentioned influences (Lim *et al.* 2015, Li *et al.* 2016, Mehralian *et al.* 2017). Also, many authors tried to investigate mechanical behaviors of nanocrystalline structures (Ebrahimi and Barati 2017b, 2018) but a work analyzing dynamic behavior of nancrystalline nanoshells is still missing. Nanoshells are thin-walled structural components which are mechanically studied by different authors to understand their static and dynamical properties using nonlocal or strain gradient theories (Zeighampour and Beni 2014, Ke *et al.* 2014, Mehralian *et al.* 2016, Farajpour *et al.* 2017, Sun *et al.* 2016).

This article investigates forced vibrational behavior of porous nanocrystalline silicon nanoshells under radial dynamic loads using strain gradient theory (SGT). This type of material contains many pores inside it and also there are nano-size grains which define the material character. The formulation for nanocrystalline nanoshell is provided by first order shell theory and a numerical approach is used in order to solve nanoshell equations. SGT gives a scale factor related to stiffness hardening provided by nano-grains. For more accurate description of size effects due to nano-grains or nano-pore, their surface energy influences have been introduced. Surface energy of inclusion exhibit extraordinary influence on dynamic response of the nanoshell. Also, dynamic response of the nanoshell is affected by the scale of nano-grain and nano-pore.

#### 2. Model of nanocrystalline nanoshells

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Figs. 1 and 2 illustrate a nanocrystalline nanoshell made of silicone under radial dynamic load with specific

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Fig. 1 Configuration of NcM nanoshell



Fig. 2 Configuration of applied radial dynamic load to the nanoshell

frequency. The figures clearly show that pores are available in the material structure and are able to change material properties (Bourada *et al.* 2019, Yahia *et al.* 2015). Elastic properties (Young's moduli and Poisson's ratio) for a nanocrystalline nanoshell can be described as functions of bulk and shear moduli ( $K_{NCM}$ ,  $\mu_{NCM}$ ) as

$$E_{NCM} = \frac{9K_{NCM}\mu_{NCM}}{3K_{NCM} + \mu_{NCM}} \tag{1}$$

$$v_{NCM} = \frac{3K_{NCM} - 2\mu_{NCM}}{2(3K_{NCM} + \mu_{NCM})}$$
(2)

So that

$$K_{NCM} \cong k_{H_1} \times k_{H_2} \times \frac{1}{\eta k_g} \tag{3}$$

$$\mu_{NCM} \cong \mu_{H_1} \times \mu_{H_2} \times \frac{1}{\eta \mu_g} \tag{4}$$

So that  $\eta = E_{in}/E_g$  and also

$$k_{H_1} = k_{eff}(k_{in} = \eta k_g, \mu_{in} = \eta \mu_g, k_g, \mu_g, f_g, k_g^s, \mu_g^s, v_{in} = v_g, R_g)$$
(5a)

$$\mu_{H_1} = \mu_{eff}(k_{in} = \eta k_g, \mu_{in} = \eta \mu_g, k_g, \mu_g, f_g, k_g^s, \mu_g^s, v_{in} = v_g, R_g)$$
(5b)

$$k_{H_2} = k_{eff}(k_{in} = \eta k_g, \mu_{in} = \eta \mu_g, k_g = 0, \mu_g = 0, f_v, k_v^s, \mu_v^s, v_v, R_v)$$
(5c)

$$\mu_{H_2} = \mu_{eff}(k_{in} = \eta k_g, \mu_{in} = \eta \mu_g, k_g = 0, \mu_g = 0, f_v, k_v^s, \mu_v^s, v_v, R_v)$$
(5d)

In above relations g denotes the nano-grains material properties. Also, v denote the porosities material properties. So,  $f_g$  and  $f_v$  are grain and pores volume fractions defined as

$$f_g = r(1 - f_v), \quad r = \frac{R_g^3}{(R_g + T_{in})^3}$$
 (6)

Here,  $R_g$ ,  $R_v$  and  $T_{in}$  respectively denote the main radiuses of nano-grain, nano-porosity and interface thickness. Above equations are employed in order to characterize all material properties including nano-porosity effect. Without including nano-porosity effect, the material properties (Bulk and shear moduli) become (Ebrahimi and Barati 2017b)

$$k_{eff} = \frac{3k_g (4f_g \mu_{in} + 3k_{in}) + 2\mu_{in} \begin{pmatrix} 4f_g \mu_{in} k_s^* \\ + 3k_{in} (2 - 2f_g + k_s^*) \end{pmatrix}}{3 \left( 3(1 - f_g)k_g + 3f_g k_{in} + 2\mu_{in} (2 + k_s^* - f_g k_s^*) \right)}$$
(7)

$$\mu_{eff} = \frac{\mu_{in} \left( 5 - 8f_g \xi_3 (7 - 5\nu_{in}) \right)}{5 - f_g (5 - 84\xi_1 - 20\xi_2)} \tag{8}$$

