

# Thermo-mechanical vibration analysis of curved imperfect nano-beams based on nonlocal strain gradient theory

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**Abstract.** In the current paper, an exact solution method is carried out for analyzing the thermo-mechanical vibration of curved FG nano-beams subjected to uniform thermal environmental conditions, by considering porosity distribution via nonlocal strain gradient beam theory for the first time. Nonlocal strain gradient elasticity theory is adopted to consider the size effects in which the stress for not only the nonlocal stress field but also the strain gradients stress field is considered. It is perceived that during manufacturing of functionally graded materials (FGMs) porosities and micro-voids can be occurred inside the material. Material properties of curved porous FG nanobeam are assumed to be temperature-dependent and are supposed to vary through the thickness direction of beam which modeled via modified power-law rule. Since variation of pores along the thickness direction influences the mechanical and physical properties, porosity play a key role in the mechanical response of curved FG nano-structures. The governing equations and related boundary condition of curved porous FG nanobeam under temperature field are derived via the energy method based on Timoshenko beam theory. An analytical Navier solution procedure is utilized to achieve the natural frequencies of porous FG curved nanobeam supposed to thermal loading. The results for simpler states are confirmed with known data in the literature. The effects of various parameters such as nonlocality parameter, porosity volume fractions, thermal effect, gradient index, opening angle and aspect ratio on the natural frequency of curved FG porous nanobeam are successfully discussed. It is concluded that these parameters play key roles on the dynamic behavior of porous FG curved nanobeam. Presented numerical results can serve as benchmarks for future analyses of curve FG nanobeam with porosity phases.

**Keywords:** curved FG beam; porous materials; thermo-mechanical vibration; nonlocal strain gradient theory

## 1. Introduction

Functionally graded materials (FGMs) as a new class of composite structures have drawn the attention of many researchers in the smart materials and structures by avoiding the cracking and delamination phenomenon, minimizing or removing stress concentrations at the interfaces of the traditional composite materials (Shen 2016, She *et al.* 2017a, b, c). These new advanced materials were proposed by a group of Japanese scientists in the mid-1980s, as thermal barrier for aerospace applications (Koizumi and Niino 1995). The material properties of FGMs vary continuously in one or more directions. Due to high strength and high temperature resistance of FGMs, they are increasingly utilized as structural components in modern industries such as heat shields of spacecraft body, nuclear reactor components, jet fighter structures and heat engine components (Pompe *et al.* 2003, Miyamoto *et al.* 2013, Mortensen and Suresh 2013). For more efficient and expand applications of nano structures, they were recently synthesized by using FGMs. Actually, functionally graded

model enables the nano materials to have the optimum properties.

Due to extensive applications of nano-structure made of FGM in advanced diverse technology, there has been intensive investigation on the caption of nano materials with functional graded.

The classical continuum theory is aptly practical in the mechanical behavior of the macroscopic structures, but it ignores the size-dependency of nano-structures, thereupon classical continuum theory is not suitable one to micro and nano scales structures. In order to bypass this drawback, two nonlocal elasticity theory namely Eringen's nonlocal elasticity theory (Eringen 1972a, b, 1983, 2002) and nonlocal strain gradient theory (Li *et al.* 2015, Ebrahimi and Barati 2016) are offered to consider size effect. According to Eringen's model, the stress state at a certain point is considered as a function of strain states of all points in its area. Numerous studies have been conducted for investigation the mechanical responses of FG nano-beams based on nonlocal elasticity theory (Zhang *et al.* 2009, Daulton *et al.* 2010, Hu *et al.* 2010, Murmu and Adhikari 2010, Eltaher *et al.* 2013). Based on nonlocal Timoshenko beam theory, Vibration characteristics of size dependent FG nanobeams is reported by Rahmani and Pedram (2014). By implementation of nonlocal Eringen theory, Nazemnezhad and Hosseini-Hashemi (2014), researched nonlinear

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vibration of functionally graded nanobeams. Furthermore, Eltaher *et al.* (2012) presented free vibration analysis of functionally graded size-dependent nanobeams. Meanwhile, Ebrahimi and Salari (2015) provided differential transform method for flexural vibrational behavior of FG nanobeams. In another survey, Ebrahimi and Barati (2016a) utilized a higher order refined beam theory for dynamic analysis of magneto electro embedded FG structures. Notwithstanding the fact that nonlocal elasticity theory is widely utilized to consider the influence of small-scale, it considers only the stiffness softening influence. Many researchists have discovered increment in the stiffness of structures, which is disregarded in Eringen's nonlocal elasticity theory (Fleck and Hutchinson 1993, Stölken and Evans 1998, Lam *et al.* 2003). According to experimental research and as well as the strain gradient theory, the Eringen's nonlocal elasticity theory is unable to anticipate the influence of stiffness-hardening and strain gradient elasticity (Lam *et al.* 2003, Ebrahimi and Barati 2016a) via introducing the length scale parameter. It is noted that, two exactly different physical Specifications were expressed by nonlocal theory and the strain gradient theory. Hereupon, nonlocal strain gradient theory is proposed to overcome the defect of nonlocal Eringen's theory with considering two length scales in a single theory (Lim *et al.* 2015). The two length scale parameters have an indeed influence on the physical and mechanical behavior of small size structures, which is considered in nonlocal strain gradient theory. By implementation of nonlocal strain gradient theory, buckling behavior of size-dependent nonlinear beams is presented by Li and Hu (2015). They reported that increasing in the nonlocal stress parameter causes the decrease in the non-dimensional frequency of FG nanobeams. However, for the length scale coefficient this behavior is vice versa. Furthermore, the flexural wave propagation response of Euler-Bernoulli FG beam was inspected by Li *et al.* (2015) within the framework of nonlocal strain gradient elasticity. Also, Ebrahimi and Barati (2017b) examined hygro thermal impact on vibration behavior embedded FG nanobeams based on nonlocal strain gradient elasticity.

Huge application of curve nanobeams and nanoring in the empirical experiments and dynamic molecular simulations (Wang and Duan 2008) led many researchers to study the mechanical characteristics of these structures.

In comparison with straight nanobeams, curved ones possess various advantages such as large strokes (Ebrahimi and Barati 2016a-e). Recently, the use of curved nanobeams has been extended in different systems as nanoswitches, nanovalves and nanofilters. Literature survey indicates that there are few researches on the vibration behavior of FGM curved nanostructures like beam, ring and arches.

Yan and Jiang (2011) investigated the electromechanical response of curved piezoelectric nanobeam with the consideration of surface effects. Eltaher *et al.* (2018a-d) analyzed the bending and vibration behaviour of piezoelectric nanobeams considering surface elasticity. Meanwhile, Eltaher and co-researchers have presented several prominent studies on the size-dependent vibrations of nanostructures (Eltaher *et al.* 2016b) including Carbon nanotube (Eltaher *et al.* 2018a, 2016b, Agwa and Eltaher

2016) and FG nanotube (Hamed *et al.* 2016). Eltaher *et al.* (2014a) assessed the vibration of nonlinear graduation of nanobeam considering neutral axis position. Eltaher *et al.* (2014a) evaluated the mechanical behaviour of higher order gradient nanobeams as well.

In addition, a new numerical technique, the differential quadrature method has been developed for dynamic analysis of the nanobeams in the polar coordinate system by Kananipour *et al.* (2014). In addition, investigating surface effects on thermomechanical behaviour of embedded circular curved nanosize beams has been studied by Ebrahimi and Daman (2016a, b). However, Ebrahimi and Daman (2016a, b) have presented the radial vibration of embedded double-curved-nanobeam-systems. As well as, investigation of mechanical behaviour of smart curved nanobeams are carried out by Ebrahimi and Daman (2017a). In addition, Wang and Duan (2008) have surveyed the free vibration problem of nanorings/arches. In this research the problem was formulated on the framework of Eringen's nonlocal theory of elasticity according to allow for the small length scale effect. Moreover, Ansari *et al.* (2013), developed vibration of FG curved microbeams with framework of modified strain gradient elasticity model. Furthermore, Out-of-plane frequency analyse of FG circular curved beams in thermal environment has been investigated by Malekzadeh *et al.* (2010). In addition, Hosseini and Rahmani (2016) presented free vibration of shallow and deep curved functionally graded nanobeam based on nonlocal Timoshenko curved beam model. Moreover, Ebrahimi and Barati (2017a/b) employed nonlocal strain gradient theory to model size-dependent buckling response of FG curve nanobeams with different boundary conditions. Very recently, Mohamed *et al.* (2018) investigated the free and forced vibration response of buckled curved nanobeams resting on nonlinear elastic foundations. She *et al.* (2019) studied the nonlinear bending behaviour of FG porous curved nanotubes under the framework of nonlocal strain gradient theory. The nonlocal strain gradient theory in conjunction with a refined beam model is employed to formulate the size-dependent model and vibration behaviors of porous nanotubes are investigated by She *et al.* (2018a). They extended their evaluation for buckling and postbuckling behaviour of nanotubes (She *et al.* 2017a), wave propagation behaviour of porous nanotubes (She *et al.* 2018b, c).

Structures made of porous materials are the latest developments in the field of FGMs, so it is vital to consider influence of porosity parameters in their dynamic analysis. The presence of pores within the microstructures of such materials is taken into account by means of the local density of the material. Typically, common composition of FGMs is ceramic-metal and there are different approaches for processing of functionally graded metal-ceramic composite components. An efficient way of manufacturing FGMs is by a sintering process, Zhu *et al.* (2001) expressed that, different shrinkage stresses between the adjacent layers of ceramic and metal phases in FGM compact can be occurs during the processes of sintering and cold compacting, which may create porosity phases inside the materials. Moreover, in the multi-step sequential infiltration method

for producing FGMs, it is observed that the most of the porosity phases appear frequently in the central areas of FGM specimens because it is hard to infiltrate the secondary material in these zones completely, whereas material infiltration at the bottom and top areas can be performed easier with less porosities. According to this work, it is indispensable to take into consideration the porosity impact when designing and analyzing structures made of FGMs. However, there are a lot of studies related to dynamic and stability behaviors of FG structures but the dynamic analysis of porous FG structures, especially for beams, are still limited in number. Eltaher *et al.* (2018b) proposed a modified porosity model in order to analyze the structural response of FG porous nanobeam. They reported that porosity volume fraction and type of porosity distribution have a significant impact on the vibrational response of the FGM plates. Recently, Mechab *et al.* (2016) developed a nonlocal elasticity model for free vibration of FG porous nanoplates resting on elastic foundations. Wattanasakulpong and Ungbhakorn (2014) investigated linear and non-linear vibration of porous FGM beams with elastically restrained ends. In another work, Wattanasakulpong and Chaikittiratana (2015) predicted flexural vibration of porous FGM beams by applying the Chebyshev collocation method (CCM). Ebrahimi and Zia (2015) investigated the large-amplitude nonlinear vibration of porous FGM beams by utilizing Galerkin and multiple scales methods.

As mentioned before the main purpose of FGM development was the extreme heat resistance especially in thermal environment, hence it is essential to assume changing material properties due to thermal environment. Ebrahimi *et al.* (2016) investigated thermo-mechanical vibration response of temperature-dependent FGM beams having porosities. Based on Timoshenko and higher order beam theory, (Ebrahimi and Barati 2016b, Ebrahimi and Jafari 2016) examined porosity effect on vibration response of FG beams subjected to thermal loadings by using differential transform and Navier solution methods. Further, a severe thermal effect may develop cracks in the structure and influence the mechanical response. In this regard, Attia *et al.* (2018) carried out thermoelastic crack analysis in FG pipelines through FE methods. The influence of unsteady pressure and temperature on the cracks was analyzed by Eltaher *et al.* (2018a-d) and Soliman *et al.* (2018). Shafiei and She (2018) predicted the vibration behaviour of two-dimensional FG nanotubes in thermal environment under the framework of higher order theory.

Reviewing the literature search in the field of vibration behavior of FG curve nanobeam indicates that there are not any published work considering small size effects, strain gradient, porosity and thermal effect on vibration characteristics of FG curve nanobeam based on Timoshenko beam theory. As a result, present research analyzes thermo-mechanical vibration of curved functionally graded porous nanobeams based on nonlocal strain gradient theory which consider both nonlocal and length scale parameter to describe size effects. Curvature rather exists in all of the real beams and nanobeams. Moreover, in previous researches in order to streamline of mathematical equations,

straight beam models have been used, whilst curved beam models are more practicable than straight ones. The modified power-law models exploited to describe gradual variation of thermo-mechanical properties of the porous FG curve nanobeam. Applying Hamilton's principle, governing equations of porous FG curve nanobeam are obtained together and they are solved applying an analytical solution method. Dimensionless natural frequencies are obtained respect to the effect of various parameters such as angle of curvature, length scale parameter, temperatures changes, mode numbers, power-law index and nonlocal parameter on vibration of curved FG porous nanobeams. Comparison between the results of present research and available data in literature reveals the accuracy of this model.

