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Exact solution for dynamic response of size dependent torsional vibration of CNT subjected to linear and harmonic loadings

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Abstract. Rotating systems concern with torsional vibration, and it should be considered in vibration analysis. To do this, the time-dependent torsional vibrations in a single-walled carbon nanotube (SWCNT) under the linear and harmonic external torque, are investigated in this paper. Eringen's nonlocal elasticity theory is considered to demonstrate the nonlocality and constitutive relations. Hamilton's principle is established to derive the governing equation of motion and consequently related boundary conditions. An analytical method, called the Galerkin method, is utilized to discretize the driven differential equations. Linear and harmonic torsional loads, along with determined amplitude, are applied to the SWCNT as the external torques. SWCNT is considered under the clamped-clamped end supports. In free vibration, analysis of small scale effect reveals the capability of natural frequencies in different modes, and this results desirably are in coincidence with another study. The forced torsional vibration in the time domain, especially for carbon nanotubes, has not been done before in the previous works. The previous forced studies were devoted to the transverse vibrations. It should be emphasized that the dynamical analysis of torsion is novel, workable, and at the beginning of the path. The variations of nonlocal parameter, CNT's thickness, and the influence of excitation frequency on time-dependent angular displacement and nondimensional angular displacement are investigated in the context.

Keywords: forced vibration; SWCNT; torsional vibration; linear and harmonic; exact solution

1. Introduction

1.1 Nanostructure and carbon nanotube

Nanoscience was discovered by physicist Richard P. Feynman in 1959 (Feynman). After that, it contained other sciences and made the researchers work on very small scale materials (Lieber 2003, Gates *et al*. 2005, Sanchez and Sobolev 2010). Nanotechnology has vast applications in medicine (Roco 2003, Wilkinson 2003, Angeli *et al*. 2008), agriculture (Baruah and Dutta 2009, Ditta 2012, Scott and Chen 2013, Mukhopadhyay 2014), surgical oncology, food packaging, food safety (Duncan 2011), biotechnology (Lee 1998, West and Halas 2000, Wang *et al*. 2011), civil engineering (Gopalakrishnan *et al*. 2011) etc. The idea of nano-science comes from utilizing atoms and molecules in order to produce structures and materials. (Adams and Barbante 2013).

CNTs have different properties such as low weight (Krishnan *et al*. 1998), small diameter (Dresselhaus *et al*. 2013), and are useable in continuous yarns (Jiang *et al*. 2002) nanobioelectronics (Katz and Willner 2004), electrochemistry (Gooding 2005), scanning probe microscopy (Dai *et al*. 1996), and sensors (Li *et al*. 2003). The invention of CNT occurred by accident when Iijima (Iijima 1991) was working on an arc-discharge evaporation method in graphite electrodes by synthesizing fullerenes. The delicacy in structure along with a hexagonal array of carbon atoms, high strength, and topology make the nanotubes individual. The property of Young's modulus in CNT is constant as a planar structure, and the elastic constant depends on the CNTs diameter (Salvetat *et al*. 1999).

1.2 Small-scale effect

There are plenty of theories to analyze the small-scale effect, including surface effect, modified coupled stress method, nonlocal elasticity theory (ET) (Kazemnia Kakhki *et al*. 2016, Murmu and Pradhan 2009b, Pradhan and Kumar 2011, Tounsi *et al*. 2013a, b, Ke *et al*. 2014, Ahouel *et al*. 2016, Bellifa *et al*. 2016, Hayati *et al*. 2016, Hosseini and Rahmani 2016a, b, c, d, Rahmani *et al*. 2016, Zarepour *et al*. 2016, Alizade Hamidi *et al*. 2019), and nonlocal strain gradient theory (SGT) (Farajpour *et al*. 2016, Ebrahimi and Barati 2017, Mehralian *et al*. 2017). In this study, we consider the nonlocal elasticity theory, which was introduced by Eringen (Eringen and Edelen 1972, Eringen 1983) for the first time. Following this theory, many researchers allocated their works to this field. Reddy (2007) used this theory to investigate the buckling and vibration of nanobeams. Şimşek (2011) investigated the effects of moving load and its velocity, ratio of length to diameter, nonlocality, and small scale-effect when DWCNT connects with an elastic medium, and compared the deflections, which analytically were obtained according to the nonlocal

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principle with those obtained by the classical continuum theory. Aydogdu and Filiz (2011) modeled an elastic SWCNT with an attached mass in two different boundary conditions using Eringen's nonlocal theory in order to analyze the axial vibration, and found that by changing the parameters such as aspect ratio, value of the mass, and nonlocal parameter, the result will be attained different.