So that

$$\xi_1 = \frac{15(1 - v_{in})(k_s^* + 2\mu_s^*)}{4H}$$
(9a)

$$-15(1 - v_{in})\left(\left(\frac{\mu_g}{\mu_{in}}\right)(7 + 5v_g)\right)$$

$$\xi_2 = \frac{-8v_g(5 + 3k_s^* + \mu_s^*) + 7(4 + 3k_s^* + 2\mu_s^*))}{4H}$$
(9b)

$$\xi_{3} = \frac{5}{16H} \left[ 2 \left( \frac{\mu_{g}}{\mu_{in}} \right)^{2} (7 + 5v_{g}) -4(7 - 10v_{g})(2 + k_{s}^{*})(1 - \mu_{s}^{*}) + \left( \frac{\mu_{g}}{\mu_{in}} \right) (7(6 + 5k_{s}^{*} + 4\mu_{s}^{*}) -v_{g}(90 + 47k_{s}^{*} + 4\mu_{s}^{*})) \right]$$

$$(9c)$$

and

$$H = -2\left(\frac{\mu_g}{\mu_{in}}\right)^2 (7 + 5v_g)(4 - 5v_{in}) +7(\frac{\mu_g}{\mu_{in}})(-39 - 20k_s^* - 16\mu_s^* +5v_g(9 + 5k_s^* + 4\mu_s^*)) +\left(\frac{\mu_g}{\mu_{in}}\right)v_g(285 + 188k_s^* + 16\mu_s^* -5v_{in}(75 + 47k_s^* + 4\mu_s^*)) +4(7 - 10v_g)(-7 - 11\mu_s^* - k_s^*(5 + 4\mu_s^*) +v_{in}(5 + 13\mu_s^* + k_s^*(4 + 5\mu_s^*)))$$
(9d)

so that  $k_s^s = k_g^s / R_g \mu_{in}$  and  $\mu_s^s = \mu_g^s / R_g \mu_{in}$  are surfaces bulks and shear moduli, respectively for which  $k_g^s = 2(\mu_a^s + \lambda_a^s)$ .

For the atoms within the material, the elastic modulus of  $E(r_0)$  has been defined. This modulus is identical to that of nano-grains  $(E_g)$ .  $r_0$  is the reference position of the atoms in which they are vibrating. Then, it is possible to define elastic modulus of interface atoms as  $E_{in} = E(r)$  at a new position r. Next, for the afore-mentioned elastic moduli there a relationship as follows

$$\frac{E_{in}}{E_g} = \frac{E(r)}{E(r_0)} = \frac{1}{n-m} \left( (n+1) \left(\frac{r_0}{r}\right)^{n+3} - (m+1) \left(\frac{r_0}{r}\right)^{m+3} \right)$$
(10)

so that

$$\frac{r_0}{r} = \left(\frac{\rho(r)}{\rho(r_0)}\right)^{1/3}$$
(11)

Above relations are applicable to silicone when m = 8

and n = 12. Then, nanoshell mass density may be defined as follows taking into account the portions of nano-grains and nano-porosity

$$\rho_{NcM} = (1 - f_g - f_v)\rho_{in} + f_g \rho_g$$
(12)

For the nanocrystalline nanoshell, we use first order shell formulation which owns three displacements (u, v, w) and two rotations ( $\varphi_x$ ,  $\varphi_\theta$ ) as (Faleh *et al.* 2018, She *et al.* 2018, Zine *et al.* 2018)

$$u_1(x,\theta,z,t) = u(x,\theta,t) + z\varphi_x(x,\theta,t)$$
(13a)

$$u_2(x,\theta,z,t) = v(x,\theta,t) + z\varphi_{\theta}(x,\theta,t)$$
(13b)

$$u_3(x,\theta,z,t) = w(x,\theta,t)$$
(13c)

Based upon first order shell formulation, we can state the strains of the nanoshell in following forms

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + z \frac{\partial \varphi_x}{\partial x}$$

$$\varepsilon_{\theta} = \frac{1}{R} \left( \frac{\partial v}{\partial \theta} + w + z \frac{\partial \varphi_{\theta}}{\partial \theta} \right)$$

$$\gamma_{x\theta} = \frac{1}{R} \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x} + \frac{z}{R} \frac{\partial \varphi_x}{\partial \theta} + z \frac{\partial \varphi_{\theta}}{\partial x}$$

$$\gamma_{zx} = \varphi_x + \frac{\partial w}{\partial x}, \quad \gamma_{z\theta} = \varphi_{\theta} + \frac{1}{R} \frac{\partial w}{\partial \theta} - \frac{v}{R}$$
(14)

Next, Hamilton's integral based on strain, kinetic and external energies (U, T, V) should be stated as

$$\int_{0}^{t} \delta(U - T - V)dt = 0$$
(15)

So that

$$\delta U = \int_{V} (\sigma_{ij} \delta \varepsilon_{ij}) R dx d\theta dz \tag{16}$$

$$\delta V = \int_{V} (q_{dynamic})) \delta w R dx d\theta dz \tag{17}$$

$$\delta K = \int_{V} \left( \left( \frac{\partial \delta u_{1}}{\partial t} \right)^{2} + \left( \frac{\partial \delta u_{2}}{\partial t} \right)^{2} + \left( \frac{\partial \delta u_{3}}{\partial t} \right)^{2} \right) R dx d\theta dz$$
(18)

Also,  $q_{dynamic}$  is radial mechanical load.

