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Nonlocal effect on the vibration of armchair and zigzag SWCNTs with bending rigidity

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Abstract. Vibration analysis of carbon nanotubes (CNTs) is very essential field owing to their many promising applications in tiny instruments. In current study, the Eringen's nonlocal elasticity theory with clamped-clamped and clamped-free end conditions is utilized for the vibration analysis of armchair and zigzag SWCNTs. The Fourier method is utilized to solve the ordinary differential equation. The motion equation for this system is developed using a novel wave propagation method. Complex exponential functions have been used and the axial model depends on BCs that has been described at the edges of CNTs. The behavior of different nonlocal parameters is considered to find the vibrational frequency of SWCNTs. It is exhibited that the effect of frequencies against aspect ratio by varying the bending rigidity. It has been investigated that by increasing the nonlocal parameter decreases the frequencies and on increasing the aspect ratio increases the frequencies for both the tubes. To generate the fundamental natural frequencies of SWCNTs, computer software MATLAB engaged. The numerical results are validated with existing open text. Since the percentage of error is negligible, the model has been concluded as valid.

Keywords: nonlocal theory; WPA; carbon nanotubes; wave propagation approach; Fourier method

1. Introduction

Carbon nanotubes (CNTs) fascinate new materials with astonishing mechanical, optical and electrical properties (Ren *et al.* 2011). They are generated by rolling of the graphene sheet (Iijima 1991, O'Connell 2006). Carbon nanotube sheets include hexagonal cells that are ideally cut to produce carbon atoms of the tube. In fact, CNTs are kinds of rolled graphene sheets, and the rolling manner shows the basic properties of the tube, and that is actually the main reason for the extraordinary feature of the CNTs (Georgantzinos *et al*. 2009). Vibrational characteristics of various nano-structures are widely investigated based on nonlocal beam model. Specifically, carbon nanotube as one of the most practical/applicable miniature structure attracts many researchers in order to analytically and experimentally probes its dynamical properties using the nonlocal beam theory (Zemri *et al*. 2015, Youcef *et al.* 2018). The nonlocal theory mostly focused on the free vibrational analysis of the nano-structure, especially, carbon nanotubes.

In addition, nano-structures can be mentioned as the important types of devices which have wide applications in a variety of technological and scientific fields (Tserpes and Papanikos 2005, Ansari *et al.* 2011, Soltani *et al.* 2012, Mouffoki *et al.* 2017, Bouadi *et al.* 2018). Nanotubes and micro-beams can be cited as one of the very applicable micro- and nano-structures in various systems, namely, sensing devices, communications and the quantum mechanics. The application of the tiny structures, specifically, carbon nanotubes in the sensors and actuators enforce the engineers to study vibrational properties of those structures experimentally and theoretically. The nonlinear forced vibration of carbon nanotubes has seldom been observed (Das *et al.* 2013, Bocko and Lengvarský 2014, Reddy and Pang 2008). However, this issue is very crucial due to the widespread application of the forced nonlinear vibration carbon nanotubes in many practical instruments.

Due to this, a new model is required to observe the nano-size structure. Some investigators studied the higher order elasticity theories (Murmu and Pradhan 2009, Civalek *et al*. 2009, Narendar and Gopalakrishnan 2011, Yayli 2013, Demir and Civalek 2016). Different non-classical elasticity theories have attracted the researcher's attention as: stress and strain theories (Mindlin and Tiersten 1962, Toupin 1964, Karami *et al.* 2018a, b), strain theories (Fleck and Hutchinson 1993), and nonlocal theory (Eringen and Edelen 1972, Eringen 1983). For the interpretation of vibrational influence of SWCNTs, the nonlocal elasticity

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theory and TBM are used (Mindlin and Tiersten 1962, Toupin 1964, Fleck and Hutchinson 1993). Many Researchers used different theories for the dynamic response, buckling, stability and instability of CNTs, pipes, and plates (Kolahchi and Bidgoli 2016, Kolahchi *et al.* 2017a, 2016a, b, Kolahchi and Cheraghbak 2017, Hajmohammad *et al.* 2017, 2018a, b, 2019, Golabchi *et al.* 2018). Lee and Chang (2008) analyzed the vibration mode shape and frequency of fluid-filled SWCNTs using nonlocal elasticity theory (NLT). It is found that mode shape and frequency are influenced significantly by the nonlocal parameters and observed that the frequency component decreases as the nonlocal parameters increase. Murmu and Pradhan (2009) investigated the vibrational frequencies with different modes along temperature change using nonlocal small scale effects. On the other side, for length scale coefficient and soft elastic medium with embedded carbon nanotube, the nonlocal frequencies are comparatively lower. It is also found that the frequencies of the nonlocal model at different stages of temperature are higher than the nonlocal with same temperature. Eringen nonlocal theory and Von-Karman geometry were fully studied by Yang *et al*. (2010). Vibration frequencies of zigzag SWCNTs (5, 0), (8, 0), (9, 0) and (11, 0), with different boundary conditions are considered and calculated numerically through MD simulation. The influence of nonlocal parameter on height and radius are studied in detail. Ansari *et al*. (2011) presented the large vibrations of the CNTs considering Eringen's nonlocal theory. They applied the Rayleigh-Ritz technique and obtained the frequency of the DWCNT association with different values of aspect ratio. The results were presented for different zigzag and armchair. Das *et al*. (2013) studied the nonlocal theories for the in-extensional vibration of SWCNTs. The in-extensional mode frequency is treated by the positive strain gradient theory with circumferential wave number. Thongyothee *et al*. (2013) investigated the Euler beam theory for SWCNTs and the classical solution yields the result and is compared with finite element method (FEM). In this study, effects of different geometrical boundary condition and tube chirality are considered. The numerical results with low aspect ratios are in good agreement with classical solution. Furthermore, in this study, the first order-ten modes for boundary conditions and different aspect ratios and repeated natural frequencies are also highlighted. Ansari and Arash (2013) investigated vibrations of DWCNTs based on NLT using differential quadrature method (DQM). The mechanical behavior of DWCNTs with geometrical parameters layer wise boundary conditions and small scale factors are fully investigated. Bocko and Lengvarský (2014) assessed the bending vibration responses of CNTs with various conditions. The fundamental natural frequency (FNF) with different nonlocal parameters as well as two distinct diameter and continuously changed length was computed by the nonlocal theory. It has been represented by them the nonlocal parameters are highly influential on the bending vibration of a carbon nanotube. It was shown that the boundary conditions with nonlocal parameter are more effective on the nanotube vibration. Moreover, it has been observed that on enhancing the length of CNT, the frequencies decrease by increasing of the nonlocal parameter.

