

## Mechanical buckling of FG-CNTs reinforced composite plate with parabolic distribution using Hamilton's energy principle

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**Abstract.** The incorporation of carbon nanotubes in a polymer matrix makes it possible to obtain nanocomposite materials with exceptional properties. It's in this scientific background that this work was based. There are several theories that deal with the behavior of plates, in this research based on the Mindlin-Reissner theory that takes into account the transversal shear effect, for analysis of the critical buckling load of a reinforced polymer plate with parabolic distribution of carbon nanotubes. The equations of the model are derived and the critical loads of linear and parabolic distribution of carbon nanotubes are obtained. With different disposition of nanotubes of carbon in the polymer matrix, the effects of different parameters such as the volume fractions, the plate geometric ratios and the number of modes on the critical load buckling are analysed and discussed. The results show that the critical buckling load of parabolic distribution is larger than the linear distribution. This variation is attributed to the concentration of reinforcement (CNTs) at the top and bottom faces for the X-CNT type which make the plate more rigid against buckling.

**Keywords:** nanotubes; shear deformation; parabolic distribution; buckling; volume fractions

### 1. Introduction

Recently, a new class of promising materials has been discovered by Iijima (1991) known as single-walled carbon nanotube (SWNT) and multi-walled carbon nanotube (MWNT) has drawn considerable attention. Multitude studies indicated that the carbon nanotubes (CNTs) have an excellent candidate for the reinforcement of polymer composites due to their extraordinary Young's modulus (Van Lier *et al.* 2000), tensile strength (Yu *et al.* 2000), electrical conductivity (Thess *et al.* 1996), thermal properties (Biercuk *et al.* 2002) and flexibility (Ma *et al.* 1998). Several studies have focused on material properties of carbon nanotube-reinforced composites (CNTRCs) and have shown that the introduction of carbon nanotubes into polymers may improve their properties (Fidelus *et al.* 2005, Han and Elliott 2007, Bonnet *et al.* 2007, Semmah *et al.* 2019).

Functionally graded materials (FGMs) are a new types of composites developed recently, namely (FGM) has high potential to use as a structural material. Therefore, by changing the properties of the material it is possible to perform a certain function of material properties (Benahmed *et al.* 2019, Bourada *et al.* 2019, Tlidji *et al.*

2019, Avcar and Mohammed 2018, Ehyaei *et al.* 2017, Fahsi *et al.* 2017, Avcar 2019, Boussoula *et al.* 2019, Avcar and Alwan 2017, Boukhari *et al.* 2016, Daouadji and Adim 2016, Sofiyev and Avcar 2010, Sekkal *et al.* 2017, Bousahla *et al.* 2016, Bounouara *et al.* 2016). Reinforced composites are made up of a combination of fibers (Fellah *et al.* 2019), or particle in a matrix material composite material are increasingly being used in aircraft primary structures because of their superior strength properties over the traditional materials. Diamanti and Soutis (2010) studied the structural health monitoring techniques for aircraft composite structures. Advanced composites have been replacing traditional structural materials to repair the aircraft structures deflection and a nonlinear bending analysis of functionally graded carbon nanotube reinforced composite are presented by (Katnam *et al.* 2013), Bonnet *et al.* (2007) studied the thermal properties and percolation in carbon nanotube-polymer composites. Thus, this topic has been fascinating many researchers for recently years (Zhang *et al.* 2015, Mirzaei and Kiani 2016, Mehar *et al.* 2017, Wu *et al.* 2016, Mehar and Panda 2017, Asadi and Beheshti 2018, Karami *et al.* 2019a).

Due to difficulties encountered in experimental methods, the molecular dynamics (MD) simulations are used to predict the elastic properties of polymer/carbon nanotube composites (Griebel and Hamaekers 2004, Han and Elliott 2007) these studies are limited by systems calculation. The continuum mechanics methods are widely used to predict the responses of reinforced composites

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structure and nanostructures (Kolahchi *et al.* 2017, Shokravi 2017a, Karami *et al.* 2017, 2018a, b, 2019b, c, Kolahchi 2017, Ould Youcef *et al.* 2015, Shokravi 2017b, Odegard *et al.* 2003, Pour *et al.* 2015, Hajmohammad *et al.* 2018, Chemi *et al.* 2015, Shahsavari and Janghorban 2017a, Bouadi *et al.* 2018, Berghouti *et al.* 2019). Vodenitcharova and Zhang (2006) developed a continuum model for pure bending of a straight nanocomposite beam with a circular cross section reinforced by a single-walled carbon nanotube.

The constitutive models and mechanical properties of carbon nanotube polymer composites have been studied analytically, experimentally, and numerically. Cooper *et al.* (2002) and Barber *et al.* (2003), which demonstrated that carbon nanotubes are effective in reinforcing a polymer due to remarkably high separation stress according to a series of pull-out tests of individual carbon nanotubes embedded within polymer matrix. (Wan *et al.* 2005 and Barati 2017) investigated the effective moduli of the CNT reinforced polymer composite, with emphasis on the influence of CNT length and CNT matrix interphase on the stiffening of the composite. The behavior of CNTRC can be considerably improved through the use of a functionally graded distribution of CNTs in the matrix (Civalek 2017, Shokravi 2017b).

The buckling instability problem is a common question in everyday life, in engineering, buckling is a sudden failure of a structure subjected to high compressive stresses and refers to loss of the load-carrying capacity of a component within a structure or of the structure itself. In reality, it is very easy to investigate the buckling instability of small systems like a single homogenous rod. Building an experimental setup one can consider different boundary conditions. There are two major categories leading to the sudden failure of a mechanical component: material failure and structural instability, (buckling). For material failures, you need to consider the yield stress for ductile materials and the ultimate stress for brittle materials. A research project on stability cannot start without recognizing the contribution of Euler to the problem of stability when he published his famous Euler's equation on the elastic stability of columns back in 1744. His original work consisted of determining the buckling load of a cantilever column that was fixed at the bottom and free at the top. Practical solutions are still not available for some types of structures.

Stability is one of the most critical limit states for structures during construction and during their service life. One of the most difficult challenges in structural stability is determining the critical load under which a structure collapses due to the loss of stability; this is because of the complexity of this phenomenon and the many material properties that are influenced by geometric and material imperfections and material nonlinearity. The critical buckling load is the force that has to be exceeded to buckle the rod. The critical force is contour length-dependent and vanishes for an infinitely long rod.

In cases of critical buckling load analyses of CNTRC structures, Mehar *et al.* (2019), investigated the buckling of graded CNT-reinforced composite sandwich shell structure

under thermal loading. Using the Multiscale modeling approach Mehar and Panda (2019) gave the solutions for thermal critical buckling of nanocomposite curved structure. The nonlinear thermal buckling behaviour of laminated composite panel structure including the stretching effect and higher-order finite element was reported by Katariya *et al.* (2017b). Thus, the topic of buckling of functionally graded materials and laminated composite has been fascinating many researchers for recently years (Meziane *et al.* 2014, Al-Basyouni *et al.* 2015, Katariya and Panda 2016, Kar *et al.* 2017, Kar and Panda 2016, 2017, Kar *et al.* 2016, Bourada *et al.* 2016, Panda and Katariya 2015, Bellifa *et al.* 2017a, b, Panda *et al.* 2017, Panda and Singh 2009, Abdelaziz *et al.* 2017, El-Haina *et al.* 2017, Menasria *et al.* 2017, Chikh *et al.* 2017, Shokravi 2017b, Kaci *et al.* 2018, Mokhtar *et al.* 2018, Yazid *et al.* 2018, Kadari *et al.* 2018, Bourada *et al.* 2018, Hellal *et al.* 2019, Karami *et al.* 2019d, Alimirzaei *et al.* 2019, Meksi *et al.* 2019).

To the best of authors' knowledge, no many reports have been found in the literature on the buckling of sandwich plates and Post-buckling of laminated composite. However, Panda and Singh (2010) used the non-linear finite element method for optimization of Thermal post-buckling of a laminated composite spherical shell panel embedded with shape memory alloy fibres. Katariya *et al.* (2017a) investigated the thermal buckling strength of laminated sandwich composite panel structure embedded with shape memory alloy fibre. Panda and Singh (2013a) gave the post-buckling analysis of laminated composite doubly curved panel embedded with SMA fibers subjected to thermal environment. Buckling analysis of SMA bonded sandwich structure—using FEM has been developed by Katariya and Panda (2018). Panda and Singh (2013b) investigated the thermal post-buckling behavior of laminated composite spherical shell panel using NFEM. Katariya and Panda (2014) studied the Thermo-Mechanical Stability of Composite Cylindrical Panels.

Elastic stability must satisfy two basic criteria, the ability of the structure to support the imposed loading (strength) and the capacity of the structure to resist distortions (stiffness) (Nethercot and Kirby 1979). Elastic buckling instability is frequently associated with large changes of geometry which often occur quickly as the structural member moves from one geometrical position of equilibrium to another. For example, when a wood I-joist is loaded in the plane of its web by a gradually increasing load, an axial deflection is the first wood I-joist response to the applied load. This axial deflection lasts until a particular load is reached. Any further increase in the applied load will cause wood I-joist instability. This instability is characterized by the presence of additional wood I-joists responses to the applied load, including a lateral wood I-joist deflection and rotation of the wood I-joist with respect to its neutral axis. This condition of instability is called lateral-torsional buckling instability. The load at which lateral-torsional buckling instability occurs is known as the critical lateral-torsional buckling load (Nethercot and Trahair 1976).

This present paper attempts to show the critical buckling

load of a functionally graded reinforced polymer plate with a parabolic distribution of carbon nanotubes using the shear deformation plate theory. This research seeks to analyze the influences of various parameters on the critical buckling load of plate such as plate thickness, aspect ratios, volume fraction and type of reinforcement.

## 2. Geometrical configuration and properties of CNTs plate

As shown in Figs. 1 and 2, consider the case of a uniform thickness, an FG-CNT reinforced polymer plate with linear and parabolic distribution of carbon nanotubes referring to coordinates (x, y, z) with length a, width b and thickness h.

Three different models of the distribution of reinforcements across the thickness are taken into consideration in this study such as uniformly distributed (referred to as UD-CNT), linear (referred to as CNT- L) and non-linear (referred to as CNT-NL) in the thickness direction (Fig. 2).

Several micromechanical models have been developed to predict the effective material properties of CNTRCs, The Mori-Tanaka model is applicable to micro particles (Seidel and Lagoudas 2006, Li *et al.* 2007) and the rule of mixture (Anumandla and Gibson 2006, Esawi and Farag 2007) is simple and convenient to predict the global material properties of the CNTRC.

