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Porosity-dependent free vibration analysis of FG nanobeam using non-local shear deformation and energy principle

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Abstract. This work focuses on the behavior of non-local shear deformation beam theory for the vibration of functionally graded (FG) nanobeams with porosities that may occur inside the functionally graded materials (FG) during their fabrication, using the non-local differential constitutive relations of Eringen. For this purpose, the developed theory accounts for the higher-order variation of transverse shear strain through the depth of the nanobeam. The material properties of the FG nanobeam are assumed to vary in the thickness direction. The equations of motion are derived from Hamilton's principle. Analytical solutions are presented for a simply supported FG nanobeam with porosities. The validity of this theory is verified by comparing some of the present results with other higher-order theories reported in the literature, the influence of material parameters, the volume fraction of porosity and the thickness ratio on the behavior mechanical P-FGM beam are represented by numerical examples.

Keywords: nanobeam; non-local elasticity theory; vibration; functionally graded; porosity coefficient

1. Introduction

Functionally graded materials (FGMs) are a new type of composite materials formed of two or more phases in which both composition and structure gradually change over gradient directions smoothly and continuously. Therefore, by changing the properties of the material it is possible to perform a certain function of material properties of mechanical strength and thermal conductivity (Kar and Panda 2015). They have great practical importance because of their vast applications in many industrial and engineering fields (Ait Yahia *et al*. 2015, Karami *et al*. 2018a, 2019a, Bennai *et al*. 2019, Bouamoud *et al*. 2019, Mahmoudi *et al*. 2019, Khiloun *et al*. 2019, Boulefrakh *et al*. 2019, Batou *et al*. 2019, Addou *et al*. 2019, Chaabane *et al*. 2019, Fellah *et al*. 2019, Salah *et al*. 2019, Fourn *et al*. 2018, Meksi *et al*. 2019, Kargani *et al*. 2013). In recent years, Karami *et al*. (2018f) used the Galerkin's approach for buckling analysis of functionally graded anisotropic nanoplates with different boundary conditions. Aydogdu *et al*. (2018) analyzed the vibration of axially functionally graded nanorods and beams with a variable nonlocal parameter. Karami *et al*. (2018e) studied the functionally graded carbon nanotube-reinforced composite plates. Avcar (2019) analyzed the free vibration of imperfect sigmoid and power-law functionally graded beams. In another hand, a variety of theoretical study analyzed a new quasi-3D shear deformation theory for functionally graded plates and beams (Achouri *et al*. 2019, Boukhlif *et al*. 2019, Boutaleb *et al*. 2019, Karami *et al*. 2018h, j, Zaoui *et al*. 2019, Zarga *et al*. 2019).

The application of FG materials has broadly been spread in nano-structures such as nano-electromechanical systems (NEMS), thin films in the form of shape memory alloys, and atomic force microscopes (AFMs) to achieve high sensitivity and desired performance. With the rapid development of technology, functionally graded (FG) beams and plates have been started to use in micro/nanoelectromechanical systems, such as the components in the form of shape memory alloy thin films with a global thickness in micro or Nano-scale (Fu *et al*. 2003, Witvrouw and Mehta 2005, Tlidji *et al*. 2019, Bouazza *et al*. 2015, Lü *et al*. 2009, Eltaher *et al*. 2013, Ehyaei *et al*. 2016), electrically actuated MEMS devices (Hasanyan *et al*. 2008, Zhang and Fu 2012), and atomic force microscopes (AFMs) (Rahaeifard *et al*. 2009). Karami and Shahsavari (2019) used a strain gradient model for the thermal stability of FG nanoplates integrated with piezoelectric layers. Akgöz and Civalek (2014) used shear deformation beam models for functionally graded microbeams with new shear correction factors. Karami *et al*. (2019b) studied the functionally graded polymer composite nanoplates reinforced with graphene nanoplatelets. Ebrahimi and Barati (2017) analyzed the vibration of embedded size dependent FG nanobeams based on third-order shear deformation beam theory. Karami *et al*. (2019c) analyzed the thermal buckling of embedded

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sandwich piezoelectric nanoplates with functionally graded core. Jandaghian and Rahmani (2017) used the third-order shear deformation theory under various boundary conditions for vibration analysis of FG nanobeams.