Ansari and Rouhi (2015) studied stability of SWCNTs under axial load and demonstrated the small scale effects of lengths based on Rayleigh-Ritz methods. The axial buckling of armchair (8, 8) SWCNTs are found with various boundary conditions by applying the molecular dynamic simulation. By adjusting the nonlocal parameter with bending rigidity and in-plane stiffness to predict the results of MD simulations. Besseghier *et al*. (2015) presented the nonlinear vibration of zigzag SWCNTs based on Winkler-type model. The energy-equivalent model was used for the derivation of general equation. Arani and Kolahchi (2016) used the nonlinear buckling of SWCNTs resting on elastic foundation. The mixture rule was employed for buckling analysis of embded CNTs with Euler and Timoshenko beam model. The influence of geometrical parameter and elastic foundation with different boundary conditions was investigated. Soltani *et al*. (2016) investigated the nonlinear vibrational characteristics of SWCNTs using the theory of nonlocal elasticity and Karman's geometric non-linearity theory. The controlling equation is derived from Donnell's shell theory and partial differential equations are converted into differential equations by invoking Galerkin's technique. The influence of aspect ratios, nonlocal parameters, nonlinear parameters and circumferential parameters are investigated. Bilouei *et al*. (2016) and Zamanian *et al*. (2017) studied the buckling behavior of concrete columns with nanofiber reinforced polymer and SiO_2 nano-particles. By using the strain-displacements, Hamilton's principles and Mori- Tanka approach, the governing equation was derived. Numerical results were presented with the variation of elastic foundations. Madani *et al*. (2016) investigated the vibration of embedded FG-CNT-reinforced piezoelectric cylindrical shells using differential quadrature method (DQM). The mixture rule of four different types of distribution was used in the thickness direction. Kolahchi (2017) and Kolahchi *et al*. (2017b, c) studied the bending and buckling of viscoelastic and non-viscoelastic sandwich nanocomposits using DQM, zigzag theory and Grey Wolf algorithm. Numerical results for volume fraction, and piezoelectric layers for the role of actuator and sensor. Akgöz and Civalek (2017) investigated the buckling analysis of SWCNTS using elastic foundation. The governing equation was obtained using several with boundary conditions. Avcar (2015, 2019) presented the vibration of FG beam and effect of rotary inertia of beam by the process of manufacturer. The thickness was controlled by the rule of mixture with volume fraction law. The governing equation was derived by classical and deformation theory with power law and sigmoid law. The frequencies for span to depth ratio with varying volume fraction index were examined in detail. Bouadi *et al*. (2018) developed the new model displacement field for the nonlocal buckling properties of single graphene sheet. The Eringen relation was used for the theoretical formation with length scale parameter. Avcar and Mohammed (2018) studied the vibration of FG beam resting on elastic foundation. The elastic foundation was linear, isotropic and homogenous. Various boundary conditions were used to find the numerical results of vibration of FG beam.

Recently, research on vibration of single-walled carbon nanotubes have been done by many material researchers (Hussain and Naeem 2017, 2018a, b, 2019a, b, c, Hussain *et al*. 2017, 2018a, b). Yazid *et al*. (2018) presented new refined plate theory by employing nonlocal small effects. By using the principle of virtual displacements, the nonlocal relation for equation of motion was obtained. The results presented here may provide a useful design for nanostructures. Civalek *et al*. (2009) and Ebrahimi and Mahmoodi (2018) presented the static analysis of SWCNTs and vibration of CNTs using Eringen's and Euler beam theory. The bending moment and function of strain were performed with different boundary conditions. Boutaleb *et al*. (2019) and Youcef *et al*. (2018) performed the dynamic analysis of nanosize rectangular plates and beams with nonlocal quasi 3D HSDT and surface elasticity theory. The effect of various parameters such as thickness-radius ratio, aspect ratio, beam thickness, material index, surface density and surface elastic constants. Karami *et al*. (2017, 2018a, b) conducted the analysis analysis of FG nanoplates, spherical nano particles and inisotropic nanoparticles using quasi-3D model, 3D elasticity theory and nonlocal strain gradient theory. They investigated the dispersion analysis of FG nanoplates nonlocal strain gradient effect and triaxial magnetic effect. Ehyaei and Daman (2017) and Eltaher *et al*. (2019) investigated the vibration characteristics of SWCNTs and DWCNTs using initial perfection and continuum mechanics approach. The general equation of motion was obtained by Hamiltonian principle and energy equivalent model. The numerical frequencies of DWCNTs and SWCNTs were determined by Navier method and finite element method. Semmah *et al*. (2019) investigated the buckling analysis of zigzag single walled boron nitride based on Winkler foundation. The governing equation was taken into account with the shear deformation theory. Effect of different nonlocal parameter was investigated with closed form solution. Many material researchers used varius methods for new results of nanocomposits (Zarei *et al.* 2017, Hajmohammad *et al.* 2018a, b, Amnieh *et al*. 2018, Fakhar *et al.* 2018, Hosseini *et al.* 2018, Jassas *et al.* 2019).