Derived from Hamilton's integral of Eq. (15) are the below governing equations (Mehralian *et al.* 2017)

$$\frac{\partial N_{xx}}{\partial x} + \frac{1}{R} \frac{\partial N_{x\theta}}{\partial \theta} = I_0 \frac{\partial^2 u}{\partial t^2} + I_1 \frac{\partial^2 \varphi_x}{\partial t^2}$$
(19a)

$$\frac{\partial N_{x\theta}}{\partial x} + \frac{1}{R} \frac{\partial N_{\theta\theta}}{\partial \theta} + \frac{Q_{z\theta}}{R} = I_0 \frac{\partial^2 v}{\partial t^2} + I_1 \frac{\partial^2 \varphi_{\theta}}{\partial t^2}$$
(19b)

$$\frac{\partial Q_{xz}}{\partial x} + \frac{1}{R} \frac{\partial Q_{z\theta}}{\partial \theta} - \frac{N_{\theta\theta}}{R} = +I_0 \frac{\partial^2 w}{\partial t^2} + q_{dynamic}$$
(19c)

$$\frac{\partial M_{xx}}{\partial x} + \frac{1}{R} \frac{\partial M_{x\theta}}{\partial \theta} - Q_{xz} = I_1 \frac{\partial^2 u}{\partial t^2} + I_2 \frac{\partial^2 \varphi_x}{\partial t^2}$$
(19d)

$$\frac{\partial M_{x\theta}}{\partial x} + \frac{1}{R} \frac{\partial M_{\theta\theta}}{\partial \theta} - Q_{\theta z} = I_1 \frac{\partial^2 v}{\partial t^2} + I_2 \frac{\partial^2 \varphi_{\theta}}{\partial t^2}$$
(19d)

for which

$$(I_0, I_1, I_2) = \int_{-h/2}^{h/2} (1, z, z^2) \rho_{NCM} dz$$
(20)

and

$$\{N_{xx}, N_{\theta\theta}, N_{x\theta}\} = \int_{-h/2}^{h/2} \{\sigma_{xx}, \sigma_{\theta\theta}, \sigma_{x\theta}\} dz \qquad (21a)$$

$$\{M_{xx}, M_{\theta\theta}, M_{x\theta}\} = \int_{-h/2}^{h/2} \{\sigma_{xx}, \sigma_{\theta\theta}, \sigma_{x\theta}\} z dz \qquad (21b)$$

$$\{Q_{xz}, Q_{z\theta}\} = \kappa_s \int_{-h/2}^{h/2} \{\sigma_{xz}, \sigma_{z\theta}\} dz$$
(21c)

In last integral,  $\kappa_s$  introduces shear correction factor. Generally, SGT (with scale factor *l*) possesses the below formulations for the relations between stresses and strains of a nanoshell

$$\begin{cases} \sigma_{xx} \\ \sigma_{\theta\theta} \\ \sigma_{x\theta} \\ \sigma_{xz} \\ \sigma_{z\theta} \end{cases} = \frac{E(z)}{1 - v^2} (1 - l^2 \nabla^2) \\ \begin{pmatrix} 1 & v & 0 & 0 & 0 \\ v & 1 & 0 & 0 & 0 \\ 0 & 0 & (1 - v)/2 & 0 & 0 \\ 0 & 0 & 0 & (1 - v)/2 & 0 \\ 0 & 0 & 0 & 0 & (1 - v)/2 \end{cases} \begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{\theta\theta} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{z\theta} \end{pmatrix}$$
(22)

Right hand side of above equation has been integrated about the thickness of nanoshell, then the following relations will be derived

$$N_{xx} = (1 - \lambda \nabla^2) \left[ A_{11} \frac{\partial u}{\partial x} + B_{11} \frac{\partial \varphi_x}{\partial x} + \frac{A_{12}}{R} (\frac{\partial v}{\partial \theta} + w) + \frac{B_{12}}{R} \frac{\partial \varphi_{\theta}}{\partial \theta} \right]$$
(23)

$$M_{xx} = (1 - \lambda \nabla^2) \left[ B_{11} \frac{\partial u}{\partial x} + D_{11} \frac{\partial \varphi_x}{\partial x} + \frac{B_{12}}{R} \left( \frac{\partial v}{\partial \theta} + w \right) + \frac{D_{12}}{R} \frac{\partial \varphi_\theta}{\partial \theta} \right]$$
(24)

$$N_{\theta\theta} = (1 - \lambda \nabla^2) \left[ A_{12} \frac{\partial u}{\partial x} + B_{12} \frac{\partial \varphi_x}{\partial x} + \frac{A_{11}}{R} \left( \frac{\partial v}{\partial \theta} + w \right) + \frac{B_{11}}{R} \frac{\partial \varphi_{\theta}}{\partial \theta} \right]$$
(25)

$$M_{\theta\theta} = (1 - \lambda \nabla^2) \left[ B_{12} \frac{\partial u}{\partial x} + D_{12} \frac{\partial \varphi_x}{\partial x} + \frac{B_{11}}{R} \left( \frac{\partial v}{\partial \theta} + w \right) + \frac{D_{11}}{R} \frac{\partial \varphi_{\theta}}{\partial \theta} \right]$$
(26)

$$N_{x\theta} = (1 - \lambda \nabla^2) \left[ A_{66} \left( \frac{1}{R} \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x} \right) + B_{66} \left( \frac{1}{R} \frac{\partial \varphi_x}{\partial \theta} + \frac{\partial \varphi_\theta}{\partial x} \right) \right]$$
(27)

$$M_{x\theta} = (1 - \lambda \nabla^2) \left[ B_{66} \left( \frac{1}{R} \frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x} \right) + D_{66} \left( \frac{1}{R} \frac{\partial \varphi_x}{\partial \theta} + \frac{\partial \varphi_\theta}{\partial x} \right) \right]$$
(28)

$$Q_{xz} = (1 - \lambda \nabla^2) \tilde{A}_{66} \left( \varphi_x + \frac{\partial w}{\partial x} \right)$$
(29)

$$Q_{\theta z} = (1 - \lambda \nabla^2) \tilde{A}_{66} \left( \varphi_{\theta} + \frac{1}{R} \frac{\partial w}{\partial \theta} - \frac{v}{R} \right)$$
(30)