## 2. Problem formulation

### 2.1 The material properties of curved FG porous nanobeams

Consider a curved FG porous nanobeam with even porosity distributions with length  $L$  in  $\theta$  direction and rectangular cross-section of width  $b$  and thickness  $h$  according to Fig. 1. Curved FG porous nanobeam is composed of  $Si_3N_4$  and SUS304 materials with the thermo-mechanical material properties presented in Table 1 and exposed to a thermal loading.

The relation between length of circular curved beam ( $\theta$ ) and the angle of curvature of beam ( $\alpha$ ) can be written as (Setoodeh *et al.* 2015)

$$\theta = R\alpha \quad (1)$$

The effective material properties of the curved FGM porous beam change continuously in the thickness direction based on modified power-law distribution. According to this model, the effective material properties ( $P_f$ ) of porous FG curve nanobeam by using the modified rule of mixture can be expressed by Wattanasakulpong and Ungbhakorn (2014)

$$P_f = P_u(V_u - \frac{\mathcal{G}}{2}) + P_l(V_l - \frac{\mathcal{G}}{2}) \quad (2)$$

where  $\mathcal{G}$  is the volume fraction of porosity and ( $P_l, P_u$ ) are the properties of materials at the lower surface and upper surface, respectively. In addition, ( $V_l, V_u$ ) are the corresponding volume fractions related by

$$V_u = \left( \frac{z}{h} + \frac{1}{2} \right)^p \quad (3)$$

$$V_u + V_l = 1 \quad (4)$$

Hence, from Eqs. (2) and (3), the material properties of the curved FG porous beam can be defined as

$$P(z) = (P_u - P_l) \left( \frac{z}{h} + \frac{1}{2} \right)^p + P_l - (P_u + P) \frac{\mathcal{G}}{2} \quad (5)$$

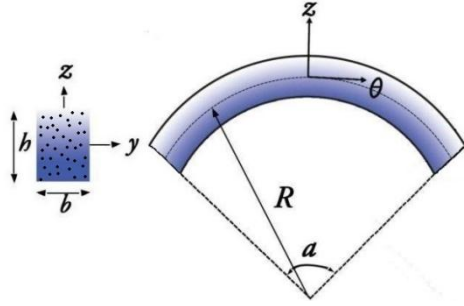


Fig. 1 Geometric of curved FG porous nanobeam

where  $p$  is the power-law exponent (non-negative variable parameter). Further, it is worthy to mention that the power-law exponent determines the material distribution profile through the thickness of the beam ( $z$  direction). Based on this distribution, the bottom surface ( $z = h/2$ ) consists of pure SUS304, while the top surface ( $z = -h/2$ ) of curved FG porous nanobeam is composed of pure  $Si_3N_4$ .

To prognosticate the treatment of curved FG materials under high temperature more accurately, it is essential to assume the temperature dependency on material properties. The nonlinear equation of thermo-elastic material properties in function of temperature  $T(K)$  can be defined as (Touloukian 1966)

$$P = P_0 \left( P_{-1} T^{-1} + 1 + P_1 T + P_2 T^2 + P_3 T^3 \right) \quad (6)$$

where  $P_0$ ,  $P_{-1}$ ,  $P_1$ ,  $P_2$  and  $P_3$  are the temperature dependent factors that accessible in the Table 1.

## 2.2 Governing equation

Based on Timoshenko beam theory, the displacement field at any point of the curved beam model can be remarked as Hosseini and Rahmani (2016)

$$u_\theta(\theta, z, t) = \left( 1 + \frac{z}{R} \right) u(\theta, t) + z\varphi(\theta, t) \quad (7a)$$

$$u_z(\theta, z, t) = w(\theta, t) \quad (7b)$$

where  $w$  and  $u$  interpret the radial and tangential displacement of curved FG porous beam. In addition,  $\varphi$  is the total bending rotation of cross sections of curved FGP beam. The nonzero strains of Timoshenko curved beam theory are expressed as

$$\varepsilon_{\theta\theta}^0 = \frac{\partial w}{\partial \theta} - \frac{u}{R} \quad (8a)$$

$$\kappa = \frac{\partial \varphi}{\partial \theta} \quad (8b)$$

$$\gamma_{\theta z} = \frac{\partial u}{\partial \theta} - \varphi + \frac{w}{R} \quad (8c)$$

Here  $\gamma$  denotes the shear strain in the curved beam model.

$$\varepsilon_{\theta\theta} = \left( \varepsilon_{\theta\theta}^0 + z\kappa \right) \quad (9)$$

Through extended Hamilton's principle, the governing equation of motion can be derived by

$$\int_0^t \delta(U_s - T + W_{ex}) \quad (10)$$

where  $U_s$ ,  $T$  and  $W_{ex}$  are strain energy, kinetic energy and work done by external exerted loads, respectively. The first variation of strain energy  $U_s$  can be written as

$$\delta U_s = \int_V (\sigma_{\theta\theta} \delta \varepsilon_{\theta\theta} + \sigma_{\theta z} \delta \gamma_{\theta z}) dV \quad (11)$$

Substituting Eqs. (6) and (7) into Eq. (21) the first variation of strain energy yields

$$\begin{aligned} \delta U_s = \int_0^L \left( N \left( -\frac{\delta u}{R} + \frac{\partial \delta w}{\partial \theta} \right) + M \left( \frac{\partial \delta \varphi}{\partial \theta} \right) \right. \\ \left. + Q \left( \frac{\partial \delta u}{\partial \theta} - \delta \varphi + \frac{\delta w}{R} \right) \right) d\theta \end{aligned} \quad (12)$$

In which the variables at the last expression are expressed as:  $M$ ,  $N$ , and  $Q$  define bending moment of cross section, axial force, and shear force, respectively. These stress resultants are expressed as

Table 1 Temperature dependent coefficients of Young's modulus, thermal expansion coefficient, mass density and thermal conductivity for SUS304 and  $Si_3N_4$

Material	Properties	$P_0$	$P_{-1}$	$P_1$	$P_2$	$P_3$
$Si_3N_4$	$E$ (Pa)	348.43e+9	0	-3.070e-4	2.160e-7	-8.946e-11
	$\lambda_1$ ( $K^{-1}$ )	5.8723e-6	0	9.095e-4	0	0
	$\rho$ (Kg/m <sup>3</sup> )	2370	0	0	0	0
	$\kappa$ (W/mk)	13.723	0	-1.032e-3	5.466e-7	-7.876e-11
SUS304	$E$ (Pa)	201.04e+9	0	3.079e-4	-6.534e-7	0
	$\lambda_1$ ( $K^{-1}$ )	12.330e-6	0	8.086e-4	0	0
	$\rho$ (Kg/m <sup>3</sup> )	8166	0	0	0	0
	$\kappa$ (W/mk)	15379	0	-1.264e-3	2.092e-6	-7.223e-10

$$N = \int_A \sigma_{\theta\theta} dA, \quad M = \int_A \sigma_{\theta\theta} z dA, \quad Q = \int_A K_{shear} \sigma_{\theta z} dA \quad (13)$$

where  $K_{shear}$  expresses the shear correction factor. The first variational of the virtual kinetic energy of present curve beam model can be written in the form as

$$T = \frac{1}{2} \int_0^L \int_A \rho(z, T) \left( \left( \frac{\partial u_\theta}{\partial t} \right)^2 + \left( \frac{\partial u_z}{\partial \theta} \right)^2 \right) dA d\theta \quad (14)$$

$$\begin{aligned} \delta T = & [I_0 \left( \frac{\partial u}{\partial t} \frac{\partial \delta u}{\partial t} \right) + (I_0 + \frac{2I_1}{R} + \frac{I_2}{R^2}) \frac{\partial w}{\partial t} \frac{\partial \delta w}{\partial t} \\ & + (I_1 + \frac{I_2}{R}) \left( \frac{\partial \varphi}{\partial t} \frac{\partial \delta w}{\partial t} + \frac{\partial w}{\partial t} \frac{\partial \delta \varphi}{\partial t} \right) + I_2 \frac{\partial \varphi}{\partial t} \frac{\partial \delta \varphi}{\partial t}] \end{aligned} \quad (15)$$

where mass moments of inertias ( $I_0, I_1, I_2$ ) are defined as follows

$$(I_0, I_1, I_2) = \int_A \rho(z, T) (1, z, z^2) dA \quad (16)$$

The first variation of work done by applied forces can be written in the form

$$\delta W_{ext} = \frac{1}{2} \int_0^L (N_T) \frac{\partial w}{\partial \theta} \delta \frac{\partial w}{\partial \theta} d\theta \quad (17)$$

where  $N_T$  is in-plane applied load namely thermal loadings, which can be given as

$$N_T = \int_{-\frac{h}{2}}^{\frac{h}{2}} E(z, T) \lambda_1(z, T) \Delta T dz \quad (18)$$

By inserting the coefficients of  $\delta u, \delta w, \delta \varphi$  and  $\delta \psi$  equal to zero, following Euler–Lagrange equations of curved FG porous nanobeam subjected to thermal loading are obtained

$$\frac{N}{R} + \frac{\partial Q}{\partial \theta} = I_0 \frac{\partial^2 u}{\partial t^2} \quad (19a)$$

$$\begin{aligned} & \frac{\partial N}{\partial \theta} - \frac{Q}{R} - N_T \frac{\partial^2 w}{\partial \theta^2} \\ & = \left( I_0 + \frac{2I_1}{R} + \frac{I_2}{R^2} \right) \frac{\partial^2 w}{\partial t^2} + \left( I_1 + \frac{I_2}{R} \right) \frac{\partial^2 \varphi}{\partial t^2} \end{aligned} \quad (19b)$$

$$\frac{\partial M}{\partial \theta} + Q = \left( I_1 + \frac{I_2}{R} \right) \frac{\partial^2 w}{\partial t^2} + I_2 \frac{\partial^2 \varphi}{\partial t^2} \quad (19c)$$

Boundary conditions that are related to equation of motions are considered as

$$N = 0 \quad \text{or} \quad w = 0 \quad \text{at} \quad \theta = 0 \quad \text{and} \quad \theta = L \quad (20a)$$

$$Q = 0 \quad \text{or} \quad u = 0 \quad \text{at} \quad \theta = 0 \quad \text{and} \quad \theta = L \quad (20b)$$

$$M = 0 \quad \text{or} \quad \varphi = 0 \quad \text{at} \quad \theta = 0 \quad \text{and} \quad \theta = L \quad (20c)$$

### 2.3 The nonlocal strain gradient elasticity model for curved FG porous nanobeam

Nonlocal strain gradient elasticity (Li and Hu 2015) accounts the stress field for both nonlocal stress and strain fields. Therefore, the stress can be given as

$$\sigma_{ij} = \sigma_{ij}^{(0)} - \frac{d\sigma_{ij}^{(1)}}{dx} \quad (21)$$

where the stresses  $\sigma_{xx}^{(0)}$  and  $\sigma_{xx}^{(1)}$  are related to strain  $\varepsilon_{xx}$  and strain gradient  $\varepsilon_{xx,x}$ , respectively and are expressed as

$$\sigma_{ij}^{(0)} = \int_0^L C_{ijkl} \alpha_0(x, x', e_0 a) \varepsilon'_{kl}(x') dx' \quad (22a)$$

$$\sigma_{ij}^{(1)} = l^2 \int_0^L C_{ijkl} \alpha_1(x, x', e_1 a) \varepsilon'_{kl,x}(x') dx' \quad (22b)$$

in which  $C_{ijkl}$  are the elastic constants and  $e_0 a$  and  $e_1 a$  take into account the effect of nonlocal stress field and  $l$  is the length scale parameter and introduces the influence of higher order strain gradient stress field. When the nonlocal functions  $\alpha_0(x, x', e_0 a)$  and  $\alpha_1(x, x', e_1 a)$  satisfy the developed conditions by Eringen (1983), the constitutive relation for a FGM nanobeam can be given as Ebrahimi and Barati (2017b)

$$\begin{aligned} & [1 - (e_1 a)^2 \nabla^2] [1 - (e_0 a)^2 \nabla^2] \sigma_{ij} \\ & = C_{ijkl} [1 - (e_1 a)^2 \nabla^2] \varepsilon_{kl} - C_{ijkl} l^2 [1 - (e_0 a)^2 \nabla^2] \nabla^2 \varepsilon_{kl} \end{aligned} \quad (23)$$

in which  $\nabla^2$  denotes the Laplacian operator. Supposing  $e_1 = e_0 = e$  and discarding terms of order  $O(\nabla^2)$ , the general constitutive relation in Eq. (21) can be rewritten as Li and Hu (2015)

$$[1 - (ea)^2 \nabla^2] \sigma_{ij} = C_{ijkl} [1 - l^2 \nabla^2] \varepsilon_{kl} \quad (24)$$