In the present study, according to the rule of mixture by introducing the CNT efficiency parameters ( $\eta_1, \eta_2, \eta_3$ ), the effective Young's modulus and shear modulus of the CNTRC layer can be expressed as (Shen 2009).

$$E_{11} = \eta_1 V_{cnt} E_{11}^{cnt} + V_p E^p \quad (1a)$$

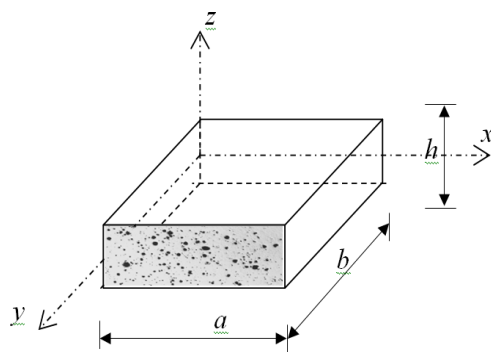


Fig. 1 Plate with nanotube of Carbone

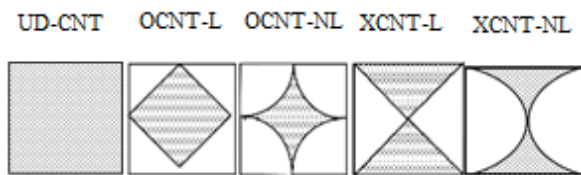


Fig. 2 The different models of the reinforcement provisions

Table 1 Distributions of reinforcements across the thickness

Uniformly distributed	UD- CNT	$V_{cnt} = V_{cnt}^*$
Linear functionally graded	O- CNT- L	$V_{cnt} = 2 \left( 1 - \frac{2 z }{h} \right) V_{cnt}^*$
	X- CNT- L	$V_{cnt} = 2 \left( \frac{2 z }{h} \right) V_{cnt}^*$
Non-linear functionally graded	O- CNT- NL	$V_{cnt} = 2 \left( 1 - \frac{2 z }{h} \right)^2 V_{cnt}^*$
	X- CNT- NL	$V_{cnt} = 2 \left( \frac{2 z }{h} \right)^2 V_{cnt}^*$

$$\frac{\eta_2}{E_{22}} = \frac{V_{cnt}}{E_{22}^{cnt}} + \frac{V_p}{E^p} \quad (1b)$$

$$\frac{\eta_3}{G_{12}} = \frac{V_{cnt}}{G_{12}^{cnt}} + \frac{V_p}{G^p} \quad (1c)$$

Where  $E_{11}^{cnt}$ ,  $E_{22}^{cnt}$  and  $G_{12}^{cnt}$  indicate the Young's moduli and shear modulus of SWCNTs, respectively, and  $E^p$  and  $G^p$  represent the properties of the isotropic matrix.  $\eta_1$ ,  $\eta_2$  and  $\eta_3$  are CNT/matrix efficiency parameters, the  $V_{cnt}$  and  $V_p$  are the volume fractions of the carbon nanotubes and matrix, respectively, and are related by  $V_{cnt} + V_p = 1$ . For other properties in terms of Poisson's ratio ( $\nu$ ) and mass density ( $\rho$ ), these can be written as

$$\nu_{12} = V_{cnt} \nu_{12}^{cnt} + V_p \nu^p, \quad \rho = V_{cnt} \rho^{cnt} + V_p \rho^p \quad (2)$$

To consider the three distributions of reinforcements across the thickness (Table 1).

Where  $V_{cnt}^*$  is the given volume fraction of CNTs, which can be obtained from the following equation (Draoui *et al.* 2019)

$$V_{cnt}^* = \frac{W_{cnt}}{W_{cnt} + (\rho^{cnt}/\rho^m)(1 - W_{cnt})} \quad (3)$$

Where  $W_{cnt}$  is the mass fraction of the carbon nanotube in the nano-composite plate. in this study, we introduce the CNT efficiency parameter to consider the small-scale effect and other effects on the material properties of CNTRCs. The CNT efficiency parameters ( $\eta$ ) associated with the given volume fraction ( $V_{cnt}^*$ ) (Zhu *et al.* 2012)

$$\eta_1 = 0.149 \text{ and } \eta_2 = \eta_3 = 0.934 \\ \text{for the case of } V_{cnt}^* = 0.11$$

$$\eta_1 = 0.150 \text{ and } \eta_2 = \eta_3 = 0.941 \\ \text{for the case of } V_{cnt}^* = 0.14$$

$$\eta_1 = 0.149 \text{ and } \eta_2 = \eta_3 = 1.381 \\ \text{for the case of } V_{cnt}^* = 0.17$$

### 3. Equations of motion

According to the high order shear deformation plate theory for describing the behaviour of the CNTRC plates. The displacements of any point in the plate along the x, y and z axes, denoted by  $u(x, y, z, t)$ ,  $v(x, y, z, t)$  and  $w(x, y, t)$  respectively is given below (Reddy 2004, Mahi *et al.* 2015)

$$\begin{Bmatrix} u(x, y, z, t) \\ v(x, y, z, t) \\ w(x, y, t) \end{Bmatrix} = \begin{Bmatrix} u_0(x, y, t) \\ v_0(x, y, t) \\ w_0(x, y, t) \end{Bmatrix} - \begin{Bmatrix} zw_{,x} \\ zw_{,y} \\ 0 \end{Bmatrix} + \psi(z) \begin{Bmatrix} \phi_x \\ \phi_y \\ 0 \end{Bmatrix} \quad (4)$$

In which  $u_0$ ,  $v_0$  and  $w_0$  are the displacements along the x, y and z directions in the mid plane of the plate, t is time and  $\phi_x$ ,  $\phi_y$  are the total bending rotation of the cross-section at any point of the reference plane. If the last term in Eq. (4) is neglected, the displacements are reduced to the classical plate theory (CPT). Also, the first order shear deformation theory (FSDT) is obtained by setting,  $\Psi(z) = z$ .

In terms of higher order shear deformation theories, the corresponding shape functions are defined as follows.

Third order shear deformation theory (TSDT):

$$\psi(z) = z \left( 1 - \frac{4z^2}{3h^2} \right) \quad (5a)$$

Sinusoidal shear deformation theory (SSDT):

$$\Psi(z) = \frac{h}{\pi} \sin\left(\frac{\pi z}{h}\right) \quad (5b)$$

The linear normal strain and transverse shear strain are associated with the displacements via

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix} - z \begin{Bmatrix} \frac{\partial^2 w_0}{\partial x^2} \\ \frac{\partial^2 w_0}{\partial y^2} \\ 2 \frac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix} \quad (6a)$$

$$+ \psi(z) \begin{Bmatrix} \frac{\partial \phi_x}{\partial x} \\ \frac{\partial \phi_y}{\partial y} \\ \left( \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \right) \end{Bmatrix}$$

$$\begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \phi_x + \frac{\partial \psi(z)}{\partial z} \\ \phi_y + \frac{\partial \psi(z)}{\partial z} \end{Bmatrix} \quad (6b)$$

The expression of normal and shear stress are written by linear elastic constitutive law as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \begin{pmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{pmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}; \quad (7a)$$

$$\begin{Bmatrix} \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{pmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{pmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}, \quad (7a)$$

Where  $Q_{ij}$  are the transformed elastic constants

$$\begin{aligned} Q_{11} &= \frac{E_{11}}{1 - \nu_{12}\nu_{21}}, & Q_{22} &= \frac{E_{22}}{1 - \nu_{12}\nu_{21}}, \\ Q_{12} &= \frac{\nu_{21}E_{11}}{1 - \nu_{12}\nu_{21}}, & Q_{66} &= G_{12} \\ Q_{55} &= G_{13} & Q_{44} &= G_{23} \end{aligned} \quad (7b)$$

The governing differential equations of motion can be derived from Hamilton's principle (Attia *et al.* 2015, Guessas *et al.* 2018, Belabed *et al.* 2018, Cherif *et al.* 2018, Fourn *et al.* 2018, Youcef *et al.* 2018, Karami *et al.* 2018c, Draiche *et al.* 2019, Chaabane *et al.* 2019).

$$\int_0^t (\delta U + \delta V) dt = 0 \quad (8)$$

Where  $\delta U$  and  $\delta V$  are the virtual variation of the strain energy and the virtual work done by external forces.

The expression of the virtual strain energy is (Beldjelili *et al.* 2016, Zine *et al.* 2018, Attia *et al.* 2018)

$$\delta U = \int_{-\frac{h}{2}}^{\frac{h}{2}} \int_A \sigma_{xx} \delta \varepsilon_{xx} + \sigma_{yy} \delta \varepsilon_{yy} + \sigma_{xy} \delta \gamma_{xy} + \sigma_{yz} \delta \gamma_{yz} + \sigma_{xz} \delta \gamma_{xz} dA dx \quad (9)$$

By substituting Eq. (6) into Eq. (9), one obtains

$$\begin{aligned} \delta U &= \int_A \{ N_{xx} \delta u_{0,x} - M_{xx} \delta w_{0,xx} + P_{xx} \delta \phi_{x,x} \\ &+ N_{yy} \delta v_{0,y} - M_{yy} \delta w_{0,yy} + P_{yy} \delta \phi_{y,y} \\ &+ N_{xy} (\delta u_{0,y} + \delta v_{0,x}) - 2M_{xy} \delta w_{0,xy} \\ &+ P_{xy} (\delta \phi_{x,y} + \delta \phi_{y,x}) + R_{yz} \delta \phi_y + R_{xz} \delta \phi_x \} dx dy \end{aligned} \quad (10)$$

Where stress resultants can be defined as follows

$$(N_{xx}, N_{yy}, N_{xy}) = \int_{-h/2}^{h/2} (\sigma_{xx}, \sigma_{yy}, \sigma_{xy}) dz \quad (11a)$$

$$(M_{xx}, M_{yy}, M_{xy}) = \int_{-h/2}^{h/2} z (\sigma_{xx}, \sigma_{yy}, \sigma_{xy}) dz \quad (11b)$$

$$\begin{aligned} (P_{xx}, P_{yy}, P_{xy}) &= \int_{-h/2}^{h/2} \psi(z) (\sigma_{xx}, \sigma_{yy}, \sigma_{xy}) dz \\ R_{yz} &= \int_{-h/2}^{h/2} \frac{\partial \psi(z)}{\partial z} \sigma_{yz} dz \\ R_{xz} &= \int_{-h/2}^{h/2} \frac{\partial \psi(z)}{\partial z} \sigma_{xz} dz \end{aligned} \quad (11c)$$

The stress resultants in form of material stiffness and displacement components are obtained by substituting Eq.