So that

$$\begin{split} A_{11} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{E_{NCM}}{1 - v^2} dz, \qquad B_{11} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{E_{NCM}}{1 - v^2} z \, dz, \\ D_{11} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{E_{NCM} z^2}{1 - v^2} dz, \qquad A_{12} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{v E_{NCM}}{1 - v^2} dz, \\ B_{12} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{v E_{NCM}}{1 - v^2} z \, dz, \qquad D_{12} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{v E_{NCM} z^2}{1 - v^2} dz, \\ A_{66} &= \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{E_{NCM}}{2(1 + v)} dz, \qquad B_{66} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{E_{NCM}}{2(1 + v)} z dz, \\ D_{66} &= \int_{-h/2}^{h/2} \frac{E_{NCM}}{2(1 + v)} z^2 \, dz \\ \tilde{A}_{66} &= k_s \int_{-h/2}^{h/2} \frac{E_{NCM}}{2(1 + v)} dz, \end{split}$$

The SGT governing equations of dynamically loaded nano-crystalline nano-sized shells might be derived by putting Eqs. (23)-(30), in Eq. (19) which are

$$(1 - \lambda \nabla^{2}) \left[ A_{11} \frac{\partial^{2} u}{\partial x^{2}} + B_{11} \frac{\partial^{2} \varphi_{x}}{\partial x^{2}} + \frac{A_{12}}{R} \left( \frac{\partial^{2} v}{\partial x \partial \theta} + \frac{\partial w}{\partial x} \right) + \frac{B_{12}}{R} \frac{\partial^{2} \varphi_{\theta}}{\partial x \partial \theta} + \frac{A_{66}}{R} \left( \frac{1}{R} \frac{\partial^{2} u}{\partial \theta^{2}} + \frac{\partial^{2} v}{\partial x \partial \theta} \right) + \frac{B_{66}}{R} \left( \frac{1}{R} \frac{\partial^{2} \varphi_{x}}{\partial \theta^{2}} + \frac{\partial^{2} \varphi_{\theta}}{\partial x \partial \theta} \right) \right] - I_{0} \frac{\partial^{2} u}{\partial t^{2}} - I_{1} \frac{\partial^{2} \varphi_{x}}{\partial t^{2}} = 0$$
(32a)

$$(1 - \lambda \nabla^{2}) \left[ A_{66} \left( \frac{1}{R} \frac{\partial^{2} u}{\partial x \partial \theta} + \frac{\partial^{2} v}{\partial x^{2}} \right) + B_{66} \left( \frac{1}{R} \frac{\partial^{2} \varphi_{x}}{\partial x \partial \theta} + \frac{\partial^{2} \varphi_{\theta}}{\partial x^{2}} \right) + \frac{A_{12}}{R} \frac{\partial^{2} u}{\partial x \partial \theta} + \frac{B_{12}}{R} \frac{\partial^{2} \varphi_{x}}{\partial x \partial \theta} + \frac{A_{11}}{R^{2}} \left( \frac{\partial^{2} v}{\partial \theta^{2}} + \frac{\partial w}{\partial \theta} \right) + \frac{B_{11}}{R^{2}} \frac{\partial^{2} \varphi_{\theta}}{\partial \theta^{2}}$$
(32b)  
$$\left( \frac{\tilde{A}_{66}}{R} \left( \varphi_{\theta} + \frac{1}{R} \frac{\partial w}{\partial \theta} - \frac{v}{R} \right) \right] - I_{0} \frac{\partial^{2} v}{\partial t^{2}} - I_{1} \frac{\partial^{2} \varphi_{\theta}}{\partial t^{2}} = 0$$

$$(1 - \lambda \nabla^{2}) \left[ \tilde{A}_{66} \left( \frac{\partial \varphi_{x}}{\partial x} + \frac{\partial^{2} w}{\partial x^{2}} \right) + \frac{\tilde{A}_{66}}{R} \left( \frac{\partial \varphi_{\theta}}{\partial \theta} + \frac{1}{R} \frac{\partial^{2} w}{\partial \theta^{2}} - \frac{1}{R} \frac{\partial v}{\partial \theta} \right) - \frac{A_{12}}{R} \frac{\partial u}{\partial x}$$
(32c)  
$$- \frac{B_{12}}{R} \frac{\partial \varphi_{x}}{\partial x} - \frac{A_{11}}{R^{2}} \left( \frac{\partial v}{\partial \theta} + w \right) - \frac{B_{11}}{R^{2}} \frac{\partial \varphi_{\theta}}{\partial \theta} \right]$$

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$$-I_0 \frac{\partial^2 w}{\partial t^2} = q_{dynamic} \tag{32c}$$

$$(1 - \lambda \nabla^{2}) \left[ B_{11} \frac{\partial^{2} u}{\partial x^{2}} + D_{11} \frac{\partial^{2} \varphi_{x}}{\partial x^{2}} + \frac{B_{12}}{R} \left( \frac{\partial^{2} v}{\partial x \partial \theta} + \frac{\partial w}{\partial x} \right) \right. \\ \left. + \frac{D_{12}}{R} \frac{\partial^{2} \varphi_{\theta}}{\partial x \partial \theta} \frac{B_{66}}{R} \left( \frac{1}{R} \frac{\partial^{2} u}{\partial \theta^{2}} + \frac{\partial^{2} v}{\partial x \partial \theta} \right) \right.$$

