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Size-dependent vibration analysis of laminated composite plates

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Abstract. The size-dependent vibration analysis of a cross-/angle-ply laminated composite plate when embedded on the Pasternak elastic foundation and exposed to an in-plane magnetic field are investigated by adopting an analytical eigenvalue approach. The formulation, which is based on refined-hyperbolic-shear-deformation-plate theory in conjunction with the Eringen Nonlocal Differential Model (ENDM), is tested against considering problems for which numerical/analytical solutions available in the literature. The findings of this study demonstrated the role of magnetic field, size effect, elastic foundation coefficients, geometry, moduli ratio, lay-up numbers and fiber orientations on the nonlocal frequency of cross-/angle-ply laminated composite plates.

Keywords: free vibration; Laminated composite plates; Pasternak foundation; Eringen nonlocal theory

1. Introduction

Over the past few years, micro-nanostructures with magnetic field effect have been attracted the attention of research community due to their fantastic characteristics and applications. Although considerable attention has been devoted to the mechanical characteristics of magneto-electro-elastic structures (Pan 2001, Pan and Heyliger 2002, Moshtagh *et al*. 2017), there are few studies concerning structures under external in-plane magnetic field. It is acknowledged that external magnetic field is able to change the stabilization factor of nanostructures without the demand of varying the material properties and geometrical parameters of them (Jalaei and Arani 2018). (Narendar *et al*. 2012) examined the critical influence of in-plane magnetic field and nonlocal parameter on the wave characteristics of single-walled carbon nanotubes (SWCNTs). The obtained results showed that the wave frequency amplifies with increasing strength of the magnetic field, whilst the nonlocal effect would decrease the wave frequency of the SWCNTs. The importance of the magnetic field on the free vibration of nanoplate on the basis of the nonlocal elasticity theory was examined by (Murmu *et al*. 2013). (Karami *et al*. 2018f) analyzed the role of magnetic field on the size-dependence wave behavior of nanoplates by considering the Lorentz magnetic force obtained from Maxwell's relation. By comparison with experimental data, their results indicated that the size effects on the magnetic field strength were not ignorable. Moreover they showed the influence of magnetic field on the results is more significant at low wave numbers when the dependence of wave frequency to material properties and size effects will be less important. (Karličić *et al*. 2017) studied the in-plane magnetic field impact on the damped-vibration of the viscoelastic orthotropic multi-nanoplate system using the nonlocal theory.

Because of requirement to have an advanced structure applicable in modern industrials, the attitude to use laminated composite structures considerably has been developed during recent years. It is due to the fact that these types of structrues present superior properties e.g., low cost, high corrosion resistance and high fatigue life (Gibson 2016). Moreover, laminated composite structures by incorporating together two (or more) constituents usually are more complex structures in comparison to isotropic ones because of growth in quantity of wrapped variables as well as the intrinsic anisotropy behavior of the particular layers (Reddy 2004). So, in order to have a safe design in these structures, firstly we need an efficient mathematical model. Since laminated composites are mainly applied as constituent of beams, shells and plates due to of their extradionay properties such as bend extensional as well as high strength to weight proportion, classical continuum theories (Farokhi and Ghayesh 2015, Gholipour *et al*. 2015, Guo *et al*. 2016, Farokhi and Ghayesh 2018a, Ghayesh *et al*. 2018) can be chosen to model them simply. Classical continuum theories have been developed for extensive studies relevant to mechancical performance of laminated composite structures recently (Sayyad and Ghugal 2015). Among these theories, higher-order-shear-deformation theories (HOSDTs), which contain variant shape functions along with fewer number of unknowns variables is involved in the equilibrium equations, satisfy the shear deformation effects on both bottom and upper surfaces of plate structures without employing any shear correction factor, if compared to the assumptions of First-Order Shear Deformation theories (FSDTs). For minimizing the total number of variables employed in the equilibrium equations, (Shimpi 2002) proposed a refined plate theory (RPT) model with two variables in order to study on isotropic plates. As such, using the RPT can reduced the strain and time of computation while it presents very accurate results.

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Afterwards, various validity studies were carried out based on the RPT models (Thai and Kim 2015, Yazid *et al*. 2018).

Given that classical continuum theories are not capable to predict the behavior of such structures at micro/ nanoscale, several non-classical continuum theories such as the nonlocal elasticty theory, strain gradient theory, nonlocal strain gradient theory and couple stress theories which contain additional small-scale parameters have been suggested to capture the size effect in terms of linear and nonlinear (Ghayesh *et al*. 2013a, b, c, d, 2014, 2016, 2017a, b, Ghayesh and Farokhi 2015, Eltaher *et al*. 2016, Zenkour 2016, Bouafia *et al*. 2017, Farokhi *et al*. 2017, Karami *et al*. 2017, Ebrahimi and Haghi 2018, Farajpour *et al*. 2018, Farokhi and Ghayesh 2018c, Ghayesh 2018a, b, c, Houari *et al*. 2018, Li *et al*. 2018, Shahsavari *et al*. 2018b, c, Wu *et al*. 2018, Karami *et al*. 2018a, b, Karami and Janghorban 2019a, b, c, Karami *et al*. 2019a, b, c, d, e, Karami and Karami 2019, Karami *et al*. 2019f, g, h, i, j, k, Salari *et al*. 2019, She *et al*. 2019, Tang *et al*. 2019a, b). As a popular model with some benefits, Eringen Nonlocal Differential Model (ENDM) (Eringen 2002) for considering the softening-stiffness mechanism of nanostructures is widely used in which the stress at a point is a function of strains at all points in the continuum body. Even though its shortcomings and limitations (Romano and Barretta 2017, Barretta *et al*. 2018), it seems this model is more thinkable and benefitical for engineering applications owing to its smoothly for comparing to the old ones. Up to data, on the basis of ENDM many studies have been conducted for investigation on size-dependent behvaior of single-walled as well as double-walled nanotubes (Zhang *et al*. 2005) nanoshells (Ghavanloo and Fazelzadeh 2013), nanobeams (Lim 2010, Aydogdu *et al*. 2018, Ebrahimi *et al*. 2018, Shahsavari *et al*. 2019), and nanoplates (Pradhan and Phadikar 2009, Chen *et al*. 2017, Shahsavari *et al*. 2017, Karami *et al*. 2018c, d, f, h, Shahsavari *et al*. 2018a, Karami and Shahsavari 2019, Karami *et al*. 2019l).

The main objective of this article is to examine applications of in-plane magnetic field on size-dependent vibration responses of laminated composite plates embedded in Pasternak foundation. A hyperbolic type of refined plate theory, which consider the triple effects of axial, bending and shear in conjunction with the ENDM which considers size effect are developed and will be discussed in Section 2. The equations of motion are derived in Section 3 according to Hamilton's principle. In Section 4, closed-form solutions based on the eigenvalue vibration response are derived for cross-ply as well as angle-ply laminated composite plates. Then, a comprehensive parametric study is carried out to show the effects of laminated composite plate geometry, foundation coefficients, nonlocal parameter, in-plane magnetic field, lay-up numbers, lay-up sequences of layers (symmetric as well as antisymmetric arrangements), fiber orientations and boundary condition (SS-1 and SS-2) on the vibration response in Section 5. Afterwards, crucial conclusions are drawn in the last section. The originality of this paper may be summarized as follows:

(1) The influence of the in-plane magnetic field has not been yet examined for the vibration response of laminated composite structures.
(2) Size-dependent vibration response of cross-ply as well as angle-ply laminated composite plates embedded in the elastic foundation are analyzed for the first time.
(3) The study covers the role of changes in the number of lay-up, fiber orientations on the size-dependent vibration of cross-ply and angle-ply laminated composite plates with symmetric as well as antisymmetric arrangements.
(4) Although the presented solution way is based on an analytical solution method, it precisely supports the results obtained of numerical methods and other analytical methods.