The foremost intension of this paper to investigate vibrations characteristics of armchair and zigzag SWCNT by means of nonlocal theory, which is our particular motivation. The suggested method to investigate the solution of fundamental relations is wave propagation approach (WPA), which is a well-known and efficient technique to develop the fundamental frequency equations. It is keenly seen from the literature, no evidence is found concerning current model where such problem has been studied so it gave impetus to conduct present work. The specific influence of four different end supports based on WPA is examined in detail.

In addition, earlier Researchers have utilized different methodologies to investigate the vibrational behavior of CNTs (single- and double-walled carbon nanotubes, for example nonlocal continuum mechanics (Wang and Varadan 2006), Timoshenko beam model (Simsek 2011), finite element method (Mohammadimehr and Alimirzaei 2016), Non-local theory of elasticity (Kolahchi *et al.* 2019) and differential quadrature method (Azmi *et al.* 2019).

A comprehensive estimation regarding nonlocal theory has been considered for vibrational behavior of the SWCNTs with distinct nonlocal parameters (ξ = 0.5, 1, 1.5, 2). Vibrations of SWCNTs for armchair indices (5, 5), (7, 7), and zigzag indices (8, 0), (15, 0) have been analyzed. We developed a new model from the combination of the nonlocal theory of elasticity with wave propagation approach. It is noted that the frequencies of C-C is higher than that of C-F. Also, WPA has been utilized for first time to consider the effects of bending rigidity on SWCNTs vibration. This modified model has less complication and has been compared with the earlier methods. The computational results indicated that there is inverse relation of nonlocal parameters and frequencies. The obtained results show that by increasing aspect ratio of carbon nanotubes, frequency value increases at all boundary conditions. In our measurement we indicated that with higher aspect ratio, the BCs have a momentous influence on vibration of CNT. It can be concluded that frequencies would increase by increasing of the bending rigidity. This means that smaller effects play an important role in predicting SWCNT frequencies, which local theory cannot capture.

2. Theoretical formulation

When a graphene sheet is rolled with its hexagonal cells, the structure can be conceptualized as SWCNTs and its circumference and quantum properties depend upon the chirality and diameter described as a pair of (n, m). In addition, the integers n and m represent the orientation of the graphene honeycomb lattice. Fig. 1 shows the orientation of the graphene sheet as, nanotubes become armchair, if n = m and the nanotubes are zigzag, if m = 0. In classical theory, physical quantities act as local behaviors but in traditional theory, the stress at this point is affected by the strain at this point. According to Eringen (2002) theory of nonlocal, the stress at applied point regarded as a functional of all points of strain field and this theory is absolutely different from all other theories as conventional theory.

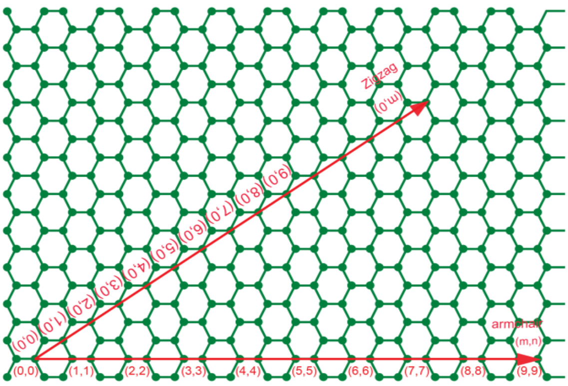


Fig. 1 Hexagonally description of armchair and zigzag SWCNTs on the graphene sheet

According to this theory, it is possible to express a nonlocal relationship based on a homogeneous isotropic beam as

$$\varepsilon_{xx} - \mu^2 {\chi_i}^2 \frac{\partial^2}{\partial x^2} \varepsilon_{xx} = E\sigma_{xx} \quad (1)$$

The factor $\xi = \mu^2 {\chi_i}^2$is called the small scale affect, where μ and χ_i are the material constant and lattice spacing length or internal characteristic length.

Eq. (1) can be stated as

$$\varepsilon_{xx} - \xi \frac{\partial^2}{\partial x^2} \varepsilon_{xx} = E\sigma_{xx} \quad (2)$$

Where ε_{xx} is the normal stress, σ_{xx} is the normal strain, E is the young's modulus and ξ is called the nonlocal parameter of SWCNTs. These parameters are used to explore the bending, buckling and vibration of beams and tubes (Thongyothee and Chucheepsakul 2008, Reddy and Pang 2008). According to Euler beam theory (Thongyothee and Chucheepsakul 2008), for the free vibration of the CNT, the controlling equation of motion including NLT, one could have

$$\tau(x)\xi \frac{\partial^2 u}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 u}{\partial t^2}\right] = 0 \quad (3)$$

Where τ, I stand for the mass per unit length and moment of inertia of CNT. Two systems of ordinary differential equations (ODEs) are derived using the Fourier method of variational separation. In this system, two terms are related to the spatial variable x and temporal variable, respectively.

