

Analysis of propagation characteristics of elastic waves in heterogeneous nanobeams employing a new two-step porosity-dependent homogenization scheme

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Abstract. The important effect of porosity on the mechanical behaviors of a continua makes it necessary to account for such an effect while analyzing a structure. motivated by this fact, a new two-step porosity dependent homogenization scheme is presented in this article to investigate the wave propagation responses of functionally graded (FG) porous nanobeams. In the introduced homogenization method, which is a modified form of the power-law model, the effects of porosity distributions are considered. Based on Hamilton's principle, the Navier equations are developed using the Euler-Bernoulli beam model. Thereafter, the constitutive equations are obtained employing the nonlocal elasticity theory of Eringen. Next, the governing equations are solved in order to reach the wave frequency. Once the validity of presented methodology is proved, a set of parametric studies are adapted to put emphasis on the role of each variant on the wave dispersion behaviors of porous FG nanobeams.

Keywords: wave propagation; porous materials; functionally graded materials (FGMs); nonlocal elasticity theory

1. Introduction

Functionally graded materials (FGMs) are a novel type of composites with either through-the-thickness or through-the-length variable mechanical properties which have been appealing enough in the researchers' point of view to be utilized as the constituent material for structural designs. These composites are better candidates for mechanical designs in comparison with laminated composites due to their improved corrosion resistance, toughness, and thermal resistance and also their lower stress concentration (Ebrahimi *et al.* 2018). Thus, many researchers analyzed the mechanical behaviors of FG structures. For example, Ebrahimi and Rastgoo (2008) surveyed the vibrational responses of FG circular plates via the Kirchhoff plate theory. Shen (2009) carried out a comparative study between the buckling and post-buckling responses of an FG plate with smart actuators. In another attempt, a new iterative method is introduced to solve the vibration problem of axially FG (AFG) beams (Huang and Li 2010). Also, Alshorbagy *et al.* (2011) performed a finite element analysis (FEA) on the dynamic behaviors of FG beams using polynomial shape functions. Şimşek *et al.* (2012) surveyed the dynamic responses of an AFG beam under a moving harmonic loading. Thai and Choi (2012) investigated the vibration problem of an FG embedded plate

based on a refined higher-order plate theory. Ebrahimi (2013) analyzed the electro-mechanical vibrational responses of FG plates. The thermo-mechanical buckling characteristics of FG plates are studied by Ghiasian *et al.* (2014). Moreover, both free and forced vibration analyses of bi-directionally FG beams are accomplished by Şimşek (2015). Ghiasian *et al.* (2015) probed the nonlinear thermo-elastic dynamic buckling behaviors of FG beams. Jafarinezhad and Eslami (2017) solved the thermo-mechanical responses of an FG plate subjected to a flexural shock. The stress analysis of FG pressure vessels is carried out by Gharibi *et al.* (2017) in the framework of the Frobenius series method. Tang and Yang (2018) investigated the post-buckling and nonlinear vibration problems of FG fluid-conveying pipes.

Besides, the influence of porosity in materials like FGMs is of high importance. To be honest with you, one of the weak points of FGMs is their high probability of possessing defects and porosity because of their critically sensitive fabrication procedure. Henceforward, the porosity effects shall be regarded once an FGM is analyzed. Originated from this fact, some of the authors preferred to account for the destructive effects of porosity in their analyses on the mechanical characteristics of FG structures. For instance, Wattanasakulpong and Ungbhakorn (2014) explored the nonlinear vibrational responses of FG porous beams with restrained ends. In a series of researches, the buckling analysis of FG plates is performed under various loadings while regarding for porosity effects (Jabbari *et al.* 2014, Mojahedin *et al.* 2014, Mojahedin *et al.* 2016). Moreover, bending, buckling and vibration analyses of FG

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porous beams are fulfilled by Chen and colleagues (Chen *et al.* 2015, 2016a, b). In addition, Rezaei and Saidi (2016) used Carrera unified formulation (CUF) to study the vibrational behaviors of FG porous plates. Wang and Wu (2017) employed a higher-order shell theory to investigate the vibrational responses of FG porous shells. In another endeavor, Atmane *et al.* (2017) considered for thickness stretching and porosity effects in their vibrational analysis on FG beams. Zenkour (2018) utilized a refined quasi-3D plate theory for the goal of analyzing the bending responses of FG porous single-layered and sandwich plates. Tian *et al.* (2018) and Stelson (2018) presented a review on academic fluid power research and the applications of shear thickening fluids. Most recently, Gupta and Talha (2018) introduced a sigmoid homogenization model on the basis of porosity effects to probe both bending and buckling behaviors of FG plates.