2. Eringen nonlocal differential model

Eringen's nonlocal elasticity model (Eringen and Edelen 1972) is assumed that the stress at a point of $\boldsymbol{x}$ depends not only on the strain there but also on strains at all other points of the body, which is in harmony with experimental remarks of atomic theory as well as lattice dynamics. So, the integral formation of nonlocal stress-strain relationship at reference point of x in the area of material is shown as

$$\sigma_{ij}^{nl} = \int_V \alpha(|x'\text{-x}|, \tau) t_{ij}(x') dv(x') \tag{1}$$

where $\alpha(|x'\text{-x}|, \tau)$ defines nonlocal kernel function, influenced by internal characteristic length, x' is any arbitrary point in the body and dv is elementary volume. $\tau = e_0 a/L$ is small-scale constant, where e_0 is a material constant, a and L are the internal and external characteristic lengths of the structure, respectively. σ_{ij}^{nl} and t_{ij} are the components of nonlocal and conventional stress tensors $\boldsymbol{\sigma}^{\boldsymbol{nl}}$ and $\boldsymbol{t}$, respectively. Classical Stress tensor $\boldsymbol{t}$ defined as

$$t = C : \varepsilon \tag{2}$$

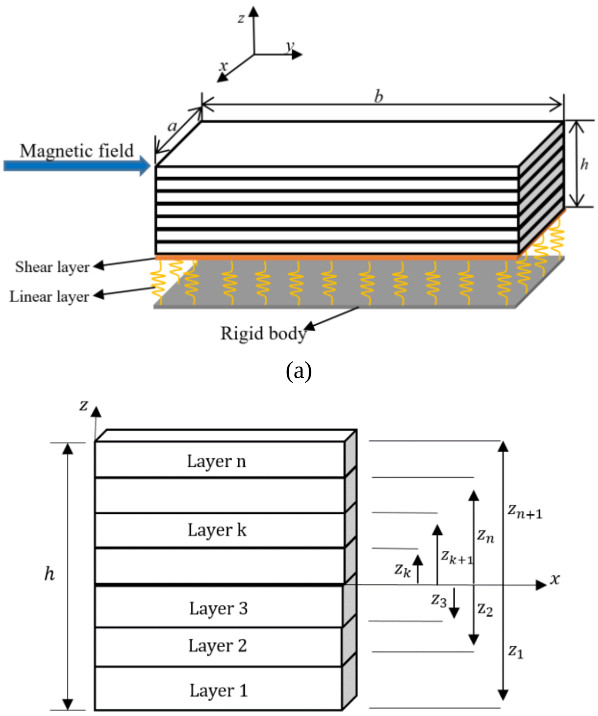
herein C is the forth order elasticity tensor and $\boldsymbol{\varepsilon}$ is strain tensor. By applying a physically admissible kernel α', integral form in Eq. (1) diminished to simplified differential formation of the nonlocal stress-strain relationship (Eringen 1983) and shown by

$$(1 - \mu \nabla^2)\sigma^{\text{nl}} = C : \varepsilon \tag{3}$$

where $\mu = (e_0 a)^2$ called the nonlocal parameter, indicating the size effect on the response of nanostructures and ∇^2 is the Laplacian operator.

3. Theoretical formulations

Consider a multilayered laminated composite plates resting on the Pasternak foundation (see Fig. 1).



(a)

(b)

Fig. 1 Geometrical shape of laminated composite plate subjected to an in-plane magnetic field (a), Layout of layers in the thickness direction (b)

3.1 Basic assumptions

The suppositions of the current theory can be shown as follow:

(1) The origin of the Cartesian coordinate system is considered at the axis of the laminated composite plates.
(2) The displacements are considered small in comparison with the thickness of plate and, hence, the involved strains are infinitesimal.
(3) The transverse displacement w contains two components of bending w_b and shear w_s. Also, both of them are functions of coordinate x only.

$$w(x,z,t) = w_b(x,t) + w_s(x,t) \tag{4}$$

(4) The displacements u and v along x- and y-directions contain extension, bending, and shear components

$$\begin{aligned} u &= u_0 + u_b + u_s \\ v &= v_0 + v_b + v_s \end{aligned} \tag{5}$$

It is assumed that the bending components u_b and v_b are similar to the displacement obtained by the classical plate theory. Hence, the expressions for u_b and v_b can be defined as

$$\begin{aligned} u_b &= -z\frac{\partial w_b}{\partial x} \\ v_b &= -z\frac{\partial w_b}{\partial y} \end{aligned} \tag{6}$$

The shear components v_b and v_s gives rise, in conjunction with w_s, to the variations of shear strains γ_{xz} and γ_{yz} hence to shear stresses τ_{xz} and τ_{yz} through the thickness of the plate in such a way that shear stresses τ_{xz} and τ_{yz} are zero at the bottom and top surfaces of the plate. Because of this, the expression of those can be given as

$$\begin{aligned} u_s &= -\psi(z)\frac{\partial w_s}{\partial x} \\ v_s &= -\psi(z)\frac{\partial w_b}{\partial y} \end{aligned} \tag{7}$$

in which the shape function can be define as below (Shahsavari *et al*. 2018d)

$$\psi(z) = -\left(\begin{array}{c} \left(\dfrac{\cosh\left(\frac{1}{2}\right)}{24\sinh\left(\frac{1}{2}\right) - 11\cosh\left(\frac{1}{2}\right)} - 1\right)\left(\dfrac{z}{h}\right) \\ -\left(\dfrac{1}{24\sinh\left(\frac{1}{2}\right) - 11\cosh\left(\frac{1}{2}\right)}\right)\sinh\left(\dfrac{z}{h}\right) \end{array} \right) h \tag{8}$$

3.2 Kinematics and fundamental equations

According to the suppositions, which made of in the previous section, the displacement field can be defined using Eqs. (4)-(8) likewise what obtained in Refs. (Benachour *et al*. 2011, Bourada *et al*. 2012, Karami *et al*. 2018e) as follow

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w_b}{\partial x} - \psi(z)\frac{\partial w_s}{\partial x} \\ v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_b}{\partial y} - \psi(z)\frac{\partial w_s}{\partial y} \\ w(x,y,t) &= w_b(x,y,t) + w_s(x,y,t) \end{aligned} \tag{9}$$