(7) into Eq. (10).

$$\begin{aligned} \begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} &= \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^{(0)}_{xx} \\ \varepsilon^{(0)}_{yy} \\ \gamma^{(0)}_{xy} \end{Bmatrix} \\ &+ \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{21} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^{(1)}_{xx} \\ \varepsilon^{(1)}_{yy} \\ \gamma^{(1)}_{xy} \end{Bmatrix} \\ &+ \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^{(\psi)}_{xx} \\ \varepsilon^{(\psi)}_{yy} \\ \gamma^{(\psi)}_{xy} \end{Bmatrix} \end{aligned} \quad (12a)$$

$$\begin{aligned} \begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} &= \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{21} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^{(0)}_{xx} \\ \varepsilon^{(0)}_{yy} \\ \gamma^{(0)}_{xy} \end{Bmatrix} \\ &+ \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{21} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^{(1)}_{xx} \\ \varepsilon^{(1)}_{yy} \\ \gamma^{(1)}_{xy} \end{Bmatrix} \\ &+ \begin{bmatrix} E_{11} & E_{12} & 0 \\ E_{21} & E_{22} & 0 \\ 0 & 0 & E_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^{(\psi)}_{xx} \\ \varepsilon^{(\psi)}_{yy} \\ \gamma^{(\psi)}_{xy} \end{Bmatrix} \end{aligned} \quad (12b)$$

$$\begin{aligned} \begin{Bmatrix} P_{xx} \\ P_{yy} \\ P_{xy} \end{Bmatrix} &= \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^{(0)}_{xx} \\ \varepsilon^{(0)}_{yy} \\ \gamma^{(0)}_{xy} \end{Bmatrix} \\ &+ \begin{bmatrix} E_{11} & E_{12} & 0 \\ E_{21} & E_{22} & 0 \\ 0 & 0 & E_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^{(1)}_{xx} \\ \varepsilon^{(1)}_{yy} \\ \gamma^{(1)}_{xy} \end{Bmatrix} \\ &+ \begin{bmatrix} F_{11} & F_{12} & 0 \\ F_{21} & F_{22} & 0 \\ 0 & 0 & F_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon^{(\psi)}_{xx} \\ \varepsilon^{(\psi)}_{yy} \\ \gamma^{(\psi)}_{xy} \end{Bmatrix} \end{aligned} \quad (12c)$$

$$\begin{Bmatrix} R_{yz} \\ R_{xz} \end{Bmatrix} = \begin{bmatrix} H_{44} & 0 \\ 0 & H_{55} \end{bmatrix} \begin{Bmatrix} \gamma^{(0)}_{yz} \\ \gamma^{(0)}_{xz} \end{Bmatrix} \quad (12d)$$

Where  $A_{ij}$ ,  $B_{ij}$ ,  $C_{ij}$ ,  $D_{ij}$ ,  $E_{ij}$ ,  $F_{ij}$ , are the plate stiffness, defined by

$$[A_{ij}, B_{ij}, D_{ij}] = \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{ij}[1, z, z^2] dz; \quad (13a)$$

$i, j = 1, 2, 6$

$$[C_{ij}, E_{ij}, F_{ij}] = \int_{-\frac{h}{2}}^{\frac{h}{2}} \psi(z) Q_{ij}[1, z, \psi(z)] dz; \quad (13b)$$

$i, j = 1, 2, 6$

For the CNTRC plates, the principle of virtual work done by external loadings is

$$\delta V = \int_A \left( N_x^0 \frac{\partial w_0}{\partial x} \frac{\partial \delta w_0}{\partial x} + N_y^0 \frac{\partial w_0}{\partial y} \frac{\partial \delta w_0}{\partial y} \right) dx dy \quad (14)$$

By substituting the equations of the strain energy and the virtual work done by external forces into Hamilton's principle, Then, integrating by parts and collecting the coefficients of  $\delta u_0$ ,  $\delta v_0$ ,  $\delta w_0$ ,  $\delta \phi_x$  and  $\delta \phi_y$ , leads to the following equations of motion.

$$\delta u_0: \quad \frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \quad (15a)$$

$$\delta v_0: \quad \frac{\partial N_{yy}}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0 \quad (15b)$$

$$\delta w_0: \quad \frac{\partial^2 M_{xx}}{\partial x^2} + \frac{\partial^2 M_{yy}}{\partial y^2} + 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + N_x^0 \frac{v^2 w_0}{v^2 x} + N_y^0 \frac{v^2 w_0}{v^2 y} = 0 \quad (15c)$$

$$\delta \phi_x: \quad \frac{\partial P_{xx}}{\partial x} + \frac{\partial P_{xy}}{\partial y} - R_{yz} = 0 \quad (15d)$$

$$\delta \phi_y: \quad \frac{\partial P_{yy}}{\partial y} + \frac{\partial P_{xy}}{\partial x} - R_{yz} = 0 \quad (15e)$$

The Navier solution procedure is employed to formulate the closed-form solutions for buckling problems of simply supported CNTRC plates (Bakhadda *et al.* 2018, Medani *et al.* 2019).

$$\begin{aligned} u_0(x, y, t) &= \sum_{M=1}^{\infty} \sum_{N=1}^{\infty} U_{MN} e^{i\omega t} \cos(\alpha x) \sin(\zeta y) \\ v_0(x, y, t) &= \sum_{M=1}^{\infty} \sum_{N=1}^{\infty} V_{MN} e^{i\omega t} \sin(\alpha x) \cos(\zeta y) \\ w_0(x, y, t) &= \sum_{M=1}^{\infty} \sum_{N=1}^{\infty} W_{MN} e^{i\omega t} \sin(\alpha x) \sin(\zeta y) \\ \phi_x(x, y, t) &= \sum_{M=1}^{\infty} \sum_{N=1}^{\infty} \theta_{xMN} e^{i\omega t} \cos(\alpha x) \sin(\zeta y) \\ \phi_y(x, y, t) &= \sum_{M=1}^{\infty} \sum_{N=1}^{\infty} \theta_{yMN} e^{i\omega t} \sin(\alpha x) \cos(\zeta y) \end{aligned} \quad (16)$$

$$\text{Where } \alpha = \frac{M\pi}{a} \text{ and } \zeta = \frac{N\pi}{b}, i = \sqrt{-1}.$$

Where  $U_{MN}$ , and  $V_{MN}$ ,  $W_{MN}$ ,  $\theta_{xMN}$  and  $\theta_{yMN}$  are arbitrary parameters and  $\omega$ , the frequency of free vibration.

Substituting the Eq. (13) into the Eq. (14), one obtains the closed-form solutions which are presented in the following matrix form.

$$\begin{pmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} \\ S_{12} & S_{22} & S_{23} & S_{24} & S_{25} \\ S_{13} & S_{23} & S_{33} & S_{34} & S_{35} \\ S_{14} & S_{24} & S_{34} & S_{44} & S_{45} \\ S_{15} & S_{25} & S_{35} & S_{45} & S_{55} \end{pmatrix} \begin{Bmatrix} U_{MN} \\ V_{MN} \\ W_{MN} \\ \theta_{xMN} \\ \theta_{yMN} \end{Bmatrix} = 0 \quad (17)$$

Where

$$\begin{aligned}
s_{11} &= -A_{11}\alpha^2 + A_{66}\zeta^2, & s_{12} &= -\alpha\zeta(A_{12} + A_{66}), \\
s_{13} &= 0, & s_{14} &= -B_{11}\alpha^3 - B_{66}\zeta^2, \\
s_{15} &= -B_{12}\alpha\zeta - B_{66}\alpha\zeta, \\
s_{21} &= s_{12}, & s_{22} &= -A_{66}\alpha^2 - A_{22}\zeta^2, \\
s_{23} &= 0, & s_{24} &= -B_{12}\alpha\zeta - B_{66}\alpha\zeta, \\
s_{25} &= -B_{66}\alpha^2 - B_{22}\zeta^2, & s_{31} &= s_{13}, \\
s_{32} &= s_{23}, \\
s_{33} &= -D_{55}\alpha^2 - D_{44}\zeta^2 - N_x\alpha^2 - N_y\zeta^2, \\
s_{34} &= -D_{55}\alpha, & s_{35} &= -D_{44}\zeta, \\
s_{41} &= s_{14}, & s_{42} &= s_{24}, \\
s_{43} &= s_{34}, \\
s_{44} &= -C_{11}\alpha^2 - C_{66}\zeta^2 - D_{55}, \\
s_{45} &= -\alpha\zeta(C_{12} + C_{66}), & s_{51} &= s_{15}, \\
s_{52} &= s_{25}, & s_{53} &= s_{35}, \\
s_{54} &= s_{45}, \\
s_{55} &= -D_{44} - C_{66}\alpha^2 - C_{22}\zeta^2
\end{aligned} \quad (18)$$

The following dimensionless parameter is used to present the numerical results for buckling analyses of CNTRC plates.

$$\bar{N}_{cr} = \frac{N_{cr}\alpha^2}{\pi D_0} \quad \text{Where} \quad D_0 = \frac{E^p h^3}{12[1 - (\nu^p)^2]} \quad (19)$$

#### 4. Results and discussions

Numerical results are presented and discussed in this section for FG-CNTRC plates. We first need to determine

the effective material characteristics of CNTRCs plates employed throughout this work are given as follows.

PMPV (Polymer) is used as the matrix in which material properties are:  $\nu^p = 0.34$ ,  $\rho^p = 1150 \text{ kg/m}^3$  and  $E^p = 2.1 \text{ GPa}$ . For reinforcement material, the armchair (10,10) SWCNTs is chosen with the following properties according to the study of Zhu *et al.* (2012)

$$\begin{aligned}
\nu_{12}^{cnt} &= 0.175; \quad \rho^{cnt} = 1400 \text{ kg/m}^3; \quad E_{11}^{cnt} = 5.6466 \text{ TPa}; \\
E_{22}^{cnt} &= 7.0800 \text{ TPa}; \quad G_{12}^{cnt} = G_{13}^{cnt} = G_{23}^{cnt} = 1.9445 \text{ TPa}
\end{aligned}$$

So as to check the mathematical formulation in previous sections of the present theory, Table 1 demonstrates a comparison between the results obtained by the present model and the results of TSDT (Wattanasakulpong and Chaikittiratana 2015) which is based on the first-order theory Mindlin- Reissner, this examination is made only for linear distribution under uniaxial and biaxial loading, with different values of carbon nanotube volume fraction and various reinforcement plate are considered in this table with thickness ratio of plate ( $a/h = 10$ ). It can be seen the good agreement between the results.

Table 2 shows a comparison between the distribution of the linear and non-linear reinforcement linear inside the polymer matrix. We note from this table that the form non linear (X-CNT-NL) gives higher critical loads than the other forms (X-CNT-L, O-CNT-L, O-CNT-NL) under uniaxial and biaxial loading. This variation shows that the nonlinear distribution of the reinforcement makes the plate stiffer that will resist better against buckling. The high variation of

Table 2 The comparison results of dimensionless critical buckling load of present square plate with (Wattanasakulpong and Chaikittiratana 2015) results under different loading

Uniaxial loading $\gamma_x = -1, \gamma_y = 0$				
Reinforcement type	Source	$V_{cnt}^* = 0.11$	$V_{cnt}^* = 0.14$	$V_{cnt}^* = 0.17$
UD-CNT	TSDT	20.6814	23.3559	32.3180
	SSDT	20.7286	23.4229	32.3890
	Present	20.6788	23.3520	32.3142
X-CNT	TSDT	24.2864	26.8941	37.6943
	SSDT	24.3943	27.0177	37.8069
	Present	24.2791	26.8860	37.6881
O-CNT	TSDT	14.4990	16.6984	22.6823
	SSDT	14.4515	16.6451	22.6276
	Present	14.5040	16.7041	22.6883
Biaxial loading $\gamma_x = -1, \gamma_y = -1$				
UD-CNT	TSDT	10.3407	11.6780	16.1590
	SSDT	10.3643	11.7115	16.1945
	Present	10.3394	11.6760	16.1571
X-CNT	TSDT	12.1432	13.4471	18.8472
	SSDT	12.1972	13.5089	18.9035
	Present	12.1396	13.4430	18.8440
O-CNT	TSDT	7.2495	8.3492	11.3411
	SSDT	7.2257	8.3225	11.3138
	Present	7.2520	8.3521	11.3442

Table 3 Dimensionless critical buckling loads of CNTRC square plates with different types of distributions (linear and nonlinear) and various (a/h) ratios