Thus, the constitutive relations for a nonlocal refined shear deformable curved FG porous nanobeam can be stated as

$$\sigma_{\theta\theta} - \mu^2 \frac{\partial^2 \sigma_{\theta\theta}}{\partial \theta^2} = E(z, T) (\varepsilon_{\theta\theta} - \lambda \frac{\partial^2 \varepsilon_{\theta\theta}}{\partial \theta^2}) \quad (25a)$$

$$\sigma_{\theta z} - \mu^2 \frac{\partial^2 \sigma_{\theta z}}{\partial \theta^2} = G(z, T) (\gamma_{\theta z} - \lambda \frac{\partial^2 \gamma_{\theta z}}{\partial \theta^2}) \quad (25b)$$

Calculating Eqs. (25a) and (25b) by integrating over cross-section area of the curved beam, force–strain and moment–strain of nonlocal curved FG porous Timoshenko beam model will be determined as

$$\begin{aligned} & N - \mu^2 \frac{\partial^2 N}{\partial \theta^2} \\ & = A \left( \frac{\partial w}{\partial \theta} - \frac{u}{R} - \lambda \left( \frac{\partial^3 w}{\partial \theta^3} - \frac{1}{R} \frac{\partial^2 u}{\partial \theta^2} \right) \right) + B \left( \frac{\partial \varphi}{\partial \theta} - \lambda \frac{\partial^3 \varphi}{\partial \theta^3} \right) \end{aligned} \quad (26a)$$

$$M - \mu^2 \frac{\partial^2 M}{\partial \theta^2} = B \left( \frac{\partial w}{\partial \theta} - \frac{u}{R} - \lambda \left( \frac{\partial^3 w}{\partial \theta^3} - \frac{1}{R} \frac{\partial^2 u}{\partial \theta^2} \right) \right) + D \left( \frac{\partial \varphi}{\partial \theta} - \lambda \frac{\partial^3 \varphi}{\partial \theta^3} \right) \quad (26b)$$

$$Q - \mu^2 \frac{\partial^2 Q}{\partial \theta^2} = K_{Shear} C \left[ \left( \frac{w}{R} + \frac{\partial u}{\partial \theta} - \varphi \right) - \lambda \left( \frac{1}{R} \frac{\partial^2 w}{\partial \theta^2} + \frac{\partial^3 u}{\partial \theta^3} - \frac{\partial^2 \varphi}{\partial \theta^2} \right) \right] \quad (26c)$$

where  $K_{Shear}$  defined as correction factor and assumed that equal 5/6. Consequently, coefficients are obtained as

$$\{A, B, D\} = \int_A E(z, T) \{1, z, z^2\} dA \quad (27a)$$

$$C = \int_A \frac{E(z, T)}{1 + \nu(z, T)} dA \quad (27b)$$

By inserting Eqs. (26) into Eqs. (19), nonlocal governing equations of curved FG porous Timoshenko nanobeam in terms of displacement can be calculated as

$$A \left( \frac{\partial w}{\partial \theta} - \frac{u}{R} \right) - A \lambda \left( \frac{\partial^3 w}{\partial \theta^3} - \frac{\partial^2 u}{R \partial \theta^2} \right) + B \frac{\partial \varphi}{\partial \theta} - B \lambda \frac{\partial^3 \varphi}{\partial \theta^3} + K_{Shear} C R \left( \frac{\partial w}{R \partial \theta} + \frac{\partial^2 u}{\partial \theta^2} - \frac{\partial \varphi}{\partial \theta} \right) - K_{Shear} C R \lambda \left( \frac{\partial^3 w}{R \partial \theta^3} + \frac{\partial^4 u}{\partial \theta^4} - \frac{\partial^3 \varphi}{\partial \theta^3} \right) \quad (28a)$$

$$= I_0 R \left( \frac{\partial^2 u}{\partial t^2} - \mu^2 \frac{\partial^4 u}{\partial \theta^2 \partial t^2} \right) + A R \left( \frac{\partial^2 w}{\partial \theta^2} - \frac{\partial u}{R \partial \theta} \right) - A R \lambda \left( \frac{\partial^4 w}{\partial \theta^4} - \frac{\partial^3 u}{R \partial \theta^3} \right) + B R \frac{\partial^2 \varphi}{\partial \theta^2} - B R \lambda \frac{\partial^4 \varphi}{\partial \theta^4} - N_T R \frac{\partial^2 w}{\partial \theta^2} + \mu^2 N_T R \frac{\partial^4 w}{\partial \theta^4} - K_{Shear} C \left( \frac{w}{R} + \frac{\partial u}{\partial \theta} - \varphi \right) - K_{Shear} C \lambda \left( \frac{\partial^2 w}{R \partial \theta^2} + \frac{\partial^3 u}{\partial \theta^3} - \frac{\partial^2 \varphi}{\partial \theta^2} \right) \quad (28b)$$

$$= \left( I_0 + \frac{2I_1}{R} + \frac{I_2}{R^2} \right) R \frac{\partial^2 w}{\partial t^2} + \left( I_1 + \frac{I_2}{R} \right) R \frac{\partial^2 \varphi}{\partial t^2} - \mu^2 \left( I_0 + \frac{2I_1}{R} + \frac{I_2}{R^2} \right) R \frac{\partial^4 w}{\partial \theta^2 \partial t^2} - \mu^2 \left( I_1 + \frac{I_2}{R} \right) R \frac{\partial^4 \varphi}{\partial \theta^2 \partial t^2} + B \left( \frac{\partial^2 w}{\partial \theta^2} - \frac{\partial u}{R \partial \theta} \right) - B \lambda \left( \frac{\partial^4 w}{\partial \theta^4} - \frac{\partial^3 u}{R \partial \theta^3} \right) + D \frac{\partial^2 \varphi}{\partial \theta^2} - D \lambda \frac{\partial^4 \varphi}{\partial \theta^4} + K_{Shear} C \left( \frac{w}{R} + \frac{\partial u}{\partial \theta} - \varphi \right) \quad (28c)$$

$$- K_{Shear} C \lambda \left( \frac{\partial^2 w}{R \partial \theta^2} + \frac{\partial^3 u}{\partial \theta^3} - \frac{\partial^2 \varphi}{\partial \theta^2} \right) = \left( I_1 + \frac{I_2}{R} \right) \frac{\partial^2 w}{\partial t^2} + I_2 \frac{\partial^2 \varphi}{\partial t^2} - \mu^2 \left( I_1 + \frac{I_2}{R} \right) \frac{\partial^4 w}{\partial \theta^2 \partial t^2} - \mu^2 I_2 \frac{\partial^4 \varphi}{\partial \theta^2 \partial t^2} \quad (28c)$$

### 3. Solution method

In this section, analytical Navier method has been developed to solve the governing equations of curved FG regarding find out free vibrational of a simply supported curved porous FG nanobeam in the thermal environment.

To satisfy simply supported boundary condition, the displacement quantities are presented in the following form as trigonometric functions

$$w(\theta, t) = \sum_{n=1}^{\infty} W_n \cos \left( \frac{n\pi}{L} \theta \right) e^{i\omega_n t} \quad (29a)$$

$$u(\theta, t) = \sum_{n=1}^{\infty} U_n \sin \left( \frac{n\pi}{L} \theta \right) e^{i\omega_n t} \quad (29b)$$

$$\varphi(\theta, t) = \sum_{n=1}^{\infty} \phi_n \cos \left( \frac{n\pi}{L} \theta \right) e^{i\omega_n t} \quad (29c)$$

where  $W_n$ ,  $U_n$  and  $\phi_n$  are the unknown coefficients which are obtained for each  $n$  value. The boundary conditions for simply supported curved FG porous nanobeam can be given as

$$w(0) = 0, \quad \frac{\partial w}{\partial \theta}(L) = 0, \quad u(0) = u(L) = 0 \quad (30)$$

$$\frac{\partial \varphi}{\partial \theta}(0) = \frac{\partial \varphi}{\partial \theta}(L) = 0$$

Inserting Eq. (29) into Eq. (28) respectively, leads to Eq. (31)

$$A \left( -\frac{U_n}{R} - \left( \frac{n\pi}{L} \right) W_n \right) + A \lambda \left( -\frac{U_n}{R} \left( \frac{n\pi}{L} \right)^2 - \left( \frac{n\pi}{L} \right)^3 W_n \right) - B \left( \frac{n\pi}{L} \right) \phi_n - B \lambda \left( \frac{n\pi}{L} \right)^3 \phi_n + K_{Shear} C R \left[ -\frac{1}{R} \left( \frac{n\pi}{L} \right) W_n - \left( \frac{n\pi}{L} \right)^2 U_n + \left( \frac{n\pi}{L} \right) \phi_n \right] \quad (31a)$$

$$+ K_{Shear} C R \lambda \left[ -\frac{1}{R} \left( \frac{n\pi}{L} \right)^3 W_n - \left( \frac{n\pi}{L} \right)^4 U_n + \left( \frac{n\pi}{L} \right)^3 \phi_n \right] = R I_0 \left( -\omega_n^2 U_n - \mu^2 \omega_n^2 \left( \frac{n\pi}{L} \right)^2 U_n \right)$$

$$A R \left[ -\frac{1}{R} \left( \frac{n\pi}{L} \right) U_n - \left( \frac{n\pi}{L} \right)^2 W_n \right] \quad (31b)$$

$$\begin{aligned}
& + AR\lambda \left[ -\frac{1}{R} \left( \frac{n\pi}{L} \right)^3 U_n - \left( \frac{n\pi}{L} \right)^4 W_n \right] \\
& - BR \left( \frac{n\pi}{L} \right)^2 \phi_n - BR\lambda \left( \frac{n\pi}{L} \right)^4 \phi_n \\
& - N_T R \left( \frac{n\pi}{L} \right)^2 W_n - \mu^2 N_T R \left( \frac{n\pi}{L} \right)^4 W_n \\
& - K_{Shear} C \left[ \frac{1}{R} W_n + \left( \frac{n\pi}{L} \right) U_n - \phi_n \right] \\
& - K_{Shear} C \lambda \left[ \frac{1}{R} \left( \frac{n\pi}{L} \right)^2 W_n + \left( \frac{n\pi}{L} \right)^3 U_n - \left( \frac{n\pi}{L} \right)^2 \phi_n \right]
\end{aligned} \quad (31b)$$

$$\begin{aligned}
& = - \left( I_0 + \frac{2I_1}{R} + \frac{I_2}{R^2} \right) R \omega_n^2 W_n - \left( I_1 + \frac{I_2}{R} \right) R \omega_n^2 \phi_n \\
& - \left( I_1 + \frac{I_2}{R} \right) R \omega_n^2 \phi_n - \mu^2 R \left( \frac{n\pi}{L} \right)^2 \left( I_0 + \frac{2I_1}{R} + \frac{I_2}{R^2} \right) \omega_n^2 W_n \\
& - \mu^2 \left( I_1 + \frac{I_2}{R} \right) R \left( \frac{n\pi}{L} \right)^2 \omega_n^2 \phi_n \\
& BR \left[ -\frac{1}{R} \left( \frac{n\pi}{L} \right) U_n + \left( \frac{n\pi}{L} \right)^2 W_n \right] \\
& + BR\lambda \left[ -\frac{1}{R} \left( \frac{n\pi}{L} \right)^3 U_n + \left( \frac{n\pi}{L} \right)^4 W_n \right] \\
& - RD \left( \frac{n\pi}{L} \right)^2 \phi_n - RD\lambda \left( \frac{n\pi}{L} \right)^4 \phi_n \\
& + K_{Shear} CR \left[ \frac{1}{R} W_n + \left( \frac{n\pi}{L} \right) U_n - \phi_n \right] \\
& + K_{Shear} CR\lambda \left[ \frac{1}{R} \left( \frac{n\pi}{L} \right)^2 W_n + \left( \frac{n\pi}{L} \right)^3 U_n - \left( \frac{n\pi}{L} \right)^2 \phi_n \right] \\
& = -R \left( I_1 + \frac{I_2}{R} \right) \omega_n^2 W_n - RI_2 \omega_n^2 \phi_n \\
& - R\mu^2 \left( I_1 + \frac{I_2}{R} \right) \left( \frac{n\pi}{L} \right)^2 \omega_n^2 W_n - R\mu^2 I_2 \left( \frac{n\pi}{L} \right)^2 \omega_n^2 \phi_n
\end{aligned} \quad (31c)$$

By finding determinant of the coefficient matrix of the following equations and setting this multinomial to zero, we can find natural frequencies  $\omega_n$ .

$$\{[K] - \omega_n^2 [M]\} \begin{Bmatrix} U_n \\ W_n \\ \phi_n \\ \psi_n \end{Bmatrix} = 0 \quad (32)$$

where  $[K]$  and  $[M]$  are stiffness and mass matrixes.