Due to the mentioned displacement field (Eq. (9)), the following non-zero strains have defined

$$\begin{aligned} &\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} \kappa_x^b \\ \kappa_x^b \\ \kappa_{xy}^b \end{Bmatrix} + \psi(z)\begin{Bmatrix} \kappa_x^s \\ \kappa_x^s \\ \kappa_{xy}^s \end{Bmatrix} \\ &\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z)\begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{yz}^s \end{Bmatrix}, \varepsilon_z = 0 \end{aligned} \tag{10}$$

where

$$\begin{aligned} &\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \begin{Bmatrix} \kappa_x^b \\ \kappa_y^b \\ \kappa_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_b}{\partial^2 x} \\ -\dfrac{\partial^2 w_b}{\partial^2 y} \\ -2\dfrac{\partial^2 w_b}{\partial x \partial y} \end{Bmatrix} \\ &\begin{Bmatrix} \kappa_x^s \\ \kappa_y^s \\ \kappa_{xy}^s \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_s}{\partial^2 x} \\ -\dfrac{\partial^2 w_s}{\partial^2 y} \\ -2\dfrac{\partial^2 w_s}{\partial x \partial y} \end{Bmatrix} \begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial w_s}{\partial y} \\ \dfrac{\partial w_s}{\partial x} \end{Bmatrix} \\ &g(z) = 1 - \frac{d\psi(z)}{dz} \end{aligned} \tag{11}$$

In the present work, laminated plate is made of advanced composite materials. The laminated plate is also made of several unidirectional plies accumulated in different orientations. Orthotropic axes in each lamina (indicated by superscript *k*) are oriented at an arbitrary angle θ to the (global) plate axis. Due to the nonlocality formulation, following constitutive relations for each layer can be written as (Raghu *et al*. 2016).

$$\begin{bmatrix}\sigma_{xx}-\mu\nabla^2(\sigma_{xx})\\ \sigma_{yy}-\mu\nabla^2(\sigma_{yy})\\ \tau_{xy}-\mu\nabla^2(\tau_{xy})\\ \tau_{yz}-\mu\nabla^2(\tau_{yz})\\ \tau_{xz}-\mu\nabla^2(\tau_{xz})\end{bmatrix} = \begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0\\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45}\\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} \tag{12}$$

Considering the rotation of the considered angle θ around *z*-axis in the *x*-*y* plane, the transformation composition for the stiffness parameters can be estimated as (Reddy 2004)

$$\begin{bmatrix}\bar{Q}_{11}\\ \bar{Q}_{12}\\ \bar{Q}_{22}\\ \bar{Q}_{16}\\ \bar{Q}_{26}\\ \bar{Q}_{66}\end{bmatrix} = \begin{bmatrix}c^4 & 2c^2s^2 & s^4 & 0 & 4c^2s^2\\ c^2s^2 & c^4+s^4 & c^2s^2 & 0 & -4c^2s^2\\ s^4 & 2c^2s^2 & c^4 & 2c^2s^2 & 0\\ c^3s & cs(-c^2-s^2) & -cs^3 & 0 & 2cs(-c^2+s^2)\\ cs^3 & cs(c^2-s^2) & -c^3s & 0 & 2cs(c^2-s^2)\\ c^2s^2 & -2c^2s^2 & c^2s^2 & 0 & c^2s^2(c^2s^2-2)\end{bmatrix}\begin{bmatrix}C_{11}\\ C_{12}\\ C_{22}\\ C_{16}\\ C_{66}\end{bmatrix}$$
$$\begin{bmatrix}\bar{Q}_{44}\\ \bar{Q}_{45}\\ \bar{Q}_{55}\end{bmatrix} = \begin{bmatrix}c^2 & s^2\\ -cs & cs\\ s^2 & c^2\end{bmatrix}\begin{bmatrix}C_{44}\\ C_{55}\end{bmatrix} \tag{13}$$

where $c = \cos(\theta)$, $s = \sin(\theta)$; C_{ij} denote stiffness coefficients as

$$C_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}, C_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}}$$
$$C_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}, C_{66} = G_{12} \tag{14}$$
$$C_{44} = G_{23}, C_{55} = G_{13}, \nu_{21} = \frac{E_2}{E_1}\nu_{12}$$

in which E_1 and E_2 are Young modulus; ν_{12}, ν_{21} are the Poisson ratios, and G_{ij} are the shear modulus.

3.3 Kinematic relations

Employing Hamilton's principle as follows, the equation of motion can be expressed as

$$\int_0^t \delta(T - U_P - U_F + V)dt = 0. \tag{15}$$

where U_P, U_F, V, and T are, respectively, the strain energy, energy owing to the elastic foundation, work done, which obtained by the external force, and kinetic energy for the plate. The variation of the strain energy for the plate can be written by

$$\begin{aligned}\delta U_P &= \int_V (\sigma_x\delta\varepsilon_x + \sigma_y\delta\varepsilon_y + \sigma_{xy}\delta\gamma_{xy} + \sigma_{xz}\delta\gamma_{xz} \\ &\qquad + \sigma_{yz}\delta\gamma_{yz})dV \\ &= \int_A \{N_x\varepsilon_x^0 + N_y\varepsilon_y^0 + N_{xy}\gamma_{xy}^0 + M_x^b\kappa_x^b + M_y^b\kappa_y^b \\ &\qquad + M_{xy}^b\kappa_{xy}^b \\ &\quad + M_x^s\kappa_x^s + M_y^s\kappa_y^s + M_{xy}^s\kappa_{xy}^s + Q_{xz}^s\gamma_{xz}^s + Q_{yz}^s\gamma_{yz}^s\}dxdy\end{aligned} \tag{16}$$

where

$$\begin{aligned}(N_x, N_y, N_{xy}) &= \sum_{k=1}^n \int_{z_k}^{z_{k+1}} (\sigma_x, \sigma_y, \tau_{xy})\, dz \\ (M_x^b, M_y^b, M_{xy}^b) &= \sum_{k=1}^n \int_{z_k}^{z_{k+1}} (\sigma_x, \sigma_y, \tau_{xy})\, z\, dz \\ (M_x^s, M_y^s, M_{xy}^s) &= \sum_{k=1}^n \int_{z_k}^{z_{k+1}} (\sigma_x, \sigma_y, \tau_{xy})\, \psi(z)\, dz \\ \{Q_{xz}^s, Q_{yz}^s\} &= \sum_{k=1}^n \int_{z_k}^{z_{k+1}} (g(z)\tau_{xz}, g(z)\tau_{yz})\, dz\end{aligned} \tag{17}$$

where Z_k and Z_{k+1} indicate, respectively, the lower and upper *z*-coordinates of the *k*th layer ($k = 1 - n$). The stress resultants are obtained as