On the other hand, it seems to be an admitted claim that engineering designs are moving fast and faster towards nanotechnology. This issue can be well perceived once comparing the increasing number of articles in the field on nanoscience and nanotechnology with the number of published articles dealing with conventional fields of interest. Therefore, it is significant to reach as more as possible knowledge about the nanosize elements because of their growing applications in various nano-electro-mechanical-systems (NEMSs). However, this purpose cannot be achieved by employing the classical continuum mechanics theories. At least, a rough modification is required to transform such theories to those which can be applied to investigate the mechanical behaviors of continua in nanoscale. Due to this obligation, Eringen (1972) presented the theory of nonlocal elasticity which denotes that stress state in any point of continua shall be considered as a multi-variable function of strains of all other adjacent points as well as the strain of the point itself. Up to now, several researchers used Eringen's theory to probe the nano-mechanical behaviors of nanostructures. For example, buckling analysis of embedded single-layered graphene sheets is carried out by Pradhan and Murmu (2010). Ansari *et al.* (2011) probed the vibration problem of a multi-layered nanoplate via nonlocal elasticity. Mahmoud *et al.* (2012) could analyze the vibration problem of a nanobeam based on the nonlocal theory. Also, Eltaher *et al.* (2013) could perform an FEA on the vibrational properties of nanobeams via nonlocal elasticity. The combined influences of moisture and temperature are included in a bending analysis performed by Alzahrani *et al.* (2013) on nanoplates. Furthermore, thermal vibration analysis of a single-walled carbon nanotube (SWCNT) is performed by Ebrahimi and Salari (2015b). Zenkour (2016) explored the transient dynamic characteristics of graphene sheets via nonlocal elasticity. The wave propagation analysis of a smart viscoelastic nanoplate is accomplished by Ghorbanpour Arani *et al.* (2017) on the basis of the nonlocal theory of Eringen. Ebrahimi and Karimiasl (2018) developed nonlocal piezoelectricity for buckling analysis of a flexoelectric nanobeam. Farajpour *et al.* (2018) performed a general mechanical analysis on the smart magneto-electro-elastic (MEE) nanofilms. One can gain more

information about the size-dependent theories studying complementary references (Ebrahimi and Dabbagh 2018a, b, Hosseini *et al.* 2018).

Furthermore, in recent years, lots of researches can be found dealing with the static and dynamic characteristics of FG nano-structures. For example, Eltaher *et al.* (2012) could investigate the vibrational responses of FG nanobeams. Later, Natarajan *et al.* (2012) studied the scale-dependent vibrational behaviors of FG nanosize plates. Rahmani and Pedram (2014) presented a nonlocal first-order shear deformable beam hypothesis for vibration analysis of FG nanobeams. In addition, the nonlinear vibrational responses of FG nanobeams are solved by Nazemnezhad and Hosseini-Hashemi (2014). Ebrahimi and Salari (2015a) studied thermo-mechanical vibration and buckling behaviors of FG nanobeams based on the Timoshenko beam theory. Ebrahimi and Barati (2016) investigated the hygro-thermo-magnetically affected stability responses of FG nanobeams. Nejad *et al.* (2016) presented a nonlocal Euler-Bernoulli beam model for vibration analysis of bi-directionally graded nanobeams. Ebrahimi and his co-workers analyzed the wave propagation problem of smart FG nano-beams and -plates (Ebrahimi *et al.* 2016a, b). Also, Ebrahimi *et al.* (2017a) probed the wave dispersion characteristics of rotating FG nanobeams. Lately, Srividhya *et al.* (2018) surveyed nonlinear deflection responses of FG nanoplates via FE method (FEM).