#### 4. Numerical results and discussion

In this section the credibility of the proposed analytical solution is justified by comparing the obtained results with those reported by Hosseini and Rahmani (2016). To this end, the free vibration problem of the perfect curve FG nanobeam regardless of temperature risings investigated by Hosseini and Rahmani (2016) is considered. As elucidated in Tables 2 and 3 for various gradient index and opening angles, it can be observed that the present results have very well agreement with the Hosseini and Rahmani (2016). Therefore, the proposed formulation can be adopted to explore the influence of nonlocality, strain gradient parameter, porosity volume fractions, temperature rising, gradient index, opening angle and aspect ratio on the natural frequencies of porous FG curve nanobeam under thermal loadings. The non-dimensional natural frequency ( $\lambda$ ) can be calculated by the relation in Eq. (33)

$$\lambda = \omega \frac{L^2}{h} \sqrt{\frac{\rho_u}{E_u}} \quad (33)$$

In Table 4, the frequency results of porous FG curved nanobeam under different thermal environment are presented. To this end, three different magnitudes of temperature gradient ( $\Delta T = 30, 60, 90$ ), opening angle ( $\alpha =$

Table 2 Comparison of dimensionless natural frequencies of S-S curved FG nanobeams for different amounts of slenderness, mode number and nonlocality where  $p = 0$  and  $\alpha = \pi/3$

$L/h$	$\omega_n$	$\mu^2 = 0$		$\mu^2 = 1$		$\mu^2 = 2$		$\mu^2 = 3$		$\mu^2 = 4$	
		Hosseini and Rahmani (2016)	Present	Hosseini and Rahmani (2016)	Present	Hosseini and Rahmani (2016)	Present	Hosseini and Rahmani (2016)	Present	Hosseini and Rahmani (2016)	Present
10	$n = 1$	8.1991	8.1989	7.8222	7.8220	7.4929	7.4927	7.2020	7.2018	6.9425	6.9423
	$n = 2$	35.7451	35.7448	30.2666	30.2663	26.7204	26.7201	24.1855	24.1853	22.2576	22.2574
	$n = 3$	77.3993	77.3991	56.3256	56.3253	46.4500	46.4500	40.4308	40.4305	36.2732	36.2730
20	$n = 1$	8.2912	8.2910	7.9101	7.9098	7.5771	7.5769	7.2829	7.2827	7.0205	7.0203
	$n = 2$	37.2875	37.2874	31.5725	31.5723	27.8733	27.8730	25.2291	25.2288	23.2180	23.2177
	$n = 3$	84.3180	84.3178	61.3605	61.3603	50.6022	50.6020	44.0449	44.0446	39.5156	39.5153
50	$n = 1$	8.3177	8.3175	7.9353	7.9351	7.6012	7.6009	7.3061	7.3059	7.0429	7.0426
	$n = 2$	37.7658	37.7654	31.9776	31.9774	28.2309	28.2306	25.5527	25.5524	23.5159	23.5157
	$n = 3$	86.7084	86.7082	63.1000	63.1000	52.0367	52.0365	45.2935	45.2933	40.6359	40.6357

Table 3 Comparison of dimensionless natural frequency of S-S curved FG nanobeams for different amounts of slenderness, mode number and nonlocality where  $p = 1$  and  $\alpha = \pi/2$ 

$L/h$	$\omega_n$	$\mu^2 = 0$		$\mu^2 = 1$		$\mu^2 = 2$		$\mu^2 = 3$		$\mu^2 = 4$	
		Hosseini and Rahmani (2016)	Present	Hosseini and Rahmani (2016)	Present	Hosseini and Rahmani (2016)	Present	Hosseini and Rahmani (2016)	Present	Hosseini and Rahmani (2016)	Present
10	$n = 1$	4.5601	4.5601	4.3504	4.3504	4.1673	4.1673	4.0055	4.0055	3.8612	3.8612
	$n = 2$	23.7375	23.7375	20.0993	20.0993	17.7444	17.7444	16.0611	16.0611	14.7808	14.7808
	$n = 3$	53.2817	53.2817	38.7745	38.7745	31.9762	31.9762	27.8325	27.8325	24.9704	24.9704
20	$n = 1$	4.6675	4.6675	4.4530	4.4530	4.2655	4.2655	4.0999	4.0999	3.9522	3.9522
	$n = 2$	25.0039	25.0039	21.1716	21.1716	18.6911	18.6911	16.9179	16.9179	15.5694	15.5694
	$n = 3$	58.3285	58.3285	42.4472	42.4472	35.0050	35.0050	30.4689	30.4689	27.3356	27.3356
50	$n = 1$	4.7208	4.7208	4.5038	4.5038	4.3142	4.3142	4.1466	4.1466	3.9972	3.9972
	$n = 2$	25.5362	25.5362	21.6223	21.6223	19.0889	19.0889	17.2780	17.2780	15.9008	15.9008
	$n = 3$	60.4005	60.4005	43.9551	43.9551	36.9551	36.9551	31.5512	31.5512	28.3067	28.3067

$\pi/2, \pi/3, \pi/4$ ), power-law indexes ( $P = 0.2, 0.5, 1$ ), porosity volume fractions ( $\mathcal{G} = 0, 0.1, 0.2$ ), strain gradient parameter ( $\lambda = 0, 2$ ) and at constant value of aspect ratio ( $L/h = 30$ ) and nonlocal parameter ( $\mu^2 = 2$ ) are considered for evaluation. It can be observed from Table 4 that, by increasing the material gradient index, the dimensionless natural frequencies decrease. This is due to the fact that the stiffness of the curved FG nanobeams decreases, as the percentage of metal phase increases with a higher power-law index. Also, improving the temperature gradient leads

to decrease in the dimensionless natural frequency. Meanwhile, it may be observed that these parameters have a significant effect on the dimensionless frequency parameter. The results of Table 4 also suggest that the significant influence of porosity parameter and gradient index prevails on the variations of the dimensionless frequencies of curved FG porous nanobeam. For example, while  $0 < p < 1$ , increasing of porosity parameter results in the higher non-dimensional natural frequency for all of opening angles and temperature risings. This trend of variation in the change of

Table 4 Temperature and gradient index effect on fundamental frequency of a S-S curved FG nanobeam with various strain gradient parameter, porosity, curvature and thermal loading  $L/h = 30, \mu^2 = 2$ 

$\mathcal{G}$	Load type		$\Delta T = 30[K]$			$\Delta T = 60[K]$			$\Delta T = 90[K]$		
			Power-law exponent			Power-law exponent			Power-law exponent		
			0.2	0.5	1	0.2	0.5	1	0.2	0.5	1
$\alpha = \pi/4$	0	$\lambda = 0$	6.5399	5.5380	4.8444	6.4502	5.4505	4.7589	6.3603	5.3568	4.6666
		$\lambda = 2$	7.1719	6.0753	5.3160	7.0904	5.9960	5.2387	7.0038	5.9111	5.1552
	0.1	$\lambda = 0$	6.8107	5.6266	4.8418	6.7297	5.5482	4.7655	6.6436	5.4642	4.6831
		$\lambda = 2$	7.4668	6.1708	5.3117	7.3961	6.0997	5.2427	7.3149	6.0235	5.1681
	0.2	$\lambda = 0$	7.1850	5.7388	4.8351	7.1153	5.6694	4.7680	7.0354	5.5950	4.6954
		$\lambda = 2$	7.8722	6.2919	5.3028	7.8088	6.2291	5.2422	7.7388	6.1616	5.1765
$\alpha = \pi/3$	0	$\lambda = 0$	5.9831	5.0550	4.4132	5.8217	4.8901	4.2513	5.6444	4.7113	4.0747
		$\lambda = 2$	6.5759	5.5597	4.8570	6.4234	5.4106	4.7108	6.2601	5.2498	4.5522
	0.1	$\lambda = 0$	6.2414	5.1446	4.4183	6.0899	4.9971	4.2741	5.9275	4.8378	4.1172
		$\lambda = 2$	6.8557	5.6549	4.8597	6.7182	5.5214	4.7294	6.5715	5.3778	4.5883
	0.2	$\lambda = 0$	6.5953	5.2559	4.4196	6.4600	5.1257	4.2929	6.3157	4.9855	4.1554
		$\lambda = 2$	7.2402	5.7738	4.8582	7.1173	5.6559	4.7437	6.9867	5.5293	4.6198
$\alpha = \pi/2$	0	$\lambda = 0$	4.4773	3.7430	3.2362	4.0290	3.2754	2.7688	3.4896	2.7039	2.1776
		$\lambda = 2$	4.9757	4.1712	3.6158	4.5595	3.7577	3.2047	4.0812	3.2717	2.7107
	0.1	$\lambda = 0$	4.7116	3.8440	3.2702	4.2967	3.4326	2.8607	3.8170	2.9434	2.3593
		$\lambda = 2$	5.2240	4.2735	3.6446	4.8533	3.9079	3.2826	4.4344	4.4862	2.8566
	0.2	$\lambda = 0$	5.0212	3.9619	3.3008	4.6564	3.6042	2.9466	4.2434	3.1890	2.5248
		$\lambda = 2$	5.5424	4.3943	3.6698	5.2279	4.0751	3.3553	4.8637	3.7130	2.9919



Table 5 Nonlocality and gradient index effect on fundamental frequency of a S-S curved FG nanobeam with various strain gradient parameter, porosity and thermal loading  $L/h = 30$ ,  $\mu^2 = 2$ 

g	Load type	$\mu^2 = 1$			$\mu^2 = 2$			$\mu^2 = 3$			
		Power-law Exponent			Power-law exponent			Power-law exponent			
		0.2	0.5	1	0.2	0.5	1	0.2	0.5	1	
$\Delta T = 30[K]$	0	$\lambda = 0$	4.7107	3.9436	3.4142	4.4773	3.7430	3.2362	4.2696	3.5640	3.0772
		$\lambda = 2$	5.2273	4.3870	3.8068	4.9757	4.1712	3.6158	4.7520	3.9791	3.4456
	0.1	$\lambda = 0$	4.9514	4.0451	3.4457	4.7116	3.8440	3.2702	4.4983	3.6649	3.1137
		$\lambda = 2$	5.4831	4.4902	3.8333	5.2240	4.2735	3.6446	4.9939	4.0807	3.4767
	0.2	$\lambda = 0$	5.2709	4.1642	3.4736	5.0212	3.9619	3.3008	4.7994	3.7819	3.1469
		$\lambda = 2$	5.8255	4.6129	3.8561	5.5551	4.3943	3.6698	5.3151	4.2001	3.5042
$\Delta T = 60[K]$	0	$\lambda = 0$	4.2869	3.5032	2.9751	4.0290	3.2754	2.7688	3.7760	3.0692	2.5809
		$\lambda = 2$	4.8329	3.9961	3.4190	4.5595	3.7577	3.2047	4.3142	3.5431	3.0112
	0.1	$\lambda = 0$	4.5584	3.6565	3.0599	4.2967	3.4326	2.8607	4.0616	3.2306	2.6802
		$\lambda = 2$	5.1313	4.1440	3.4911	4.8533	3.9079	3.2826	4.6046	3.6959	3.0949
	0.2	$\lambda = 0$	4.9247	3.8257	3.1392	4.6564	3.6042	2.9466	4.4163	3.4052	2.7729
		$\lambda = 2$	5.5144	4.3101	3.5583	5.2279	4.0751	3.3553	4.9721	3.8647	3.1731
$\Delta T = 90[K]$	0	$\lambda = 0$	3.7845	2.9758	2.4347	3.4896	2.7039	2.1776	3.1818	2.4498	1.9330
		$\lambda = 2$	4.3846	3.5430	2.9610	4.0812	3.2717	2.7107	3.8051	3.0227	2.4787
	0.1	$\lambda = 0$	4.1095	3.2018	2.5975	3.8170	2.9434	2.3593	3.5502	2.7050	2.1368
		$\lambda = 2$	4.7370	3.7490	3.0940	4.4344	3.4862	2.8566	4.1606	3.2468	2.6386
	0.2	$\lambda = 0$	4.5362	3.4374	2.7473	4.2434	3.1890	2.5248	3.9783	2.9621	2.3196
		$\lambda = 2$	5.1704	3.9696	3.2180	4.8637	3.7130	2.9919	4.5876	3.4808	2.7859

dimensionless frequency is reversed for  $p > 1$ . Further, it is also witnessed that as the change in temperature increases, the dimensionless natural frequency decreases. It also can be seen that with increasing in opening angle from  $\pi/4$  to  $\pi/2$ , the natural frequency decreases significantly. Furthermore, increasing length scale parameter gives larger natural frequencies as it can be observed in Table 4. This behavior reveals that, while increasing in length scale parameter, the curved FG porous nanobeam becomes stiffer. Table 5 illustrates the variation of fundamental natural frequencies with different nonlocality and length scale parameters. To this end, a constant aspect ratio ( $L/h = 30$ ) and opening angle ( $\alpha = \pi/2$ ) is adopted for curved FG porous nanobeam with simply supported boundary condition at both ends.