$$\begin{bmatrix}\begin{Bmatrix}(1-\mu\nabla^2)N_x\\ (1-\mu\nabla^2)N_y\\ (1-\mu\nabla^2)N_{xy}\end{Bmatrix}\\ \begin{Bmatrix}(1-\mu\nabla^2)M_x^b\\ (1-\mu\nabla^2)M_y^b\\ (1-\mu\nabla^2)M_{xy}^b\end{Bmatrix}\\ \begin{Bmatrix}(1-\mu\nabla^2)M_x^s\\ (1-\mu\nabla^2)M_y^s\\ (1-\mu\nabla^2)M_{xy}^s\end{Bmatrix}\end{bmatrix} = \begin{bmatrix}\begin{bmatrix}A_{11} & A_{12} & A_{16}\\ A_{12} & A_{22} & A_{26}\\ A_{16} & A_{26} & A_{66}\end{bmatrix} & \begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{12} & B_{22} & B_{26}\\ B_{16} & B_{26} & B_{66}\end{bmatrix} & \begin{bmatrix}B_{11}^s & B_{12}^s & B_{16}^s\\ B_{12}^s & B_{22}^s & B_{26}^s\\ B_{16}^s & B_{26}^s & B_{66}^s\end{bmatrix}\\ \begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{12} & B_{22} & B_{26}\\ B_{16} & B_{26} & B_{66}\end{bmatrix} & \begin{bmatrix}D_{11} & D_{12} & D_{16}\\ D_{12} & D_{22} & D_{26}\\ D_{16} & D_{26} & D_{66}\end{bmatrix} & \begin{bmatrix}D_{11}^s & D_{12}^s & D_{16}^s\\ D_{12}^s & D_{22}^s & D_{26}^s\\ D_{16}^s & D_{26}^s & D_{66}^s\end{bmatrix}\\ \begin{bmatrix}B_{11}^s & B_{12}^s & B_{16}^s\\ B_{12}^s & B_{22}^s & B_{26}^s\\ B_{16}^s & B_{26}^s & B_{66}^s\end{bmatrix} & \begin{bmatrix}D_{11}^s & D_{12}^s & D_{16}^s\\ D_{12}^s & D_{22}^s & D_{26}^s\\ D_{16}^s & D_{26}^s & D_{66}^s\end{bmatrix} & \begin{bmatrix}H_{11}^s & H_{12}^s & H_{16}^s\\ H_{12}^s & H_{22}^s & H_{26}^s\\ H_{16}^s & H_{26}^s & H_{66}^s\end{bmatrix}\end{bmatrix}\begin{bmatrix}\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}\\ \begin{Bmatrix}\kappa_x^b\\ \kappa_y^b\\ \kappa_{xy}^b\end{Bmatrix}\\ \begin{Bmatrix}\kappa_x^s\\ \kappa_y^s\\ \kappa_{xy}^s\end{Bmatrix}\end{bmatrix} \tag{18}$$

$$\begin{bmatrix}(1-\mu\nabla^2)Q_{yz}^s\\ (1-\mu\nabla^2)Q_{xz}^s\end{bmatrix} = \begin{bmatrix}A_{44}^s & A_{45}^s\\ A_{45}^s & A_{55}^s\end{bmatrix}\begin{Bmatrix}\gamma_{yz}^s\\ \gamma_{xz}^s\end{Bmatrix} \tag{19}$$

where

$$\begin{aligned}(A_{ij}, B_{ij}, D_{ij}) &= \int_{\frac{-h}{2}}^{\frac{h}{2}} \bar{Q}_{ij}\,(1, z, z^2)dz \\ (B_{ij}^s, D_{ij}^s, H_{ij}^s, A_{ij}^s) &\\ &= \int_{\frac{-h}{2}}^{\frac{h}{2}} \bar{Q}_{ij}(\psi(z), z\psi(z), \psi^2(z), g^2(z))\, dz\end{aligned} \tag{20}$$

The energy corresponding to the elastic foundation can be defined by

$$U_F = -\int_A \int_{-\frac{h}{2}}^{\frac{h}{2}} U_{\text{Pasternak}}\, dAdz = -q_{\text{Pasternak}} \tag{21}$$

in which

$$q_{\text{Pasternak}} = K_w(w_b + w_s) - K_p \nabla^2 (w_b + w_s) \tag{22}$$

where K_P is shear layer and an K_W refers to linear spring layer. The work done by external force is given by

$$\delta V = -\int_A q\, wdA \tag{23}$$

where $w = w_b = w_s$; q is the transverse force obtained via distribution of in-plane magnetic field, which will be defined below

$$q_{\text{Lorentz}} = \sum_{k=1}^{n} \int_{z_k}^{z_{k+1}} f_z dz \tag{24}$$

where f_z is Lorentz force and acts only on the z-direction (Karami and Janghorban 2016, Karami *et al*. 2018g). The variation of kinetic energy is determined by

$$\begin{aligned}
\delta T &= \iint_R \sum_{k=1}^{n} \int_{z_k}^{z_{k+1}} \rho u_i \delta u_i dV \\
&= \int_0^a \int_0^b \sum_{k=1}^{n} \int_{z_k}^{z_{k+1}} \rho \frac{\partial u_i}{\partial t} \frac{\partial \delta u_i}{\partial t} dzdydx \\
\delta K &= \int_A \Big\{ I_0 (\frac{\partial u_0}{\partial t}\frac{\partial \delta u_0}{\partial t} + \frac{\partial v_0}{\partial t}\frac{\partial \delta v_0}{\partial t} + (\frac{\partial w}{\partial t} +)(\frac{\partial \delta w}{\partial t})) \\
&-I_1(\frac{\partial u_0}{\partial t}\frac{\partial^2 \delta w_b}{\partial t \partial x} + \frac{\partial^2 w_b}{\partial t \partial x}\frac{\partial \delta u_0}{\partial t} + \frac{\partial v_0}{\partial t}\frac{\partial^2 \delta w_b}{\partial t \partial y} \\
&\qquad + \frac{\partial^2 w_b}{\partial t \partial y}\frac{\partial \delta v_0}{\partial t}) \\
&-J_1(\frac{\partial u_0}{\partial t}\frac{\partial^2 \delta w_s}{\partial t \partial x} + \frac{\partial^2 w_s}{\partial t \partial x}\frac{\partial \delta u_0}{\partial t} + \frac{\partial v_0}{\partial t}\frac{\partial^2 \delta w_s}{\partial t \partial y} \\
&\qquad + \frac{\partial^2 w_s}{\partial t \partial y}\frac{\partial \delta v_0}{\partial t}) \\
&+I_2(\frac{\partial^2 w_b}{\partial t \partial x}\frac{\partial^2 \delta w_b}{\partial t \partial x} + \frac{\partial^2 w_b}{\partial t \partial y}\frac{\partial^2 \delta w_b}{\partial t \partial y}) + K_2(\frac{\partial^2 w_s}{\partial t \partial x}\frac{\partial^2 \delta w_s}{\partial t \partial x} \\
&\qquad + \frac{\partial^2 w_s}{\partial t \partial y}\frac{\partial^2 \delta w_s}{\partial t \partial y}) \\
&+J_2(\frac{\partial^2 w_b}{\partial t \partial x}\frac{\partial^2 \delta w_b}{\partial t \partial x} + \frac{\partial^2 w_s}{\partial t \partial x}\frac{\partial^2 \delta w_s}{\partial t \partial x} + \frac{\partial^2 w_b}{\partial t \partial y}\frac{\partial^2 \delta w_b}{\partial t \partial y} \\
&\qquad + \frac{\partial^2 w_s}{\partial t \partial y}\frac{\partial^2 \delta w_s}{\partial t \partial y}) \Big\} dA
\end{aligned} \tag{25}$$

herein ρ is the mass density; (I_0, I_1, J_1, I_2, J_2, K_2) are mass inertias as below

$$\begin{aligned}
&\{I_0, I_1, J_1, I_2, J_2, K_2\} \\
&= \sum_{k=1}^{n} \int_{z_k}^{z_{k+1}} \{1, z, \psi, z^2, z\psi, \psi^2\} \rho(z) dz
\end{aligned} \tag{26}$$