$$u(x,t) = \gamma(x)S(t) \quad (4)$$

$$\tau\xi \frac{\partial^2}{\partial t^2}\gamma(x)S(t) + \frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2}{\partial x^2}\gamma(x)S(t)\right] = 0 \quad (5)$$

$$\tau\xi\gamma(x)\frac{d^2 S}{dt^2} + EIS(t)\frac{d^4\gamma}{dx^4} = 0 \quad (6)$$

$$EIS(t)\frac{d^4\gamma}{dx^4} = -\tau\xi\gamma(x)\frac{d^2 S}{dt^2} \quad (7)$$

For harmonic response

$$S(t) = e^{iwt} \text{ or } cos\omega t \text{ or } sin\omega t \quad (8)$$

Substitute Eq. (8) into Eq. (7), the relation can be written as

$$\tau\xi\gamma(-\omega^2 cos\omega t) + EIcos\omega t\frac{d^4\gamma}{dx^4} = 0 \quad (9)$$

$$\frac{d^4\gamma}{dx^4} - \frac{\tau\xi\omega^2}{EI}\gamma(x) = 0 \quad (10)$$

$$\frac{d^4\gamma}{dx^4} - \lambda^4\gamma(x) = 0 \quad (11)$$

Here $\gamma(x)$ denotes the mode shape (Eigen shape)

For parameter λ

$$\lambda^4 = \frac{\tau\xi\omega^2}{EI} \quad (12)$$

The general solution of fourth order ODE is postulated as

$$\gamma(x) = q_1 sin\lambda x + q_2 cos\lambda x + q_3 sin\lambda x + q_4 cosh\lambda x \quad (13)$$

where q_1, q_2, q_3, and q_4 are the unknown constants.

Eq. (11), becomes as

$$\gamma^{iv}(x) - \lambda^4\gamma(x) = 0 \quad (14)$$

3. Numerical technique

Wave propagation approach is used to study the vibrational behavior of SWCNTs. Before this work current approach was successfully used for vibration and buckling analysis of cylindrical shell (Hussain *et al.* 2018a, c, Hussain and Naeem 2018a, b) and plates vibrations (Hussain and Naeem 2018b). The Galerkin's method (Hussain *et al*. 2018b, Hussain and Naeem 2019a, b) has been used for the frequencies calculations of SWCNTs. This technique contains many integrals of these functions and a long process to solve these integrals. A new exponential form for the deformed axial functions $\gamma(x)$ can be written as

$$\gamma(x) = e^{-i\Gamma_m} \quad (15)$$

For vibrating carbon nanotubes, the axial wavenumber Γ_m related to support conditions applied on both sides of SWCNTs and m denotes axial half-wave number.

$$\gamma^{iv}(x) = {\Gamma^4}_m e^{-i\Gamma_m x} \quad (16)$$

Substituting Eq. (16) into Eq. (14), we have

$${\Gamma^4}_m e^{-i\Gamma_m x} - \lambda^4 e^{-i\Gamma_m x} = 0 \quad (17)$$

$$\lambda^4 = {\Gamma^4}_m \quad (18)$$

By using the Eq. (12), we can write as

$$\frac{\tau\xi\omega^2}{EI} = {\Gamma^4}_m \quad (19)$$

4. Nonlocal boundary conditions

Appropriate material properties and boundary conditions is applied and then the model is solved for natural frequencies SWCNTs of different indices (5, 5), (7, 7), (9, 9) for armchair and for zigzag indices (8, 0), (15, 0), (20, 0) SWCNTs.

From Eq. (19)

Table 1 Comparison of nondimensional frequencies $\Delta = \omega R\sqrt{\rho/E}$ (L/R = 1, n = 1)

m	n = 1		n = 2	
	Alibeigloo and Shaban (2013)	Present	Alibeigloo and Shaban (2013)	Present
0	0.97087	0.97063	0.99351	0.99289
1	0.59721	0.59698	0.88357	0.88301
2	0.34025	0.34019	0.68072	0.68013
3	0.20145	0.20099	0.50059	0.5003
4	0.12886	0.12872	0.36918	0.36897
5	0.09105	0.09087	0.27671	0.27662

For C-C

$$\frac{\tau\xi\omega^2}{EI} = \left(\frac{(2n+1)\pi}{2L}\right)^4 \tag{20}$$

Here $\Gamma_m = \frac{(2n+1)\pi}{2L}$ (C-C boundary condition)

For C-F

$$\frac{\tau\xi\omega^2}{EI} = \left(\frac{(2n-1)\pi}{2L}\right)^4 \tag{21}$$

Where $\Gamma_m = \frac{(2n-1)\pi}{2L}$ (C-F boundary condition)

From Eq. (19) the fundamental natural frequencies are calculated where $\omega = 2\pi f$. There exists uncertainty in defining the nanotube thickness. Here, we apply relations from (Tserpes and Papanikos 2005).

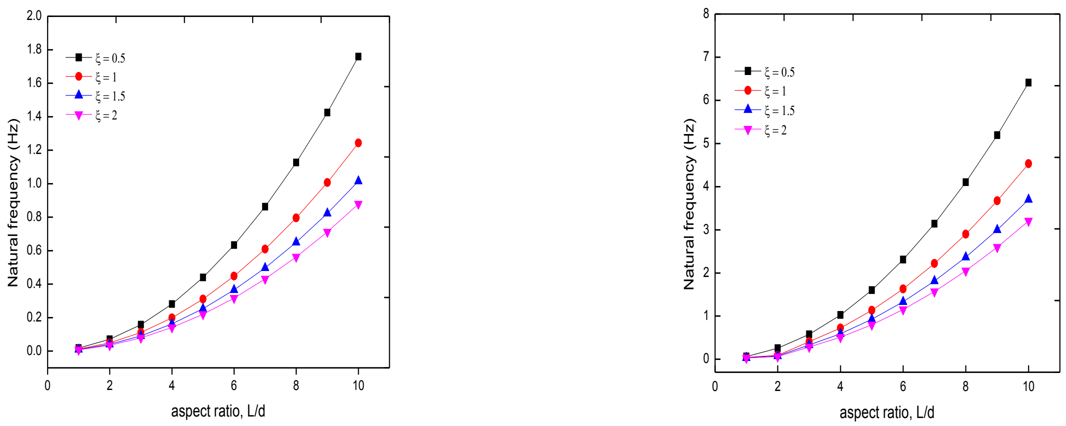
$$m = \rho A = 2.4 \times 10^{-24} d[kg/nm] \tag{22}$$

$$EI = 428.48d^2 - 397.08d + 109.24[kgnm^3/s^2] \tag{23}$$

Diameter of nanotubes is indicated by d and can be calculated from translation indices (n, m) by relation.