1.3 Vibration analysis

The ratio of nonlinear to linear frequencies versus nondimensional amplitude and nonlocal parameter for different boundary conditions based on Eringen's elasticity theory were indicated by Şimşek (2014) using Galerkin method to calculate the approximate nonlinear responses. Aydogdu (2009b) developed a generalized nanobeam along with other presented nonlocal beam theories in other researches to investigate the nonlocal effect and dimensions on the buckling, bending, and natural frequencies of the nanobeam. Ke *et al*. (2012) studied the nanobeams when a piezoelectric, is added to nanobeam, and the nonlinear responses and mode shapes are measured by the existence of piezoelectricity effect. Eltaher *et al*. (2013) utilized FEM based on nonlocal theory for Euler Bernoulli beam model to obtain the responses of the system by changing the aspect ratio and nonlocal parameter. Zhang *et al*. (2015) used FG nanocomposite with CNTs as reinforcement embedded in the triangular composite layers, and applied the first-order shear deformation and element free Ritz method, to evaluate the nondimensional natural frequency when the geometrical parameters and boundary conditions are various. Sharabiani and Yazdi (2013) cooperated to obtain the nonlinear nondimensional frequencies of the FG nanobeam by the existence of the surface effect considering the beam as Euler-Bernoulli one and used geometrically von-Kármán theory to derive the equation of motion. They also showed the responses in various amplitude and volume fraction. Kiani (2014) investigated the produced axial vibration of size-dependent current-carry nanowires when exposed by a magnetic field. Assadi (2013) recommended an analytical method for a rectangular nanoplate when an external load applies, and the surface effect is non-negligible. Aydogdu (2009a) established a nanorod, which deflects only in axial direction based on the nonlocal theory with different boundary conditions and determined the effects of the external characteristic length in fundamental frequencies and compared them with local theory. Aydogdu (Aydogdu and Elishakoff 2014) investigated the nanorods, which linearly were connected with spring, and they encountered a diversity of frequencies in different modes by changing the position of the spring. Murmu and Adhikari (2010) utilized double nanorod to obtain the axial response of the system using nonlocal theory and showed the influence of nonlocality. Murmu and Pradhan (2009a) used nanoplate to show the effect of nonlocal theory on the in-plane frequencies and showed its necessity. Wang and Feng (2009, 2010) modeled nanowires as a Timoshenko beam, which a compressive force is applied as a buckling force with a rectangular and circular cross-section. They showed the surface effects in compressive force and response of the system by changing the value of aspect ratio, and studied the surface effect on the natural frequencies and buckling when a piezoelectric is connected to Euler-Bernoulli nanowire with a rectangular cross section, and the load is applied transversely. Fatahi-Vajari and Imam (2016) investigated the natural frequency in SWCNT due to the torsion, based on DM method for two kind of nanotubes. They indicated the small scale effect and chirality and also the influence of the variation in geometrical CNT's parameter on the responses and compared the results with local and nonlocal theories. Gheshlaghi and Hasheminejad (2010) developed the normalized natural frequencies in nanotubes based on the torsional response, and used analytical method to show the geometrical dimension's effect (e.g., size-dependency) on the normalized natural frequencies. Nazemnezhad and Fahimi (2017) studied nanobeams when the surface effect is considerable and contains crack. Therefore, surface elasticity is added to the classical elasticity theory. They counted the crack as the spring and evaluated its severity and position in the various boundary condition. Adeli *et al*. (2017) investigated the free torsional responses in a homogeneous and one-directional nano-cone by nonlinear variant axial cross-section based on strain gradient theory and discussed the effect of cross section change and length scale. Arda and Aydogdu (2014) investigated the torsional response of the CNT surrounded by an elastic medium based on the nonlocal theory, and showed the effect of the external length of CNT, stiffness of an elastic medium on the nondimensional frequencies statically. Apuzzo *et al*. (2017) used an enhanced nonlocal model for torsional responses of nanocantilever and nanobeam with various boundary conditions based on Eringen's theory when external torque is applied uniformly. The research's filed of Hao *et al*. (2010) was investigating the small-scale effect on the torsional behavior of the MWCNT embedded in an elastic medium, critical temperature change, critical shear force, and the buckling force. Also, the temperature loads coupled between the tubes. El-Borgi *et al*. (2018) obtained the nondimensional frequencies of the viscoelastic nanorod, which enclosed by an elastic medium with various end supports by mixing strain and velocity gradient methods. Furthermore, the effects of the stiffness, damping coefficient, and size-dependency on different modes were investigated. Lim *et al*. (2015) considered the cylindrical nanostructures under the constant and distributed torques. For integrating the derived equations, applied an innovative FEM method based on nonlocal theory. The effects of the boundary conditions in the nanoscale on varied nonlocal parameters, angular displacement, and nondimensional angular displacement were evaluated.

1.4 Novelty of the work

By the deep studying of references, it can be understood that the time-dependent forced torsional vibration in carbon nanotubes has not been done before. There are some studies in forced axial and remarkably in transverse vibration. The forced torsional vibration in the time domain, especially for carbon nanotubes, has not been done before in the previous works. It should be emphasized that the dynamical analysis of torsional vibration is novel, useful, and the main aim of this study is devoted to filling this gap.

1.5 Present work

This study technically is paid to time-dependent torsional vibration of a SWCNT under the linear and harmonic external torque. Clamped-clamped boundary condition is considered. The Hamilton's principle is established to derive the equation of motion and boundary conditions. A Galerkin method is investigated to discretize the derived equations. For the free torsional vibration, the effects of the nonlocal parameter and mode number on the first three natural frequencies have been shown, and the results are compared with another study to evaluate the accuracy. In dynamic analysis, the effects of nonlocal parameter, CNT's thickness, and excitation frequency on angular displacement and nondimensional angular displacement for harmonic loading are investigated.

2. Theroy and formulation

Firstly, we investigate theory of the nonlocal elasticity, which includes the effect of long range interatomic forces, and because the scale is in nanometer, it can be used as a continuum model of the atomic lattice dynamics. According to the nonlocal Eringen's elasticity, the stress at a reference point$x$is regarded as a function of strains of all around reference points. This theory is based on experimental observations on phonon scattering and the atomic theory of lattice dynamics.

2.1 Eringen's nonlocal elasticity theory

The basic equations for an isotropic linear homogenous neglecting the body force for a homogeneous and isotropic, the nonlocal stress-tensor at point$x$can be defined as

$$\sigma_{ij}(x) = \int_V K(|x' - x|, \tau)\, t_{ij}(x')dV \tag{1}$$

In which $t_{ij}(x')$, $K(|x' - x|, \tau)$, and τ represent the classical local stress tensor at point$x$, the nonlocal modulus, and material constant, respectively. Also, the nonlocal kernel $K(|x' - x|, \tau)$ reflects the impact of the strain at the point x'on the stress at the point$x$; $|x' - x|$ denotes the Euclidean distance. The stress tensor and material constant are expressed as follows

$$t_{ij}(x) = C_{ijkl}(x) : \varepsilon_{kl}(x) \tag{2}$$

$$\tau = e_0 a/l \tag{3}$$

From above equations, $C_{ijkl}(x)$ represents the fourth-order elasticity tensor, which represents the "double-dot product", and τ is the ratio of the characteristic internal length, a (e.g., Lattice parameter) per external length, l (e.g., crack length, wavelength) and $e_0$is a constant, which