The governing equations for a refined four-variable laminated composite plate in terms of the displacement can be derived by substituting Eqs. (16)-(26) into Eq. (15) as follows

$$\begin{aligned}
&\delta u_0 : \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = \\
&(1 - \mu \nabla^2)(I_0 \frac{\partial^2 u_0}{\partial t^2} - I_1 \frac{\partial^3 w_b}{\partial x \partial t^2} - J_1 \frac{\partial^3 w_s}{\partial x \partial t^2})
\end{aligned} \tag{27}$$

$$\begin{aligned}
&\delta v_0 : \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = \\
&(1 - \mu \nabla^2)(I_0 \frac{\partial^2 v_0}{\partial t^2} - I_1 \frac{\partial^3 w_b}{\partial y \partial t^2} - J_1 \frac{\partial^3 w_s}{\partial y \partial t^2})
\end{aligned} \tag{28}$$

$$\begin{aligned}
&\delta w_b : \left[\frac{\partial^2 M_x^b}{\partial x^2} + 2 \frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} \right] \\
&+(1 - \mu \nabla^2)(q_{\text{Lorentz}} - q_{\text{Pasternak}}) \\
&= (1 - \mu \nabla^2)(I_0 (\frac{\partial^2 w_b}{\partial t^2} + \frac{\partial^2 w_s}{\partial t^2}) + I_1 (\frac{\partial^3 u_0}{\partial x \partial t^2} + \frac{\partial^3 v_0}{\partial y \partial t^2}) \\
&-I_2 (\frac{\partial^4 w_b}{\partial x^2 \partial t^2} + \frac{\partial^4 w_b}{\partial y^2 \partial t^2}) - J_2 (\frac{\partial^4 w_s}{\partial x^2 \partial t^2} + \frac{\partial^4 w_s}{\partial y^2 \partial t^2}))
\end{aligned} \tag{29}$$

$$\begin{aligned}
&\delta w_s : \left[\frac{\partial^2 M_x^s}{\partial x^2} + 2 \frac{\partial^2 M_{xy}^s}{\partial x \partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial Q_{xz}^s}{\partial x} + \frac{\partial Q_{yz}^s}{\partial y} \right] \\
&+(1 - \mu \nabla^2)(q_{\text{Lorentz}} - q_{\text{Pasternak}}) \\
&= (1 - \mu \nabla^2)(I_0 (\frac{\partial^2 w_b}{\partial t^2} + \frac{\partial^2 w_s}{\partial t^2}) + J_1 (\frac{\partial^3 u_0}{\partial x \partial t^2} + \frac{\partial^3 v_0}{\partial y \partial t^2}) \\
&-J_2 (\frac{\partial^4 w_b}{\partial x^2 \partial t^2} + \frac{\partial^4 w_b}{\partial y^2 \partial t^2}) - J_2 (\frac{\partial^4 w_s}{\partial x^2 \partial t^2} + \frac{\partial^4 w_s}{\partial y^2 \partial t^2}))
\end{aligned} \tag{30}$$

According to the mentioned relations, the equations of motion defined for the size-dependent laminated composite plate can be shown in terms of fourfold displacements (u_0, v_0, w_b, w_s) as follow

$$\begin{aligned}
&[A_{11} d_{11} + 2 A_{16} d_{12} + A_{66} d_{22}] u_0 \\
&+[A_{16} d_{11} + (A_{12} + A_{66}) d_{12} + A_{26} d_{22}] v_0 \\
&- \begin{bmatrix} B_{11} d_{111} + 3 B_{16} d_{112} \\ +(B_{12} + 2 B_{66}) d_{122} + B_{26} d_{222} \end{bmatrix} w_b \\
&- \begin{bmatrix} B_{11}^s d_{111} + 3 B_{16}^s d_{112} \\ +(B_{12}^s + 2 B_{66}^s) d_{122} + B_{26}^s d_{222} \end{bmatrix} w_s \\
&= \{1 - \mu(d_{11} + d_{22})\} \\
&\quad \left(I_0 \frac{\partial^2 u_0}{\partial t^2} - I_1 d_1 \frac{\partial^2 w_b}{\partial t^2} - J_1 d_1 \frac{\partial^2 w_s}{\partial t^2} \right)
\end{aligned} \tag{31}$$

$$\begin{aligned}
&[A_{16} d_{11} + (A_{12} + A_{66}) d_{12} + A_{26} d_{22}] u_0 \\
&+[A_{66} d_{11} + 2 A_{26} d_{12} + A_{22} d_{22}] v_0 \\
&- \begin{bmatrix} B_{16} d_{111} + (B_{12} + 2 B_{66}) d_{112} \\ +3 B_{26} d_{122} + B_{22} d_{222} \end{bmatrix} w_b \\
&- \begin{bmatrix} B_{16}^s d_{111} + (B_{12}^s + 2 B_{66}^s) d_{112} \\ +3 B_{26}^s d_{122} + B_{22}^s d_{222} \end{bmatrix} w_s \\
&= \{1 - \mu(d_{11} + d_{22})\} \\
&\quad \left(I_0 \frac{\partial^2 v_0}{\partial t^2} - I_1 d_2 \frac{\partial^2 w_b}{\partial t^2} - J_1 d_2 \frac{\partial^2 w_s}{\partial t^2} \right)
\end{aligned} \tag{32}$$

$$\begin{aligned}
&\begin{bmatrix} B_{11}d_{111} + 3B_{16}d_{112} \\ +(B_{12}+2B_{66})d_{122} + B_{26}d_{222} \end{bmatrix} u_0 \\
&+ \begin{bmatrix} B_{16}d_{111} + (B_{12}+2B_{66})d_{112} \\ +3B_{26}d_{122} + B_{22}d_{222} \end{bmatrix} v_0 \\
&- \begin{bmatrix} D_{11}d_{1111} + 4D_{16}d_{1112} + 2(D_{12}+2D_{66})d_{1122} \\ +4D_{26}d_{1222} + D_{22}d_{2222} \end{bmatrix} w_b \\
&- \begin{bmatrix} D_{11}^s d_{1111} + 4D_{16}^s d_{1112} + 2(D_{12}^s+2D_{66}^s)d_{1122} \\ +4D_{26}^s d_{1222} + D_{22}^s d_{2222} \end{bmatrix} w_s \\
&+ \begin{bmatrix} K_w\{1-\mu(d_{11}+d_{22})\} + (\eta h H_x^2 - K_P) \\ \{(d_{11}+d_{22}) - \mu(d_{1111}+2d_{1122}+d_{2222})\} \end{bmatrix} w \\
&= \{1-\mu(d_{11}+d_{22})\}(I_0(\frac{\partial^2 w_b}{\partial t^2} + \frac{\partial^2 w_s}{\partial t^2}) \\
&+ I_1\left(d_1\frac{\partial^2 u_0}{\partial t^2} + d_2\frac{\partial^2 v_0}{\partial t^2}\right) \\
&\qquad - I_2\left(d_{11}\frac{\partial^2 w_b}{\partial t^2} + d_{22}\frac{\partial^2 w_b}{\partial t^2}\right) \\
&- J_2(d_{11}\frac{\partial^2 w_s}{\partial t^2} + d_{22}\frac{\partial^2 w_s}{\partial t^2}))
\end{aligned} \tag{33}$$