Up to now, several analyses have been carried out by the authors for the goal of investigating the static and dynamic behaviors of FG nano-structures. However, the influences of porosity on the mechanical responses of FG nanosize beams and plates are studied in just a couple of articles. In the only available work on the FG nanobeams, Ebrahimi and Dabbagh (2017) probed the wave propagation behaviors of a porous nanostructure. Also, the nonlinear wave propagation analysis of FG porous nanobeams is fulfilled by Barati (2017). Herein, a modified saturated porosity based two-step model is introduced which is made from a power-law model incorporated with a saturated porosity based model. The beam is modeled as a Euler beam and on the basis of the infinitesimal strains inside a beam, the Navier equations are achieved. Size-dependency is prescribed according to nonlocal elasticity and the governing equations are solved via an analytical method to reach the frequency and velocity of propagated waves.

2. Theory and formulation

2.1 Two-step homogenization method for porous FGMs

Based on the power-law method for FGMs, the effective Young's moduli and mass density of FGMs can be formulated in the following form

$$E(z) = (E_c - E_m) \left(\frac{z}{h} + \frac{1}{2} \right)^p + E_m, \quad (1)$$

$$\rho(z) = (\rho_c - \rho_m) \left(\frac{z}{h} + \frac{1}{2} \right)^p + \rho_m \quad (2)$$

where c and m subscripts denote ceramic and metal phases, respectively. In addition, p stands for the gradient index which is proposed to control the volume of constituent materials through the thickness direction. Afterward, porosity influences should be employed. Herein, both uniform and graded porosity distributions are implemented. Therefore, the effective material properties can be modified as follows

$$\begin{cases} E_{eff}(z) = E(z)(1 - e_0 \cos(\pi z/h)) \\ \rho_{eff}(z) = \rho(z)(1 - e_m \cos(\pi z/h)) \end{cases} \quad (3)$$

$$\begin{cases} E_{eff}(z) = E(z)(1 - e_0 \lambda) \\ \rho_{eff}(z) = \rho(z) \sqrt{1 - e_m \lambda} \end{cases} \quad (4)$$

where Eq. (3) is implemented for symmetric porosity distribution and Eq. (4) is employed once the uniform distribution of porosity is regarded. In the above equations, e_0 and e_m are porosity and mass density coefficients, respectively. The term e_0 is defined as $1 - E_2/E_1$ where E_2 and E_1 correspond with minimum and maximum amounts of Young's moduli in the continua. Herein, the maximum and minimum values are related to ceramic and metal, respectively. Also, the relation between e_0 and e_m can be considered as (Chen *et al.* 2016a)

$$e_m = 1 - \sqrt{1 - e_0} \quad (5)$$

Moreover, the coefficient λ in Eq. (4) can be calculated as

$$\lambda = \frac{1}{e_0} - \frac{1}{e_0} \left(\frac{2}{\pi} \sqrt{1 - e_0} - \frac{2}{\pi} + 1 \right)^2 \quad (6)$$

The effect of implementation of the discussed porosity-based homogenization method on the stiffness behaviors of the FGM across the thickness direction is depicted in the framework of Fig. 1. It can be seen that the stiffness of the FGM can be decreased dramatically while the porosity coefficient is considered to be a nonzero value. So, it will be natural to see a reduction in the mechanical responses of the nanobeam while a porous material is selected instead of a non-porous one. It must be noticed that among two undertaken models, the modulus of the symmetrically distributed porous FGMs experiences a more remarkable decrease in the bigger dimensionless thicknesses in comparison with the uniformly distributed porous ones.

2.2 Euler-Bernoulli beam theory

The equations of motion for the FG beam are modeled in the present research according to the classical beam theory. The displacement field of this theory can be written as

$$u_x(x, z, t) = u(x, t) - z \frac{\partial w(x, t)}{\partial t} \quad (7)$$

$$u_z(x, z, t) = w(x, t) \quad (8)$$

in which, u and w correspond with the axial displacement and bending deflection of the beam. Therefore, the nonzero strains of the beam can be defined as

$$\varepsilon_{xx} = \varepsilon_{xx}^0 - z \kappa_x^0 \quad (9)$$

where

$$\varepsilon_{xx}^0 = \frac{\partial u}{\partial x}, \quad \kappa_x^0 = \frac{\partial^2 w}{\partial x^2} \quad (10)$$