Uniaxial loading: $\gamma_x = -1, \gamma_y = 0$					
a/h	UD-CNT	X-CNT	O-CNT	X-CNTNL	O-CNTNL
5	13.9179	14.8082	11.5783	15.3194	10.2369
10	32.3142	37.6881	22.6883	40.3459	17.3376
20	51.8827	68.6094	30.9110	77.5797	21.3800
40	61.5705	87.3715	34.0940	102.3821	22.7389
Biaxial loading: $\gamma_x = -1, \gamma_y = -1$					
5	6.9590	7.4041	5.7892	7.6597	5.1184
10	16.1571	18.8440	11.3442	20.1730	8.6688
20	25.9414	34.3047	15.4555	38.7898	10.6900
40	30.7852	43.6858	17.0470	51.1910	11.3695

Table 4 The effect of different values of the volume fraction on the critical buckling load for CNTRC plate

Uniaxial loading: $\gamma_x = -1, \gamma_y = 0$				Biaxial loading: $\gamma_x = -1, \gamma_y = -1$		
Volume fraction	UD-CNT	X-CNT-NL	O-CNT-NL	UD-CNT	X-CNT-NL	O-CNT-NL
$V_{cnt}^* = 0.11$	20.6788	26.0727	11.1411	10.3394	13.0363	5.5706
$V_{cnt}^* = 0.14$	23.3520	28.6197	12.9088	11.6760	14.3098	6.4544
$V_{cnt}^* = 0.17$	32.3142	40.3459	17.3376	16.1571	20.1730	8.6688

Table 5 Effect of mode number and type of loading on the variation of critical buckling load for different types of CNTRC plate

		Uniaxial loading: $\gamma_x = -1, \gamma_y = 0$			Biaxial loading: $\gamma_x = -1, \gamma_y = -1$		
(n; m)	$V_{cnt}^* = 0.17$	UD-CNT	X-CNT-NL	O-CNT-NL	UD-CNT	X-CNT-NL	O-CNT-NL
(1,1)		32.3142	40.3459	17.3376	16.1571	20.1730	8.6688
(1,2)		56.5915	60.7802	41.4465	50.9324	54.7022	37.3014
(1,3)		70.6534	77.1926	49.6396	67.9360	74.2237	47.7287
(n; m)	$V_{cnt}^* = 0.14$						
(1,1)		23.3520	28.6197	12.9088	11.6760	14.3098	6.4544
(1,2)		38.9258	42.3454	27.8884	35.0333	38.1109	25.0996
(1,3)		49.7685	55.5404	32.7098	47.8543	53.4043	31.4517
(n; m)	$V_{cnt}^* = 0.11$						
(1,1)		20.6788	26.0727	11.1411	10.3394	13.0363	5.5706
(1,2)		35.9232	39.6244	25.1726	32.3309	35.6619	22.6553
(1,3)		44.9805	50.3565	29.8728	43.2505	48.4197	28.7237

critical buckling load is esteemed at the high values of (a/h) ratios.

The dimensionless buckling critical loads of square reinforced plate under uniaxial and biaxial loads for various reinforcements are presented in Table 3 with different values of volume fraction. It is seen that the dimensionless critical load increases if the volume fraction of CNTs increases for all reinforcement type. On the other hand, the X-CNT-NL reinforced plate has a high resistance against buckling compared to other types of reinforcement because there is a concentration of the reinforcement at the top and

bottom face of reinforced plate.

The effect of various mode number on the dimensionless critical buckling load of square reinforced plate are also presented in the Table 4 under two types of compressive load, uniaxial ( $\gamma_x = -1, \gamma_y = 0$ ) and biaxial ( $\gamma_x = -1, \gamma_y = -1$ ). It is deduced that the dimensionless critical buckling load increase by increasing of the number of modes. The increase of dimensionless critical buckling load is attributed to the deformation configuration concerning the vibration mode when the number of modes increases the wave length decreases and the plate supports better under

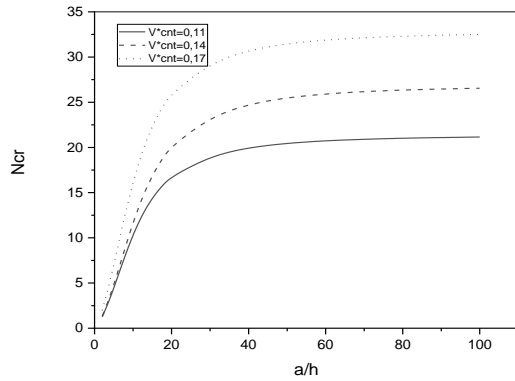


Fig. 3 The effect of aspect ratio ( $a/h$ ) and values of volume fraction on the dimensionless critical buckling load of UD-NT under biaxial load

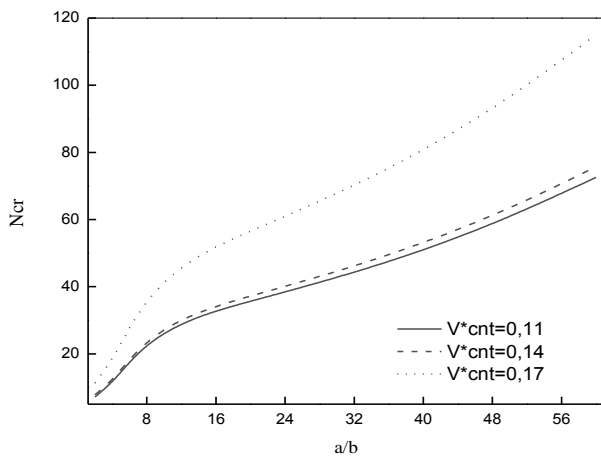


Fig. 4 The effect of geometric ratio ( $a/b$ ) and values of volume fraction on the dimensionless critical buckling load of UD-CNT under biaxial load with ( $a/h = 10$ )

the applied axial load. In addition, the results reveal that the dimensionless critical buckling load results increase as the volume fraction increase.

Fig. 3 shows the effect of ratio  $a/h$  on the critical buckling load for different values of the volume fraction of CNT. Note that the increase in the ratio  $a/h$  leads to an increase in the critical buckling load. The increase in the ratio  $a/h$  makes the plate thin and diminish the transversal shear effect taken into consideration by the Mindlin-Reissner theory. In addition, the critical load with a volume fraction equal 0.17 gives the largest load compared to the other fractions of carbon nanotube. The increase in the dimensionless critical buckling load is appropriate to the quantity of carbon nanotubes in the reinforced plate.

The effect of geometric ratio  $a/b$  on the dimensionless critical buckling load for different values of the volume fraction of CNT is presented in Fig. 4. Note that the increase in the ratio  $a/b$  leads to an increase in the critical buckling load. The effect of geometric ratio  $a/b$  is attributed to the increase of reinforced plate dimensions. Also, the dimensionless critical buckling load increase with increasing of the CNTs volume fraction.

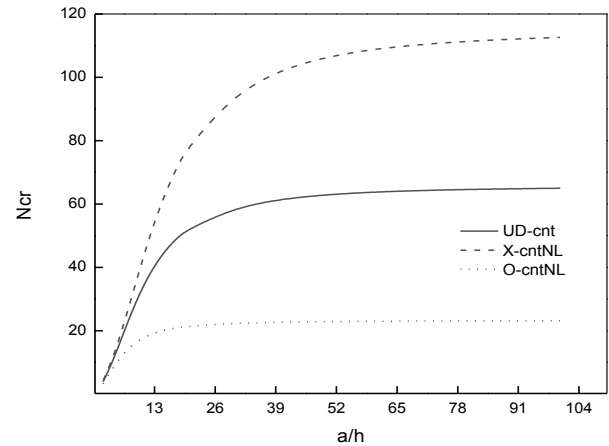


Fig. 5 The dimensionless critical buckling load of a square plate for different form of reinforcement distribution and geometric ratio with  $V_{cnt}^* = 0.17$

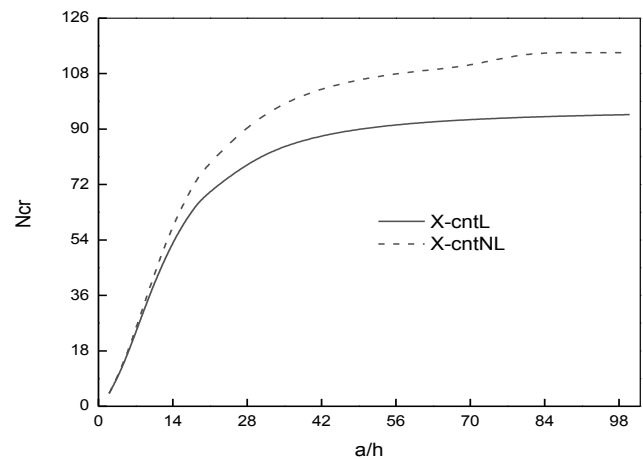


Fig. 6 The comparison between the linear X-CNT-L and nonlinear X-CNT-NL distribution for the plate type X-CNT

Fig. 5 shows the influence of the geometric ratio parameter  $a/h$  and the distribution of carbon nanotubes in the polymer matrix on the critical buckling load. We observe that the critical buckling load increase with increasing of ratio  $a/h$ . Furthermore, the plate with distribution nonlinear X-CNTNL gives a Larger critical buckling load because has a high resistance against buckling compared to other reinforcement types UD -CNT and O-CNT-NL. The high resistance is attributing to the concentration of the CNTs reinforcement at the top and bottom face of reinforced plate.

The linear X-CNT-L and nonlinear X-CNT-NL distribution for the X-CNT reinforcement plate type is illustrated in Fig. 6. We see that the critical buckling load for both distribution increase when the ratio  $a/h$  increases. This increase is estimated on the little values of  $a/h$  ratio. We also note that the critical load of buckling of nonlinear distribution X-CNT-NL is larger than the linear distribution. This variation is attributed to the concentration of reinforcement (CNTs) at the top and bottom faces of the plate, which makes it more rigid against buckling.



## 5. Conclusions

In this paper, the influence of different parameters on the dimensionless critical buckling load of carbon nanotube-reinforced composite plates using the shear deformation theory is studied and discussed. The overseeing differential equations incorporate the various parameters that are solved by implementing Hamilton's principle and the dimensionless critical load analyses of linear and nonlinear distribution of CNTs are gotten. The exactness of the outcomes is analyzed utilizing the available data in the literature. Finally, through some parametric investigated study the results showed the dependence of buckling behavior on the different parameters such as aspect ratios, volume fraction, plate thickness and linear and nonlinear distribution.

From the numerical outcomes, it is presumed that the concentration of the nanotubes in the nonlinear distribution at the top and bottom face of plate conduce to high resistance against buckling compared with different types of reinforcement. In terms of critical buckling load, the results show:

- The form nonlinear (X-CNT-NL) gives higher critical loads than the other forms (UD -CNT, X-CNT-L, O-CNT-L, O-CNT-NL) under uniaxial and biaxial loading.
- It is seen that the dimensionless critical load increases if the volume fraction of CNTs increases for all reinforcement type.
- It is deduced that the dimensionless critical buckling load increase by increasing of the number of modes
- As the number of modes increases, the wavelength decreases and the plate supports better under the applied axial load.
- The results reveal that the dimensionless critical buckling load results increase as the volume fraction increase.
- The increase in the ratio  $a/h$  makes the plate thin and diminish the transversal sheer effect taken into consideration by the Mindlin-Reissner theory.
- The high resistance of the reinforced plate is attributing to the concentration of the CNTs reinforcement at the top and bottom face.