$$\left. + \frac{D_{66}}{R} \left( \frac{1}{R} \frac{\partial^{2} \varphi_{x}}{\partial \theta^{2}} + \frac{\partial^{2} \varphi_{\theta}}{\partial x \partial \theta} \right) - \tilde{A}_{66} \left( \varphi_{x} + \frac{\partial w}{\partial x} \right) \right]$$

$$\left. - I_{1} \frac{\partial^{2} u}{\partial t^{2}} - I_{2} \frac{\partial^{2} \varphi_{x}}{\partial t^{2}} = 0$$

$$(32d)$$

$$(1 - \lambda \nabla^{2}) \left[ B_{66} \left( \frac{1}{R} \frac{\partial^{2} u}{\partial x \partial \theta} + \frac{\partial^{2} v}{\partial x^{2}} \right) + D_{66} \left( \frac{1}{R} \frac{\partial^{2} \varphi_{x}}{\partial x \partial \theta} + \frac{\partial^{2} \varphi_{\theta}}{\partial x^{2}} \right) + \frac{B_{12}}{R} \frac{\partial^{2} u}{\partial x \partial \theta} + \frac{D_{12}}{R} \frac{\partial^{2} \varphi_{x}}{\partial x \partial \theta} + \frac{B_{11}}{R^{2}} \left( \frac{\partial^{2} v}{\partial \theta^{2}} + \frac{\partial w}{\partial \theta} \right) + \frac{D_{11}}{R^{2}} \frac{\partial^{2} \varphi_{\theta}}{\partial \theta^{2}} - \tilde{A}_{66} \left( \varphi_{\theta} + \frac{1}{R} \frac{\partial w}{\partial \theta} - \frac{v}{R} \right) \right] - I_{1} \frac{\partial^{2} v}{\partial t^{2}} - I_{2} \frac{\partial^{2} \varphi_{\theta}}{\partial t^{2}} = 0$$

$$(32e)$$

# 3. Method of solution

As mentioned there are five displacements based on considered shell theory. So, the first step to define and approximate these displacements in the following forms (Saidi *et al.* 2016, Merazi *et al.* 2015)

$$u = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} U_{mn} \frac{\partial X_m(x)}{\partial x} \cos(n\theta) \sin(\omega_{ex} t)$$
(33)

$$v = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} V_{mn} X_m(x) \sin(n\theta) \sin(\omega_{ex} t)$$
(34)

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} X_m(x) \cos(n\theta) \sin(\omega_{ex} t)$$
(35)

$$\varphi_{x} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \varphi_{mn} \frac{\partial X_{m}(x)}{\partial x} \cos(n\theta) \sin(\omega_{ex}t) \quad (36)$$

$$\varphi_{\theta} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Theta_{mn} X_m(x) \sin(n\theta) \sin(\omega_{ex} t)$$
(37)

The maximum values of displacements are denoted by  $U_{mn}, V_{mn}, W_{mn}, \Phi_{mn}, \Theta_{mn}$  and  $X_m$  is a function based on the type of the boundary condition. Here are the boundary conditions at x = 0, L of nanoshell

$$w = \frac{\partial^2 w}{\partial x^2} = \frac{\partial^4 w}{\partial x^4} = 0 \quad for \quad S - S$$
  

$$w = \frac{\partial w}{\partial x} = \frac{\partial^3 w}{\partial x^3} = 0 \quad for \quad C - C$$
(38)

By putting Eqs. (33)-(37) in Eq. (32) and taking into account the Galerkin's concept, we obtain

$$\left\{ [K] + \omega_{ex}^{2} [M] \right\} \begin{cases} U_{mn} \\ V_{mn} \\ W_{mn} \\ \Phi_{mn} \\ \Theta_{mn} \end{cases} = \begin{cases} 0 \\ 0 \\ Q_{dynamic} \\ 0 \\ 0 \end{cases}$$
(39)

So that  $\omega_{ex}$  is external frequency and

$$k_{1,1} = A_{11} \left( Y_{31} - \lambda \left( Y_{51} - \frac{n^2}{R} Y_{31} \right) \right) -n^2 \frac{A_{66}}{R^2} \left( Y_{11} - \lambda \left( Y_{31} - \frac{n^2}{R} Y_{11} \right) \right)$$
(40)

$$k_{2,1} = n\left(\frac{A_{12}}{R} + \frac{A_{66}}{R}\right) \left(Y_{11} - \lambda\left(Y_{31} - \frac{n^2}{R}Y_{11}\right)\right),$$
  

$$k_{1,2} = -n\left(\frac{A_{12}}{R} + \frac{A_{66}}{R}\right) \left(Y_{20} - \lambda\left(Y_{40} - \frac{n^2}{R}Y_{20}\right)\right)$$
(41)

$$k_{3,1} = +\frac{A_{12}}{R} \left( Y_{11} - \lambda \left( Y_{31} - \frac{n^2}{R} Y_{11} \right) \right),$$
  

$$k_{1,3} = -\frac{A_{12}}{R} \left( Y_{20} - \lambda \left( Y_{40} - \frac{n^2}{R} Y_{20} \right) \right)$$
(42)

$$k_{4,1} = +B_{11} \left( Y_{31} - \lambda \left( Y_{51} - \frac{n^2}{R} Y_{31} \right) \right) -n^2 \frac{B_{66}}{R^2} \left( Y_{11} - \lambda \left( Y_{31} - \frac{n^2}{R} Y_{11} \right) \right),$$

$$k_{1,4} = B_{11} \left( Y_{31} - \lambda \left( Y_{51} - \frac{n^2}{R} Y_{31} \right) \right)$$