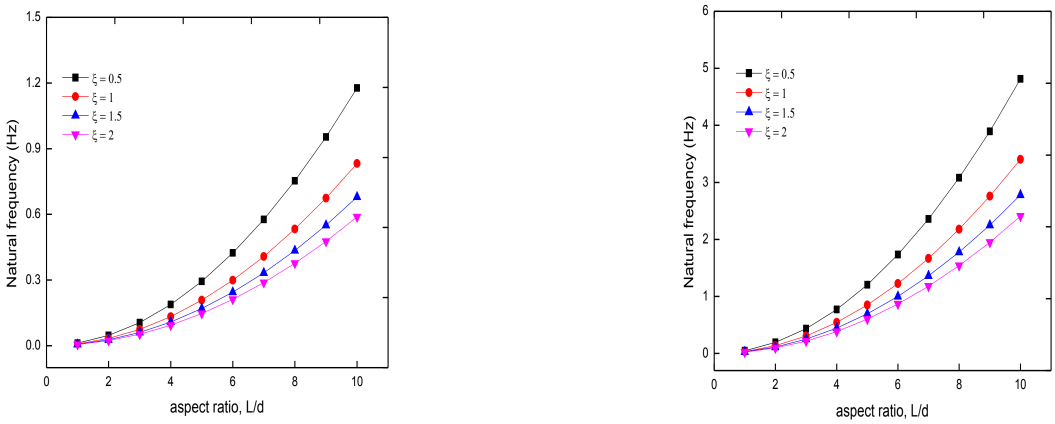
$$d = 2R = a_0\sqrt{3(m^2 + n^2 + nm)}/\pi \tag{24}$$

Where the carbon-carbon bond length (a_0 = 1.42 Å). It can be seen that the error percentage is negligible, hence showing high rate of convergence. The results of nondimensional frequency are computed for two different values of n = 1, 2 with circumferential wave number (***m*** = 0, 1, 2, 3, 4, 5) as shown in Table 1. Alibeigloo **and** Shaban (2013) investigated the impact of nonlocal parameters on the vibration of CNTs by using the three-dimensional elastic theory based on the Fourier series expansion. It was concluded that the frequency decreased when nonlocal parameters increased. The proposed model based on WPA can incorporate in order to accurately predict the acquired results of material data point. Figs. 2-3 show the natural frequencies of armchair C-C and C-F SWCNTs versus aspect ratio with nonlocal parameters. The natural frequencies are calculated of C-C armchair (5, 5) SWCNTs against ratio of length-to-diameter with different nonlocal parameters (ξ = 0.5, 1, 1.5, 2). It is observed that the frequency values (L/d = 1~10) decreases on increasing the nonlocal parameter (ξ = 0.5, 1, 1.5, 2). The corresponding frequency modes have been sketched in Fig. 2. It is indicated that the frequencies with different nonlocal parameters are nearly equal at L/d = 1, but at L/d = 5, the gap between frequencies is visible and as the frequencies increase and at L/d = 6 ~ 10, the gap between frequencies curve also increases. Fig. 2, shows the variation of the frequencies of C-C armchair (7, 7) with same nonlocal parameters. The frequency value at L/d = 1 (10) are 0.0641 (6.4093), at ξ = 1, 0.0453 (4.5321), at ξ = 1.5, and 0.0370 (3.2046) at ξ = 2, respectively. We can see that the frequency curves increases with the increases of indices, wherein the same trend of the curves are seen. These patterns get excited at greater frequencies than C-C (5, 5) also maintain regularity in the wave form patterns. It can be depicted from these figures that the natural frequencies decrease by increasing the nonlocal parameters (ξ = 0.5, 1, 1.5, 2). It is also concluded that with same parameters and bending rigidity, frequency values become larger and larger as we increases the indices. Fig. 3 show that the effect of fundamental natural frequencies against length-to-diameter ratio of C-F armchair (5, 5), (7, 7) with different nonlocal parameters (ξ = 0.5, 1, 1.5, 2). When (L/d, ξ = 1, 0.5 ~ 2)



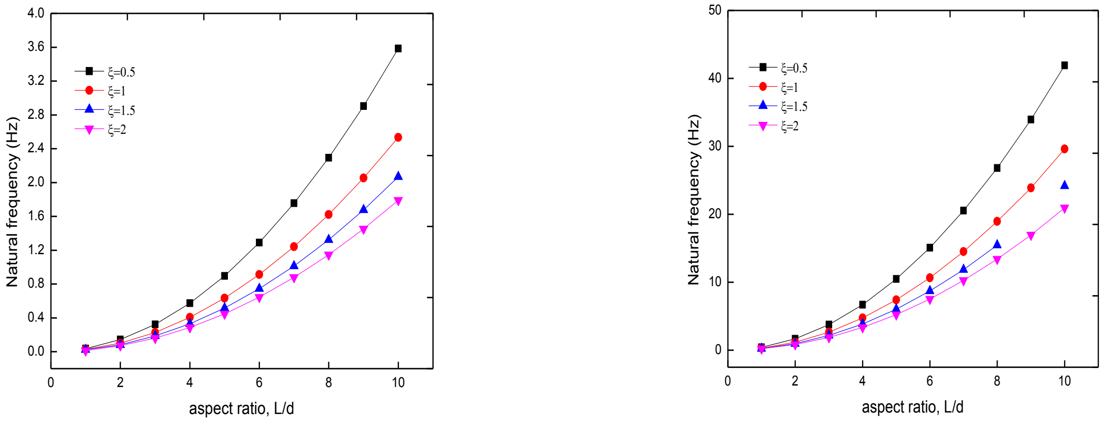
(a) (b)