Since the dimension of these structural devices typically falls below micron- or nano-scale in at least one direction, an essential feature triggered in these devices is that their mechanical properties such as Young's modulus, flexural rigidity, and so on are size-dependent. So far, only a few works have been reported for FG nano and micro-structure based on the non-local elasticity theory (Akbas 2018, Chemi *et al*. 2015, Zidour *et al*. 2015, Rakrak *et al*. 2016, Belmahi *et al*. 2018, Dihaj *et al*. 2018, Bensaid 2017, Shahsavari *et al*. 2017, Bensaid *et al*. 2018). Shahsavari *et al*. (2018a) used a new size-dependent quasi-3D shear deformation theory for the shear buckling of porous nanoplates. (Pisano *et al*. 2009) exploited the non-local finite element method for analyzing homogeneous and nonhomogeneous non-local elastic 2D problems. (Janghorban and Zare 2011) investigated non-local free vibration axially FG nanobeams by using differential quadrature method. Karami and Karami (2019) analyzed the buckling of nanoplate. (Eltaher *et al*. 2012) studied free vibration of FG nanobeam based on the non-local Euler-Bernoulli beam theory. Recently, (Larbi Chaht *et al*. 2015) studied the bending and buckling of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect. (Kolahchi *et al*. 2016) analyzed the stability of embedded piezoelectric nanoplates based on visco-non-local-piezoelasticity theories. (Kolahchi *et al*. 2017) studied the buckling of laminated nanoplates using differential cubature-Bolotin methods. (Avcar 2015) studied the effects of rotary inertia shear deformation and non-homogeneity on frequencies. Reddy (2011) developed non-local models for bending, free vibration and buckling of the functionally graded beam according to Euler–Bernoulli and Timoshenko beam theories based on the modified couple stress theory. Algebraic relationships between the bending solutions of Timoshenko beam theory (TBT) and homogeneous Bernoulli–Euler beams for microstructure dependent FGM beams has developed by Reddy and Arbind (2012). a variety of theoretical study analyzed a nano and micro-structure (Balubaid *et al*. 2019, Hussain *et al*. 2019, Bouadi *et al*. 2018, Alimirzaei *et al*. 2019, Cherif *et al*. 2018, Kadari *et al*. 2018, Karami *et al*. 2018i, Semmah *et al*. 2019, Zemri *et al*. 2015, Chemi *et al*. 2018, Hamidi *et al*. 2018, Bensattalah *et al*. 2018a, Hamza-Cherif *et al*. 2018, Bensattalah *et al*. 2018b, Belmahi *et al*. 2019, Bensattalah *et al*. 2018c, 2019).

The material characteristics of the FG beam change across the thickness direction, the neutral surface of such beam may not coincide with its geometric middle surface. Thus, stretching and bending deformations of FG beam are coupled. Some works (Dai *et al*. 2005, Ebrahimi and Barati 2018) have shown that there is no stretching-bending coupling in constitutive equations if the reference surface is properly selected. Şimşek and Reddy (2013) presented a unified higher-order beam theory for an FGM micro-beam embedded in elastic Pasternak medium. Also, in FGM fabrication, micro-voids or porosities can occur within the materials during the process of sintering. This is because of the large difference in solidification temperatures between material constituents (Zhu *et al*. 2001, Karami *et al*. 2018c). It is important to take into account the porosity effect when designing FGM structures subjected to dynamic loadings.

In the same way, (Ait Yahia *et al*. 2015) investigated the wave propagation of an infinite FG plate having porosities by using various simple higher-order shear deformation theories. Karami *et al*. (2018d) analyzed the thermal buckling of smart porous functionally graded nanobeam rested on Kerr foundation. In another hand, a variety of theoretical study analyzed variety structures with porosity (Karami *et al*. 2018b, e, g, Ait Atmane *et al*. 2017, Bourada *et al*. 2019, Benahmed *et al*. 2019, Guessas *et al*. 2018, Ehyaei *et al*. 2017, Li *et al*. 2018a). Karami and Janghorban (2019) studied the dynamics of porous nanotubes with variable material properties and variable thickness. Shahsavari *et al*. (2018b) used a high-order gradient model for wave propagation analysis of porous FG nanoplates. Karami *et al*. (2019d) analyzed the wave propagation of porous nanoshells.