$$-n^2 \frac{B_{66}}{R^2} \left( Y_{11} - \lambda \left( Y_{31} - \frac{n^2}{R} Y_{11} \right) \right)$$
(43)

$$k_{5,1} = n\left(\frac{B_{12}}{R} + \frac{B_{66}}{R}\right) \left(Y_{11} - \lambda\left(Y_{31} - \frac{n^2}{R}Y_{11}\right)\right), \quad (44)$$

$$k_{1,5} = -n(\frac{B_{66}}{R} + \frac{B_{12}}{R})(\gamma_{20} - \lambda(\gamma_{40} - \frac{n^2}{R}\gamma_{20}))$$
(45)

$$k_{2,2} = A_{66} \left( Y_{20} - \lambda \left( Y_{40} - \frac{n^2}{R} Y_{20} \right) \right) -n^2 \frac{A_{11}}{R^2} \left( Y_{00} - \lambda \left( Y_{20} - \frac{n^2}{R} Y_{00} \right) \right)$$
(46)  
$$- \frac{\tilde{A}_{66}}{R^2} \left( Y_{00} - \lambda \left( Y_{20} - \frac{n^2}{R} Y_{00} \right) \right)$$
(47)  
$$k_{3,2} = -n \left( \frac{A_{11}}{R^2} + \frac{\tilde{A}_{66}}{R^2} \right) \left( Y_{00} - \lambda \left( Y_{20} - \frac{n^2}{R} Y_{00} \right) \right),$$
(47)

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$$k_{2,3} = -n\left(\frac{\tilde{A}_{66}}{R^2} + \frac{A_{11}}{R^2}\right) \left(\Upsilon_{00} - \lambda\left(\Upsilon_{20} - \frac{n^2}{R}\Upsilon_{00}\right)\right)$$
(47)  
$$k_{4,2} = -n\left(\frac{B_{12}}{R} + \frac{B_{66}}{R}\right) \left(\Upsilon_{20} - \lambda\left(\Upsilon_{40} - \frac{n^2}{R}\Upsilon_{20}\right)\right),$$
  
$$k_{2,4} = +n\left(\frac{B_{12}}{R} + \frac{B_{66}}{R}\right) \left(\Upsilon_{11} - \lambda\left(\Upsilon_{31} - \frac{n^2}{R}\Upsilon_{11}\right)\right)$$
(48)

$$k_{5,2} = B_{66} \left( \Upsilon_{20} - \lambda \left( \Upsilon_{40} - \frac{n^2}{R} \Upsilon_{20} \right) \right) -n^2 \frac{B_{11}}{R^2} \left( \Upsilon_{00} - \lambda \left( \Upsilon_{20} - \frac{n^2}{R} \Upsilon_{00} \right) \right) + \frac{\tilde{A}_{66}}{R} \left( \Upsilon_{00} - \lambda \left( \Upsilon_{20} - \frac{n^2}{R} \Upsilon_{00} \right) \right)$$
(49)

$$k_{3,3} = \tilde{A}_{66} \left( Y_{20} - \lambda \left( Y_{40} - \frac{n^2}{R} Y_{20} \right) \right) - n^2 \frac{\tilde{A}_{66}}{R^2} \left( Y_{00} - \lambda \left( Y_{20} - \frac{n^2}{R} Y_{00} \right) \right)$$
(50)  
$$- \frac{A_{11}}{R^2} \left( Y_{00} - \lambda \left( Y_{20} - \frac{n^2}{R} Y_{00} \right) \right)$$

$$k_{4,3} = \left(\tilde{A}_{66} - \frac{B_{12}}{R}\right) \left(Y_{20} - \lambda \left(Y_{40} - \frac{n^2}{R}Y_{20}\right)\right),$$
  

$$k_{3,4} = + \left(\frac{B_{12}}{R} - \tilde{A}_{66}\right) \left(Y_{11} - \lambda \left(Y_{31} - \frac{n^2}{R}Y_{11}\right)\right)$$
(51)

$$k_{5,3} = n\left(+\frac{\tilde{A}_{66}}{R} - \frac{B_{11}}{R^2}\right) \left(Y_{00} - \lambda\left(Y_{20} - \frac{n^2}{R}Y_{00}\right)\right),$$
  

$$k_{3,5} = -n\left(+\frac{B_{11}}{R^2} - \frac{\tilde{A}_{66}}{R}\right) \left(Y_{00} - \lambda\left(Y_{20} - \frac{n^2}{R}Y_{00}\right)\right)^{(52)}$$

$$k_{4,4} = +D_{11}\left(Y_{31} - \lambda\left(Y_{51} - \frac{n^2}{R}Y_{31}\right)\right) -n^2 \frac{D_{66}}{R^2}\left(Y_{11} - \lambda\left(Y_{31} - \frac{n^2}{R}Y_{11}\right)\right)$$
(53)  
$$-\tilde{A}_{66}\left(Y_{11} - \lambda\left(Y_{31} - \frac{n^2}{R}Y_{11}\right)\right)$$

$$k_{5,4} = +n\left(\frac{D_{12}}{R} + \frac{D_{66}}{R}\right) \left(Y_{11} - \lambda\left(Y_{31} - \frac{n^2}{R}Y_{11}\right)\right),$$
  

$$k_{4,5} = -n\left(\frac{D_{66}}{R} + \frac{D_{12}}{R}\right) \left(Y_{20} - \lambda\left(Y_{40} - \frac{n^2}{R}Y_{20}\right)\right)$$
(54)