$$\begin{aligned}
&\begin{bmatrix} B_{11}^s d_{111} + 3B_{16}^s d_{112} \\ +(B_{12}^s+2B_{66}^s)d_{122} + B_{26}^s d_{222} \end{bmatrix} u_0 \\
&+ \begin{bmatrix} B_{16}^s d_{111} + (B_{12}^s+2B_{66}^s)d_{112} \\ +3B_{26}^s d_{122} + B_{22}^s d_{222} \end{bmatrix} v_0 \\
&- \begin{bmatrix} D_{11}^s d_{1111} + 4D_{16}^s d_{1112} + 2(D_{12}^s+2D_{66}^s)d_{1122} \\ +4D_{26}^s d_{1222} + D_{22}^s d_{2222} \end{bmatrix} w_b \\
&- [H_{11}^s d_{1111} + 4H_{16}^s d_{1112} + 2(H_{12}^s+2H_{66}^s)d_{1122} \\
&\qquad +4H_{26}^s d_{1222} + H_{22}^s d_{2222} \\
&- A_{55}^s d_{11} - A_{44}^s d_{22} - 2A_{45}^s d_{12}] w_s \\
&+ \begin{bmatrix} K_w\{1-\mu(d_{11}+d_{22})\} + (\eta h H_x^2 - K_P) \\ \{(d_{11}+d_{22}) - \mu(d_{1111}+2d_{1122}+d_{2222})\} \end{bmatrix} w \\
&= \{1-\mu(d_{11}+d_{22})\}(I_0(\frac{\partial^2 w_b}{\partial t^2} + \frac{\partial^2 w_s}{\partial t^2}) \\
&+ J_1\left(d_1\frac{\partial^2 u_0}{\partial t^2} + d_2\frac{\partial^2 v_0}{\partial t^2}\right) \\
&\qquad - J_2\left(d_{11}\frac{\partial^2 w_b}{\partial t^2} + d_{22}\frac{\partial^2 w_b}{\partial t^2}\right) \\
&- K_2(d_{11}\frac{\partial^2 w_s}{\partial t^2} + d_{22}\frac{\partial^2 w_s}{\partial t^2}))
\end{aligned} \tag{34}$$

in which d_{ij}, d_{ijk}, and d_{ijkl} denote differential operators defined below

$$d_{ij} = \frac{\partial^2}{\partial x_i x_j}, d_{ijk} = \frac{\partial^3}{\partial x_i x_j x_k}, d_{ijkl} = \frac{\partial^4}{\partial x_i x_j x_k x_l} \tag{35}$$

4. Solution procedure

In this section, Navier solution is used for a simply supported rectangular laminated plate. In this study, two models of simply supported boundary conditions namely SS-1 and SS-2 are considered for the rectangular laminated plate when is influenced by magnetic field along the x-axis.

4.1 Antisymmetric cross-ply laminates with SS-1 boundary condition

For this model, the following elements of plate stiffness are considered equal to zero

$$\begin{aligned}
&A_{16} = A_{26} = D_{16} = D_{26} = D_{16}^s \\
&= D_{26}^s = H_{16}^s = H_{26}^s = 0, \\
&B_{12} = B_{16} = B_{26} = B_{66} = B_{12}^s \\
&= B_{16}^s = B_{26}^s = B_{66}^s = A_{45} \\
&= A_{45}^s = D_{45} = 0, B_{22} = -B_{11} = B_{22}^s = -B_{11}^s,
\end{aligned} \tag{36}$$

The following SS-1 boundary conditions are inflicted at the side edges:

(1) On edges $x = 0, a$

$$\left\{v_0, w_b, w_s, \frac{\partial w_b}{\partial y}, \frac{\partial w_s}{\partial y}, N_x, M_x^b, M_x^s\right\} = 0 \tag{37}$$

(2) On edges $y = 0, b$

$$\left\{u_0, w_b, w_s, \frac{\partial w_b}{\partial x}, \frac{\partial w_s}{\partial x}, N_y, M_y^b, M_y^s\right\} = 0 \tag{38}$$

To satisfy the mentioned boundary conditions, following Navier series are adopted as

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_b \\ w_s \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn}\cos(\alpha x)\sin(\beta y)e^{i\omega t} \\ V_{mn}\sin(\alpha x)\cos(\beta y)e^{i\omega t} \\ W_{bmn}\sin(\alpha x)\sin(\beta y)e^{i\omega t} \\ W_{smn}\sin(\alpha x)\sin(\beta y)e^{i\omega t} \end{Bmatrix} \tag{39}$$

where ω is the eigen-frequency related to the (m,n)-th eigen-mode, and $\alpha = m\pi/a$, $\beta = n\pi/b$.

4.2 Antisymmetric angle-ply laminates with SS-2 boundary condition

For this model, the following elements of plate stiffness are considered equal to zero

$$\begin{aligned}
&A_{16} = A_{26} = D_{16} = D_{26} = D_{16}^s = D_{26}^s = H_{16}^s = H_{26}^s \\
&= 0, \\
&B_{12} = B_{16} = B_{26} = B_{66} = B_{12}^s = B_{16}^s = B_{26}^s = B_{66}^s \\
&= 0, \\
&A_{45} = A_{45}^s = D_{45} = 0
\end{aligned} \tag{40}$$

The following SS-2 boundary conditions are inflicted at the side edges:

(1) On edges $x = 0, a$

$$\left\{u_0, w_b, w_s, \frac{\partial w_b}{\partial y}, \frac{\partial w_s}{\partial y}, N_{xy}, M_x^b, M_x^s\right\} = 0 \tag{41}$$

(2) On edges $y = 0, b$

$$\left\{v_0, w_b, w_s, \frac{\partial w_b}{\partial x}, \frac{\partial w_s}{\partial x}, N_{xy}, M_y^b, M_y^s\right\} = 0 \tag{42}$$

Likewise to the cross-ply laminate case model, the solution satisfies the boundary conditions using following series

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_b \\ w_s \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn}\sin(\alpha x)\cos(\beta y)e^{i\omega t} \\ V_{mn}\cos(\alpha x)\sin(\beta y)e^{i\omega t} \\ W_{bmn}\sin(\alpha x)\sin(\beta y)e^{i\omega t} \\ W_{smn}\sin(\alpha x)\sin(\beta y)e^{i\omega t} \end{Bmatrix} \tag{43}$$

Table 1 Material properties used in the laminated composite plates

	E_2 (GPa)	E_1	G_{12}	G_{13}	G_{23}	ν_{12}	P (kg/m^3)
Material 1[*a]	1	(3,10,20,30,40)E_2	0.6E_2	0.6E_2	0.5E_2	0.25	1
Material 2[*b]	1	25E_2	0.5E_2	0.5E_2	0.2E_2	0.25	1

*[a] Ref. (Noor 1975), *[b] Ref. (Phan and Reddy 1985)