The continuum mechanics methods are widely used to predict the responses of various structures and problems such as, nano beam and nano plates (Bouazza *et al*. 2015). buckling analyses (Belkacem *et al*. 2018), static and free vibration (Avcar 2016), Bending analysis (Draoui *et al*. 2019), thermal stability and Hygrothermo-mechanical (Bensattalah *et al*. 2016). A variety of theoretical study used the continuum mechanics methods (Djedid *et al*. 2019, Abdelmalek *et al*. 2019, Guenaneche *et al*. 2019, Belbachir *et al*. 2019, Medani *et al*. 2019, Draiche *et al*. 2019, Sahla *et al*. 2019, Abualnour *et al*. 2019, Karami *et al*. 2019d). In structural analysis of nano beam, two models are usually employed, namely Euler–Bernoulli and Timoshenko beam models. Both models assume that plane sections remain plane. But in Euler beam model, the sections remain perpendicular to the neutral axis whereas this assumption is removed in Timoshenko beam model to account for the effect of shear and rotary effect, but at the same time these shear stresses are constant across the thickness, which does not represent reality. For this, the researchers found corrective coefficients to solve this problem. Unlike classical theory and Timoshenko's theory of acceptances of the linear distribution of displacement across thickness, the higher order theory is based on a nonlinear distribution of fields in thickness. Therefore, the effects of transverse shear deformation and / or transverse normal deformation are taken into account. These models do not require correction factors. Several authors offer some important contributions to the development of high order models that have distinguished themselves in the literature by the expression of the shear function. These models are based on a non-linear distribution.

In this paper, we make the first attempt to investigate the free vibration problems of functionally graded (FG) nanobeams with porosities based on the higher-order transverse shear deformation model and Eringen's non-local theory. The equations of motion are derived from Hamilton's principle. Analytical solutions are presented for a simply supported FG nanobeam including porosities. To illustrate the accuracy of the present theory, the obtained results are compared with those predicted by other higher-order theories reported in the literature. Finally, the influences of non-local parameter, power-law index, and aspect ratio on the vibration responses of FG nanobeam with porosities are discussed.

2. Theoretical formulations

Consider a uniform FG beam with porosities of length L, width b, andtotal depth h, according to Fig. 1 Cartesian coordinate system O (x, y, z) is located on the central axis of the beam, as x-axis is matched with neutral axis of the beam in the undeflected position, the y-axis in the width direction, and the z-axis in the depth direction.

2.1 Functionally graded materials

It is assumed that material properties of the FG nanobeam, such as Young's modulus (E), shear modulus (G), mass density (ρ) and thermal expansions (α) are assumed to vary continuously in the depth direction according to power-law. z is the distance from the mid-plane of the FGM beam and k is the non-negative variable parameter (power-law exponent) which determine the material distribution through the thickness of the beam. For the even distribution of porosities (FGM-I), the effective material properties are obtained as Wattanasakulpong and Ungbhakorn (2014).

$$P(z) = (P_t - P_b)\left(\frac{z}{h} + \frac{1}{2}\right)^k + P_b - (P_t - P_b)\frac{\alpha}{2} \qquad (1)$$

Where P_t and P_b are the corresponding material property at the top and bottom surfaces of the nanobeam, respectively. k is a non-negative number that dictates the material variation profile through the thickness of the nanobeam. and α is the volume fraction of porosities.

2.2 Basic assumptions

The displacement field of the proposed theory is chosen based on the following assumptions:

(1) The displacements are small in comparison with the FG nanobeam thickness and, therefore, strains involved are infinitesimal.
(2) The transverse displacement w includes two components of bending w_b and shear w_s. These components are functions of coordinate x only.

$$w(x,z) = w_b(x) + w_s(x) \qquad (2)$$

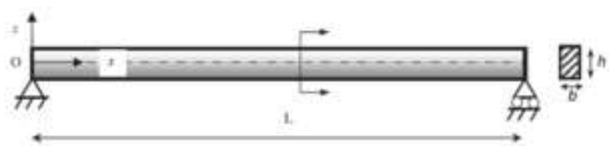

Fig. 1 Gradation of material properties through the thickness of the FG beam

(3) The displacements u in x-direction consist of extension, bending, and shear components.