Finally, the results demonstrate the dependence of critical buckling load on the different parameters and the concentration of reinforcement (CNTs) at the top and bottom faces of nonlinear distribution X-CNT-NL plate make it more rigid compared with the linear distribution. An improvement of present formulation will be considered in the future work to consider the thickness stretching effect by using quasi-3D shear deformation models (Draiche *et al.* 2016, Ait Atmane *et al.* 2017, Abualnour *et al.* 2018, Benchohra *et al.* 2018, Younsi *et al.* 2018, Bouhadra *et al.* 2018, Karami *et al.* 2018d, e, Addou *et al.* 2019, Boukhelif *et al.* 2019, Boutaleb *et al.* 2019, Mahmoudi *et al.* 2019, Zarga *et al.* 2019, Bouanati *et al.* 2019, Zaoui *et al.* 2019, Khiloun *et al.* 2019, Boulefrakh *et al.* 2019).

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## References

- Abdelaziz, H.H., Ait Amar Meziane, M., Bousahla, A.A., Tounsi, A., Mahmoud, S.R. and Alwabri, A.S. (2017), "An efficient hyperbolic shear deformation theory for bending, buckling and free vibration of FGM sandwich plates with various boundary conditions", *Steel Compos. Struct., Int. J.*, **25**(6), 693-704. <https://doi.org/10.12989/scs.2017.25.6.693>
- Abualnour, M., Houari, M.S.A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2018), "A novel quasi-3D trigonometric plate theory for free vibration analysis of advanced composite plates", *Compos. Struct.*, **184**, 688-697. <https://doi.org/10.1016/j.compstruct.2017.10.047>
- Addou, F.Y., Meradjah, M., M.A.A. Bousahla, Benachour, A., Bourada, F., Tounsi, A. and Mahmoud, S.R. (2019), "Influences of porosity on dynamic response of FG plates resting on Winkler/Pasternak/Kerr foundation using quasi 3D HSDT", *Comput. Concrete, Int. J.*, **24**(4), 347-367. <https://doi.org/10.12989/cac.2019.24.4.347>
- Ait Atmane, H., Tounsi, A. and Bernard, F. (2017), "Effect of thickness stretching and porosity on mechanical response of a functionally graded beams resting on elastic foundations", *Int. J. Mech. Mater. Des.*, **13**(1), 71-84. <https://doi.org/10.1007/s10999-015-9318-x>
- Al-Basyouni, K.S., Tounsi, A. and Mahmoud, S.R. (2015), "Size dependent bending and vibration analysis of functionally graded micro beams based on modified couple stress theory and neutral surface position", *Compos. Struct.*, **125**, 621-630. <https://doi.org/10.1016/j.compstruct.2014.12.070>
- Alimirzaei, S., Mohammadimehr, M. and Tounsi, A. (2019), "Nonlinear analysis of viscoelastic micro-composite beam with geometrical imperfection using FEM: MSGT electro-magneto-elastic bending, buckling and vibration solutions", *Struct. Eng. Mech., Int. J.*, **71**(5), 485-502. <https://doi.org/10.12989/sem.2019.71.5.485>
- Anumandla, V. and Gibson, R.F. (2006), "A comprehensive closed form micromechanics model for estimating the elastic modulus of nanotube-reinforced composites", *Compos. Part A: Appl. Sci. Manuf.*, **37**(12), 2178-2185. <https://doi.org/10.1016/j.compositesa.2005.09.016>
- Asadi, H. and Beheshti, A.R. (2018), "On the nonlinear dynamic responses of FG-CNTRC beams exposed to aerothermal loads using third- order piston theory", *Acta Mechanica*, **229**(6), 2413-2430. <https://doi.org/10.1007/s00707-018-2121-7>
- Attia, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "Free vibration analysis of functionally graded plates with temperature-dependent properties using various four variable refined plate theories", *Steel Compos. Struct., Int. J.*, **18**(1), 187-212. <https://doi.org/10.12989/scs.2015.18.1.187>
- Attia, A., Bousahla, A.A., Tounsi, A., Mahmoud, S.R. and Alwabri, A.S. (2018), "A refined four variable plate theory for thermoelastic analysis of FGM plates resting on variable elastic foundations", *Struct. Eng. Mech., Int. J.*, **65**(4), 453-464. <https://doi.org/10.12989/sem.2018.65.4.453>
- Avcar, M. (2019), "Free vibration of imperfect sigmoid and power law functionally graded beams", *Steel Compos. Struct., Int. J.*, **30**(6), 603-615. <https://doi.org/10.12989/scs.2019.30.6.603>
- Avcar, M. and Alwan, A.S. (2017), "Free vibration of functionally graded Rayleigh beam", *Int. J. Eng. Appl. Sci.*, **9**(2), 127-127.

- <https://doi.org/10.24107/ijeas.322884>
- Avcar, M. and Mohammed, W.K.M. (2018), "Free vibration of functionally graded beams resting on Winkler-Pasternak foundation", *Arab. J. Geosci.*, **11**(10), 232. <https://doi.org/10.1007/s12517-018-3579-2>
- Bakhadda, B., Bachir Bouiadjra, M., Bourada, F., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2018), "Dynamic and bending analysis of carbon nanotube-reinforced composite plates with elastic foundation", *Wind Struct., Int. J.*, **27**(5), 311-324. <https://doi.org/10.12989/was.2018.27.5.311>
- Barati, M.R. (2017), "Nonlocal-strain gradient forced vibration analysis of metal foam nanoplates with uniform and graded porosities", *Adv. Nano Res., Int. J.*, **5**(4), 393-414. <https://doi.org/10.12989/anr.2017.5.4.393>
- Barber, A.H., Cohen, S.R. and Wagner, H.D. (2003), "Measurement of carbon nanotube-polymer interfacial strength", *Appl. Phys. Lett.*, **82**, 4140-4142. <https://doi.org/10.1063/1.1579568>
- Beik Mohammadlou, H. and Ekhteraei Tounsi, H. (2017), "Parametric studies on elastoplastic buckling of rectangular FGM thin plates", *Aerosp. Sci. Technol.*, **69**, 513-525. <https://doi.org/10.1016/j.ast.2017.07.015>
- Belabed, Z., Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2018), "A new 3-unknown hyperbolic shear deformation theory for vibration of functionally graded sandwich plate", *Earthq. Struct., Int. J.*, **14**(2), 103-115. <https://doi.org/10.12989/eas.2018.14.2.103>
- Beldjelili, Y., Tounsi, A. and Mahmoud, S.R. (2016), "Hygro-thermo-mechanical bending of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory", *Smart Struct. Syst., Int. J.*, **18**(4), 755-786. <https://doi.org/10.12989/sss.2016.18.4.755>
- Bellifa, H., Bakora, A., Tounsi, A., Bousahla, A.A. and Mahmoud, S.R. (2017a), "An efficient and simple four variable refined plate theory for buckling analysis of functionally graded plates", *Steel Compos. Struct., Int. J.*, **25**(3), 257-270. <https://doi.org/10.12989/scs.2017.25.3.257>
- Bellifa, H., Benrahou, K.H., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2017b), "A nonlocal zeroth-order shear deformation theory for nonlinear postbuckling of nanobeams", *Struct. Eng. Mech., Int. J.*, **62**(6), 695-702. <https://doi.org/10.12989/sem.2017.62.6.695>
- Benahmed, A., Fahsi, B., Benzair, A., Zidour, M., Bourada, F. and Tounsi, A. (2019), "Critical buckling of functionally graded nanoscale beam with porosities using nonlocal higher-order shear deformation", *Struct. Eng. Mech., Int. J.*, **69**(4), 457-466. <https://doi.org/10.12989/sem.2019.69.4.457>
- Benchohra, M., Driz, H., Bakora, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2018), "A new quasi-3D sinusoidal shear deformation theory for functionally graded plates", *Struct. Eng. Mech., Int. J.*, **65**(1), 19-31. <https://doi.org/10.12989/sem.2018.65.1.019>
- Berghouti, H., Adda Bedia, E.A., Benkhedda, A. and Tounsi, A. (2019), "Vibration analysis of nonlocal porous nanobeams made of functionally graded material", *Adv. Nano Res., Int. J.*, **7**(5), 351-364. <https://doi.org/10.12989/anr.2019.7.5.351>
- Biercuk, M.J., Llaguno, M.C., Radosavljevic, M., Hyun, J.K., Johnson, A.T. and Fischer, J.E. (2002), "Carbon nanotube composites for thermal management", *Appl. Phys. Lett.*, **80**(15), 2767-2769. <https://doi.org/10.1063/1.1469696>
- Bonnet, P., Sireude, D., Garnier, B. and Chauvet, O. (2007), "Thermal properties and percolation in carbonnanotube-polymer composites", *J. Appl. Phys.*, **91**, 2019-2010. <https://doi.org/10.1063/1.2813625>
- Bouadi, A., Bousahla, A.A., Houari, M.S.A., Heireche, H. and Tounsi, A. (2018), "A new nonlocal HSDT for analysis of stability of single layer graphene sheet", *Adv. Nano Res., Int. J.*, **6**(2), 147-162. <https://doi.org/10.12989/anr.2018.6.2.147>
- Bouanati, S., Benrahou, K.H., Ait Atmane, H., Ait Yahia, S., Bernard, F., Tounsi, A. and Adda Bedia, E.A. (2019), "Investigation of wave propagation in anisotropic plates via quasi 3D HSDT", *Geomech. Eng., Int. J.*, **18**(1), 85-96. <https://doi.org/10.12989/gae.2019.18.1.085>
- Bouhadra, A., Tounsi, A., Bousahla, A.A., Benyoucef, S. and Mahmoud, S.R. (2018), "Improved HSDT accounting for effect of thickness stretching in advanced composite plates", *Struct. Eng. Mech., Int. J.*, **66**(1), 61-73. <https://doi.org/10.12989/sem.2018.66.1.061>
- Boukhari, A., Ait Atmane, H., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2016), "An efficient shear deformation theory for wave propagation of functionally graded material plates", *Struct. Eng. Mech., Int. J.*, **57**(5), 837-859. <https://doi.org/10.12989/sem.2016.57.5.837>
- Boukhilif, Z., Bouremana, M., Bourada, F., Bousahla, A.A., Bourada, M., Tounsi, A. and Al-Osta, M.A. (2019), "A simple quasi-3D HSDT for the dynamics analysis of FG thick plate on elastic foundation", *Steel Compos. Struct., Int. J.*, **31**(5), 503-516. <https://doi.org/10.12989/scs.2019.31.5.503>
- Boulefrakh, L., Hebali, H., Chikh, A., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2019), "The effect of parameters of visco-Pasternak foundation on the bending and vibration properties of a thick FG plate", *Geomech. Eng., Int. J.*, **18**(2), 161-178. <https://doi.org/10.12989/gae.2019.18.2.161>
- Bounouara, F., Benrahou, K.H., Belkorissat, I. and Tounsi, A. (2016), "A nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic foundation", *Steel Compos. Struct., Int. J.*, **20**(2), 227-249. <https://doi.org/10.12989/scs.2016.20.2.227>
- Bourada, F., Amara, K. and Tounsi, A. (2016), "Buckling analysis of isotropic and orthotropic plates using a novel four variable refined plate theory", *Steel Compos. Struct., Int. J.*, **21**(6), 1287-1306. <https://doi.org/10.12989/scs.2016.21.6.1287>
- Bourada, F., Amara, K., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2018), "A novel refined plate theory for stability analysis of hybrid and symmetric S-FGM plates", *Struct. Eng. Mech., Int. J.*, **68**(6), 661-675. <https://doi.org/10.12989/sem.2018.68.6.661>
- Bourada, F., Bousahla, A.A., Bourada, M., Azzaz, A., Zinata, A. and Tounsi, A. (2019), "Dynamic investigation of porous functionally graded beam using a sinusoidal shear deformation theory", *Wind Struct., Int. J.*, **28**(1), 19-30. <https://doi.org/10.12989/was.2019.28.1.019>
- Bousahla, A.A., Benyoucef, S., Tounsi, A. and Mahmoud, S.R. (2016), "On thermal stability of plates with functionally graded coefficient of thermal expansion", *Struct. Eng. Mech., Int. J.*, **60**(2), 313-335. <https://doi.org/10.12989/sem.2016.60.2.313>
- Boussoula, A., Boucham, B., Bourada, M., Bourada, F., Tounsi, A., Bousahla, A.A. and Tounsi, A. (2019), "A simple nth-order shear deformation theory for thermomechanical bending analysis of different configurations of FG sandwich plates", *Smart Struct. Syst., Int. J.*, **25**(2), 197-218. <https://doi.org/10.12989/sss.2020.25.2.197>
- Boutaleb, S., Benrahou, K.H., Bakora, A., Algarni, A., Bousahla, A.A., Tounsi, A., Mahmoud, S.R. and Tounsi, A. (2019), "Dynamic analysis of nanosize FG rectangular plates based on simple nonlocal quasi 3D HSDT", *Adv. Nano Res., Int. J.*, **7**(3), 191-208. <https://doi.org/10.12989/anr.2019.7.3.191>
- Chaabane, L.A., Bourada, F., Sekkal, M., Zerouati, S., Zaoui, F.Z., Tounsi, A., Derras, A., Bousahla, A.A. and Tounsi, A. (2019), "Analytical study of bending and free vibration responses of functionally graded beams resting on elastic foundation", *Struct. Eng. Mech., Int. J.*, **71**(2), 185-196. <https://doi.org/10.12989/sem.2019.71.2.185>
- Chemi, A., Heireche, H., Zidour, M., Rakrak, K. and Bousahla, A.A. (2015), "Critical buckling load of chiral double-walled