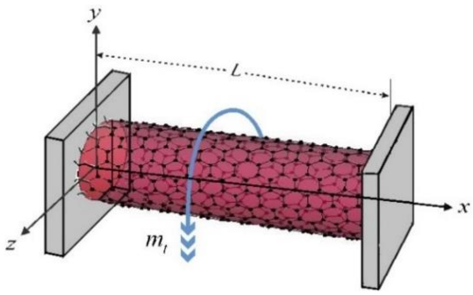


Fig. 1 Schematic of SWCNT (12,6) under the coupling of torsional vibration

variates for each material, and is obtained experimentally. The integral constitutive relations in an equivalent differential form of Eq. (1) can be rewritten as

$$(1 - \mu^2 \nabla^2)\sigma_{ij}(x) = t_{ij}(x) \tag{4}$$

Where ∇^2 represents the Laplacian operator, and μdenotes the nonlocal parameter and can be defined as

$$\mu = \tau l = (e_0 a) \tag{5}$$

Considering a CNT, a simple one-dimensional nonlocal model, which undergoes torsional vibration with the length of l as shown in Fig. 1, the constitutive relation for shear stress in a differential form (Eq. (4)) can be rewritten as

$$\sigma_{xz} - \mu^2 \frac{\partial^2 \sigma_{xz}}{\partial x^2} = G\varepsilon_{xz} \tag{6}$$

$$\sigma_{xy} - \mu^2 \frac{\partial^2 \sigma_{xy}}{\partial x^2} = G\varepsilon_{xy} \tag{7}$$

Where G represents the shear modulus; σ_{xy}, σ_{xz} and ε_{xz}, ε_{xz} are the nonlocal shear stresses and strains of CNT, respectively. The torque resultant (Rao 2019) of CNT can be expressed as

$$T = \int_A (y\sigma_{xz} - z\sigma_{xy})dA \tag{8}$$

u, v, and w denote the displacements of any point of the CNT parallel to x-,y-, and z- axes, respectively, and can be expressed as

$$\begin{aligned} u(x,t) &= 0 \\ v(x,t) &= -z\theta(x,t) \\ w(x,t) &= y\theta(x,t) \end{aligned} \tag{9}$$

Where $\theta(x,t)$ is the angular displacement about the$x$axis, which is the center of twist. The strains of the CNT are defined as

$$\begin{aligned} \varepsilon_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -z\frac{\partial \theta}{\partial x} \\ \varepsilon_{xz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = y\frac{\partial \theta}{\partial x} \\ \varepsilon_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = 0 \\ \varepsilon_{xx} &= \varepsilon_{yy} = \varepsilon_{zz} = 0 \end{aligned} \tag{10}$$

2.2 Deriving equation and boundary condition

According to the Hamilton's principle the governing equation can be expressed as

$$\int_0^t \delta\big(U-(T_k+V_{ext})\big)dt=0 \tag{11}$$

Here U, T_k and, V denote the strain energy, kinetic energy and the external works, respectively. The virtual strain energy for CNT can be stated as

$$\delta U=\int_V \sigma_{ij}\delta\varepsilon_{ij}dV=\int_V(\sigma_{xy}\delta\varepsilon_{xy}+\sigma_{xz}\delta\varepsilon_{xz})dV \tag{12}$$

Replacing Eqs. (10) into Eq. (12) leads to

$$\delta U=\int_V\left(-z\sigma_{xy}\left(\frac{\partial\delta\theta}{\partial x}\right)+y\sigma_{xz}\left(\frac{\partial\delta\theta}{\partial x}\right)\right)dV \tag{13}$$

Substituting Eq. (8) into Eq. (12) is given by

$$\delta U=\int_0^L T\frac{\partial\delta\theta}{\partial x}dx \tag{14}$$

The kinetic energy of the CNT can be defined as

$$T_K=\frac{1}{2}\int_V\rho\left[\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial v}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right]dV \tag{15}$$

By substituting the first derivative of Eq. (10) into Eq. (15), the following equation will be obtained

$$\begin{aligned}T_K&=\frac{1}{2}\int_V\rho\left[\left(-z\frac{\partial\theta}{\partial t}\right)^2+\left(y\frac{\partial\theta}{\partial t}\right)^2\right]dV\\&=\frac{1}{2}\rho I_p\int_0^l\left(\frac{\partial\theta}{\partial t}\right)^2dx=\frac{1}{2}I_0\int_0^l\left(\frac{\partial\theta}{\partial t}\right)^2dx\end{aligned} \tag{16}$$

Where I_p,I_0, andρrepresent the polar moment of inertia, mass inertia and density, respectively, and are equivalent to the following statements

$$I_p=\int_A(y^2+z^2)dA \tag{17}$$

$$I_0=\rho I_p \tag{18}$$

Since the cross-section is uniform and circular, and x-axis is coincident with the CNT's axis, it is obvious that the element's distance in cross section from the origin of coordinate ($\sqrt{y^2+z^2}$) is equal to r. Therefore, the polar moment of inertia for CNT changes into

$$I_p=\int_A r^2dA=\frac{\pi}{2}({r^4}_{out}-{r^4}_{in}) \tag{19}$$

From Eq. (19), r_{out} and r_{in} represent the external and internal radii of CNT, respectively. The first variation of kinetic energy can be expressed as

$$\delta T_K=I_0\int_0^l\left(\frac{\partial\theta}{\partial t}\frac{\partial\delta\theta}{\partial t}\right)dx \tag{20}$$