2.3 Hamilton's principle

Now, Hamilton's principle is applied to obtain the Navier equations of FG beam as follows

$$\int_0^t \delta(U - T) dt = 0 \quad (11)$$

where U and T account for strain energy and kinetic energy, respectively. Now, the variation of strain energy can be formulated as

$$\delta U = \int_V (\sigma_{xx} \delta \varepsilon_{xx}) dV = \int_0^L (N (\delta \varepsilon_{xx}^0) - M (\delta \kappa_x^0)) dx \quad (12)$$

in above equation, the axial force (N) and bending moment (M) can be defined as

$$N = \int_A \sigma_{xx} dA, \quad M = \int_A \sigma_{xx} z dA \quad (13)$$

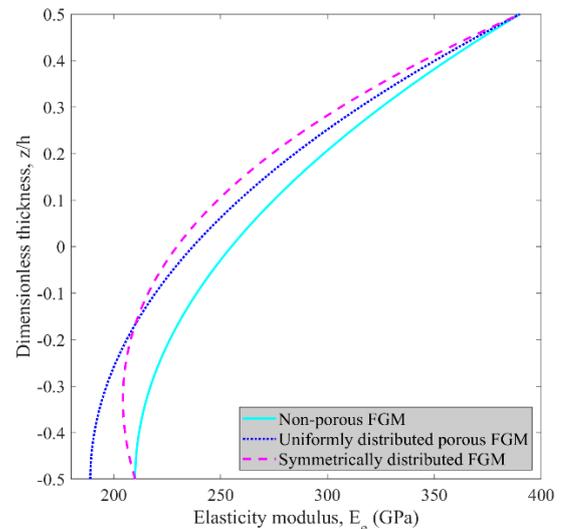


Fig. 1 Effect of various types of porous and non-porous FGMs on the elasticity modulus of the composites ($p = 2$)

Also, the variation of kinetic energy can be expressed as follows

$$\delta T = \int_0^L \left(I_0 \left(\frac{\partial u}{\partial t} \frac{\partial \delta u}{\partial t} + \frac{\partial w}{\partial t} \frac{\partial \delta w}{\partial t} \right) - I_1 \left(\frac{\partial u}{\partial t} \frac{\partial^2 \delta w}{\partial x \partial t} + \frac{\partial^2 w}{\partial x \partial t} \frac{\partial \delta u}{\partial t} \right) + I_2 \left(\frac{\partial^2 w}{\partial x \partial t} \frac{\partial^2 \delta w}{\partial x \partial t} \right) \right) dx \quad (14)$$

in which, the mass moments of inertia are defined as

$$(I_0, I_1, I_2) = \int_A (1, z, z^2) \rho(z) dA \quad (15)$$

Herein, once Eqs. (12) and (14) are substituted in Eq. (11) and the coefficients of δu and δw are set to zero, the Navier equations of FG beam can be written as follows

$$\frac{\partial N}{\partial x} = I_0 \frac{\partial^2 u}{\partial t^2} - I_1 \frac{\partial^3 w}{\partial x \partial t^2}, \quad (16)$$

$$\frac{\partial^2 M}{\partial x^2} = I_0 \frac{\partial^2 w}{\partial t^2} + I_1 \frac{\partial^3 u}{\partial x \partial t^2} - I_2 \frac{\partial^4 w}{\partial t^2 \partial x^2} \quad (17)$$

2.3 Nonlocal elasticity theory

Based upon the nonlocal constitutive equations, the stress state of a point inside a nano-structure is a function of strain of all adjacent points in addition to that point's strain. So, the stress-strain relationship can be described in the following form

$$(1 - \mu^2 \nabla^2) \boldsymbol{\sigma} = \mathbf{C} : \boldsymbol{\varepsilon} \quad (18)$$

where $\boldsymbol{\sigma}$, $\boldsymbol{\varepsilon}$ and \mathbf{C} are stress, strain and elasticity tensors, respectively. Once extending Eq. (18) and integrating from it over the beam's cross-section, the following relations can be achieved for axial force and bending moment

$$(1 - \mu^2 \nabla^2) N = A \frac{\partial u}{\partial x} - B \frac{\partial^2 w}{\partial x^2}, \quad (19)$$

$$(1 - \mu^2 \nabla^2) M = B \frac{\partial u}{\partial x} - D \frac{\partial^2 w}{\partial x^2} \quad (20)$$