Table 2 Comparison of the normalized natural frequency $\bar{\omega}$ of simply supported square nanoplates $\left(\nu_{12} = \nu_{21} = 0.3, b = a, E_1 = E_2 = 30(\text{MPa}), \rho = 1(\frac{kg}{m^3})\right)$

a/h	μ	NFSDT[*a]	NTSDT[*a]	NTVRPT[*b]	Present
10	0	0.0930	0.0935	0.09303	0.09303
	1	0.0850	0.0854	0.08502	0.08502
	2	0.0788	0.0791	0.07877	0.07877
20	0	0.0239	0.0239	0.02386	0.02386
	1	0.0218	0.0218	0.0218	0.02181
	2	0.0202	0.0202	0.0202	0.02021

*[a] Ref. (Aghababaei and Reddy 2009),
*[b] Ref. (Malekzadeh and Shojaee 2013)

4.3 Eigenvalue approach for vibration problems

By substituting Eqs. (39)-(43) into Eqs. (31)-(34), the following eigenvalue equations for any fixed value of m and n, for the free vibration case can be obtained

$$([K] - \omega^2[M])\{\Delta\} = \{0\} \tag{44}$$

where $[\boldsymbol{K}]$, $[\boldsymbol{M}]$, and $\{\Delta\}$ are the stiffness matrix, matrix of mass, and the unknown amplitude vector, respectively.

5. Numerical results

In this section, in order to find the numerical results and make a comparison with published works, the following factors are defined (Shen *et al*. 2003, Murmu *et al*. 2013)

$$\bar{\omega} = \omega h\sqrt{\frac{\rho}{G}}, \quad \widetilde{\omega} = \omega a^2\sqrt{\frac{\rho}{E_2h^2}}, \quad MP = \frac{\eta h H_x^2 a^2}{D_0},$$
$$\bar{K}_W = \frac{K_W a^4}{E_2h^3}, \quad \bar{K}_P = \frac{K_P a^2}{E_2h^3}, \quad G = \frac{E_1}{2(1+\nu_{12})},$$
$$D_0 = \frac{E_2h^3}{12(1-\nu_{12}\nu_{21})}$$

To investigate the vibrational behavior of laminated composite plates, two different types of lamina are considered in which these material properties are tabulated in in Table 1. It is note that, in the current work the length of the plate is fixed.

5.1 Validation

In Table 2, the normalized natural frequency of simply supported isotropic square nanoplate with various quantities of the nonlocal parameter and thickness ratio are compared with the results in the open literature (Aghababaei and Reddy 2009, Malekzadeh and Shojaee 2013). As can be observed from this table, the results from different methods (Nonlocal First Order Shear Deformation Theory (NFSDT), Nonlocal Third Order Shear Deformation Theory (NTSDT), and Nonlocal Two Variable Refined Plate Theory (NTVRPT)) including the present one all agree well with each other and that the frequencies of nanoplate in the presence of nonlocality are lower than the ones without a nonlocal parameter. This is expected since when larger nanoplates are considered, the effect of the nonlocal parameter will decrease.

5.2 Free vibration analysis of laminated structures

5.2.1 Effect of moduli ratio and magnetic field

In Table 3, the natural frequencies of the antisymmetric cross-ply laminated plates with two-layer [0/90]and six-layer $[0/90]_3$ are given for fixed aspect a/h = 5. For different moduli ratios, our results compare well with the 3D model by (Noor 1973) and the results by other methods (Higher Order Shear Deformation Theory (HSDT) and Hyperbolic Shear Deformation Theory (HSDT)) (Reddy 1984, Akavci 2007), especially when the ratios are E_1/E_2 = 3, 10, 20. Under the combined effects of the nonlocal parameter and magnetic force, it is observed that while the natural frequencies of the plate increase with increasing moduli ratio, increasing magnetic strength, or increasing number of layers, they decrease with increasing nonlocal parameter.

5.2.2 Effect of thickness ratio and elastic foundations

The natural frequencies of the laminated plates containing the cross-ply [0/90/0] with three layers and angle-ply [45/-45] with four layers rested on Winkler-Pasternak foundation are listed in Table 4 for diverse values of thickness ratio a/h = 5,10,20,50 and as further compared to those using other method (higher order shear deformation theory (HSDT)) (Shen *et al*. 2003). It can be observed that for the reduced case (without the nonlocal parameter), the results by the present theory are in excellent agreement with those from other methods for the plate ranging from very thin to moderately thick. As listed in the Table, (1) existence of the elastic foundation raises the frequency of the laminated composite plate. This increment is relevant to the hardness effect of the foundation. Hence, a laminated composite plate with a foundation is stiffer than the one without foundation. (2) the natural frequency of the laminated plate on the Pasternak foundation with extra shear layer is higher than that of the plate on the Winkler foundation. (3) it is observed that the influence of the elastic foundation is more obvious in the low values of the

Table 3 Normalized natural frequencies $\tilde{\omega}$ of antisymmetric cross-ply square laminated composite plates (a = b) with two and six layers for various values of moduli ratio, magnetic field, and nonlocal parameter (Material 1, a/h = 5, and SS-1)

	MP	μ	Method	E_1/E_2 3	10	20	30	40
[0/90]	0	0	3D Model[*a]	6.2578	6.9845	7.6745	8.1763	8.5625
			HSDT[*b]	6.2169	6.9887	7.8210	8.5050	9.0871
			HSDT[*c]	6.2181	6.9939	7.8324	8.5228	9.1114
			Present	6.2168	6.9881	7.8197	8.5028	9.0841
		1	Present	5.6813	6.3862	7.1461	7.7704	8.3016
		2	Present	5.2640	5.9171	6.6212	7.1996	7.6918
	50	0	Present	7.0466	7.8762	8.7902	9.5460	10.1910
		1	Present	6.7345	7.5137	8.3794	9.0971	9.7105
		2	Present	6.5424	7.2860	8.1200	8.8134	9.4068
	100	0	Present	7.7634	8.6443	9.6332	10.4555	11.1595
		1	Present	7.6005	8.4417	9.4003	10.2008	10.8879
		2	Present	7.5407	8.3556	9.2994	10.0914	10.7729
$[0/90]_3$	0	0	3D Model[*a]	6.61	8.4143	9.8398	10.695	11.272
			HSDT[*b]	6.5552	8.4041	9.9175	10.854	11.500
			HSDT[*c]	6.5563	8.4057	9.9188	10.856	11.503
			Present	6.5558	8.4053	9.9181	10.8546	11.5009
		1	Present	5.9911	7.6813	9.0638	9.9196	10.5103
		2	Present	5.5510	7.1170	8.3980	9.1910	9.7382
	50	0	Present	7.4533	9.5586	11.2907	12.3630	13.1014
		1	Present	7.1318	9.1490	10.8125	11.8425	12.5515
		2	Present	6.9379	8.9041	10.5296	11.5368	12.2297
	100	0	Present	8.2338	10.5679	12.4981	13.6940	14.5168
		1	Present	8.0792	10.3787	12.2881	13.4725	14.2870
		2	Present	8.0364	10.3360	12.2542	13.4461	14.2657

[*a] Ref. (Noor 1973), [*b] Ref. (Reddy 1984), [*c] Ref. (Akavci 2007)