Fig. 2 Aspect ratios against frequencies of C-C armchair (a) (5, 5); (b) (7, 7) with ξ = 0.5, 1, 1.5, 2



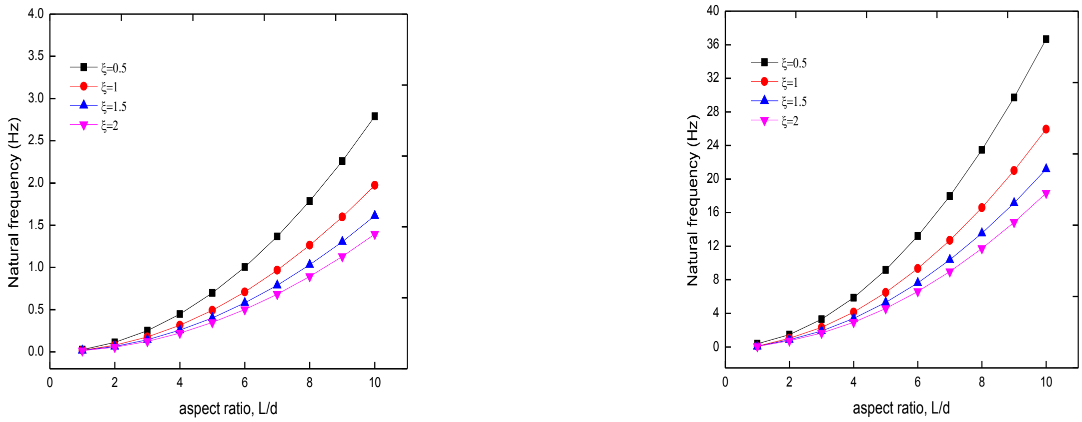
(a) (b)

Fig. 3 Aspect ratios against frequencies of C-F armchair (a) (5, 5); (b) (7, 7) with ξ = 0.5, 1, 1.5, 2



(a) (b)

Fig. 4 Aspect ratios against frequencies of C-C zigzag (a) (8, 0); (b) (15, 0) with ξ = 0.5, 1, 1.5, 2



(a) (b)

Fig. 5 Aspect ratios against frequencies of C-F zigzag (a) (8, 0), (b) (15, 0) with ξ = 0.5, 1, 1.5, 2

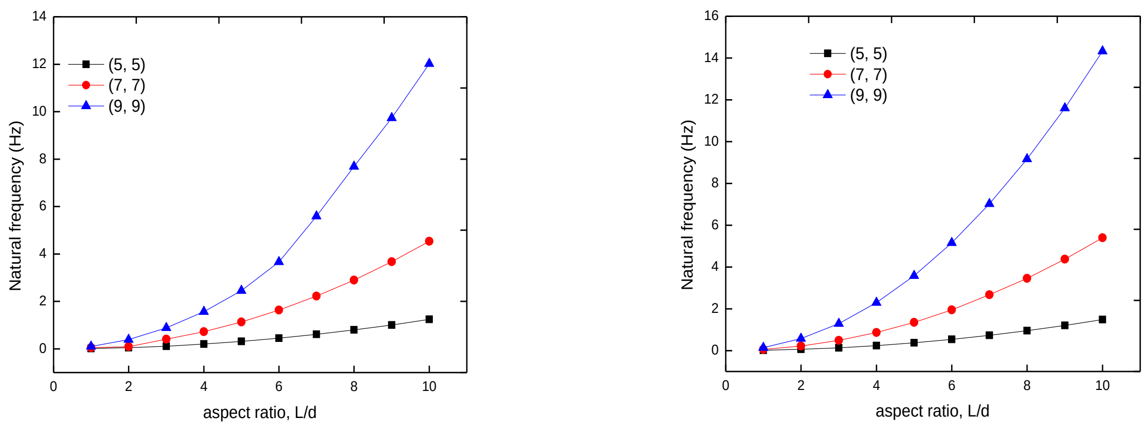
for C-F armchair (5, 5) are 0.0118 ~ 0.0059 and (*L/d*, ξ = 1, 0.5 ~ 2) is 1.1772 ~ 0.5886 as shown in the Fig. 3. With same parameters, for armchair (7, 7) are 0.0481~0.0241 and 4.8141~2.4070, respectively, has been shown in the Fig. 3. The trend of the frequencies is same as the C-C [= (5, 5), & (7, 7)], but it is noted that with each index the C-F values is lower than those of corresponding C-C frequencies. It is also observed that these frequencies have a paramount impact on the vibration of CNTs and this is due to the constraints which are applied on the edges of CNT. Fig. 4 show the variation of the frequency with zigzag indices (8, 0) and (15, 0) with different nonlocal parameters

(ξ = 0.5, 1, 1.5, 2). It can be seen in Fig. 4, when the index changes from zigzag (8, 0) to (15, 0), the frequency becomes larger. It is observed that in the zigzag case, the frequencies are higher than that of armchair case. The reason for this is that the zigzag of the main carbon structure has several elements parallel to the tube axis, while the armchair tube does not have these characteristics. For this case, the frequency curves are much lower than that of above clamped-clamped CNTs. In deepness, to understand the vibration characteristics of carbon nanotubes, namely zigzag carbon nanotubes (8, 0), and (15, 0) with bending rigidity, different nonlocal parameters and aspect ratio of 1~10 are considered and the results are discussed. Fig. 5 show the C-F frequencies of different zigzag indices with different nonlocal parameters. Next, the frequency values with C-F zigzag (8, 0) at (*L/d*, ξ) = (1, 0.5 and 2) are *f* (Hz) ~ 0.0279, 0.0140 and at (*L/d*, ξ) = (10, 0.5and 2) are *f* (Hz) ~ 2.7905, 1.3952 as shown in the Fig. 5. For the same parameter with C-F zigzag (15, 0), the computed values are *f* (Hz) ~ 36.6680, 18.3340 as shown in Fig. 5. It is observed that in the zigzag case, the frequencies are higher than that of armchair case. The reason for this is that the zigzag of the main carbon structure has several elements parallel to the tube axis, while the armchair tube does not have these characteristics. As a result, zigzag type CNTs are expected to have greater bending and longitudinal stability than that armchair CNT.