where

$$(A, B, D) = \int_A (1, z, z^2) dA \quad (21)$$

2.3 Governing equations

Now, the final governing equations of FG porous nanobeams can be achieved by substituting Eqs. (19) and (20) in Eqs. (16) and (17)

$$A \frac{\partial^2 u}{\partial x^2} - B \frac{\partial^3 w}{\partial x^3} + (1 - \mu^2 \nabla^2) \left(-I_0 \frac{\partial^2 u}{\partial t^2} + I_1 \frac{\partial^3 w}{\partial x \partial t^2} \right) = 0, \quad (22)$$

$$B \frac{\partial^3 u}{\partial x^3} - D \frac{\partial^4 w}{\partial x^4} + (1 - \mu^2 \nabla^2) \left(-I_0 \frac{\partial^2 w}{\partial t^2} - I_1 \frac{\partial^3 u}{\partial x \partial t^2} + I_2 \frac{\partial^4 w}{\partial x^2 \partial t^2} \right) = 0 \quad (23)$$

3. Solution procedure

Here, an analytical solution method is applied to solve the governing equations. The displacement field's components are supposed to be

$$\begin{Bmatrix} u \\ w \end{Bmatrix} = \begin{Bmatrix} U \exp[i(\beta x - \omega t)] \\ W \exp[i(\beta x - \omega t)] \end{Bmatrix} \quad (204)$$

where, U and W are wave amplitudes, β is wave number and ω is the circular frequency of dispersed waves. Substituting for u and w from Eq. (24) in the Eqs. (22) and (23), the following equation is obtained

$$([K] - \omega^2 [M]) \begin{Bmatrix} U \\ W \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (25)$$

Once the above eigenvalue equation is solved for ω , the wave frequency of propagated waves can be reached. Moreover, the phase velocity can be computed by dividing wave frequency to wave number as

$$c_p = \frac{\omega}{\beta} \quad (26)$$

Table 1 Comparison of the natural frequencies of FG nanobeams

μ	p = 0.1		p = 0.5		p = 1	
	Eltaher <i>et al.</i> (2012)	Present	Eltaher <i>et al.</i> (2012)	Present	Eltaher <i>et al.</i> (2012)	Present
0	9.2129	9.1887	7.8061	7.7377	7.0904	6.9885
1	8.7889	8.7663	7.4458	7.3820	6.7631	6.6672
2	8.4166	8.3972	7.1312	7.0712	6.4774	6.3865
3	8.0887	8.0712	6.8533	6.7966	6.2251	6.1386