$$u = u_0 + u_b + u_s \qquad (3)$$

The bending component u_b is assumed to be similar to the displacement given by the classical beam theory. Therefore, the expression for $u_b$can be given as

$$u_b = -z\frac{\partial w_b}{\partial x} \qquad (4)$$

The shear component u_s is the variations of shear strain γ_{xz}through the thickness of the beam in such a way that shear stress $\tau_{xz}$is zero at the top and bottom faces of the beam. Consequently, the expression for u_s can be given as

$$u_s = -f(z)\frac{\partial w_s}{\partial x} \qquad (5)$$

Where

$$f(z) = \left(z - \frac{h}{\pi}sin\frac{\pi z}{h}\right) \qquad (6)$$

2.3 Kinematics and constitutive relations

Based on the assumptions made in the preceding section, the displacement field can be obtained using Eqs. (2)-(5) as

$$u(x,z,t) = u_0(x,t) - z\frac{\partial w_b}{\partial x} + f(z)\frac{\partial w_s}{\partial x} \qquad (7a)$$

$$w(x,z,t) = w_b(x,t) + w_s(x,t) \qquad (7b)$$

The strains associated with the displacements in Eqs. (7a)-(7b)

$$\varepsilon_x = \varepsilon_x^0 + zk_x^b + f(z)k_x^s, \qquad \gamma_{xz} = g(z)\gamma_{xz}^s \qquad (8)$$

Where

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x}, \quad k_x^b = -\frac{\partial^2 w_b}{\partial x^2}, \quad k_x^s = -\frac{\partial^2 w_s}{\partial x^2} \qquad (9a)$$

$$\gamma_{xz}^s = \frac{\partial w_s}{\partial x}, \quad f = \left(z - \frac{h}{\pi}sin\frac{\pi z}{h}\right), \quad g = cos\left(\frac{\pi z}{h}\right) \qquad (9b)$$

2.4 Constitutive relations

The response of materials at the nanoscale is different from those of their bulk counterparts. Non-local elasticity is first considered by Eringen (1972, 1983). He assumed that the stress at a reference point is a function of the strain field at every point of the continuum. Eringen (1972, 1983) proposed a differential form of the non-local constitutive relation as

$$\sigma_x - \mu\frac{d^2\sigma_x}{dx^2} = E\varepsilon_x \qquad (10a)$$

$$\tau_{xz} - \mu\frac{d^2\tau_{xz}}{dx^2} = G\gamma_{xz} \qquad (10b)$$

Where E and G are the Young's and shear moduli, respectively, and γ is the shear strain. $\mu = (e_0 a)^2$ is the non-local parameter, e_0 is a constant appropriate to each material and a is an internal characteristic length. In general, a conservative estimate of the non-local parameter is $e_0 a$ < 2.0 nm for a single wall carbon nanotube (Wang 2005).

2.5 Equations of motion

Using the dynamic version of the principle of virtual work (Lv and Liu 2018), variationally consistent governing differential equations for the FG nanobeam under consideration is obtained. The principle of virtual work, when applied to the FG nanobeam leads to

$$\int_0^L \int_A (\sigma_x \delta\varepsilon_x + \tau_{zx}\delta\gamma_{zx})\, dAdx - \int_0^L \int_A \rho \left[\dot{u}_0 \delta \dot{u}_0 + (\dot{w}_b + \dot{w}_s)\delta(\ddot{w}_b + \ddot{w}_s)\right] dAdx = 0 \quad (11)$$

Collecting the coefficients of δu_0, δw_b and δw_s in Eq. (11), equations of motion are obtained as

$$\delta u_0: \quad \frac{dN}{dx} = I_0 u_0 \quad (12a)$$

$$\delta w_b: \quad \frac{d^2 M_b}{dx^2} = I_0(w_b + w_s) - I_2 \frac{d^2 w_b}{dx^2} \quad (12b)$$

$$\delta w_s: \quad \frac{d^2 M_s}{dx^2} + \frac{dQ}{dx} = I_0(w_b + w_s) - \frac{I_2}{84}\frac{d^2 w_s}{dx^2} \quad (12c)$$