The work done by an external torque $m_t(x,t)$can be expressed as

$$V_{ext}=\int_0^l\big(m_t(x,t)\big)\theta dx \tag{21}$$

The first variation of the external works is given by

$$\delta V_{ext}=\int_0^l\big(m_t(x,t)\big)\delta\,\theta dx \tag{22}$$

By replacing Eqs. (14), (20), and (22) into Eq. (11), we will have

$$\begin{aligned}&\int_0^t\left(\left(\int_0^l T\frac{\partial\delta\theta}{\partial x}dx-I_0\int_0^l\left(\frac{\partial\theta}{\partial t}\frac{\partial\delta\theta}{\partial t}\right)dx\right.\right.\\&\left.\left.-\left(\int_0^l m_t(x,t)\delta\,\theta dx\right)\right)\right)dt\\&=\int_0^t\left(T\delta\theta\Big|_0^l-\int_0^l\frac{\partial T}{\partial x}\delta\theta dx-I_0\frac{\partial\theta}{\partial t}\delta\theta\Big|_0^l\right.\\&\left.+\int_0^l I_0\frac{\partial^2\theta}{\partial t^2}\delta\theta dx-m_t(x,t)\delta\theta dx\right)dt\end{aligned} \tag{23}$$

In which T is the induced torque. The equation of motion for torsional CNT can be obtained as

$$\frac{\partial T}{\partial x}=I_0\frac{\partial^2\theta}{\partial t^2}-m_t(x,t) \tag{24}$$

Multiplying Eq. (6) by y and Eq. (7) by z, then subtracting Eq. (7) from Eq. (6), and integrating with respect to the cross section by consideration of Eq. (17), the constitutive equation can be written as

$$T-\mu^2\frac{\partial^2T}{\partial x^2}=GI_p\frac{\partial\theta}{\partial x} \tag{25}$$

By applying the first derivative of Eq. (24) into Eq. (25) induced torque (T) can be expressed as

$$T=\mu^2\left(I_0\frac{\partial^3\theta}{\partial x\partial t^2}-\frac{\partial m_t(x,t)}{\partial x}\right)+GI_p\frac{\partial\theta}{\partial x} \tag{26}$$

As a final step, by substituting the first derivative of Eq. (26) into Eq. (24), the following equation can be concluded

$$\begin{aligned}&m_t(x,t)-I_0\frac{\partial^2\theta}{\partial t^2}+\mu^2\left(I_0\frac{\partial^4\theta}{\partial x^2\partial t^2}\right)\\&+GI_p\frac{\partial^2\theta}{\partial x^2}-\mu^2\left(\frac{\partial^2m_t(x,t)}{\partial x^2}\right)=0\end{aligned} \tag{27}$$

2.3 Analytical solution

2.3.1 Free torsional vibration analysis

In order to obtain the natural frequencies, caused by torsion in each modes, the expression corresponded to the loading should be removed. Therefore, $m_t(x,t)$ is omitted from Eq. (27), and can be rewritten as

$$GI_p \frac{\partial^2\theta}{\partial x^2} - I_0 \frac{\partial^2\theta}{\partial t^2} + \mu^2\left(I_0 \frac{\partial^4\theta}{\partial x^2\partial t^2}\right) = 0 \tag{28}$$

By assuming the time-dependent term of discretized angular displacement to be harmonic, it can be defined as follows

$$\theta(x,t) = \sum_{n=1}^{\infty} \Theta_n(x) e^{i\omega t} \tag{29}$$

Substituting Eq. (29) into Eq. (28) leads to

$$GI_p\Theta_n'' + I_0\Theta_n\omega^2 - \mu^2(I_0\omega^2\Theta_n'') = 0 \tag{30}$$

Where Θ_n and Θ_n'' express the nth corresponding mode shape and second derivative of it and for the clamped-clamped boundary condition can be written as

$$\Theta_n(x) = C_n \sin\left(\frac{n\pi}{l}x\right) \tag{31}$$

$$\Theta_n''(x) = -C_n\left(\frac{n\pi}{l}\right)^2 \sin\left(\frac{n\pi}{l}x\right) \tag{32}$$

Replacing Eqs. (31) and (32) into Eq. (30) results in

$$-GI_p\left(\frac{n\pi}{l}\right)^2 + I_0\omega^2 + \mu^2\left(\frac{n\pi}{l}\right)^2 (I_0\omega^2) = 0 \tag{33}$$

Consequently, the natural frequency can be obtained as follows

$$\omega_n = \sqrt{\frac{GI_p\left(\frac{n\pi}{l}\right)^2}{I_0\left(1+\mu^2\left(\frac{n\pi}{l}\right)^2\right)}} \tag{34}$$

The dimensionless natural frequency can be written as

$$\bar{\omega}_n = \sqrt{\frac{(n\pi)^2}{\left(1+\mu^2\left(\frac{n\pi}{l}\right)^2\right)}} \tag{35}$$

2.3.2 Dynamic torsional vibration

In order to solve Eq. (27) dynamically, an analytical solution based on expansion theorem is utilized. For this purpose, we should discretize mode shape $\Theta_n(x)$ as a known coefficient and nth generalized coordinate $\eta_n(t)$ as an unknown function. Therefore, the angular displacement of a CNT in torsional vibration can be expressed as

$$\theta(x,t) = \sum_{n=1}^{\infty} \Theta_n(x)\eta_n(t) \tag{36}$$

Replacing Eq. (36) into Eq. (27) leads to

$$\begin{aligned} &m_t(x,t) - \sum_{n=1}^{\infty} I_0\Theta_n(x)\ddot{\eta}_n(t) + \sum_{n=1}^{\infty} \mu^2 I_0\Theta_n''(x)\ddot{\eta}_n(t) \\ &+ \sum_{n=1}^{\infty} GI_p\Theta_n''(x)\eta_n(t) - \mu^2\left(\frac{\partial^2 m_t(x,t)}{\partial x^2}\right) = 0 \end{aligned} \tag{37}$$

By substituting Eqs. (31) and (32) into Eq. (37), the following relation can be inferred

$$\begin{aligned} &m_t(x,t) - \sum_{n=1}^{\infty} I_0\Theta_n(x)\ddot{\eta}_n(t) \\ &- \sum_{n=1}^{\infty} \mu^2 I_0\left(\frac{n\pi}{l}\right)^2 \Theta_n(x)\ddot{\eta}_n(t) \\ &- \sum_{n=1}^{\infty} GI_p\left(\frac{n\pi}{l}\right)^2 \Theta_n(x)\eta_n(t) - \mu^2(m_t''(x,t)) = 0 \end{aligned} \tag{38}$$