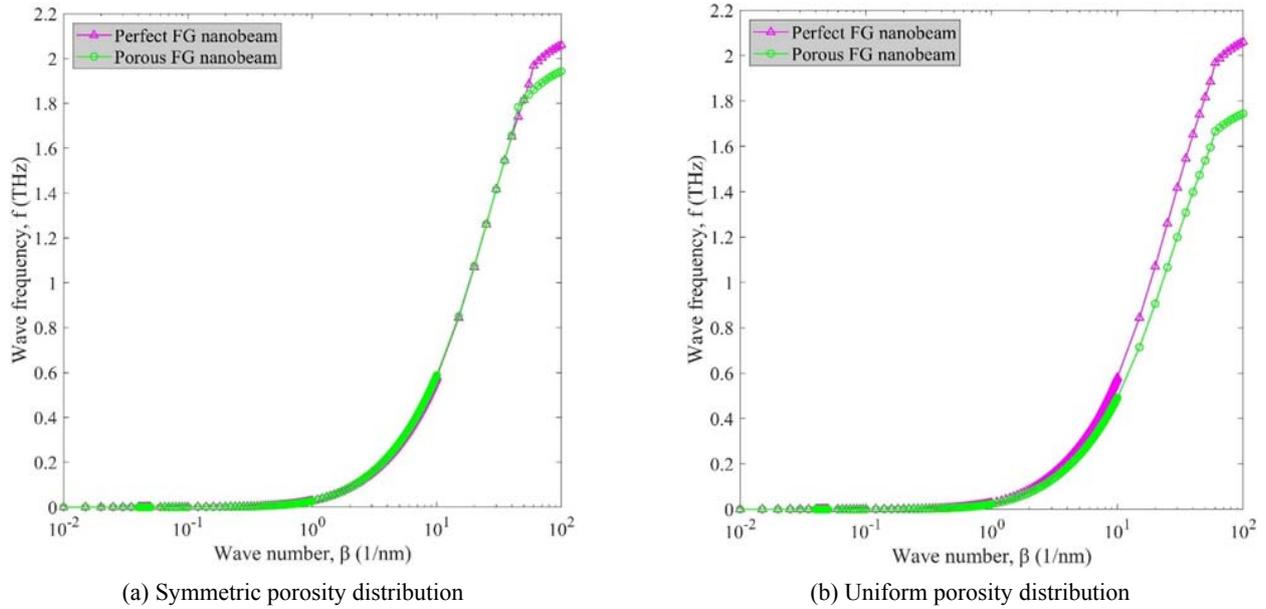


Fig. 2 Variation of wave frequency versus wave number for both perfect and porous materials by considering the influence of different porosity distributions ($\mu = 0.5$ nm, $p = 2$)

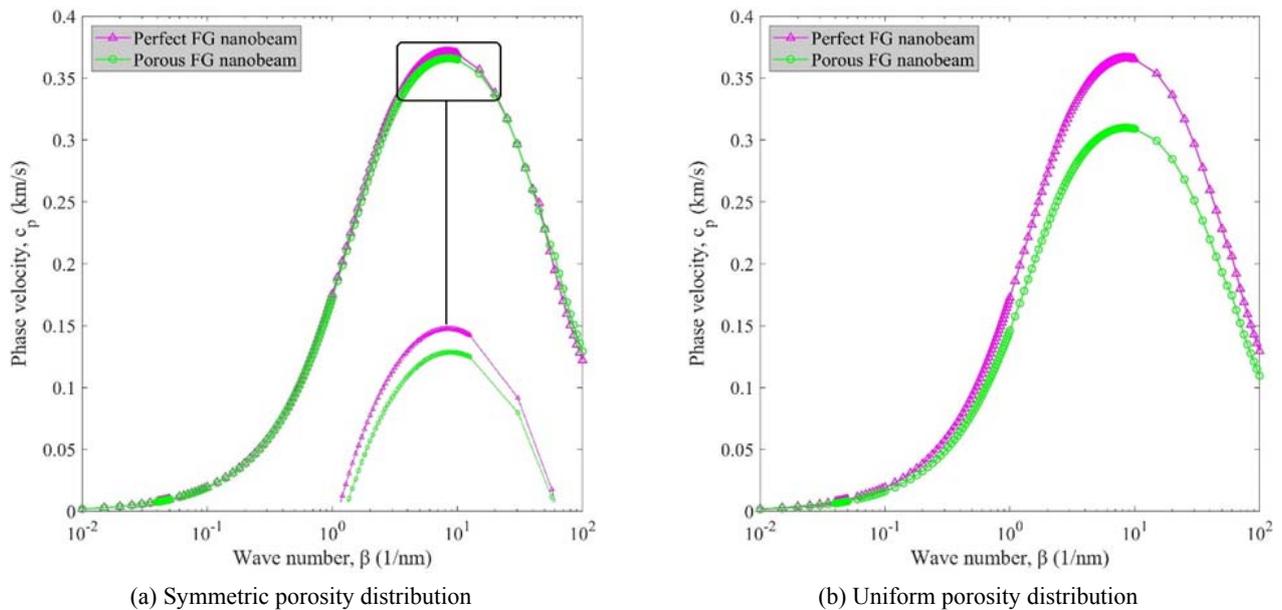


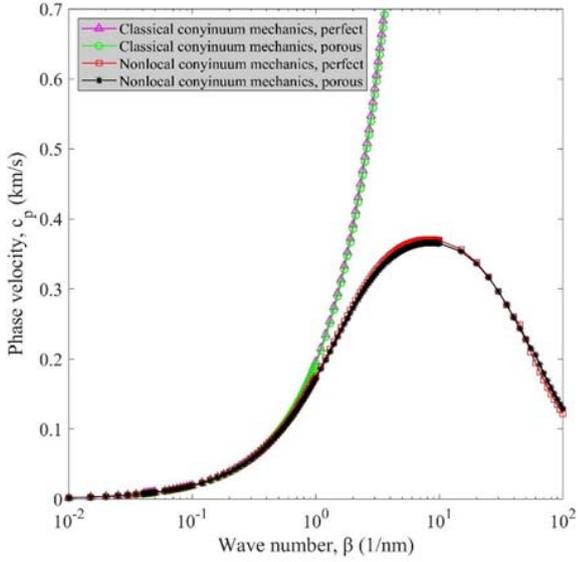
Fig. 3 Variation of phase velocity versus wave number for both perfect and porous materials by considering the influence of different porosity distributions ($\mu = 0.5$ nm, $p = 2$)