Where N, M_b, M_s and Q are the stress resultants defined as

$$(N, M_b, M_S) = \int_A (1, z, f)\, \sigma_x dA \quad (13a)$$

$$Q = \int_A g\tau_{xz}\, dA \quad (13b)$$

and (I_0, I_2) are mass inertias defined as

$$(I_0, I_2) = \int_A (1, z^2)\rho(z) dA \quad (14)$$

When the shear deformation effect is neglected (w_s = 0), the equilibrium equations in Eq. (12) recover those derived from the Euler-Bernoulli beam theory. By substituting Eq. (8) into Eqs. (10a)-(10b) and the subsequent results into Eqs. (13a)-(13b), the stress resultants are obtained as

$$N - \mu\frac{d^2 N}{dx^2} = A\frac{d^2 u_0}{dx} - B\frac{d^2 W_b}{dx^2} - B_s\frac{d^2 W_s}{dx^2} \quad (15a)$$

$$M_b - \mu\frac{d^2 M_b}{dx^2} = B\frac{du_0}{dx} - D\frac{d^2 W_b}{dx^2} - D_s\frac{d^2 W_s}{dx^2} \quad (15b)$$

$$M_s - \mu\frac{d^2 M_s}{dx^2} = B\frac{du_0}{dx} - D\frac{d^2 W_b}{dx^2} - H_s\frac{d^2 W_s}{dx^2} \quad (15c)$$

$$Q - \mu\frac{d^2 Q}{dx^2} = A_S\frac{dw_s}{dx} \quad (15d)$$

Where the stiffness components are given as

$$\begin{aligned}
&\{A, B, D, \overline{E, F, H}\} = \int_A \{1, z, z^2, z^3, z^4, z^6\}\, E(z) dA,\\
&B_S = -\frac{1}{4}B + \frac{5}{3h^2}\overline{E}, D_S = -\frac{1}{4}D + \frac{5}{3h^2}F,\\
&H_S = -\frac{1}{16}D - \frac{5}{6h^2}F + \frac{25}{9h^4}H,\\
&\{A_{55}, D_{55}, F_{55}\} = \int_A \{1, z^2, z^4\}\, G(z) dA,\\
&A_S = \frac{25}{16}A_{55} - \frac{25}{2h^2}D_{55} + \frac{25}{h^4}F_{55},
\end{aligned} \quad (16)$$

By substituting Eqs. (15a)-(15d) into Eqs. (12a)-(12c), the non-local equations of motion can be expressed in terms of displacements (u_0, w_b, w_s) as

$$A\frac{d^2 u_0}{dx^2} - B\frac{d^3 w_b}{dx^3} - B_S\frac{d^3 w_S}{dx^3} = I_0\left(u_0 - \mu\frac{d^2 u_0}{dx^2}\right) \quad (17a)$$

$$\begin{aligned}
&B_S\frac{d^3 u_0}{dx^3} - D_S\frac{d^4 w_b}{dx^4} - H_S\frac{d^4 w_S}{dx^4} + A_S\frac{d^2 w_S}{dx^2}\\
&= I_0\left((w_b + w_s) - \mu\frac{d^2(w_b + w_s)}{dx^2}\right)\\
&-\frac{I_2}{84}\left(\frac{d^2 w_s}{dx} - \mu\frac{d^4 w_s}{dx^4}\right)
\end{aligned} \quad (17b)$$

$$\begin{aligned}
&B\frac{d^3 u_0}{dx^3} - D\frac{d^4 w_b}{dx^4} - D_S\frac{d^4 w_S}{dx^4}\\
&= I_0\left((w_b + w_s) - \mu\frac{d^2(w_b + w_s)}{dx^2}\right)\\
&-I_2\left(\frac{d^2 w_b}{dx^2} - \mu\frac{d^4 w_b}{dx^4}\right)
\end{aligned} \quad (17c)$$

The equations of motion of local beam theory can be obtained from Eqs. (17a)-(17c) by setting the non-local parameter μ equal to zero.

3. Analytical solution

The above equations of motion are analytically solved for free vibration problems. The Navier solution procedure is used to determine the analytical solutions for a simply supported FG nanobeam. The solution is assumed to be of the form

$$\begin{Bmatrix} u_0 \\ w_b \\ w_s \end{Bmatrix} = \sum_{m=1}^{\infty} \begin{Bmatrix} u_n \cos(\alpha x) e^{i\omega t} \\ W_{bn} \sin(\alpha x) e^{i\omega t} \\ W_{sn} \sin(\alpha x) e^{i\omega t} \end{Bmatrix} \quad (18)$$

Where u_n, w_{bn}, and w_{sn} are arbitrary parameters to be determined, ω is the eigenfrequency associated with n th eigenmode, and $\alpha = n\pi/L$.