By multiplying Eq. (38) by $\Theta_m(x)$, the following equation can be expressed

$$\begin{aligned} &\int_0^l [m_t(x,t) - \mu^2(m_t''(x,t))]\Theta_m(x)dx \\ &-I_0\ddot{\eta}_n(t)\int_0^l \Theta_n(x)\Theta_m(x)dx \\ &-\mu^2 I_0\left(\frac{n\pi}{l}\right)^2 \ddot{\eta}_n(t)\int_0^l \Theta_n(x)\Theta_m(x)dx \\ &-GI_p\left(\frac{n\pi}{l}\right)^2 \eta_n(t)\int_0^l \Theta_n(x)\Theta_m(x)dx = 0 \end{aligned} \tag{39}$$

In which

$$\int_0^l \Theta_n(x)\Theta_m(x)dx = \begin{cases} 1 & n = m \\ 0 & n \neq m \end{cases} \tag{40}$$

Applying Eq. (31) into Eq. (40) results in

$$C_n^2 \int_0^l \sin^2\left(\frac{n\pi}{l}\right)dx = 1 \tag{41}$$

C_n can be obtained

$$C_n = \sqrt{\frac{2}{l}} \quad n = 1,2,\ldots \tag{42}$$

Eqs. (31) and (32) changes into

$$\Theta_n(x) = \sqrt{\frac{2}{l}}\sin\left(\frac{n\pi}{l}x\right) \tag{43}$$

$$\Theta_n''(x) = -\sqrt{\frac{2}{l}}\left(\frac{n\pi}{l}\right)^2 sin\left(\frac{n\pi}{l}x\right) \tag{44}$$

For orthogonality condition and considering Eq. (42), Eq. (39) reduces to

$$\begin{aligned} &I_0\left(1+\mu^2\left(\frac{n\pi}{l}\right)^2\right)\ddot{\eta}_n(t) + GI_p\left(\frac{n\pi}{l}\right)^2\eta_n(t) \\ &= \int_0^l [m_t(x,t) - \mu^2(m_t''(x,t))]\Theta_m(x)dx \end{aligned} \tag{45}$$

Simplifying Eq. (45) leads to the following equation

$$\ddot{\eta}_n(t) + {\lambda_n}^2\eta_n(t) = \Psi_n Q_n(t) \tag{46}$$

Where the nonlocal natural frequency and coefficient Ψ_n, respectively, are equivalent to

$$\lambda_n = \frac{\omega_n}{\sqrt{1+\mu^2\left(\frac{n\pi}{l}\right)^2}} \tag{47}$$

$$\Psi_n = \frac{1}{I_0\left(1+\mu^2\left(\frac{n\pi}{l}\right)^2\right)} \tag{48}$$

$Q_n(t)$, which is called generalized force in$n$thmode, is defined as

$$Q_n(t) = \Psi_n\int_0^l [m_t(x,t) - \mu^2(m_t''(x,t))]\Theta_m(x)dx \tag{49}$$

The complete solution of Eq. (46) can be specified as

$$\begin{aligned} \eta_n(t) &= A_n\,cos(\lambda_n t) + B_n\,sin(\lambda_n t) \\ &+ \frac{1}{\lambda_n}\int_0^{t\int(\lambda_n(t-\tau))} Q_n(\tau)sin \end{aligned} \tag{50}$$

Where A_n and B_n are constants, which can be determined from initial conditions. The steady state of CNT regardless of initial conditions can be obtained

$$\eta_n(t) = \frac{\Psi_n}{\lambda_n}\int_0^t Q_n(\tau)\,sin\big(\lambda_n(t-\tau)\big)\,d\tau \tag{51}$$

Linear external torque

If the external torque is concentrated at point $x = l/2$, and assumed to be linear, $m_t(x,t)$ can be expressed as

$$m_t(x,t) = M_t\delta\left(x-\frac{l}{2}\right) = a_0 t\delta\left(x-\frac{l}{2}\right) \tag{52}$$

Where $\delta(\cdot)$ denotes the Dirac delta function. It is clear that the concentrated external torque only depends on the time. As the time is linear, subsequently, the external torque changes linearly over time. By using Eq. (52) into Eq. (49), the normalized force can be rewritten as

$$\begin{aligned} Q_n(t) &= \Psi_n\int_0^l a_0 t\left(\delta\left(x-\frac{l}{2}\right)\right. \\ &\left.-(\mu)^2\delta''\left(x-\frac{l}{2}\right)\right)\Theta_n(x)dx \\ &= \Psi_n a_0 t\left(\Theta_n\left(\frac{l}{2}\right) - (\mu)^2\Theta_n''\left(\frac{l}{2}\right)\right) \end{aligned} \tag{53}$$

Setting $x = l/2$ in Eqs. (31) and (32), leads to

$$\begin{aligned} \Theta_n\left(\frac{l}{2}\right) &= \sqrt{\frac{2}{l}}sin\left(\frac{n\pi}{2}\right) \\ \Theta_n''\left(\frac{l}{2}\right) &= -\sqrt{\frac{2}{l}}\left(\frac{n\pi}{l}\right)^2 sin\left(\frac{n\pi}{2}\right) \end{aligned} \tag{54}$$

By substituting Eq. (54) into Eq. (53), Eq. (53) can be rewritten as

$$Q_n(t) = \Psi_n a_0 t\sqrt{\frac{2}{l}}sin\left(\frac{n\pi}{2}\right)\left(1+(\mu)^2\left(\frac{n\pi}{l}\right)^2\right) \tag{55}$$