5. Vibration of SWCNTs with bending rigidity

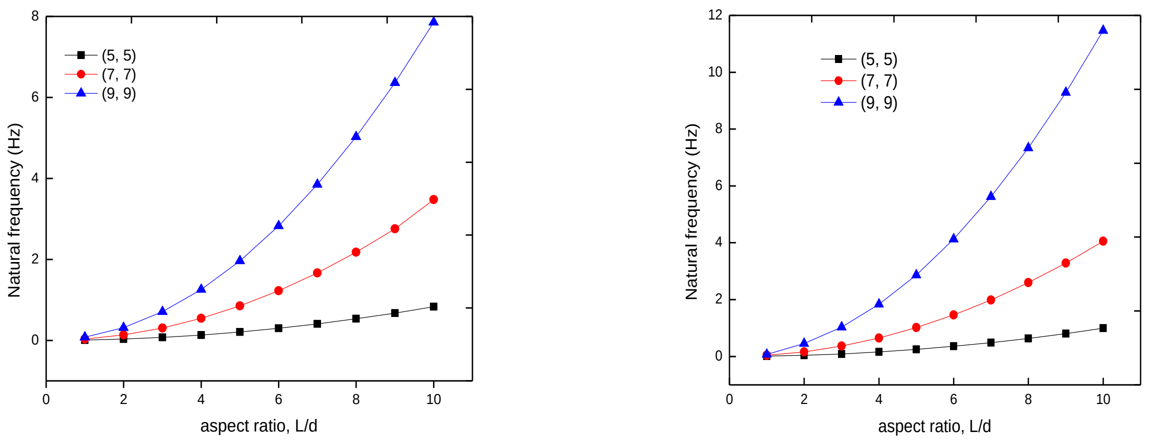
5.1 Effect of bending rigidity on the vibration of armchair SWCNTs

Figs. 6-7 show FNFs with (5, 5), (7, 7) and (9, 9) armchair type SWCNTs calculated with WPA based NLT under boundary conditions C-C and C-F. As seen in these figures, with the increase of bending rigidity (*EI* = 5.1122 e^{-9} nm~7.2617e^{-9} nm) the frequencies increase, and with increasing *L/d* its value also increases, as C-C = (5, 5) *f* (Hz): 0.0124 ~ 0.0148 [C-F (5, 5) *f* (Hz): 0.0083 ~ 0.0099] and C-C = (7, 7) *f* (Hz): 0.0453~0.0540 [C-F (7, 7) *f* (Hz): 0.0340~0.0406] and C-C = (9, 9) *f* (Hz): 0.1202~ 0.1433[C-F (9, 9) *f* (Hz): 0.0962~0.0694] at *L/d* = 1. The fundamental



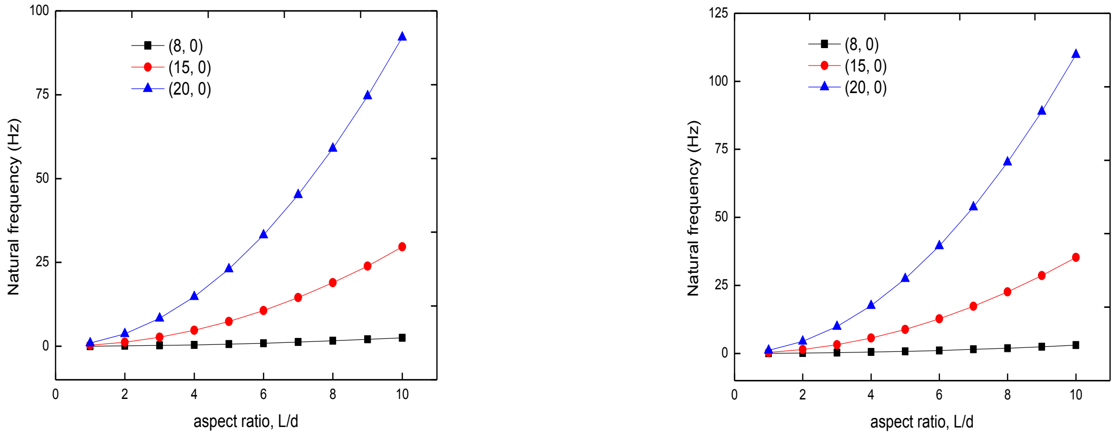
(a) (b)

Fig. 6 Aspect ratios against frequencies of C-C (5, 5), (7, 7) and (9, 9) armchair SWCNTs with (a) $EI = 5.1122e^{-9}$ nm; (b) $EI = 7.2617e^{-9}$ nm and ξ = 1