It can be observed that from the results of table 5 that the non-dimensional fundamental frequencies of curved FG porous nanobeam decrease with the increase of temperature. Also, it may be observed that, the size effect parameter has a significant effect on vibration of nonlocal curved FG porous beam. Therefore, by increasing nonlocal parameter, dimensionless natural frequency decreases for all porosity parameter. However, increasing the length scale parameter results in larger dimensionless natural frequencies. These phenomena reveal that the curved FG nanobeam exerts a stiffness-softening effect when nonlocal parameter increases and exerts a stiffness-hardening effect length scale parameter increases.

The numerical evaluation is extended to clearly understand the porosity effect on the vibration of S-S FG

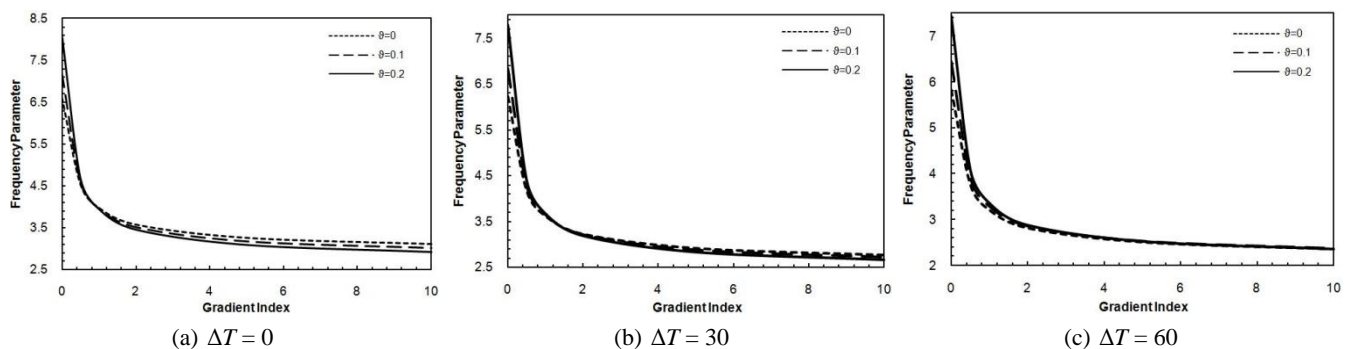


Fig. 2 The variation of the fundamental dimensionless frequency of S-S curved FG porous nanobeam respect to gradient index and porosity for different temperature changes values ( $\alpha = \pi/2$ ,  $\mu^2 = 2$ ,  $L/h = 30$ ,  $\lambda = 2$ )

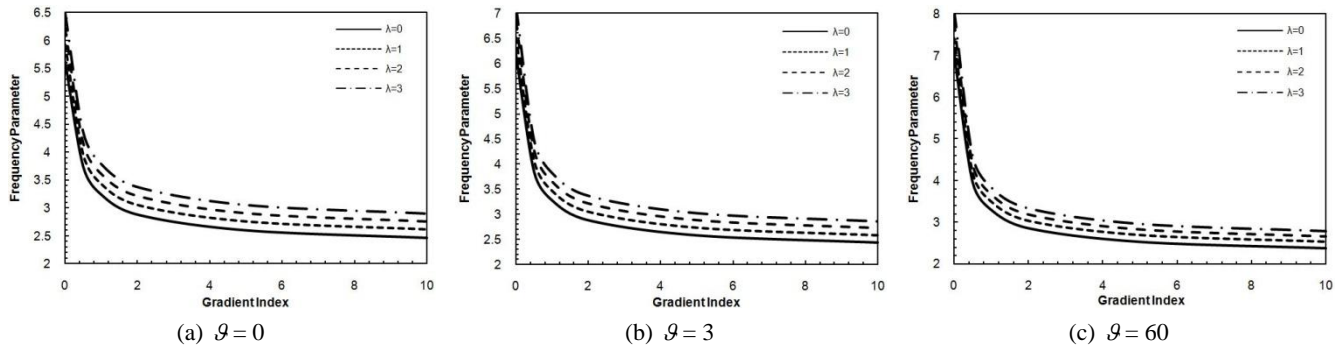


Fig. 3 The variation of the frequency of S-S curved FG porous nanobeam respect to gradient index and strain gradient and porosity parameters ( $\alpha = \pi/2$ ,  $\lambda = 2$ ,  $L/h = 30$ ,  $\mu^2 = 2$ )

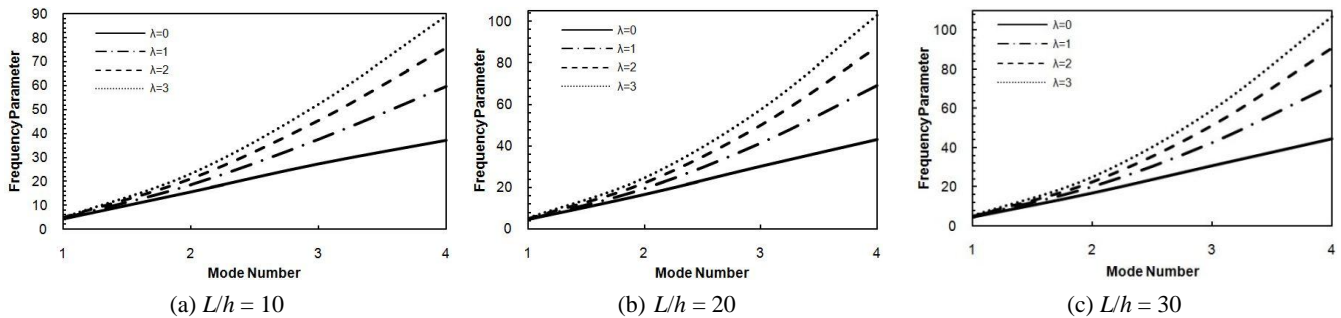


Fig. 4 Effects of strain gradient parameter on nondimensional frequency of curved FG porous nanobeam with respect to mode number for different slenderness values ( $\Delta T = 30$ ,  $\alpha = \pi/3$ ,  $g = 0.1$ ,  $\mu^2 = 2$ ,  $p = 1$ )

curve nanobeam subjected to thermal loading. Fig. 2 demonstrates the variation of the natural frequency with material gradation, temperature changings and porosity parameter at a constant values of aspect ratio ( $L/h = 30$ ), nonlocal parameter ( $\mu^2 = 2$ ), length scale parameter ( $\lambda = 2$ ) and opening angle ( $\alpha = \pi/2$ ). It is observed from this figure that the porosity effect according to the even distribution depends on the value of power-law index and temperature changings. For example, at ( $\Delta T = 0$ ) by increasing the porosity parameter, the natural frequency of curved FG porous nanobeam first increases at lower gradient indexes. However, an opposite behavior is observed from a certain value of the power index. In other words, from a certain value of power-law index, increasing porosity volume fraction leads to lower non-dimensional frequencies. Moreover, it is observed that this certain value of the power-law index is dependent on the value of temperature gradient. In addition, it is found that temperature changing indicates reducing impacts on the natural frequency of porous FG curved nanobeam when their values change from zero to positive one which highlights the saliency of the role of temperature environment. Also, it is beheld that dimensional frequency of curved FG nanobeam decrease as the power-law exponent increases for all values of temperature gradients and porosity parameters. While the power exponent is in the range of 0 to 2, the natural frequencies decrease with high pace compared to those when the power-law index is in the realm of between 2 and 10.

Fig. 3 shows the variation of natural frequency of curved FG porous nanobeam with various porosity

parameters ( $g = 0, 0.1, 0.2$ ) versus gradient index ( $p$ ), respectively, for different values of length scale parameter ( $\lambda$ ), at  $\Delta T = 30$ ,  $\alpha = \pi/2$ ,  $L/h = 30$  and  $\mu^2 = 2$ . It can be observed that for all the values of porosity parameters, the frequency reduces as the gradient index increases. This reduction in the frequency is significant for smaller gradient index. In addition, it can be witnessed from this figure that increasing the length scale parameter ( $\lambda$ ) improves the nanobeam stiffness and dimensionless frequencies. Therefore, it can be realized that, both scale parameters (nonlocality and length scale parameter) have remarkable effect on the vibration response of curved FG porous nanobeams and contribute significantly for their accurate analysis.

Fig. 4 illustrates the variation of frequency parameter of curved FG porous nanobeam versus mode number for different values of length scale parameter at  $\alpha = \pi/3$ ,  $p = 1$ ,  $\Delta T = 30$ ,  $g = 0.1$  and  $\mu^2 = 2$ . For various slenderness values, a higher mode gives larger natural frequency. It is seen that the effect of length scale parameter ( $\lambda$ ) on vibration frequency of curved FG nanobeam is more sensitivity at higher modes. For all mode numbers, increasing length scale parameter leads to enlargement of frequency parameter. As expected, the largest and the smallest values of slenderness have the greatest and smallest frequencies, respectively. In fact, smaller aspect ratio makes the nanobeam more rigid and lead to larger frequencies.

The variations of fundamental natural frequencies of FG porous curved nanobeam subjected to uniform temperature rise, against gradient index for different temperature gradients are illustrated in Fig. 5 at constant values of ( $L/h$

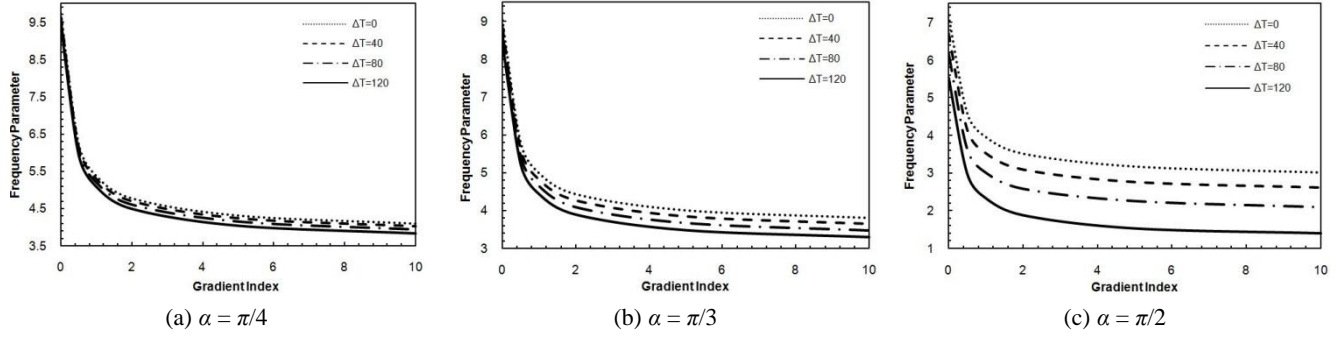


Fig. 5 The variation of the frequency of S-S curved FG porous nanobeam respect to gradient index and temperature changes with various opening angles ( $\lambda = 2$ ,  $\vartheta = 0.1$ ,  $L/h = 30$ ,  $\mu^2 = 2$ )