By applying Eq. (55) into Eq. (51) the following equation can be concluded

$$\begin{aligned} \eta_n(t) &= \Phi_n\int_0^t \tau\,sin\big(\lambda_n(t-\tau)\big)\,d\tau \\ &= \Phi_n\left(t-\frac{1}{\lambda_n}sin(\lambda_n t)\right) \end{aligned} \tag{56}$$

$$\Phi_n = \frac{\Psi_n a_0\sqrt{\frac{2}{l}}sin\left(\frac{n\pi}{2}\right)\left(1+(\mu)^2\left(\frac{n\pi}{l}\right)^2\right)}{\lambda_n^2} \tag{57}$$

By substituting Eqs. (56) and (31) into Eq. (36), the angular displacement can be expressed as

$$\theta_n(x,t) = \frac{a_0}{I_0}\sum_{n=1}^{\infty}\frac{\Theta_n(\vartheta)\Theta_n(x)}{{\lambda_n}^2}\left(t-\frac{1}{\lambda_n}sin(\lambda_n t)\right) \tag{58}$$

By using Eq. (54), and considering the external torque exists at point of $\vartheta = l/2$, the multiplication of the normal modes in Eq. (58) can be expressed as

$$\Theta_n(\vartheta)\Theta_n\left(\frac{l}{2}\right) = \begin{cases} 0 & n = 0 \\ 2/l & n = 1,3,5,\dots \end{cases} \tag{59}$$

Finally, Eq. (58) can be rewritten as

$$\theta_n(x,t) = \frac{2a_0}{lI_0}\sum_{n=1,3,5,..}^{\infty}\frac{1}{{\lambda^2}_n}\left(t-\frac{1}{\lambda_n}sin(\lambda_n t)\right) \tag{60}$$

Harmonic external torque

In this section, the external torque is concentrated at point $x = l/2$ and assumed to be Harmonic, $m_t(x,t)$ is given by

$$m_t(x,t) = M_t\delta\left(x-\frac{l}{2}\right) = (M_0\,sin(\Omega t))\delta\left(x-\frac{l}{2}\right) \quad (61)$$

Where Ω denotes the excitation frequency, and $m_t(x,t)$ only depends on the harmonic time. By using Eq. (61) into Eq. (49), the normalized force can be specified as

$$\begin{aligned} Q_n(t) &= \Psi_n \int_0^l M_0\,sin(\Omega t)\left(\delta\left(x-\frac{l}{2}\right)\right. \\ &\left.-(\mu)^2\delta''\left(x-\frac{l}{2}\right)\right)\Theta_n(x)dx \\ &= \Psi_n M_0\,sin(\Omega t)\left(\Theta_n\left(\frac{l}{2}\right)-(\mu)^2\Theta_n''\left(\frac{l}{2}\right)\right) \end{aligned} \quad (62)$$

Replacing Eq. (54) into Eq. (62) leads to

$$\begin{aligned} Q_n(t) &= \Psi_n M_0\,sin(\Omega t)\sqrt{\frac{2}{l}}sin\left(\frac{n\pi}{2}\right) \\ &\times\left(1+(\mu)^2\left(\frac{n\pi}{l}\right)^2\right) \end{aligned} \quad (63)$$

By applying Eq. (63) into Eq. (51), the following equation can be concluded

$$\begin{aligned} \eta_n(t) &= \Phi_n\int_0^t sin(\Omega\tau)\,sin(\lambda_n(t-\tau))\,d\tau \\ &= \Phi_n\left(sin(\Omega t)-\frac{\Omega}{\lambda_n}sin(\lambda_n t)\right) \end{aligned} \quad (64)$$

In which coefficient of Φ_n is equal to

$$\Phi_n = \frac{\Psi_n M_0\sqrt{\frac{2}{l}}sin\left(\frac{n\pi}{2}\right)\left(1+(\mu)^2\left(\frac{n\pi}{l}\right)^2\right)}{{\lambda_n}^2-\Omega^2} \quad (65)$$

By replacing Eqs. (64) and (31) into Eq. (36), the angular displacement can be expressed as

$$\begin{aligned} \theta_n(x,t) &= \frac{M_0}{I_0}\sum_{n=1}^{\infty}\frac{\Theta_n(\vartheta)\Theta_n(x)}{(\lambda^2{}_n-\Omega^2)} \\ &\times\left(sin(\Omega t)-\frac{\Omega}{\lambda_n}sin(\lambda_n t)\right) \end{aligned} \quad (66)$$

By using Eq. (54), for the external Harmonic torque, which exists at point $l/2$, the angular displacement of the CNT is given by

$$\theta_n(x,t) = \frac{2M_0}{lI_0}\sum_{n=1,3,5,..}^{\infty}\frac{\left(sin(\Omega t)-\frac{\Omega}{\lambda_n}sin(\lambda_n t)\right)}{(\lambda^2{}_n-\Omega^2)} \quad (67)$$

3. Results and discussion

In this section, several numerical examples are utilized. The angular displacement by different external torques as linear and harmonic, are investigated from derived expressions for torsional vibration. Table 1 shows the characteristics of the CNT. For this purpose, a chiral structure single-walled carbon nanotube has been considered (SWCNT(12,6)) as a rolled-up rectangular plane of the graphene layer in the form of a hollow cylindrical tube (Zakeri and Shayanmehr 2013, Eatemadi *et al*. 2014). The geometrical parameters and mechanical properties of CNT are indicated in Table 1.