- carbon nanotube using non-local theory elasticity", *Adv. Nano Res., Int. J.*, **3**(4), 193-206.  
<https://doi.org/10.12989/anr.2015.3.4.193>
- Cherif, R.H., Meradjah, M., Zidour, M., Tounsi, A., Belmahi, H. and Bensattalah, T. (2018), "Vibration analysis of nano beam using differential transform method including thermal effect", *J. Nano Res.*, **54**, 1-14.  
<https://doi.org/10.4028/www.scientific.net/JNanoR.54.1>
- Chikh, A., Tounsi, A., Hebali, H. and Mahmoud, S.R. (2017), "Thermal buckling analysis of cross-ply laminated plates using a simplified HSDT", *Smart Struct. Syst., Int. J.*, **19**(3), 289-297.  
<https://doi.org/10.12989/ss.2017.19.3.289>
- Civalek, Ö. (2017), "Free vibration of carbon nanotubes reinforced (CNTR) and functionally graded shells and plates based on FSDT via discrete singular convolution method", *Compos. Part B: Eng.*, **111**, 45-59.  
<https://doi.org/10.1016/j.compositesb.2016.11.030>
- Cooper, C.A., Cohen, S.R., Barber, A.H. and Wagner, H.D. (2002), "Detachment of nanotubes from a polymer matrix", *Appl. Phys. Lett.*, **81**, 3873-3875. <https://doi.org/10.1063/1.1521585>
- Daouadji, T.H. and Adim, B. (2016), "An analytical approach for buckling of functionally graded plates", *Adv. Mater. Res., Int. J.*, **5**(3), 141-169. <https://doi.org/10.12989/amr.2016.5.3.141>
- Diamanti, K. and Soutis, C. (2010), "Structural health monitoring techniques for aircraft composite structures", *Prog. Aerosp. Sci.*, **46**(8), 342-352. <https://doi.org/10.1016/j.paerosci.2010.05.001>
- Draiche, K., Tounsi, A. and Mahmoud, S.R. (2016), "A refined theory with stretching effect for the flexure analysis of laminated composite plates", *Geomech. Eng., Int. J.*, **11**(5), 671-690. <https://doi.org/10.12989/gae.2016.11.5.671>
- Draiche, K., Bousahla, A.A., Tounsi, A., Alwabri, A.S., Tounsi, A. and Mahmoud, S.R. (2019), "Static analysis of laminated reinforced composite plates using a simple first-order shear deformation theory", *Comput. Concrete, Int. J.*, **24**(4), 369-378. <https://doi.org/10.12989/cac.2019.24.4.369>
- Draoui, A., Zidour, M., Tounsi, A. and Adim, B. (2019), "Static and dynamic behavior of nanotubes-reinforced sandwich plates using (FSDT)", *J. Nano Res.*, **57**, 117-135.
- Ebrahimi, F. and Jafari, A. (2016), "A four-variable refined shear-deformation beam theory for thermo-mechanical vibration analysis of temperature-dependent FGM beams with porosities", *Mech. Adv. Mater. Struct.*, **25**(3), 12-224.  
<http://dx.doi.org/10.1080/15376494>
- Ehyaie, J., Akbarshahi, A. and Shafiei, N. (2017), "Influence of porosity and axial preload on vibration behavior of rotating FG nanobeam", *Adv. Nano Res., Int. J.*, **5**(2), 141-169.  
<https://doi.org/10.12989/anr.2017.5.2.141>
- El-Haina, F., Bakora, A., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2017), "A simple analytical approach for thermal buckling of thick functionally graded sandwich plates", *Struct. Eng. Mech., Int. J.*, **63**(5), 585-595.  
<https://doi.org/10.12989/sem.2017.63.5.585>
- Esawi, A.M.K. and Farag, M.M. (2007), "Carbon nanotube reinforced composites: potential and current challenges", *Mater. Des.*, **28**, 2394-2401. <https://doi.org/10.1016/j.matdes.2006.09.022>
- Fahsi, A., Tounsi, A., Hebali, H., Chikh, A., Adda Bedia, E.A. and Mahmoud, S.R. (2017), "A four variable refined nth-order shear deformation theory for mechanical and thermal buckling analysis of functionally graded plates", *Geomech. Eng., Int. J.*, **13**(3), 385-410. <https://doi.org/10.12989/gae.2017.13.3.385>
- Fellah, M., Draiche, K., Houar, M.S.A., Tounsi, A., Saeed, T., Alhodaly, M.S. and Benguediab, M. (2019), "A novel refined shear deformation theory for the buckling analysis of thick isotropic plates", *Struct. Eng. Mech., Int. J.*, **69**(3), 335-345.  
<https://doi.org/10.12989/sem.2019.69.3.335>
- Fidelus, J.D., Wiesel, E., Gojny, F.H., Schulte, K. and Wagner, H.D. (2005), "Thermo-mechanical properties of randomly oriented carbon/epoxy nanocomposites", *Compos. Part A: Appl. Sci. Manuf.*, **36**(11), 1555-1561.  
<https://doi.org/10.1016/j.compositesa.2005.02.006>
- Fourn, H., Ait Atmane, H., Bourada, M., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2018), "A novel four variable refined plate theory for wave propagation in functionally graded material plates", *Steel Compos. Struct., Int. J.*, **27**(1), 109-122.  
<https://doi.org/10.12989/scs.2018.27.1.109>
- Griebel, M. and Hamaekers, J. (2004), "Molecular dynamics simulations of the elastic moduli of polymer carbon nanotube composites", *Comput. Meth. Appl. Mech. Eng.*, **193**, 1773-1788.  
<https://doi.org/10.1016/j.cma.2003.12.025>
- Guessas, H., Zidour, M., Meradjah, M. and Tounsi, A. (2018), "The critical buckling load of reinforced nanocomposite porous plates", *Struct. Eng. Mech., Int. J.*, **67**(2), 115-123.  
<https://doi.org/10.12989/sem.2018.67.2.115>
- Hajmohammad, M.H., Zarei, M.S., Farrokhan, A. and Kolahchi, R. (2018), "A layerwise theory for buckling analysis of truncated conical shells reinforced by CNTs and carbon fibers integrated with piezoelectric layers in hygrothermal environment", *Adv. Nano Res., Int. J.*, **6**(4), 299-321.  
<https://doi.org/10.12989/anr.2018.6.4.299>
- Han, Y. and Elliott, J. (2007), "Molecular dynamics simulations of the elastic properties of polymer/carbon nanotube composites", *Comput. Mater. Sci.*, **39**, 315-323.  
<https://doi.org/10.1016/j.commatsci.2006.06.011>
- Hellal, H., Bourada, M., Hebali, H., Bourada, F., Tounsi, A., Bousahla, A.A. and Mahmoud, S.R. (2019), "Dynamic and stability analysis of functionally graded material sandwich plates in hygro-thermal environment using a simple higher shear deformation theory", *J. Sandw. Struct. Mater.*  
<https://doi.org/10.1177/1099636219845841>
- Hwang, J.T. and Steeves, C.A. (2015), "Optimization of 3D lattice cores in composite sandwich structures", *J. Compos. Mater.*, **49**(17), 2041-2055. <https://doi.org/10.1177/0021998314541537>
- Iijima, S. (1991), "Helical microtubules of graphitic carbon", *Nature*, **354**, 56-58. <https://doi.org/10.1038/354056a0>
- Kaci, A., Houari, M.S.A., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2018), "Post-buckling analysis of shear-deformable composite beams using a novel simple two-unknown beam theory", *Struct. Eng. Mech., Int. J.*, **65**(5), 621-631.  
<https://doi.org/10.12989/sem.2018.65.5.621>
- Kadari, B., Bessaim, A., Tounsi, A., Heireche, H., Bousahla, A.A. and Houari, M.S.A. (2018), "Buckling analysis of orthotropic nanoscale plates resting on elastic foundations", *J. Nano Res.*, **55**, 42-56.  
<https://doi.org/10.4028/www.scientific.net/JNanoR.55.42>
- Kar, V.R. and Panda, S.K. (2016), "Post-buckling behaviour of shear deformable functionally graded curved shell panel under edge compression", *Int. J. Mech. Sci.*, **115**, 318-324.  
<https://doi.org/10.1016/j.ijmecsci.2016.07.014>
- Kar, V.R. and Panda, S.K. (2017), "Postbuckling analysis of shear deformable FG shallow spherical shell panel under nonuniform thermal environment", *J. Thermal Stress.*, **40**(1), 25-39.  
<https://doi.org/10.1080/01495739.2016.1207118>
- Kar, V.R., Panda, S.K. and Mahapatra, T.R. (2016), "Thermal buckling behaviour of shear deformable functionally graded single/doubly curved shell panel with TD and TID properties", *Adv. Mater. Res., Int. J.*, **5**(4), 205-221.  
<https://doi.org/10.12989/amr.2016.5.4.205>
- Kar, V.R., Mahapatra, T.R. and Panda, S.K. (2017), "Effect of different temperature load on thermal postbuckling behaviour of functionally graded shallow curved shell panels", *Compos. Struct.*, **160**, 1236-1247.  
<https://doi.org/10.1016/j.compstruct.2016.10.125>
- Karami, B., Janghorban, M. and Tounsi, A. (2017), "Effects of triaxial magnetic field on the anisotropic nanoplates", *Steel*