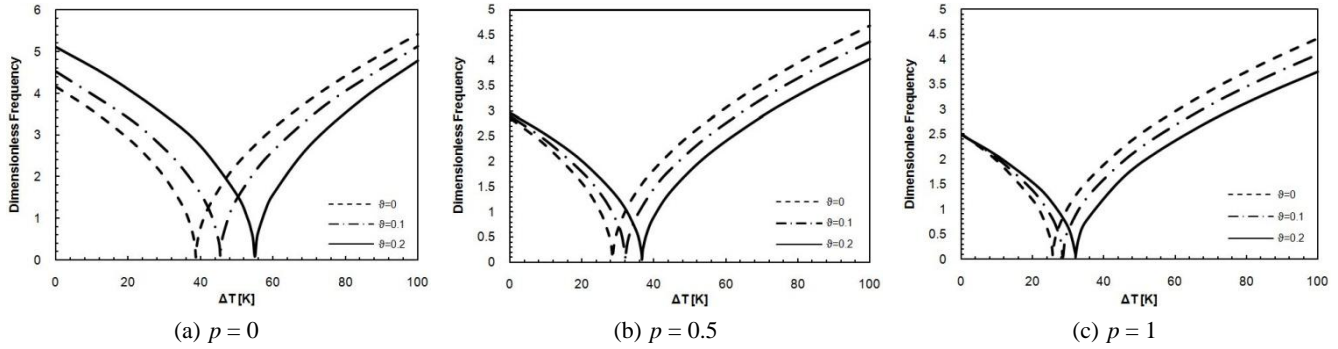


Fig. 6 Variations of the first dimensionless natural frequency of the S-S curved FG porous nanobeam with respect to uniform temperature change for different values of porosity parameter and gradient indexes without strain gradient effect ( $L/h = 40$ ,  $\alpha = 2\pi/3$ ,  $\mu^2 = 2$ , 0 ( $L/h = 40$ ,  $\alpha = 2\pi/3$ ,  $\mu^2 = 2$ ,  $\lambda = 0$ ))

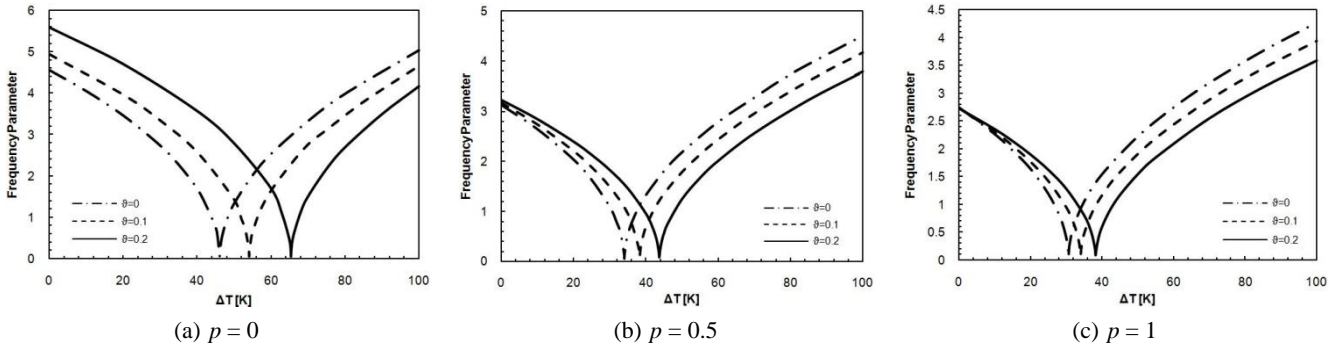


Fig. 7 Variations of the first dimensionless natural frequency of the curved FG porous nanobeam with respect to uniform temperature change for different values of porosity parameter and gradient indexes with strain gradient effect ( $L/h = 40$ ,  $\alpha = 2\pi/3$ ,  $\mu^2 = 2$ ,  $\lambda = 2$ )

$= 30$ ,  $\mu^2 = 2$ ,  $\lambda = 2$ ,  $\vartheta = 0.1$ ). Uniform temperature rise with different temperature gradients ( $\Delta T = 0, 40, 80, 120$ ) are considered for evaluation. It can be clearly understood from the results of Fig. 5 that the fundamental natural frequencies of curved FG porous nanobeam decrease with the increase of power-law index for all values of temperature gradients. In addition, the effect of uniform temperature changes is clearly noticed in Fig. 5. The dimensionless natural frequencies decrease by rising the magnitude of temperature gradient, for all values of power-law exponents and opening angles. Thus, temperature rises have a prominent effect on the natural frequency of the curved FG porous nanobeams. Moreover, it should be noted that, the dimensionless natural

frequencies of curved FG porous nanobeam decrease with the increase of opening angles as elucidated in Fig. 5. Also, increasing the opening angles of curved FG porous nanobeams results in making the natural frequencies more sensitive to the temperature changes.

The non-dimensional frequency of porous FG curved nanobeam with and without length scale parameter as a function of temperature changings and porosity volume fractions for different power-law indexes ( $p = 0, 0.5, 1$ ) at  $L/h = 40$ ,  $\mu^2 = 2$  and  $\alpha = 2\pi/3$  is depicted respectively in Figs. 6 and 7.

It may be observed that the frequencies of curved FG porous nano beam decrease with the increase in the

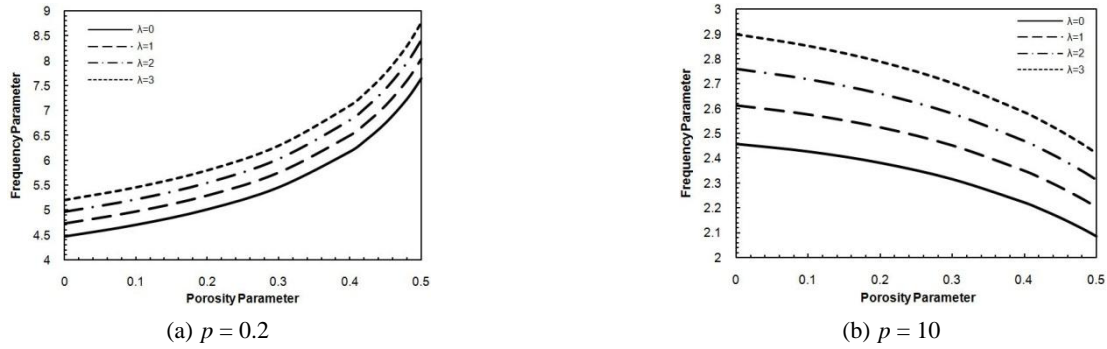


Fig. 8 Variations of the first dimensionless natural frequency of the S-S curved FG porous nanobeam with respect to porosity parameter for different values of strain gradient and gradient indexes values ( $L/h = 30$ ,  $\alpha = \pi/2$ ,  $\mu^2 = 2$ ,  $\Delta T = 30$ )

temperature change until it reaches to zero at the critical temperature point. The reason of this phenomenon is reduction in total stiffness of the beam, since stiffness of the curved FG porous nanobeam decreases when temperature gradient increases. The important seeing within the realm of temperature before the critical temperature is that the curved FG porous nanobeams with greater values of porosity parameter usually provide higher values of the dimensionless frequency results. Furthermore, this treatment is vice versa in the realm of temperature after the critical temperature. Also, it can be observed that curved FG porous nanobeam without considering a length scale parameter has a softening effect on curved FG porous nanobeam at pre-buckling realm and a rise in temperature increases this effect. It is also objective that the ramified point of the curved FG porous nanobeam is postponed by assumption of the lower porosity parameter due to the fact that the lower porosity parameter results in the decrease of stiffness of the curved FG porous nanobeam. Also, it can be noted that the buckling temperatures decrease depending on an increasing in power-law index and porosity parameter.

It should be noted that for all values of power-law indexes increasing of the temperature change is cause of reduction in natural frequency until frequency reaches to zero at the critical temperature point. However, it can be concluded that critical temperature points with considering length scale parameter are higher in contrast to the critical temperature points without length scale parameter, for all gradient index and porosity parameters.

In order to display the porosity effect on the vibration of the curved FG porous nanobeam with length scale parameter, variations of the non-dimensional natural frequency of S-S curved FG porous nanobeam as a function of porosity parameter for different values of length scale parameter at a constant value of slenderness ( $L/h = 30$ ), opening angle ( $\alpha = \pi/2$ ), nonlocality ( $\mu^2 = 2$ ) and uniform temperature rise ( $\Delta T = 30$ ), is plotted at Fig. 8. It can be concluded that the greatest frequency of curved FG porous nanobeam is obtained for the maximum amount of length scale parameter ( $\lambda = 4$ ). It should be noted that the effect of porosity volume fraction on the natural frequency of curved FG porous nanobeam with length scale parameter is similar previous conclusions for values of strain gradient parameter. On the other hand, growing of porosity volume fraction with and without existence of length scale parameter and

power-law index value of ( $p = 0.2$ ) provide higher natural frequencies for curved FG porous nanobeam. So, existence of length scale parameter has significant role on vibration behavior of curved FG porous nanobeam. However, as it can be seen in Fig. 8, porosity volume fraction treatment is vice versa for power-law index value of ( $p = 10$ ). Furthermore, growing of porosity volume fraction with and without existence of length scale parameter and power-law index value of ( $p = 10$ ) provide lower natural frequencies for curved FG porous nanobeam.

## 5. Conclusions

In the proposed investigation, vibration characteristics of porous FG curved nanobeams exposed to thermal loading with different opening angles is implemented within the framework of nonlocal strain gradient elasticity theory. This study also considers the influence of length scale and nonlocality parameters. Thermo-Mechanical properties of porous FG curved nanobeams are temperature-dependent and vary in the radial direction based on modified power-law model for approximation of material properties with even distribution of porosities. The governing differential equations of motion and related boundary condition are derived by using Hamilton principle and then solved by applying an analytical exact solution method for Simply-Simply supported boundary condition. The accuracy of the results is examined using available data in the literature. It is indicated that the thermo-mechanical vibration characteristics of curved FG porous nanobeam is significantly affected by various parameters such as material graduation index, porosity parameter, temperature gradient, length scale parameter, nonlocal parameter, angle of curvature and gradient index. The numerical results show that:

- By increasing the power-law index value and nonlocal parameter, the non-dimensional frequencies of porous FG curve nanobeams diminishes regardless of opening angle and porosity values.
- Effect of porosity volume fraction on natural frequency of porous FG curve nanobeams depends on material graduation index value and temperature gradient.
- For porous FG curve nanobeams, increasing the



volume fraction of porosity first yields an increasing natural frequency, then this trend reverses for upper values of gradient index. In other words, increasing of porosity volume fraction decreases the non-dimensional frequency from a certain value of power-law index which depends on the temperature gradients.

- Increasing temperature gradient reduces the natural frequency of porous FG curve nanobeams. However, after critical temperature this behavior is reversed. Also, it is observable that effect of temperature gradient on the curved FG nanobeams with higher opening angles is more prominent in contrast to the curved FG nanobeams with lower opening angles.
- Increasing strain gradient parameter improves the non-dimensional frequency of curved FGM porous nanobeam. However, for the nonlocal parameter this behavior is opposite. Also, the impact of strain gradient parameter on frequencies corresponding to the higher mode number is more prominent than lower mode numbers.
- As the slenderness ratio increases, the non-dimensional frequencies of porous FG curve nanobeams increase. But for the opening angle, this trend is vice versa.