$$k_{5,5} = +D_{66} \left( Y_{20} - \lambda \left( Y_{40} - \frac{n^2}{R} Y_{20} \right) \right) -n^2 \frac{D_{11}}{R^2} \left( Y_{00} - \lambda \left( Y_{20} - \frac{n^2}{R} Y_{00} \right) \right)$$
(55)

$$-\tilde{A}_{66}\left(\Upsilon_{00} - \lambda\left(\Upsilon_{20} - \frac{n^2}{R}\Upsilon_{00}\right)\right)$$
(55)

$$m_{1,1} = +I_0 \Upsilon_{11} \tag{56}$$

$$m_{2,2} = m_{3,3} = m_{5,5} = +I_0 \Upsilon_{00} \tag{57}$$

$$m_{4,1} = +I_1 \Upsilon_{11}, \ m_{4,4} = +I_2 \Upsilon_{11}, \tag{58}$$

$$m_{5,2} = m_{2,5} = +I_1 \Upsilon_{00} \tag{59}$$

$$Q_{dynamic} = Q_n \Upsilon_{00} \tag{60}$$

where

$$Y_{00} = \int_0^L X_m X_m dx \tag{61}$$

$$Y_{20} = \int_0^L \frac{d^2 X_m}{dx^2} X_m dx$$
 (62)

$$\Upsilon_{11} = \int_0^L \frac{dX_m}{dx} \frac{dX_m}{dx} dx \tag{63}$$

$$Y_{31} = \int_0^L \frac{d^3 X_m}{dx^3} \frac{dX_m}{dx} dx$$
(64)

$$Y_{40} = \int_0^L \frac{d^4 X_m}{dx^4} X_m dx$$
 (65)

In the following, the normalized parameters and also suitable forms of function  $X_{\rm m}$  have been introduced

$$\varpi = 100\omega_n h \sqrt{\frac{\rho_g}{E_g}}, \quad \lambda = \frac{l}{L},$$

$$\bar{W}_{uniform} = W \frac{10E_c h^3}{L^4 q_0}$$
(66)

$$X_m(x) = \sin\left(\frac{m\pi}{L}x\right)$$
 for  $S - S$  (67)

$$X_m(x) = \sin^2\left(\frac{m\pi}{L}x\right) \quad \text{for} \quad C - C \tag{68}$$

The dynamical loading acted in the nanoshell may be defined as

$$q_{dynamic} = \sum_{n=1}^{\infty} Q_n s in \left[\frac{m\pi}{a}x\right] cos[n\theta] sin \,\omega_{ex} t \quad (69)$$

 $Q_n$ 

$$= \frac{1}{2\pi L} \int_{x_0 - 0.5L_0}^{x_0 + 0.5L_0} \int_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} s in\left[\frac{m\pi}{L}x\right] cos[n\theta] q(x) dx d\theta$$
(70)  
$$= \frac{8q_0}{mn\pi^2} sin\left[\frac{m\pi}{L}x_0\right] sin\left[\frac{m\pi L_0}{2L}\right]$$

So that  $q(\mathbf{x}) = q_0$  defines the magnitude of uniform loading and  $x_0$  is load position.

Table 1 Comparison of natural frequencies of strain gradient shells (l = h)

	$\omega_{11}$		$\omega_{22}$	
h/R	SGT (Zeighampour and Beni 2014)	Present	SGT (Zeighampour and Beni 2014)	Present
0.02	0.1980	0.1980	0.2795	0.2795
0.05	0.2110	0.2111	0.3953	0.3954

 Table 2 Material properties of nanocrystalline nanoshell

Phase-1 (Interface)	$\begin{split} E_{in} = 45.56 \ GPa,  v_{in} = 0.064, \\ \rho_{in} = 2004.3 \ kg/m^3 \end{split}$
Phase-2 (Si-nanograins)	$\begin{split} E_g &= 169 \text{ GPa, } v_g = 0.064, \\ \rho_g &= 2300 \text{ kg/m}^3 \end{split}$
Phase-3 (nanovoids)	$E_{\rm v}=0$
Surface coefficients of grains and voids	$\lambda_s = -4.488 \text{ N/m}, \ \mu_s = -2.774 \text{ N/m}$

## 4. Numerical results and discussions

Present chapter explores forced vibrational behaviors of porous nanocrystalline nano-sized shells in the context of strain gradient theory. One strain gradient factor is employed to describe stiffness enhancement of NcM nanoshells. According to Table 1, we found that vibrational frequency of nanoshells in the context of first order shell theory and SGT is very close to that presented by Zeighampour and Beni (2014). For comparison, SGT factor is set to l = h. Moreover, the material properties of NcM nanoshells have been prepared in Table 2.

Fig. 3 examines the effects of SGT factor on dynamical deflections and resonance frequencies of NcM nanoshell having S-S and C-C edge condition at R/h = 20 and  $f_v = 0.1$ . One is able to derive the frequencies of classical elasticity

theory discarding SGT factor by assuming  $\lambda = 0$ . It is obvious that dynamical deflections of SGT nanoshell are prominently influenced by the value of external frequency of dynamical load. Actually, dynamical deflections increase smoothly by increasing of external frequency. In a special magnitude of external frequency, a remarkable increment in deflections of SGT nanoshells will be seen. Note that the external frequency of dynamical loading matches with the natural frequencies of SGT nanoshells causing the resonance phenomena. According to SGT, increase of SGT factors yields greater vibrational frequencies. Such result demonstrates the rigidity enhancement influence owning to incorporation of strain gradients.