From Table 1 the radius, length, thickness, shear modulus, density, linear external torque amplitude, and harmonic external torque amplitude are shown with r, l, t, G, ρ, a_0 and M_0, respectively, in the context. $r+t/2$ and $r-t/2$ represent the outer and inner radii of the rolled-up SWCNT, and are denoted in Eq. (19) as r_{out} and r_{in}, respectively. Considering the numerical values of

Table 1 Characteristic of graphene SWCNT (12,6)

Radius (nm)	Length (nm)	Thickness (nm)	Shear modulus (Gpa)	Densit (Kg/m^3)	Linear torque amplitude (nN.mn/ns)	Harmonic torque amplitude (nN.nm)
0.621	30	0.34	298	1400	1	1

Table 2 First three dimensionless natural frequencies$\left(\omega_n\times l\times\sqrt{\rho/G}\right)$for clamped-clamped SWCNT (12,6) and different values of the nonlocal parameter ($e_0a\times$ nm) (ρ =1400 kg/m^3, l =30 nm, t =0.34 nm, G =298 Gpa , ν =0.25, R =0.621 nm)

e_0a (nm)	Frequency mode number					
	n = 1		n = 2		n = 3	
	Present	(Adeli *et al*. 2017)	Present	(Adeli *et al*. 2017)	Present	(Adeli *et al*. 2017)
0	3.1415	3.1416	6.2831	6.2832	9.4247	9.4248
1.5	3.1035	3.1035	5.9943	5.9943	8.5255	8.5256
3	2.9971	2.9972	5.3201	5.3202	6.8586	6.8587
4.5	2.8418	2.8419	4.5724	2.5724	5.4426	5.4427

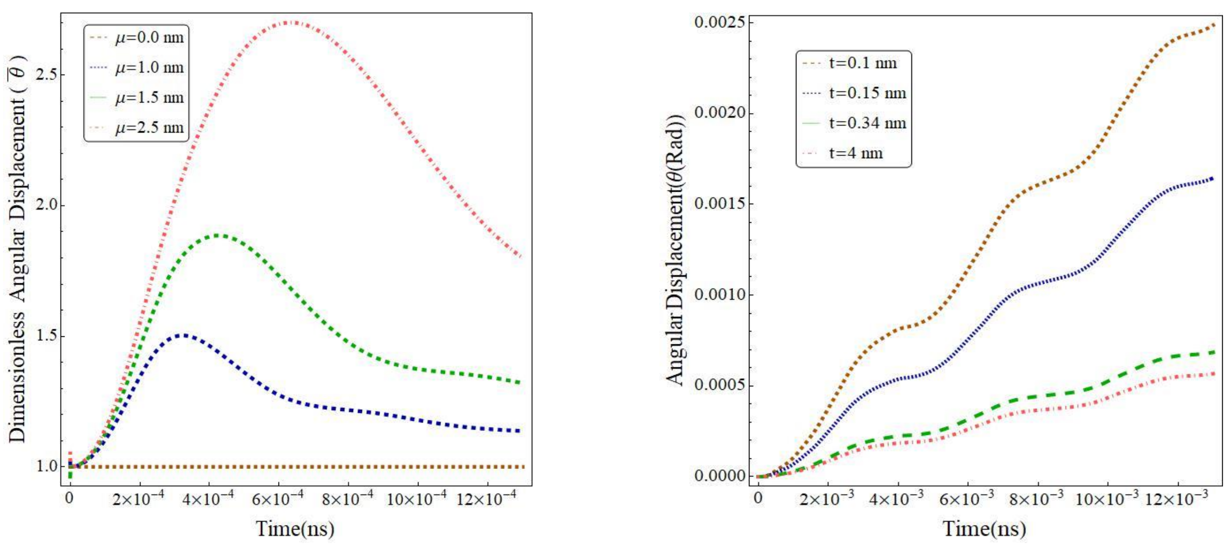
Table 1, Eqs. (19) and (20) can be determined as $I_p = 0.5499\ \mathrm{nm}^4$ and $I_0 = 0.7699 \times 10^{-24}$N.m.s^2, respectively.

For the results, the dimensionless natural frequency is investigated. The influences of the nonlocal parameter and mode shape on dimensionless natural frequency are indicated. Moreover, the frequencies are compared with another study, and the results are in good agreement with reference. It is comprehensible form Table 2, as the nonlocal parameter increases, the value of dimensionless natural frequency decreases for different modes. It is because of decrement in the stiffness. The increment in the dimensionless natural frequency occurs with higher rates by increasing the values of the nonlocal parameter. Also, the amount of the dimensionless natural frequency increases, as the value of mode shape increases.

Afterward, the angular displacements (θ) and dimensionless angular displacements $(\bar{\theta})$ of SWCNT versus time under external torque (m_t) are analyzed. The effects of the nonlocal parameter, CNT's thickness, and the excitation frequency for harmonic external torque case on the angular displacement are investigated. The time is based on nanosecond; the torsion occurs at point $x = l/2$. In Fig. 2(a), the variation of $\bar{\theta}$ versus time with linear external torque is illustrated. It is obvious, when the nonlocal parameter (μ) set to zero, the dimensionless angular displacement becomes constant, and equal to one, and by setting μ to zero, Eq. (27) changes into the classical form. When μ is not zero ($\mu =$ 1, 1.5 and 2.5 nm), as the value of μ increases, the value of $\bar{\theta}$ increases by the time and after a while, it will possess fluctuating behavior (e.g., decreasing and increasing of $\bar{\theta}$). It is normal because of the existence of the sinusoidal term in Eq. (54). When $\mu =$ 1 nm, the value of $\bar{\theta}$ is almost constant over time and has just fluctuating with small amplitudes. As μ increases, this amplitude becomes more and more. In Fig. 2(b), the variation of θ versus time with linear m_t and the effect of thickness of CNT with a constant nonlocal parameter, which is equal to 1.5 nm, and thicknesses ($t =$ 0.1, 0.15, 0.34 and 0.4) has been indicated.