## References

- Agwa, M.A. and Eltaher, M.A. (2016), "Vibration of a carbyne nanomechanical mass sensor with surface effect", *Appl. Phys. A*, **122**(4), 1-8. <https://doi.org/10.1007/s00339-016-9934-9>
- Akbas, S.D. (2018), "Forced vibration analysis of cracked functionally graded microbeams", *Adv. Nano. Res., Int. J.*, **6**(1), 39-55. <https://doi.org/10.12989/anr.2018.6.1.039>
- Altabay, W.A. (2017), "An exact solution for mechanical behavior of BFRP Nano-thin films embedded in NEMS", *Adv. Nano Res., Int. J.*, **5**(4), 337-357. <https://doi.org/10.12989/anr.2017.5.4.337>
- Ansari, R., Gholami, R. and Sahmani, S. (2013), "Size-dependent vibration of functionally graded curved microbeams based on the modified strain gradient elasticity theory", *Arch. Appl. Mech.* **83**(10), 1439-1449. <https://doi.org/10.1007/s00419-013-0756-3>
- Attia, M.A., Eltaher, M.A., Soliman, A., Abdelrahman, A. and Alshorbagy, A.E. (2018), "Thermoelastic Crack Analysis in Functionally Graded Pipelines Conveying Natural Gas by a FEM", *Int. J. Appl. Mech.*, **10**(4), 1850036. <https://doi.org/10.1142/S1758825118500369>
- Barati, M.R. (2017), "Nonlocal-strain gradient forced vibration analysis of metal foam nanoplates with uniform and graded porosities", *Adv. Nano. Res., Int. J.*, **5**(4), 393-414. <https://doi.org/10.12989/anr.2017.5.4.393>
- Bessegghier, A., Heireche, H., Bousahla, A.A., Tounsi, A. and Benzair, A. (2015), "Nonlinear vibration properties of a zigzag single-walled carbon nanotube embedded in a polymer matrix", *Adv. Nano. Res., Int. J.*, **3**(1), 29-37. <http://dx.doi.org/10.12989/anr.2015.3.1.029>
- Bouadi, A., Bousahla, A.A., Houari, M.S.A., Heireche, H. and Tounsi, A. (2018), "A new nonlocal HSDT for analysis of stability of single layer graphene sheet", *Adv. Nano. Res., Int. J.*, **6**(2), 147-162. <https://doi.org/10.12989/anr.2018.6.2.147>
- Castrucci, P. (2014), "Carbon nanotube/silicon hybrid heterojunctions for photovoltaic devices", *Adv. Nano. Res., Int. J.*, **2**(1), 23-56. <http://dx.doi.org/10.12989/anr.2014.2.1.023>
- Chemi, A., Heireche, H., Zidour, M., Rakrak, K. and Bousahla, A.A. (2015), "Critical buckling load of chiral double-walled carbon nanotube using non-local theory elasticity", *Adv. Nano. Res., Int. J.*, **3**(4), 193-206. <http://dx.doi.org/10.12989/anr.2015.3.4.193>
- Daulton, T.L., Bondi, K.S. and Kelton, K.F. (2010), "Nanobeam diffraction fluctuation electron microscopy technique for structural characterization of disordered materials—Application to Al 88– x Y 7 Fe 5 Ti x metallic glasses", *Ultramicroscopy*, **110**(10), 1279-1289. <https://doi.org/10.1016/j.ultramic.2010.05.010>
- Ebrahimi, F. and Barati, M.R. (2016a), "Buckling analysis of nonlocal third-order shear deformable functionally graded piezoelectric nanobeams embedded in elastic medium", *J. Brazil. Soc. Mech. Sci. Eng.*, **39**(3), 937-952. <https://doi.org/10.1007/s40430-016-0551-5>
- Ebrahimi, F. and Barati, M.R. (2016b), "Magneto-electro-elastic buckling analysis of nonlocal curved nanobeams", *Eur. Phys. J. Plus*, **131**(9), 346. <https://doi.org/10.1140/epjp/i2016-16346-5>
- Ebrahimi, F. and Barati, M.R. (2016c), "A nonlocal higher-order refined magneto-electro-viscoelastic beam model for dynamic analysis of smart nanostructures", *Int. J. Eng. Science*, **107**, 183-196. <https://doi.org/10.1016/j.ijengsci.2016.08.001>
- Ebrahimi, F. and Barati, M.R. (2016d), "Size-dependent dynamic modeling of inhomogeneous curved nanobeams embedded in elastic medium based on nonlocal strain gradient theory", *Proceedings of the Institution of Mechanical Engineers, Part C: J. Mech. Eng. Sci.*, 0954406216668912. <https://doi.org/10.1177/0954406216668912>
- Ebrahimi, F. and Barati, M.R. (2016e), "Thermal environment effects on wave dispersion behavior of inhomogeneous strain gradient nanobeams based on higher order refined beam theory", *J. Thermal Stress*, **39**(12), 1560-1570. <https://doi.org/10.1080/01495739.2016.1219243>
- Ebrahimi, F. and Barati, M.R. (2017a), "Hygrothermal effects on vibration characteristics of viscoelastic FG nanobeams based on nonlocal strain gradient theory", *Compos. Struct.*, **159**, 433-444. <https://doi.org/10.1016/j.compstruct.2016.09.092>
- Ebrahimi, F. and Barati, M.R. (2017b), "A nonlocal strain gradient refined beam model for buckling analysis of size-dependent shear-deformable curved FG nanobeams", *Compos. Struct.*, **159**, 174-182. <https://doi.org/10.1016/j.compstruct.2016.09.058>
- Ebrahimi, F. and Daman, M. (2016a), "Investigating Surface Effects on Thermomechanical Behavior of Embedded Circular Curved Nanosize Beams", *J. Eng.*, **2016**. <http://dx.doi.org/10.1155/2016/9848343>
- Ebrahimi, F. and Daman, M. (2016b), "An Investigation of Radial Vibration Modes of Embedded Double-Curved-Nanobeam-Systems", *Cankaya Univ. J. Sci. Eng.*, **13**, 58-79.
- Ebrahimi, F. and Daman, M. (2016a), "Nonlocal thermo-electro-mechanical vibration analysis of smart curved FG piezoelectric Timoshenko nanobeam", *Smart Struct. Syst., Int. J.*, **20**(3), 351-368. <http://dx.doi.org/10.12989/ss.2017.20.3.351>
- Ebrahimi, F. and Daman, M. (2017b), "Dynamic characteristics of curved inhomogeneous nonlocal porous beams in thermal environment", *Struct. Eng. Mech., Int. J.*, **64**(1), 121-133. <http://dx.doi.org/10.12989/sem.2017.64.1.121>
- Ebrahimi, F. and Jafari, A. (2016), "Thermo-mechanical vibration analysis of temperature-dependent porous FG beams based on Timoshenko beam theory", *Struct. Eng. Mech., Int. J.*, **59**(2), 343-371. <http://dx.doi.org/10.12989/sem.2016.59.2.343>
- Ebrahimi, F. and Salari, E. (2015), "Size-dependent free flexural vibrational behavior of functionally graded nanobeams using semi-analytical differential transform method", *Compos. Part B: Eng.*, **79**, 156-169. <https://doi.org/10.1016/j.compositesb.2015.04.010>

- Ebrahimi, F. and Zia, M. (2015), "Large amplitude nonlinear vibration analysis of functionally graded Timoshenko beams with porosities", *Acta Astronautica*, **116**, 117-125.  
<https://doi.org/10.1016/j.actaastro.2015.06.014>
- Ebrahimi, F., Ghasemi, F. and Salari, E. (2016), "Investigating thermal effects on vibration behavior of temperature-dependent compositionally graded Euler beams with porosities", *Meccanica*, **51**(1), 223-249.  
<https://doi.org/10.1007/s11012-015-0208-y>
- Ebrahimi, F., Daman, M. and Jafari, A. (2017), "Nonlocal strain gradient-based vibration analysis of embedded curved porous piezoelectric nano-beams in thermal environment", *Smart Struct. Syst., Int. J.*, **20**(6), 709-728.  
<http://dx.doi.org/10.12989/sss.2017.20.6.709>
- Ehyaei, J., Ebrahimi, F. and Salari, E. (2016), "Nonlocal vibration analysis of FG nano beams with different boundary conditions", *Adv. Nano. Res., Int. J.*, **4**(2), 85-111.  
<https://doi.org/10.12989/anr.2016.4.2.085>
- Eltaher, M.A., Emam, S.A. and Mahmoud, F.F. (2012), "Free vibration analysis of functionally graded size-dependent nanobeams", *Appl. Math. Comput.*, **218**(14), 7406-7420.  
<https://doi.org/10.1016/j.amc.2011.12.090>
- Eltaher, M.A., Alshorbagy, A.E. and Mahmoud, F.F. (2013), "Vibration analysis of Euler-Bernoulli nanobeams by using finite element method", *Appl. Math. Model.*, **37**(7), 4787-4797.  
<https://doi.org/10.1016/j.amc.2011.12.090>
- Eltaher, M.A., Abdelrahman, A.A., Al-Nabawy, A., Khater, M. and Mansour, A. (2014a), "Vibration of nonlinear graduation of nano-Timoshenko beam considering the neutral axis position", *Appl. Math. Comput.*, **235**, 512-529.  
<https://doi.org/10.1016/j.amc.2014.03.028>
- Eltaher, M.A., Hamed, M.A., Sadoun, A.M. and Mansour, A. (2014b), "Mechanical analysis of higher order gradient nanobeams", *Appl. Math. Comput.*, **229**, 260-272.  
<https://doi.org/10.1016/j.amc.2013.12.076>
- Eltaher, M.A., El-Borgi S. and Reddy J.N. (2016a), "Nonlinear analysis of size-dependent and material-dependent nonlocal CNTs", *Compos. Struct.*, **153**, 902-913.  
<https://doi.org/10.1016/j.compstruct.2016.07.013>
- Eltaher, M.A., Khater, M.E. and Emam, S.A. (2016b), "A review on nonlocal elastic models for bending, buckling, vibrations, and wave propagation of nanoscale beams", *Appl. Math. Model.*, **40**(5-6), 4109-4128.  
<https://doi.org/10.1016/j.apm.2015.11.026>
- Eltaher, M.A., Attia, M.A., Soliman, A.E. and Alshorbagy, A.E. (2018a), "Analysis of crack occurs under unsteady pressure and temperature in a natural gas facility by applying FGM", *Struct. Eng. Mech., Int. J.*, **66**(1), 97-111.  
<https://doi.org/10.12989/sem.2018.66.1.097>
- Eltaher, M.A., Fatema-Alzahraa Omar, Abdalla W.S. and E.H. Gad (2018b), "Bending and vibrational behaviors of piezoelectric nonlocal nanobeam including surface elasticity", *Waves Random Complex Media*, **29**(2), 264-280.  
<https://doi.org/10.1080/17455030.2018.1429693>
- Eltaher, M.A., Agwa, M. and Kabeel, A. (2018c), "Vibration analysis of material size-dependent CNTs using energy equivalent model", *J. Appl. Computat. Mech.*, **4**(2), 75-86.  
<https://doi.org/10.22055/JACM.2017.22579.1136>
- Eltaher, M.A., Fouda, N., El-midany, T. and Sadoun, A.M. (2018d), "Modified porosity model in analysis of functionally graded porous nanobeams", *J. Brazil. Soc. Mech. Sci. Eng.*, **40**(3), 141.  
<https://doi.org/10.1007/s40430-018-1065-0>
- Eringen, A.C. (1972a), "Linear theory of nonlocal elasticity and dispersion of plane waves", *Int. J. Eng. Sci.*, **10**(5), 425-435.  
[https://doi.org/10.1016/0020-7225\(72\)90050-X](https://doi.org/10.1016/0020-7225(72)90050-X)
- Eringen, A.C. (1972b), "Nonlocal polar elastic continua", *Int. J. Eng. Sci.*, **10**(1), 1-16.  
[https://doi.org/10.1016/0020-7225\(72\)90070-5](https://doi.org/10.1016/0020-7225(72)90070-5)
- Eringen, A.C. (1983), "On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves", *J. Appl. Phys.*, **54**(9), 4703-4710.  
<https://doi.org/10.1063/1.332803>
- Eringen, A.C. (2002), *Nonlocal Continuum Field Theories*, Springer Science & Business Media.
- Fleck, N. and Hutchinson, J. (1993), "A phenomenological theory for strain gradient effects in plasticity", *J. Mech. Phys. Solids*, **41**(12), 1825-1857.  
[https://doi.org/10.1016/0022-5096\(93\)90072-N](https://doi.org/10.1016/0022-5096(93)90072-N)
- Hamed, M.A. and Eltaher, M.A., Sadoun, A.M. and Almitani, K.H. (2016), "Free vibration of symmetric and sigmoid functionally graded nanobeams", *Appl. Phys. A*, **122**(9), 829.  
<https://doi.org/10.1007/s00339-016-0324-0>
- Hosseini, S. and Rahmani, O. (2016), "Free vibration of shallow and deep curved FG nanobeam via nonlocal Timoshenko curved beam model", *Appl. Phys. A*, **122**(3), 1-11.  
<https://doi.org/10.1007/s00339-016-9696-4>
- Hu, B., Ding, Y., Chen, W., Kulkarni, D., Shen, Y., Tsukruk, V.V. and Wang, Z.L. (2010), "External-Strain Induced Insulating Phase Transition in VO<sub>2</sub> Nanobeam and Its Application as Flexible Strain Sensor", *Adv. Mater.*, **22**(45), 5134-5139.  
<https://doi.org/10.1002/adma.201002868>
- Kananipour, H., Ahmadi, M. and Chavoshi, H. (2014), "Application of nonlocal elasticity and DQM to dynamic analysis of curved nanobeams", *Latin Am. J. Solids Struct.*, **11**(5), 848-853.  
<http://dx.doi.org/10.1590/S1679-78252014000500007>
- Koizumi, M. and Niino, M. (1995), "Overview of FGM Research in Japan", *Mrs Bulletin*, **20**(1), 19-21.  
<https://doi.org/10.1557/S0883769400048867>
- Kumar, B.R. (2018), "Investigation on mechanical vibration of double-walled carbon nanotubes with inter-tube Van der waals forces", *Adv. Nano. Res., Int. J.*, **6**(2), 135-145.  
<https://doi.org/10.12989/anr.2018.6.2.135>
- Lam, D.C., Yang, F., Chong, A.C.M., Wang, J. and Tong, P. (2003), "Experiments and theory in strain gradient elasticity", *J. Mech. Phys. Solids*, **51**(8), 1477-1508.  
[https://doi.org/10.1016/S0022-5096\(03\)00053-X](https://doi.org/10.1016/S0022-5096(03)00053-X)
- Li, L. and Hu, Y. (2015), "Buckling analysis of size-dependent nonlinear beams based on a nonlocal strain gradient theory", *Int. J. Eng. Sci.*, **97**, 84-94.  
<https://doi.org/10.1016/j.ijengsci.2015.08.013>
- Li, L. and Hu, Y. (2016), "Nonlinear bending and free vibration analysis of nonlocal strain gradient beams made of functionally graded material", *Int. J. Eng. Sci.*, **102**, 77-97.  
<https://doi.org/10.1016/j.ijengsci.2016.07.011>
- Li, L., Hu, Y. and Ling, L. (2015), "Flexural wave propagation in small-scaled functionally graded beams via a nonlocal strain gradient theory", *Compos. Struct.*, **133**, 1079-1092.  
<https://doi.org/10.1016/j.compstruct.2015.08.014>
- Lim, C.W., Zhang, G. and Reddy, J.N. (2015), "A higher-order nonlocal elasticity and strain gradient theory and its applications in wave propagation", *J. Mech. Phys. Solids*, **78**, 298-313.  
<https://doi.org/10.1016/j.jmps.2015.02.001>
- Malekzadeh, P., Haghighi, M.G. and Atashi, M.M. (2010), "Out-of-plane free vibration of functionally graded circular curved beams in thermal environment", *Compos. Struct.*, **92**(2), 541-552.  
<https://doi.org/10.1016/j.compstruct.2009.08.040>
- Marani, R. and Perri, A.G. (2017), "An approach to model the temperature effects on IV characteristics of CNTFETs", *Adv. Nano. Res., Int. J.*, **5**(1), 61-67.  
<https://doi.org/10.12989/anr.2017.5.1.061>
- Mechab, I., Mechab, B., Benaissa, S., Serier, B. and Bouiadjra, B.B. (2016), "Free vibration analysis of FGM nanoplate with