Fig. 4 studies the effects of nangrains/nanopores average radius and surface layer on resonance frequency of NcM nanoshell with S-S edge conditions at R/h = 20 and  $f_v = 0.1$ . It is obvious that vibration behaviors of NcM nanoshells are relied on the surface energies of nano-grains and nanoporosities. After ignoring the surface energies of nanograins and nano-porosities, increase of average radius yields lower resonance frequencies. After incorporating the surface energy of nano-grains and nano-porosities, the greatest and lowest resonance frequencies have been achieved n the cases of  $R_g = R_v = 100 \ nm$  and 20 nm, respectively. Actually, resonance frequencies at  $R_{ave} = 0.5$ *nm* are higher than in the case  $R_g=20$  *nm*. Such result is owning to the hardening influences of surface layers and strain gradients on the nanoshell structure at small size of nano-grain. Accordingly, influences of the surface layers of nano-grains and nano-porosities become more remarkable as their sizes decline.

Pore percentage effects on forced vibration characteristic of NcM nanoshells with various edge conditions are depicted in Fig. 5 for  $R_g = 20$  nm. Achieved result shows that increment of pore coefficient is corresponding to lower resonance frequencies at a fixed strain gradient parameter. Such result is owning to a remarkable decline in rigidity of nanoshells in the existence of nano-pores within the material texture. Thus, one may



Fig. 3 Dimensionless amplitude of NCM nanoshell versus excitation frequency for different strain gradient parameters  $(R/h = 20, R_g = 20 \text{ nm}, f_v = 0.1)$ 



Fig. 4 Dimensionless amplitude of NCM nanoshell versus excitation frequency for different inhomogeneity sizes  $(R/h = 20, f_v = 0.1, \lambda = 0.5)$ 



Fig. 5 Dimensionless amplitude of NCM nanoshell versus excitation frequency for different strain gradient parameters  $(R/h = 20, R_g = 20 \text{ nm}, \lambda = 0.5)$ 

conclude that both nano-pores scale (size) and amount are crucial to describe the behaviors of NcM nanoshells.

Fig. 6 indicates the effect of radius-to-thickness ratio (R/h) of SGT nanoshells on dynamic deflection and resonance frequencies when  $\lambda = 0.5$ ,  $R_g = 20 \text{ nm}$ ,  $f_v = 0.1$ . One can observe that resonance frequency is notably declined with the increasing in radius-to-thickness ratios. This is because the nanoshell with higher radius-to-thickness ratios is more flexible leading to smaller resonance frequency. Accordingly, dynamic deflection of the nanoshell is increased with the increment in the magnitude of radius-to-thickness ratio at a certain value of excitation frequency.

Impacts of dynamical loading areas  $(L_0/L)$  and positions  $(x_0/L)$  on dynamical deflections and resonance frequencies

of NcM nanoshell have been depicted in Figs. 7 and 8, respectively. For these figures, values of R/h = 20,  $\lambda = 0.5$  are supposed. It is clear that as the radial dynamical loadings move away from the boundary, the dynamical deflections become greater. Moreover, when the areas of radial dynamical loadings become wider, the dynamical deflections of SGT nanoshell gets bigger. However, the resonance frequencies or natural frequencies remain fixed by changes of dynamical loading areas and locations.

Fig. 9 depicts 1<sup>st</sup> longitudinal mode shape of SGT nanoshell based on various pore coefficient at R/h = 20,  $\lambda = 0.5$ , and  $\theta = \pi/6$ . For this figure,  $\Omega = 0.1$  is supposed for external frequency. One can observe that nano-pores within the material may notably affect the modes shapes of the nanoshell. The radial deflections of a nanoshell increase by



Fig. 6 Dimensionless amplitude of NCM nanoshell versus excitation frequency for different radius-to-thickness ratios  $(L/h = 20, \lambda = 0.5, R_g = 20 \text{ nm}, f_v = 0.1)$ 



Fig. 7 Dimensionless amplitude of NCM nanoshell versus excitation frequency for various area of uniform dynamic load (R/h = 20,  $\lambda = 0.5$ ,  $R_g = 20$  nm,  $f_v = 0.1$ ,  $x_0 = 0.5$  L)

increase of nano-pore coefficients. Thus, configuration of SGT nanoshell under dynamical loadings relies on the magnitude of nano-pore percentage.

# 5. Conclusions

In this paper, forced vibration behavior of strain gradient porous nanocrystalline silicon shells was explored by employing first order shell model. Micromechanical modeling of nanocrystalline materials accounting for the size of nano-grains and nano-porosities and their volume



Fig. 8 Dimensionless amplitude of NCM nanoshell versus excitation frequency for different locations of uniform dynamic load (R/h = 20,  $\lambda = 0.5$ ,  $R_g = 20$  nm,  $f_v = 0.1$ ,  $L_0 = 0.3L$ )

fractions was presented. It was found that increasing the strain gradient parameter results in enlargement of the resonance frequencies of NcM nanoshell. The porosity percentage had a great influence on resonance frequencies of NcM nanoshells. However, all of these observations were dependent on the surface layer of nano-grains and nanoporosities. As the size of nano-grains and nano-porosities declined, the effect of their surface layer on resonance frequencies became more important. Therefore, configuration of a NcM nanoshell under dynamical loadings relies on the value of nano-pore percentage.



Fig. 9 First mode shape of NCM nanoshell for different porosity percentages (R/h = 20,  $\lambda = 0.5$ ,  $\Omega = 0.1$ ,  $\theta = \pi/6$ ,  $R_g = 20$  nm)

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