- Compos. Struct., Int. J.*, **25**(3), 361-374.  
<https://doi.org/10.12989/scs.2017.25.3.361>
- Karami, B., Shahsavari, D., Li, L., Karami, M. and Janghorban, M. (2018a), "Thermal buckling of embedded sandwich piezoelectric nanoplates with functionally graded core by a nonlocal second-order shear deformation theory", *Proceedings of the Institution of Mechanical Engineers, Part C: J. Mech. Eng. Sci.*, **233**(1), 287-301. <https://doi.org/10.1177/0954406218756451>
- Karami, B., Shahsavari, D. and Janghorban, M. (2018b), "Wave propagation analysis in functionally graded (FG) nanoplates under in-plane magnetic field based on nonlocal strain gradient theory and four variable refined plate theory", *Mech. Adv. Mater. Struct.*, **25**(12), 1047-1057.  
<https://doi.org/10.1080/15376494.2017.1323143>
- Karami, B., Janghorban, M. and Tounsi, A. (2018c), "Variational approach for wave dispersion in anisotropic doubly-curved nanoshells based on a new nonlocal strain gradient higher order shell theory", *Thin-Wall. Struct.*, **129**, 251-264.  
<https://doi.org/10.1016/j.tws.2018.02.025>
- Karami, B., Janghorban, M., Shahsavari, D. and Tounsi, A. (2018d), "A size-dependent quasi-3D model for wave dispersion analysis of FG nanoplates", *Steel Compos. Struct., Int. J.*, **28**(1), 99-110. <https://doi.org/10.12989/scs.2018.28.1.099>
- Karami, B., Janghorban, M. and Tounsi, A. (2018e), "Nonlocal strain gradient 3D elasticity theory for anisotropic spherical nanoparticles", *Steel Compos. Struct., Int. J.*, **27**(2), 201-216.  
<https://doi.org/10.12989/scs.2018.27.2.201>
- Karami, B., Shahsavari, D., Janghorban, M. and Tounsi, A. (2019a), "Resonance behavior of functionally graded polymer composite nanoplates reinforced with graphene nanoplatelets", *Int. J. Mech. Sci.*, **156**, 94-105.  
<https://doi.org/10.1016/j.ijmecsci.2019.03.036>
- Karami, B., Janghorban, M. and Tounsi, A. (2019b), "On exact wave propagation analysis of triclinic material using three dimensional bi-Helmholtz gradient plate model", *Struct. Eng. Mech., Int. J.*, **69**(5), 487-497.  
<https://doi.org/10.12989/sem.2019.69.5.487>
- Karami, B., Janghorban, M. and Tounsi, A. (2019c), "Wave propagation of functionally graded anisotropic nanoplates resting on Winkler-Pasternak foundation", *Struct. Eng. Mech., Int. J.*, **7**(1), 55-66. <https://doi.org/10.12989/sem.2019.70.1.055>
- Karami, B., Janghorban, M. and Tounsi, A. (2019d), "Galerkin's approach for buckling analysis of functionally graded anisotropic nanoplates/different boundary conditions", *Eng. Comput.*, **35**, 1297-1316. <https://doi.org/10.1007/s00366-018-0664-9>
- Katariya, P.V. and Panda, S.K. (2014), "Thermo-Mechanical Stability Analysis of Composite Cylindrical Panels", *Proceedings of ASME 2013 Gas Turbine India Conference*, American Society of Mechanical Engineers Digital Collection.
- Katariya, P.V. and Panda, S.K. (2016), "Thermal buckling and vibration analysis of laminated composite curved shell panel", *Aircraft Eng. Aerosp. Technol.: Int. J.*, **88**(1), 97-107.  
<https://doi.org/10.1108/AEAT-11-2013-0202>
- Katariya, P.V., Panda, S.K., Hirwani, C.K., Mehar, K. and Thakare, O. (2017a), "Enhancement of thermal buckling strength of laminated sandwich composite panel structure embedded with shape memory alloy fibre", *Smart Struct. Syst., Int. J.*, **20**(5), 595-605. <https://doi.org/10.12989/ss.2017.20.5.595>
- Katariya, P.V., Panda, S.K. and Mahapatra, T.R. (2017b), "Nonlinear thermal buckling behaviour of laminated composite panel structure including the stretching effect and higher-order finite element", *Adv. Mater. Res., Int. J.*, **6**(4), 349-361.  
<https://doi.org/10.12989/amr.2017.6.4.349>
- Katariya, P.V., Das, A. and Panda, S.K. (2018), "Buckling analysis of SMA bonded sandwich structure—using FEM", *Proceedings of IOP Conference Series: Materials Science and Engineering* (Vol. 338, No. 1, p. 012035), IOP Publishing.
- Katnam, K.B., Da Silva, L.F.M. and Young, T.M. (2013), "Bonded repair of composite aircraft structures: A review of scientific challenges and opportunities", *Prog. Aerosp. Sci.*, **61**, 26-42.  
<https://doi.org/10.1016/j.paerosci.2013.03.003>
- Khiloun, M., Bousahla, A.A., Kaci, A., Bessaim, A., Tounsi, A. and Mahmoud, S.R. (2019), "Analytical modeling of bending and vibration of thick advanced composite plates using a four-variable quasi 3D HSDT", *Eng. Comput.*  
<https://doi.org/10.1007/s00366-019-00732-1>
- Kolahchi, R. (2017), "A comparative study on the bending, vibration and buckling of viscoelastic sandwich nanoplates based on different nonlocal theories using DC, HDQ and DQ methods", *Aerosp. Sci. Technol.*, **66**, 235-248  
<https://doi.org/10.1016/j.ast.2017.03.016>
- Kolahchi, R., Keshtegar, B. and Fakhar, M.H. (2017), "Optimization of dynamic buckling for sandwich nanocomposite plates with sensor and actuator layer based on sinusoidal-viscopiezoelectricity theories using grey wolf algorithm", *J. Sandw. Struct. Mater.*, **22**(1), 3-27.  
<https://doi.org/10.1177/1099636217731071>
- Lei, Z.X., Liew, K.M. and Yu, J.L. (2013), "Buckling analysis of functionally graded carbon nanotube reinforced composite plates using the element-free kp-Ritz method", *Compos. Struct.*, **98**, 160-168. <https://doi.org/10.1016/j.compstruct.2012.11.006>
- Li, X., Gao, H., Scrivens, W.A., Fei, D., Xu, X., Sutton, M.A., Reynolds, A.P. and Myrick, M.L. (2007), "Reinforcing mechanisms of single-walled carbon nanotube-reinforced polymer composites", *J. Nanosci. Nanotech.*, **7**(7), 2309-2317.  
<https://doi.org/10.1166/jnn.2007.410>
- Ma, R.Z., Wu, J., Wei, B.Q., Liang, J. and Wu, D.H. (1998), "Processing and properties of carbon nanotubes-nano SiC ceramics", *J. Mater. Sci.*, **33**, 5243-5246.  
<https://doi.org/10.1023/A:1004492106337>
- Mahi, A., Adda Bedia, E.A. and Tounsi, A. (2015), "A new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates", *Appl. Math. Model.*, **39**(9), 2489-2508. <https://doi.org/10.1016/j.apm.2014.10.045>
- Mahmoudi, A., Benyoucef, S., Tounsi, A., Benachour, A., Adda Bedia, E.A. and Mahmoud, S.R. (2019), "A refined quasi-3D shear deformation theory for thermo-mechanical behavior of functionally graded sandwich plates on elastic foundations", *J. Sandw. Struct. Mater.*, **21**(6), 1906-1929.  
<https://doi.org/10.1177/1099636217727577>
- Medani, M., Benahmed, A., Zidour, M., Heireche, H., Tounsi, A., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2019), "Static and dynamic behavior of (FG-CNT) reinforced porous sandwich plate", *Steel Compos. Struct., Int. J.*, **32**(5), 595-610.  
<https://doi.org/10.12989/scs.2019.32.5.595>
- Mehar, K. and Panda, S.K. (2017), "Thermoelastic analysis of FG-CNT reinforced shear deformable composite plate under various loading", *Int. J. Computat. Methods*, **14**(2), 1750019.  
<https://doi.org/10.1142/S0219876217500190>
- Mehar, K. and Panda, S.K. (2019), "Multiscale modeling approach for thermal buckling analysis of nanocomposite curved structure", *Adv. Nano Res., Int. J.*, **7**(3), 181-190.  
<https://doi.org/10.12989/anr.2019.7.3.181>
- Mehar, K., Panda, S.K. and Patle, B.K. (2017), "Thermoelastic vibration and flexural behavior of FG-CNT reinforced composite curved panel", *Int. J. Appl. Mech.*, **9**(4), 1750046.  
<https://doi.org/10.1142/S1758825117500466>
- Mehar, K., Panda, S.K., Devarajan, Y. and Choubey, G. (2019), "Numerical buckling analysis of graded CNT-reinforced composite sandwich shell structure under thermal loading", *Compos. Struct.*, **216**, 406-414.  
<https://doi.org/10.1016/j.compstruct.2019.03.002>
- Meksi, R., Benyoucef, S., Mahmoudi, A., Tounsi, A., Adda Bedia,