- porosities resting on Winkler Pasternak elastic foundations based on two-variable refined plate theories", *J. Brazil. Soc. Mech. Sci. Eng.*, **38**(8), 2193-2211.  
<https://doi.org/10.1007/s40430-015-0482-6>
- Miyamoto, Y., Kaysser, W.A., Rabin, B.H., Kawasaki, A. and Ford, R.G. eds. (2013), *Functionally Graded Materials: Design, Processing and Applications*, Springer Science & Business Media.
- Mohamed, N., Eltaher, M.A., Mohamed, S.A. and Seddek, L.F. (2018), "Numerical analysis of nonlinear free and forced vibrations of buckled curved beams resting on nonlinear elastic foundations", *Int. J. Non-Linear Mech.*, **101**, 157-173.  
<https://doi.org/10.1016/j.ijnonlinmec.2018.02.014>
- Mortensen, A. and Suresh, S. (2013), "Functionally graded metals and metal-ceramic composites: Part I Processing", *Int. Mater. Rev.*, **40**(6), 239-265.  
<https://doi.org/10.1179/imr.1995.40.6.239>
- Murmu, T. and Adhikari, S. (2010), "Nonlocal transverse vibration of double-nanobeam-systems", *J. Appl. Phys.*, **108**(8), 083514.  
<https://doi.org/10.1063/1.3496627>
- Nazemnezhad, R. and Hosseini-Hashemi, S. (2014), "Nonlocal nonlinear free vibration of functionally graded nanobeams", *Compos. Struct.*, **110**, 192-199.  
<https://doi.org/10.1016/j.compstruct.2013.12.006>
- Pompe, W., Worch, H., Epple, M., Friess, W., Gelinsky, M., Greil, P., Hempel, U., Scharnweber, D. and Schulte, K. (2003), "Functionally graded materials for biomedical applications", *Mater. Sci. Eng.: A*, **362**(1), 40-60.  
[https://doi.org/10.1016/S0921-5093\(03\)00580-X](https://doi.org/10.1016/S0921-5093(03)00580-X)
- Rahmani, O. and Pedram, O. (2014), "Analysis and modeling the size effect on vibration of functionally graded nanobeams based on nonlocal Timoshenko beam theory", *Int. J. Eng. Sci.*, **77**, 55-70. <https://doi.org/10.1016/j.ijengsci.2013.12.003>
- Setoodeh, A., Derahaki, M. and Bavi, N. (2015), "DQ thermal buckling analysis of embedded curved carbon nanotubes based on nonlocal elasticity theory", *Latin Am. J. Solids Struct.*, **12**(10), 1901-1917.  
<http://dx.doi.org/10.1590/1679-78251894>
- Shafiei, N. and She, G.L. (2018), "On vibration of functionally graded nano-tubes in the thermal environment", *Int. J. Eng. Sci.*, **133**, 84-98. <https://doi.org/10.1016/j.ijengsci.2018.08.004>
- She, G.L., Shu, X. and Ren, Y.R. (2017a), "Thermal buckling and postbuckling analysis of piezoelectric FGM beams based on high-order shear deformation theory", *J. Thermal Stress.*, **40**(6), 783-797. <https://doi.org/10.1080/01495739.2016.1261009>
- She, G.L., Yuan, F.G. and Ren, Y.R. (2017b), "Research on nonlinear bending behaviors of FGM infinite cylindrical shallow shells resting on elastic foundations in thermal environments", *Compos. Struct.*, **170**, 111-121.  
<https://doi.org/10.1016/j.compstruct.2017.03.010>
- She, G.L., Yuan, F.G. and Ren, Y.R. (2017c), "Thermal buckling and post-buckling analysis of functionally graded beams based on a general higher-order shear deformation theory", *Appl. Math. Model.*, **47**, 340-357.  
<https://doi.org/10.1016/j.apm.2017.03.014>
- She, G.L., Yuan, F.G., Ren, Y.R. and Xiao, W.S. (2017d), "On buckling and postbuckling behavior of nanotubes", *Int. J. Eng. Sci.*, **121**, 130-142.  
<https://doi.org/10.1016/j.ijengsci.2017.09.005>
- She, G.L., Ren, Y.R., Yuan, F.G. and Xiao, W.S. (2018a), "On vibrations of porous nanotubes", *Int. J. Eng. Sci.*, **125**, 23-35.  
<https://doi.org/10.1016/j.ijengsci.2017.12.009>
- She, G.L., Yuan, F.G. and Ren, Y.R. (2018b), "On wave propagation of porous nanotubes", *Int. J. Eng. Sci.*, **130**, 62-74.  
<https://doi.org/10.1016/j.ijengsci.2018.05.002>
- She, G.L., Yan, K.M., Zhang, Y.L., Liu, H.B. and Ren, Y.R. (2018c), "Wave propagation of functionally graded porous nanobeams based on non-local strain gradient theory", *Eur. Phys. J. Plus*, **133**, 368.  
<https://doi.org/10.1140/epjp/i2018-12196-5>
- She, G.L., Yuan, F.G., Ren, Y.R., Liu, H.B. and Xiao, W.S. (2018d), "Nonlinear bending and vibration analysis of functionally graded porous tubes via a nonlocal strain gradient theory", *Composite Structures*, **203**, 614-623.  
<https://doi.org/10.1016/j.compstruct.2018.07.063>
- She, G.L., Yuan, F.G., Karami, B., Ren, Y.R. and Xiao, W.S. (2019), "On nonlinear bending behavior of FG porous curved nanotubes", *Int. J. Eng. Sci.*, **135**, 58-74.  
<https://doi.org/10.1016/j.ijengsci.2018.11.005>
- Shen, H.-S. (2016), *Functionally Graded Materials: Nonlinear Analysis of Plates and Shells*, CRC press.
- Soliman, A.E., Eltaher, M.A., Attia, M.A. and Alshorbagy, A.E. (2018), "Nonlinear transient analysis of FG pipe subjected to internal pressure and unsteady temperature in a natural gas facility", *Struct. Eng. Mech., Int. J.*, **66**(1), 85-96.  
<https://doi.org/10.12989/sem.2018.66.1.085>
- Stölken, J. and Evans, A. (1998), "A microbend test method for measuring the plasticity length scale", *Acta Materialia*, **46**(14), 5109-5115. [https://doi.org/10.1016/S1359-6454\(98\)00153-0](https://doi.org/10.1016/S1359-6454(98)00153-0)
- Tounsi, A., Benguediab, S., Adda, B., Semmah, A. and Zidour, M. (2013), "Nonlocal effects on thermal buckling properties of double-walled carbon nanotubes", *Adv. Nano. Res., Int. J.*, **1**(1), 1-11. <https://doi.org/10.12989/anr.2013.1.1.001>
- Touloukian, Y.S. (1966), "Thermophysical Properties of High Temperature Solid Materials" 6. Intermetallics, *Ceramics, Polymers, and Composite Systems. Part II. Ceramics, Polymers, Composite Systems, DTIC Document*.
- Wang, C.M. and Duan, W. (2008), "Free vibration of nanorings/arches based on nonlocal elasticity", *J. Appl. Phys.*, **104**(1), 014303. <https://doi.org/10.1063/1.2951642>
- Wattanasakulpong, N. and Chaikittiratanana, A. (2015), "Flexural vibration of imperfect functionally graded beams based on Timoshenko beam theory: Chebyshev collocation method", *Meccanica*, **50**(5), 1331-1342.  
<https://doi.org/10.1007/s11012-014-0094-8>
- Wattanasakulpong, N. and Ungbhakorn, V. (2014), "Linear and nonlinear vibration analysis of elastically restrained ends FGM beams with porosities", *Aerosp. Sci. Technol.*, **32**(1), 111-120.  
<https://doi.org/10.1016/j.ast.2013.12.002>
- Yahia, S.A., Atmane, H.A., Houari, M.S.A. and Tounsi, A. (2015), "Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories", *Struct. Eng. Mech., Int. J.*, **53**(6), 1143-1165.  
<http://dx.doi.org/10.12989/sem.2015.53.6.1143>
- Yan, Z. and Jiang, L. (2011), "Electromechanical response of a curved piezoelectric nanobeam with the consideration of surface effects", *J. Phys. D: Appl. Phys.*, **44**(36), 365301.  
<https://doi.org/10.1088/0022-3727/44/36/365301>
- Youcef, D.O., Kaci, A., Houari, M.S.A., Tounsi, A., Benzair, A. and Heireche, H. (2015), "On the bending and stability of nanowire using various HSDTs", *Adv. Nano. Res., Int. J.*, **3**(4), 177-191. <https://doi.org/10.12989/anr.2015.3.4.177>
- Zhang, Y.Y., Wang, C.M. and Challamel, N. (2009), "Bending, buckling, and vibration of micro/nanobeams by hybrid nonlocal beam model", *J. Eng. Mech.*, **136**(5), 562-574.  
[https://doi.org/10.1061/\(ASCE\)EM.1943-7889.0000107](https://doi.org/10.1061/(ASCE)EM.1943-7889.0000107)
- Zhu, J., Lai, Z., Yin, Z., Jeon, J. and Lee, S. (2001), "Fabrication of ZrO<sub>2</sub>-NiCr functionally graded material by powder metallurgy", *Mater. Chem. Phys.*, **68**(1), 130-135.  
[https://doi.org/10.1016/S0254-0584\(00\)00355-2](https://doi.org/10.1016/S0254-0584(00)00355-2)