Substituting the expansions of u_0, w_b, and w_s from Eq. (18) into Eqs. (17), the analytical solutions can be obtained from the following equations for bending problem

$$\left(\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{12} & s_{22} & s_{23} \\ s_{13} & s_{23} & s_{33} \end{bmatrix} - \lambda\omega^2 \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{23} & m_{33} \end{bmatrix}\right) \begin{Bmatrix} u_n \\ W_{bn} \\ Wsn \end{Bmatrix} = 0 \quad (19)$$

Where

$$\begin{aligned} & s_{11} = A\alpha^2, \quad s_{12} = -B\alpha^3, \quad s_{13} = -B_S\alpha^3, \\ & s_{22} = D\alpha^4, \quad s_{23} = D_s\alpha^4, \quad s_{33} = H_s\alpha^4 + a_s\alpha^2, \\ & m_{11} = m_{23} = I_0, \qquad m_{22} = I_0 + I_2\alpha^2, \\ & m_{33} = I_0 + \frac{I_2}{84}\alpha^2, \qquad \lambda = 1 + \mu\alpha^2 \end{aligned} \quad (20)$$

4. Results and discussions

In this study, free vibration analysis of simply supported FG nanobeams by the present method is suggested for investigation. The FG nanobeam is composed of a steel (St) and alumina (Al_2O_3). The bottom surface of the beam is pure steel, whereas the top surface of the beam is pure alumina. The material properties are as follows:

E_t = 390 GPa, E_b = 210 GPa, ρ_t = 3960 kg/m^3, ρ_b = 7800 kg/m^3, $v_t = v_b = 0.3$ (Eltaher *et al.* 2012). For convenience, the following non-dimensionalizations are used: frequency parameter $\overline{\omega} = \omega L^2 \sqrt{\frac{\rho_t A}{E_t I}}$

To validate the accuracy of nanoscale beam with porosities, a comparisons was made between the present results and the available results obtained by (Larbi Chaht *et al*. 2015) and (Zemri *et al*. 2015)

Table 2 present the comparisons of the non-dimensional natural frequencies with results of (Larbi *et al*. 2015) and (Zemri *et al*. 2015) for two different values of the thickness-to-length ratio. As can be seen, the new results are in good agreement with previous ones, particularly for long beams (L/h = 30 and 100). A slight difference appears for the short beams (L/h = 10); As we can see perfect FG nanobeam and takes maximum values for the imperfect FG nanobeam (α = 0.1 and α = 0.2). This is expected because the imperfect FG nanobeam has the lowest stiffness while the perfect FG nanobeam has the highest stiffness. In addition, the comparisons show the effect of the porosity on the frequencies of FG nanobeam with two different value of porosity. The results reveal that the frequencies results decrease as the volume fraction of porosity (α) increases.

Fig. 2 shows the effect of the aspect ratio on free vibration responses of FG nanobeam. The local and non-local results are given for e_0a = 0 and e_0a = 2 nm, respectively. The power-law index is assumed to be constant, k = 0.3. It is observed from this figure that the non-local solution of the fundamental frequency is smaller than the local one due to the small-scale effects. This result indicates that the used of non-local parameter bring to

Table 1 Comparisons of dimensionless frequencies (ϖ) of isotropic homogenous nanobeam (k = 1, e_0a = 1 nm)

	Larbi Chaht *et al*. 2015	Zemri *et al*. 2015	Present		
L/h	α = 0	α = 0	α = 0	α = 0.1	α = 0.2
10	6.5699	6.5699	6.5697	6.3613	6.1592
20	6.8838	6.8838	6.8838	6.6653	6.4532

Table 2 Dimensionless fundamental frequency (ϖ) of the FG nanobeam with porosities