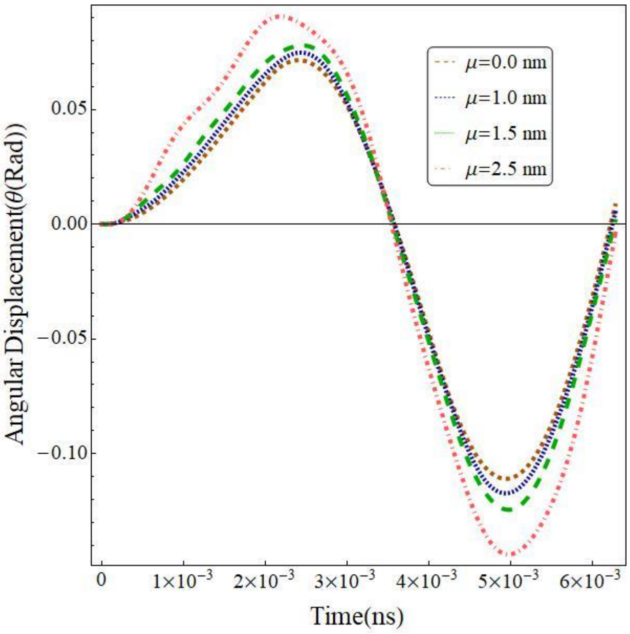
It can be found from Fig. 3(b), as the value of thickness increases, θ decreases in relative to the specified time, and increases for each case (e.g., different thicknesses). The behaviors of the diagrams are almost similar, and is fluctuating. The amplitude of the vibration becomes less for more values of thicknesses, and the slope of the diagrams decreases by increasing the value of thickness. All the diagrams are ascending. Fig. 3(a) indicates the variation of angular displacement with respect to the time, based on harmonic m_t, which is equivalent to $M_0 \sin \Omega t$; the excitation frequency and the amplitude of m_t are equal to $\Omega = 1000$ GHz and $M_0 = 1$ nN.nm, respectively. The variation of the nonlocal parameter is also equal to μ =0.2, 1, 1.5 and 2.5 nm. It is Comprehensible from Fig. 3(a) that the magnitude of the angular displacement increases by increasing the value of the nonlocal parameter. The angular displacement regardless of the value of the nonlocal parameter becomes zero at the specified time. CNT acts like a spring, and an increase in the values of the nonlocal parameter causes the stiffness to increase.

Consequently, the magnitude of the angular displacement closes to zero at a specified time for a constant value of m_t and different values of the nonlocal parameter (due to the applied harmonic load). The dimensionless angular displacement versus time is plotted in Fig. 4b; M_0 is also equal to 1nN.nm; the nonlocal parameter is μ =1nm. It is clear that for all cases disregarding the value of ratio, $\bar{\theta}$ increases first and then decreases with steep slope over time. After that, it continues fluctuating behavior, and when the time arrives at $t = 2 \times 10^{-3}$ ns, this ratio becomes infinity for those cases that contain ratio $\Omega/\omega =$ 2, 3 and 4. It is just because of the

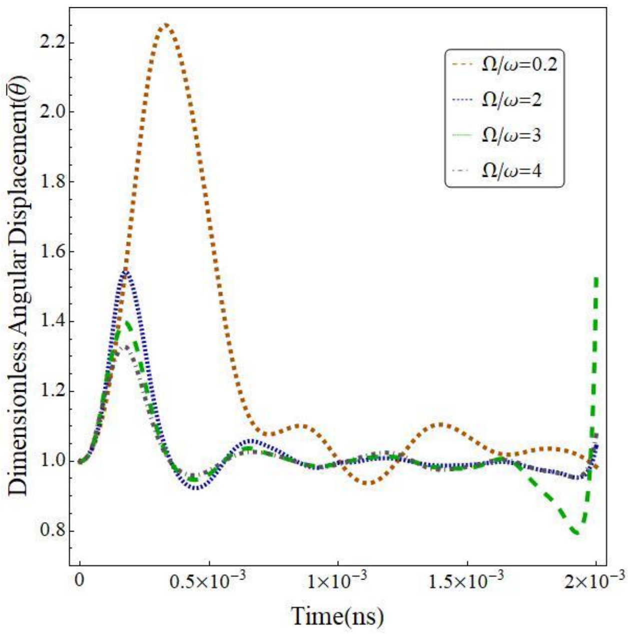


(a) (b)

Fig. 2 (a)Variation of dimensionless angular displacement versus time for four different nonlocal parameters; (b) Variation of angular displacement versus time for four different CNT's thicknesses and constant nonlocal parameter $\mu = 1.5$ nm. Linear external torque is equal to $a_0 = 1$ nN.nm/ns for (a) and (b)



(a)



(b)

Fig. 3 (a)Variation of angular displacement versus time for four different nonlocal parameters; (b) Variation of dimensionless angular displacement versus time for four different excitation frequency(ratio of excitation frequency to natural frequency) and constant nonlocal parameter $\mu = 1.5$ nm. Harmonic external torque is equal to $M_0 = 1$ nN.nm for (a) and (b)

angular displacement for classical form at this time becomes zero. For the case that $\Omega/\omega = 0.2$, it will take more times to become infinity. As the ratio increases, the fluctuating amplitude becomes less.

4. Conclusions

In this present work, the torsional vibration of the chiral SWCNT (12,6) under the time-dependent linear and harmonic external torques was analyzed. The governing equation of motions and boundary conditions were derived using Hamilton's principle. A Galerkin method was established for clamped-clamped boundary condition to discretize of derived differential equation. The Eringen's nonlocal elasticity theory was utilized to justify the small-scale effect. It was proved:

- The increasing of the nonlocal parameter has an inverse effect on the stiffness and subsequently the natural frequencies for free vibration.
- The obtained natural frequencies were in good agreement with the results of another study.
- The variation of the time-dependent angular displacement and nondimensional angular displacement versus time in SWCNT was investigated for the first time.
- It was discerned from the results that an increase in the value of the nonlocal parameter causes an increase in magnitude of the angular displacement and nondimensional angular displacement regardless type of loading.
- Raising of thickness leads to a reduction in angular displacement under the linear and harmonic loading, and raising of the excitation frequency to natural frequency ratio first increases, and then decreases the nondimensional angular displacement with steep slope, and then fluctuates in less amplitude till the resonance happens.

As mentioned, the study of the forced torsional vibration in the time domain is workable, and it can be developed. Considering this fact, the size dependency on the torsional behavior of the MWCNT, FG nanobeam, as well as the forced vibration of CNT under the multiple loadings with different boundary conditions, namely clamped-attached mass and clamped-torsional spring, can be worked and focused. Furthermore, different theories can be used to continue this study.

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