- E.A. and Mahmoud, S.R. (2019), "An analytical solution for bending, buckling and vibration responses of FGM sandwich plates", *J. Sandw. Struct. Mater.*, **21**(2), 727-757. <https://doi.org/10.1177/1099636217698443>
- Menasria, A., Bouhadra, A., Tounsi, A., Bousahla, A.A. and Mahmoud, S.R. (2017), "A new and simple HSDT for thermal stability analysis of FG sandwich plates", *Steel Compos. Struct.*, **Int. J.**, **25**(2), 157-175. <https://doi.org/10.12989/scs.2017.25.2.157>
- Meziane, M.A.A., Abdelaziz, H.H. and Tounsi, A. (2014), "An efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions", *J. Sandw. Struct. Mater.*, **16**(3), 293-318. <https://doi.org/10.1177/1099636214526852>
- Mirzaei, M. and Kiani, Y. (2016), "Thermal buckling of temperature dependent FG-CNT reinforced composite plates", *Meccanica*, **51**(9), 2185-2201. <https://doi.org/10.1007/s11012-015-0348-0>
- Mokhtar, Y., Heireche, H., Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2018), "A novel shear deformation theory for buckling analysis of single layer graphene sheet based on nonlocal elasticity theory", *Smart Struct. Syst.*, **Int. J.**, **21**(4), 397-405. <https://doi.org/10.12989/sss.2018.21.4.397>
- Nethercot, D.A. and Kirby, P.A. (1979), *Design for Structural Stability*, John Wiley and Sons, New York.
- Nethercot, D.A. and Trahair, N.S. (1976), "Lateral buckling approximations for elastic beams", *Struct. Engr.*, **54**, 197-204.
- Odegard, G.M., Gates, T.S., Wise, K.E., Park, C. and Siochi, E.J. (2003), "Constitutive modelling of nanotube-reinforced polymer composites", *Compos. Sci. Technol.*, **63**, 1671-1687. [https://doi.org/10.1016/S0266-3538\(03\)00063-0](https://doi.org/10.1016/S0266-3538(03)00063-0)
- Ould Youcef, D., Kaci, A., Houari, M.S.A., Tounsi, A., Benzair, A. and Heireche, H. (2015), "On the bending and stability of nanowire using various HSDTs", *Adv. Nano Res.*, **Int. J.**, **3**(4), 177-191. <https://doi.org/10.12989/anr.2015.3.4.177>
- Panda, S.K. and Katariya, P.V. (2015), "Stability and free vibration behaviour of laminated composite panels under thermo-mechanical loading", *Int. J. Appl. Comput. Mathe.*, **1**(3), 475-490. <https://doi.org/10.1007/s40819-015-0035-9>
- Panda, S.K. and Singh, B.N. (2009), "Thermal post-buckling behaviour of laminated composite cylindrical/hyperboloid shallow shell panel using nonlinear finite element method", *Compos. Struct.*, **91**(3), 366-374. <https://doi.org/10.1016/j.compstruct.2009.06.004>
- Panda, S.K. and Singh, B.N. (2010), "Thermal post-buckling analysis of a laminated composite spherical shell panel embedded with shape memory alloy fibres using non-linear finite element method", *Proceedings of the Institution of Mechanical Engineers, Part C: J. Mech. Eng. Sci.*, **224**(4), 757-769.
- Panda, S.K. and Singh, B.N. (2013a), "Post-buckling analysis of laminated composite doubly curved panel embedded with SMA fibers subjected to thermal environment", *Mech. Adv. Mater. Struct.*, **20**(10), 842-853. <https://doi.org/10.1080/15376494.2012.677097>
- Panda, S.K. and Singh, B.N. (2013b), "Thermal Postbuckling Behavior of Laminated Composite Spherical Shell Panel Using NFEM#", *Mech. Based Des. Struct. Mach.*, **41**(4), 468-488. <https://doi.org/10.1080/15397734.2013.797330>
- Panda, S.K., Mahapatra, T.R. and Kar, V.R. (2017), Nonlinear Finite Element Solution of Post-buckling Responses of FGM Panel Structure under Elevated Thermal Load and TD and TID Properties.
- Pour, H.R., Vossough, H., Heydari, M.M., Beygipoor, G. and Azimzadeh, A. (2015), "Nonlinear vibration analysis of a nonlocal sinusoidal shear deformation carbon nanotube using differential quadrature method", *Struct. Eng. Mech.*, **Int. J.**, **54**(6), 1061-1073. <https://doi.org/10.12989/sem.2015.54.6.1061>
- Reddy, J.N. (2004), *Mechanics of Laminated Composite Plates and Shells: Theory and Analysis*, (2nd Edition), Taylor & Francis eBooks, CRC Press.
- Sahmani, S., Aghdam, M.M. and Rabczuk, T. (2018), "Nonlinear bending of functionally graded porous micro/nano-beams reinforced with graphene platelets based upon nonlocal strain gradient theory", *Compos. Struct.*, **186**, 68-78. <https://doi.org/10.1016/j.compstruct.2017.11.082>
- Seidel, G.D. and Lagoudas, D.C. (2006), "Micromechanical analysis of the effective elastic properties of carbon nanotube reinforced composites", *Mech. Mater.*, **38**(8-10), 884-907. <https://doi.org/10.1016/j.mechmat.2005.06.029>
- Sekkal, M., Fahsi, B., Tounsi, A. and Mahmoud, S.R. (2017), "A new quasi-3D HSDT for buckling and vibration of FG plate", *Struct. Eng. Mech.*, **Int. J.**, **64**(6), 737-749. <https://doi.org/10.12989/sem.2017.64.6.737>
- Semmah, A., Heireche, H., Bousahla, A.A. and Tounsi, A. (2019), "Thermal buckling analysis of SWBNNT on Winkler foundation by non local FSDT", *Adv. Nano Res.*, **Int. J.**, **7**(2), 89-98. <https://doi.org/10.12989/anr.2019.7.2.089>
- Shen, H.S. (2009), "Nonlinear bending of functionally graded carbon nanotube reinforced composite plates in thermal environments", *Compos. Struct.*, **91**, 9-19. <https://doi.org/10.1016/j.compstruct.2009.04.026>
- Shahsavari, D. and Janghorban, M. (2017), "Bending and shearing responses for dynamic analysis of single-layer graphene sheets under moving load", *J. Brazil. Soc. Mech. Sci. Eng.*, **39**(10), 3849-3861. <https://doi.org/10.1007/s40430-017-0863-0>
- Shokravi, M. (2017a), "Buckling of sandwich plates with FG-CNT reinforced layers resting on orthotropic elastic medium using Reddy plate theory", *Steel Compos. Struct.*, **Int. J.**, **23**(6), 623-631. <https://doi.org/10.12989/scs.2017.23.6.623>
- Shokravi, M. (2017b), "Buckling analysis of embedded laminated plates with agglomerated CNT-reinforced composite layers using FSDT and DQM", *Geomech. Eng.*, **Int. J.**, **12**(2), 327-346.
- Sobhy, M. (2015), "A comprehensive study on FGM nanoplates embedded in an elastic medium", *Compos. Struct.*, **134**, 966-980. <https://doi.org/10.1016/j.compstruct.2015.08.102>
- Sofiyev, A.H. and Avcar, M. (2010), "The stability of cylindrical shells containing an FGM layer subjected to axial load on the Pasternak foundation", *Engineering*, **2**(4), 228-236. <https://doi.org/10.4236/eng.2010.24033>
- Thess, A., Lee, R., Nikolaev, P., Dai, H., Petit, P., Robert, J., Xu, C., Lee, Y.H., Kim, S.G., Rinzler, A.G. and Colbert, D.T. (1996), "Crystalline ropes of metallic carbon nanotubes", *Science*, **273**(5274), 483-487. <https://doi.org/10.1126/science.273.5274.483>
- Tlidji, Y., Zidour, M., Draiche, K., Safa, A., Bourada, M., Tounsi, A., Bousahla, A.A. and Mahmoud, S.R. (2019), "Vibration analysis of different material distributions of functionally graded microbeam", *Struct. Eng. Mech.*, **Int. J.**, **69**(6), 637-649. <https://doi.org/10.12989/sem.2019.69.6.637>
- Tounsi, A., Houari, M.S.A. and Bessaim, A. (2016), "A new 3-unknowns non-polynomial plate theory for buckling and vibration of functionally graded sandwich plate", *Struct. Eng. Mech.*, **Int. J.**, **60**(4), 547-565. <https://doi.org/10.12989/sem.2016.60.4.547>
- Van Lier, G., Van Alsenoy, C., Van Doren, V. and Geerlings, P. (2000), "Ab initio study of the elastic properties of single-walled carbon nanotubes and graphene", *Chem. Phys. Lett.*, **326**(1-2), 181-185. [https://doi.org/10.1016/S0009-2614\(00\)00764-8](https://doi.org/10.1016/S0009-2614(00)00764-8)
- Vodenitcharova, T. and Zhang, L.C. (2006), "Bending and local buckling of a nanocomposite beam reinforced by a single-walled carbon nanotube", *Int. J. Solids Struct.*, **43**, 3006-3024. <https://doi.org/10.1016/j.ijsolstr.2005.05.014>
- Wan, H., Delale, F. and Shen, L. (2005), "Effect of CNT length and CNT-matrix inter phase in carbon nanotube (CNT)

- reinforced composites”, *Mech. Res. Commun.*, **32**, 481-489.  
<https://doi.org/10.1016/j.mechrescom.2004.10.011>
- Wang, Z.X. and Shen, H.S. (2011), “Nonlinear vibration of nanotube-reinforced composite plates in thermal environments”, *Comput. Mater. Sci.*, **50**, 2319-2330.  
<https://doi.org/10.1016/j.commatsci.2011.03.005>
- Wattanasakulpong, N. and Chaikittiratana, A. (2015), “Exact solutions for static and dynamic analyses of carbon nanotube-reinforced composite plates with Pasternak elastic foundation”, *Appl. Mathe. Model.*, **39**(18), 5459-5472.  
<https://doi.org/10.1016/j.apm.2014.12.058>
- Wu, H., Kitipornchai, S. and Yang, J. (2016), “Thermo-electro-mechanical postbuckling of piezoelectric FG-CNTRC beams with geometric imperfections”, *Smart Mater. Struct.*, **25**(9), 095022. <https://doi.org/10.1088/0964-1726/25/9/095022>
- Yazid, M., Heireche, H., Tounsi, A., Bousahla, A.A. and Houari, M.S.A. (2018), “A novel nonlocal refined plate theory for stability response of orthotropic single-layer graphene sheet resting on elastic medium”, *Smart Struct. Syst., Int. J.*, **21**(1), 15-25. <https://doi.org/10.12989/sss.2018.21.1.015>
- Youcef, D.O., Kaci, A., Benzair, A., Bousahla, A.A. and Tounsi, A. (2018), “Dynamic analysis of nanoscale beams including surface stress effects”, *Smart Struct. Syst., Int. J.*, **21**(1), 65-74.  
<https://doi.org/10.12989/sss.2018.21.1.065>
- Younsi, A., Tounsi, A., Zaoui, F.Z., Bousahla, A.A. and Mahmoud, S.R. (2018), “Novel quasi-3D and 2D shear deformation theories for bending and free vibration analysis of FGM plates”, *Geomech. Eng., Int. J.*, **14**(6), 519-532.  
<https://doi.org/10.12989/gae.2018.14.6.519>
- Yu, M.F., Lourie, O., Dyer, M.J., Moloni, K., Kelly, T.F. and Ruoff, R.S. (2000), “Strength and breaking mechanism of multiwalled carbon nanotubes under tensile load”, *Science*, **287**(5453), 637-640. <https://doi.org/10.1126/science.287.5453.637>
- Zaoui, F.Z., Ouinas, D. and Tounsi, A. (2019), “New 2D and quasi-3D shear deformation theories for free vibration of functionally graded plates on elastic foundations”, *Compos. Part B*, **159**, 231-247. <https://doi.org/10.1016/j.compositesb.2018.09.051>
- Zarga, D., Tounsi, A., Bousahla, A.A., Bourada, F. and Mahmoud, S.R. (2019), “Thermomechanical bending study for functionally graded sandwich plates using a simple quasi-3D shear deformation theory”, *Steel Compos. Struct., Int. J.*, **32**(3), 389-410. <https://doi.org/10.12989/scs.2019.32.3.389>
- Zhang, L.W., Lei, Z.X. and Liew, K.M. (2015), “Buckling analysis of FG-CNT reinforced composite thick skew plates using an element-free approach”, *Compos. Part B: Eng.*, **75**, 36-46.  
<https://doi.org/10.1016/j.compositesb.2015.01.033>
- Zhu, P., Lei, Z.X. and Liew, K.M. (2012), “Static and free vibration analyses of carbon nanotube reinforced composite plates using finite element method with first order shear deformation plate theory”, *Compos. Struct.*, **94**, 1450-1460.  
<https://doi.org/10.1016/j.compstruct.2011.11.010>
- Zine, A., Tounsi, A., Draiche, K., Sekkal, M. and Mahmoud, S.R. (2018), “A novel higher-order shear deformation theory for bending and free vibration analysis of isotropic and multilayered plates and shells”, *Steel Compos. Struct., Int. J.*, **26**(2), 125-137.  
<https://doi.org/10.12989/scs.2018.26.2.125>