Table 4 Normalized natural frequencies $\tilde{\omega}$ of angle-ply and cross-ply square laminated composite plates with diverse values of thickness ratios, magnetic field, and nonlocal parameter, (Material 1 with E_1/E_2 = 40 and SS-1)

	$\bar{K}_W$	$\bar{K}_P$	μ	Method	a/h 5	10	20	50
$[45/-45]_2$	0	0	0	HSDT[*]	12.544	18.333	21.812	23.225
				Present	12.534	18.324	21.807	23.224
			1	Present	11.455	16.746	19.929	21.224
			2	Present	10.613	15.516	18.465	19.664
	100	0	0	HSDT[*]	16.022	20.868	23.989	25.285
				Present	16.009	20.857	23.983	25.284
			1	Present	15.178	19.485	22.289	23.460
			2	Present	14.556	18.438	20.990	22.059
		10	0	HSDT[*]	21.278	25.132	27.789	28.924
				Present	17.207	25.118	27.783	28.923
			1	Present	15.725	23.990	26.334	27.343
			2	Present	14.570	23.148	25.244	26.151

[*]Ref. (Shen *et al*. 2003)

Table 4 Continued

	$\bar{K}_W$	$\bar{K}_P$	μ	Method	a/h 5	10	20	50
[0/90/0]	0	0	0	HSDT*	10.263	14.702	17.483	18.689
				Present	11.770	15.940	17.994	18.738
			1	Present	10.756	14.567	16.444	17.124
			2	Present	9.966	13.497	15.236	15.866
	100	0	0	HSDT*	14.244	17.753	20.132	21.152
				Present	15.401	18.795	20.578	21.238
			1	Present	14.650	17.645	19.237	19.829
			2	Present	14.080	16.773	18.215	18.753
		10	0	HSDT*	19.879	22.596	24.536	25.390
				Present	20.801	23.423	24.903	25.462
			1	Present	20.244	22.518	23.807	24.299
			2	Present	19.835	21.841	22.989	23.429

*Ref. (Shen *et al*. 2003)

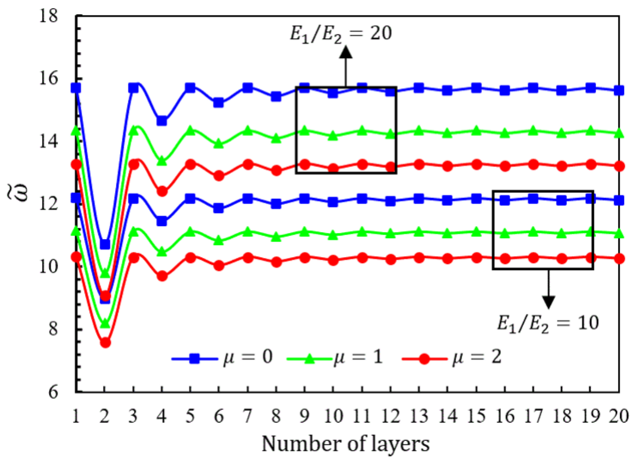


Fig. 2 Variation of normalized natural frequency $\tilde{\omega}$ vs. the number of layers of the laminated composite square plate ($a = b$) with different nonlocal parameters (Material 1 with a/h = 10 and SS-2)

thickness ratios than that in the high ones. Again, for a fixed moduli ratio, the natural frequency decreases with increasing nonlocal parameter. (4) it is concluded further that the natural frequency increases with increasing moduli ratios.

5.2.3 Effect of lay-up numbers

The natural frequencies vs. layer number of the antisymmetric angle-ply [45/-45] laminated composite square plate (a = b = 10 h) are shown in Fig. 2. It is observed: (1) For a fixed nonlocal parameter, the frequency reaches a minimum when the number of layer is 2 and the frequency corresponding to an even number of layers is usually smaller than those in its adjacent odd ones; (2) The frequency reaches its asymptotic value when the number of layers is close to 20; (3) a large nonlocal parameter corresponds to a small natural frequency. (4) It is noteworthy that, the maximum natural frequencies attribute to the laminated plate containing higher moduli ratios.

Table 5 Higher-order normalized natural frequencies $\tilde{\omega}_{mn}$ of cross-ply laminated composite plates, (Material 1 with E_1/E_2 = 40 and [0/90/90/0])

		MP = 0			MP = 100		
$(\bar{K}_W, \bar{K}_P)$	μ	$\tilde{\omega}_{11}$	$\tilde{\omega}_{22}$	$\tilde{\omega}_{33}$	$\tilde{\omega}_{11}$	$\tilde{\omega}_{22}$	$\tilde{\omega}_{33}$
(0,0)	0	17.9938	63.7613	123.395	18.0786	64.0594	123.9680
	1	16.4439	47.6632	74.0539	16.5549	48.3730	76.6606
	2	15.2359	39.7027	57.8292	15.3754	40.9210	63.1476
(100,0)	0	20.5776	64.5341	123.7960	20.6517	64.8286	124.366
	1	19.237	48.6922	74.7189	19.3319	49.3870	77.3026
	2	18.2152	40.9323	58.6784	18.3318	42.1144	63.9246
(100,10)	0	24.9030	70.3386	130.7020	24.9641	70.6084	131.241
	1	23.8072	56.1586	85.6764	23.8837	56.7603	87.9310
	2	22.9894	49.5814	72.1155	23.0817	50.5575	76.4210

5.2.4 Effect of eigen-modes

Some higher-order natural frequencies ((m, n) = (1,1), (2,2), (3,3)) are listed in Table 5 for the laminated composite plates resting on elastic foundation with distinct values of foundation parameters, mode numbers and nonlocal parameter, and in the absence and presence of the in-plane magnetic field. It can be observed that while a higher frequency corresponds to a higher-order mode number (m, n), the frequency increases with increasing magnetic field and hardness of elastic foundation (or decreasing nonlocal parameter). This feature is consistent with the results observed in other examples of this paper. Moreover, it is noted that at the high mode numbers, the nonlocal parameter has a dominant role and leads to sharp change in the frequency responses. For soft and hard foundations with and without magnetic effect, the trend of higher frequency changes remains relatively constant.

6. Conclusions

Size-dependent free vibrations of the symmetric as well as antisymmetric laminated composite plates on Pasternak foundation are presented considering the in-plane magnetic field effects. A refined plate theory and ENDM is adopted to obtain the governing equations where the virtual work of Hamilton's principle is used. Then, an analytical technique based on Navier series is employed to find the natural frequencies of the structure. Further, the impacts of two-different simply supported lateral boundary conditions including both of angle-ply and cross-ply laminated plates are analyzed. The reported numerical results show that:

- The present methodology with only four variables is accurate and comparable in compression with those of other higher-order-shear-deformation theories for size-dependent analysis.
- The impact of the nonlocality on the frequency rises monotonically with growing aspect ratio and mode numbers.
- The natural size-dependent frequencies of the laminated composite plate increase with growth in moduli ratio, magnetic strength, number of layers.
- In all values of moduli ratio, the applied magnetic field plays an important role on the natural frequencies of the plates with different lay-up numbers.
- The elastic foundation raises the frequency of size-dependent laminated composite plate, although, its influence is more obvious in the low values of the thickness ratio.

For the laminated composite plate consisted of an odd number of layers, the frequency is higher than that consisted of an even number of layers adjacent to the odd number.

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