4. Numerical results

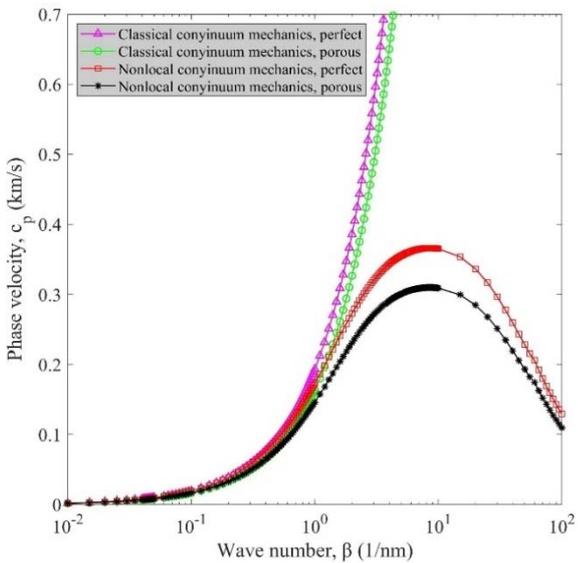
Present part is devoted to study the influences of various parameters on wave propagation behaviors of FG nanobeams. In this study, the material properties of ceramic are $E_c = 390$ GPa, $\rho_c = 3960$ kg/m³; also, the metal's mechanical properties are $E_m = 210$ GPa, $\rho_m = 7800$ kg/m³. The thickness of the studied nanobeam is 0.1 nm and the length to thickness ratio is assumed to be 100. Also, the nanobeam's width is as same as its thickness. It is worth mentioning that the results of the present research are obtained free from consideration of the shear deformation effects. Indeed, the length to thickness ratio of the

nanobeam is selected to be big enough so that the error generated from employment of the Euler-Bernoulli theory is negligible. However, once a nanobeam with slenderness ratio smaller than 10 or 15 is employed, the researchers must employ higher order beam models to consider for the deflection produced from the shear strain. The presented formulation is validated by comparing the natural frequency responses of ours with those of Eltaher *et al.* (2012). The results of this verification are presented in Table 1.

Fig. 2 shows the variation of wave frequency versus wave number for different porosity distributions for the goal of showing the effect of porosity on the mechanical response of FG nanobeams. It is clear that porous



(a) Symmetric porosity distribution



(b) Uniform porosity distribution

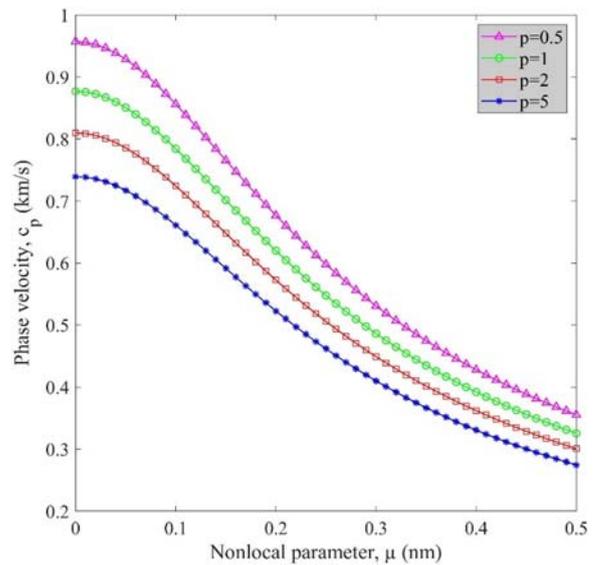
Fig. 4 Variation of phase velocity versus wave number with respect to both classical and nonlocal continuum theories and porosity effects ($p = 2$)

nanobeams are a little weaker than perfect ones in enduring wave frequencies in a similar desirable wave number. This trend can be well justified once pointing to the fact that porous materials have lower rigidities once compared with perfect ones. However, this phenomenon can be better perceived in high values of wave number. Also, it is shown that porous beams with uniformly distributed porosities support lower frequencies in comparison with those with symmetric porosity distribution.