e_0a		0	0	0			2	2	2		
		Larbi Chaht *et al*. 2015	Zemri *et al*. 2015	Present			Larbi Chaht *et al*. 2015	Zemri *et al*. 2015	Present		
	K	α = 0	α = 0	α = 0	α = 0.1	α = 0.2	α = 0	α = 0	α = 0	α = 0.1	α = 0.2
L/h	0	9.7075	9.7075	9.7076	9.3705	9.0522	8,2196	8,2197	8.2198	7.9343	7.6648
	0,3	8.1700	8.1709	8.1727	7.9074	7.6532	6,9178	6,9185	6.9201	6.6954	6.4802
	1	6.8814	6.8814	6.8862	6.6679	6.4560	5,8267	5,8267	5.8308	5.6459	5.4665
	3	6.0784	6.0755	6.0799	5.8898	5.7036	5.1468	5.1443	5.1480	4.9870	4.8295
10	10	5.5794	5.5768	5.5783	5.4033	5.2314	4.7242	4.7221	4.7233	4.5751	4.4296
	0	9.8511	9.8511	9.8511	9.5089	9.1860	9.6419	9.6419	9.6419	9.3070	8.9909
	0,3	8.2901	8.2902	8.2904	8.0210	7.7630	8.1140	8.1141	8.1143	7.8506	7.5981
	1	6.9832	6.9832	6.9838	6.7620	6.5469	6.8349	6.8349	6.8354	6.6184	6.4078
	3	6.1715	6.1712	6.1716	5.9786	5.7897	6.0405	6.0401	6.0406	5.8517	5.6668
30	10	5.6858	5.6855	5.6656	5.4881	5.3137	5.5455	5.5452	5.5453	5.3716	5.2009
	0	9.8679	9.8679	9.8679	9.5252	9.2017	9.8485	9.8485	9.8485	9.5064	9.1835
	0,3	8.3042	8.3042	8.3041	8.0343	7.7759	8.2878	8.2878	8.2878	8.0185	7.7606
	1	6.9952	6.9952	6.9952	6.7731	6.5575	6.9814	6.9814	6.9814	6.7597	6.5446
100	3	6.1825	6.1824	6.1824	5.9891	5.7998	6.1703	6.1703	6.1703	5.9773	5.7884
	10	5.6760	5.6759	5.6759	5.4981	5.3234	5.6648	5.6648	5.6648	5.4873	5.3129

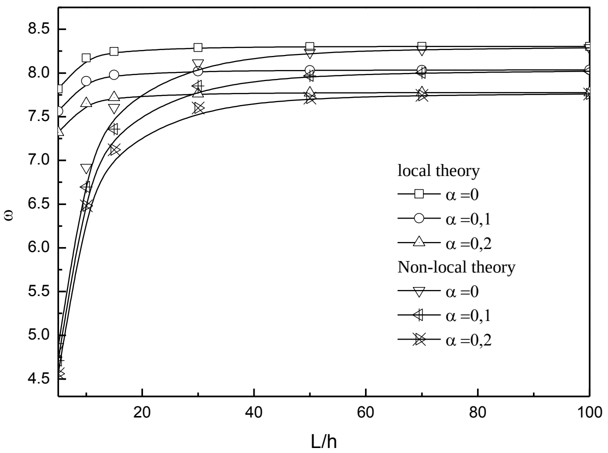

Fig. 2 Variation of the non-dimensional fundamental frequency with the aspect ratio and porosity coefficient α

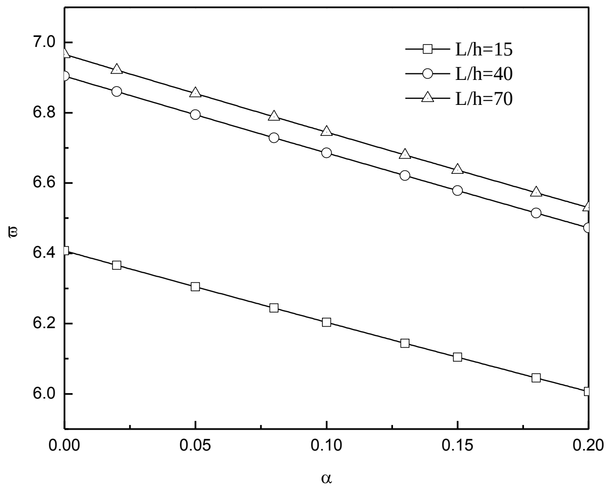

Fig. 4 Effect of porosity coefficient α on the non-dimensional fundamental frequency with $e_0 a$ = 2 nm and k = 1