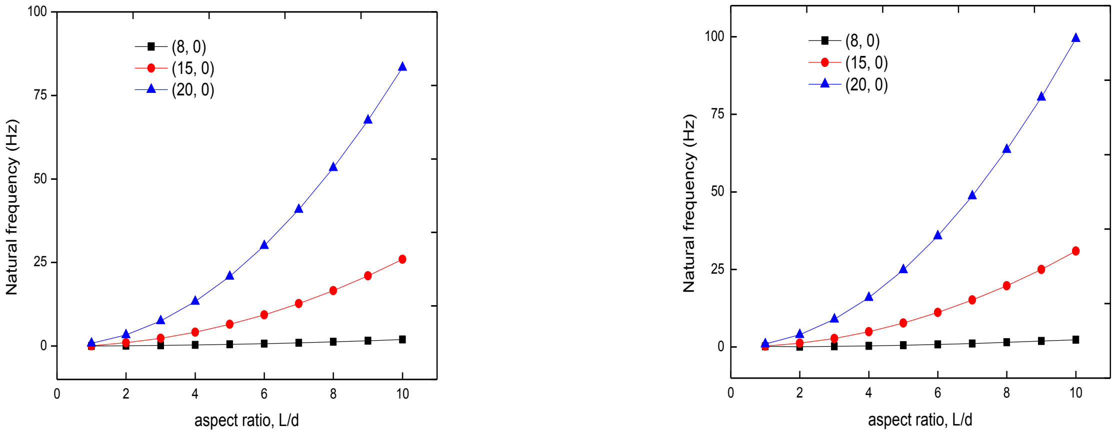
(a) (b)

Fig. 7 Aspect ratios against frequencies of C-F (5, 5), (7, 7) and (9, 9) armchair SWCNTs with (a) $EI = 5.1122e^{-9}$ nm; (b) $EI = 7.2617e^{-9}$ nm and ξ = 1



(a) (b)

Fig. 8 Aspect ratios against frequencies of C-C (8, 0), (15, 0) and (20, 0) zigzag SWCNTs with (a) $EI = 5.1122e^{-9}$ nm; (b) $EI = 7.2617e^{-9}$ nm and $\xi = 1$



(a) (b)

Fig. 9 Aspect ratios against frequencies of C-F (8, 0), (15, 0) and (20, 0) zigzag SWCNTs with (a) $EI = 5.1122e^{-9}$ nm; (b) $EI = 7.2617e^{-9}$ nm and $\xi = 1$

natural frequencies at *L/d* = 10 as C-C = (5, 5) *f* (Hz): 1.2435~1.4820 [C-F (5, 5) *f* (Hz): 1.1772~0.9921] and C-C = (7, 7) *f* (Hz): 4.5321~4.0571 [C-F (7, 7) *f* (Hz): 4.8141~ 4.0571] and C-C = (9, 9) *f* (Hz): 12.0201~35.3114 [C-F (9, 9) *f* (Hz): 9.6227~11.4687]. It can be mentioned that the FNFs decrease with the increase of bending rigidity (*EI*), for C-C (Fig. 6) and C-F (Fig. 7), and FNFs increase with the increasing of aspect ratio, *L/d*. These figures refer to the case when *EI* varying from $5.1122e^{-9}$ nm to $7.2617e^{-9}$ nm.

5.2 Effect of bending rigidity on the vibration of zigzag SWCNTs

Fig. 8-9 show the natural frequency behavior of the calculated SWCNT system under bending rigidity (*EI*). These figures show the frequencies of zigzag (8, 0), (15, 0) and (20, 0) SWCNTs, computed with nonlocal parameter ξ = 1 based on WPA. It can be seen that the fundamental natural frequency increases with the increase of bending rigidity, ($EI = 5.1122e^{-9}$ nm ~ $7.2617e^{-9}$ nm) and with the increasing of aspect ratio, frequency value increases, as C-C = (8, 0) *f* (Hz): 0.0297~0.0302 [C-F (8, 0) *f* (Hz: 0.0197~0.0235)] and C-C = (15, 0) *f* (Hz): 0.2963~0.0738 [C-F (15, 0) *f* (Hz): 0.0738~0.300)] and C-C = (20, 0) *f* (Hz): 0.9213 ~0.1.0981 [C-F (20, 0) *f* (Hz): 0.8336~0.9936)] at *L/d* = 1. The frequencies at *L/d* = 10 as C-C = (8, 0) *f* (Hz): 2.5344~ 3.0206 [C-F (8, 0) *f* (Hz): 1.9732~2.3517] and C-C = (15, 0) *f* (Hz): 29.6278~35.3114 [C-F (15, 0) *f* (Hz): 25.9282~ 30.902] and C-C = (20, 0) *f* (Hz): 92.1345~ 109.8089 [C-F (20, 0) *f* (Hz): 83.3650~99.3571]. It is evident from these figures that the FNF C-C = (8, 0), (15, 0) values are lower than C-C = (20, 0). As indicated by the figures that the fundamental frequencies increase with the increase of aspect ratio and its value increases with the bending rigidity.

6. Conclusions

The conclusion in this current paper shows the vibration of armchair and zigzag SWCNTs using nonlocal theory based on wave propagation approach with different end conditions. Frequency spectra of armchair (5, 5), (7, 7), (9, 9) and zigzag (8, 0), (15, 0), 20, 0) SWCNTs have been analyzed with proposed model. We developed a new model from the combination of the nonlocal elasticity theory with WPA. The governing equation has been developed for the vibrations of SWCNTs considering the nonlocal parameter with C-C and C-F boundary conditions. Effects of nonlocal parameters and bending rigidity have been fully investigated on the natural frequency against aspect ratios. It has been shown that frequency curves decrease as an increment in the nonlocal parameter increases by increasing of the aspect ratio. Additionally, it can be seen that by increasing in-plane rigidity, the frequencies are increased. Also, the frequency curves for C-F are lower throughout the computation than that of C-C curves. Accordingly, it would be interesting to consider the above model for the vibration of nonlocal rotating SWCNTs in future research.

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