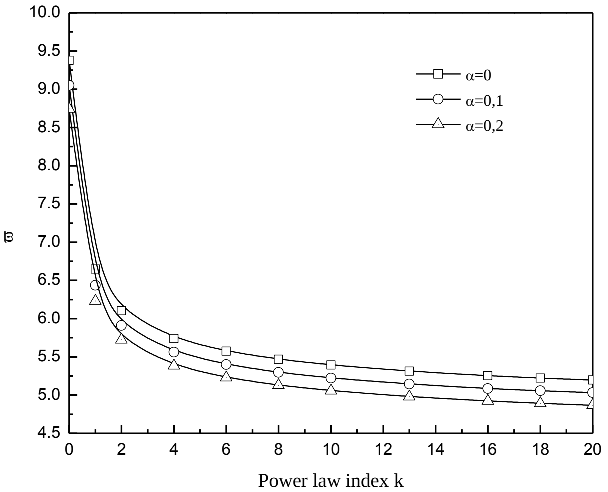

Fig. 3 Variation of the non-dimensional fundamental frequency in function of the Power-law index with L/h = 20, $e_0 a$ = 2 nm and porosity coefficient α

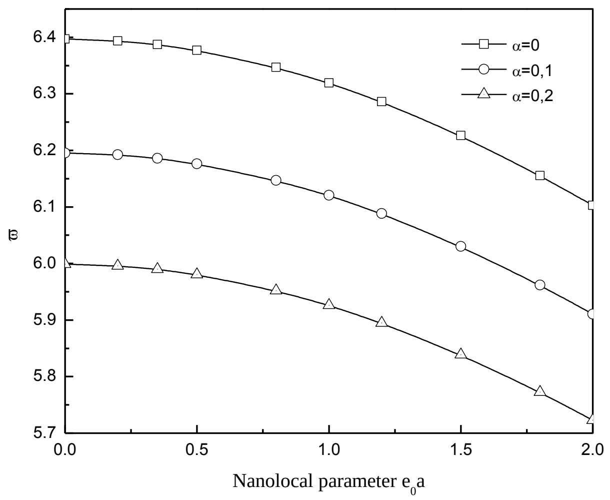

Fig. 5 Variation of the non-dimensional fundamental frequency in function of the Non-local parameter with L/h = 10, k = 2 and porosity coefficient α

a high accuracy. As also explained, that the effect of the porosity on the natural frequency became very clear for the biggest values of the thickness-to-length ratio increases.

Fig. 3 presents the variation of the non-dimensional fundamental frequency in function of the power-law index for three values of the porosity coefficient α = 0, 0.1 and 0.2. It can be deduced from this curve that the frequency parameter decreases when the power-law index increases. Furthermore, an increase of the porosity coefficient leads to a decrease in the frequency parameter.

Fig. 4 presents the variation of the non-dimensional fundamental frequency parameter in the function of the porosity coefficient$\alpha$and for three values of the aspect ratio, L/h = 15, 40 and 70. It can be deduced from this curve that the frequency decreases linearly when the porosity coefficients increases. However, an increase of the porosity coefficients leads to decrease the rigidity of beam and decrease in the frequency parameter.

In Fig. 5, the non-dimensional fundamental frequency in function of the non-local parameter with the aspect ratio L/h = 10, power-law index k = 2, and porosity coefficient α = 0, 0.1 and 0.2. This figure show that the dimensionless fundamental frequencies of FG nanobeams decrease with increasing the non-local parameter. The effect of non-local parameter is clearly for the higher values of e_0a.

5. Conclusions

The aim of the paper is to studies the free vibration of functionally graded nanoscale beam with porosities based on the non-local shear deformation beam theory. The central idea studied and discussed in this paper is supported by the various parameters analyses such as material parameter, the volume fraction, aspect ratios, the degree of porosity and beam thickness. The formulations and the governing equations are solved and the values of frequencies are obtained. The material properties are assumed to vary across the thickness direction of the beam according to the rule of mixture, which is reformulated to assess the material characteristics with the porosity phases.

For this study, the results showed the dependence of non-dimensional frequency with the different parameters. The results obtained from the analysis of frequencies indicated in first teams the good agreement between the present results and the existing ones in the literature. In terms of porosity analyses, it is found that the frequencies results decrease as the volume fraction of porosity (α) increases. The following conclusions were noticed from the results obtained for different parameters.

- This result indicates that the used of non-local parameter bring to a high accuracy.
- The effect of the porosity on the natural frequency became very clear for the biggest values of the thickness-to-length ratio increase.
- An increase of the porosity coefficient leads to decrease the rigidity of beam and decrease in the frequency parameter.
- It can be deduced from the results that the frequency parameter decreases when the power-law index increases.
- The dimensionless fundamental frequencies of FG nanobeams decrease with increasing the non-local parameter.

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