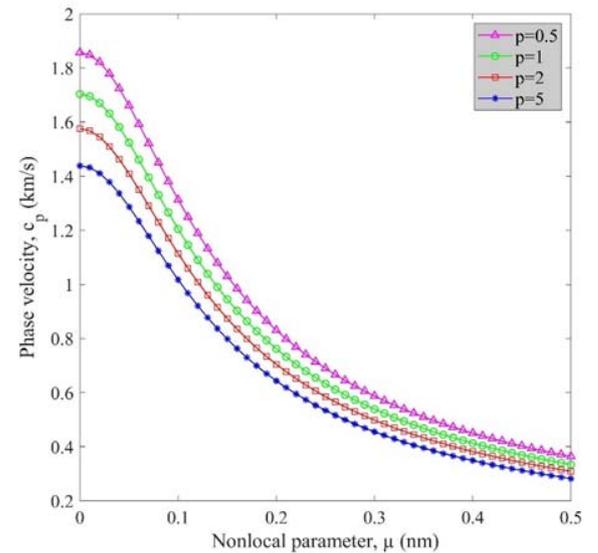
A same illustration is presented in Fig. 3 plotting the variation of phase velocity of both perfect and imperfect FG nanobeams against wave number for different types of porosity distribution. Similarly, it can be found that perfect materials are better relative candidates to be employed as a constituent material due to their greater capacity of passing

waves with higher speeds while compared with porous materials. In addition, it can be well observed that symmetric porosity distribution is better than uniform distribution because it does not affect the rigidity in the inner regions inside the beam, whereas, uniform distribution affects all of the beam in a same manner.

Furthermore, in Fig. 4, the influence of using classical or nonlocal continuum mechanics are covered as well as porosity effects while investigating the variation of phase velocity against wave number. On the basis of this diagram, utilization of nonlocal continuum mechanics corresponds with a decrease in the value of wave speed which is exactly the stiffness-softening influence of nonlocal parameter. Also, it can be seen that differences between porous and perfect materials can be better observed once uniform

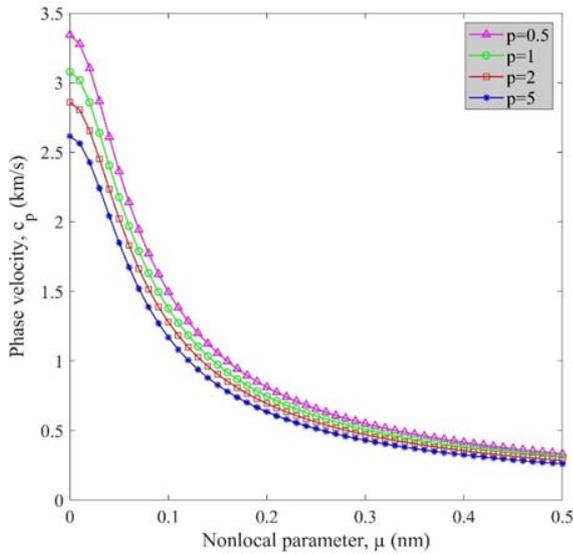


(a) $\beta = 5$ (1/nm)



(b) $\beta = 10$ (1/nm)

Fig. 5 Variation of phase velocity versus nonlocal parameter for various gradient indices for uniformly porous FG nanobeams with respect to the influence of wave number ($\mu = 0.5$ nm)



(c) $\beta = 20$ (1/nm)

Fig. 5 Continued

distribution is selected. Indeed, the bending rigidity's decrease in uniform distribution of porosities is greater than its decrease in the case of choosing symmetric distribution of porosities.

Fig. 5 is presented to put emphasize on the importance of wave number and gradient index while drawing the variation of phase velocity versus nonlocal parameter for porous FG nanobeams. According to the figure, it can be understood that the higher is the chosen nonlocal parameter, the lower is the corresponding phase velocity. On the other hand, it is observed that in a desired amount of nonlocal parameter phase velocity of porous FG nanobeam can be diminished selecting a higher gradient index. Besides, the effect of wave number can be seen. Indeed, the decreasing

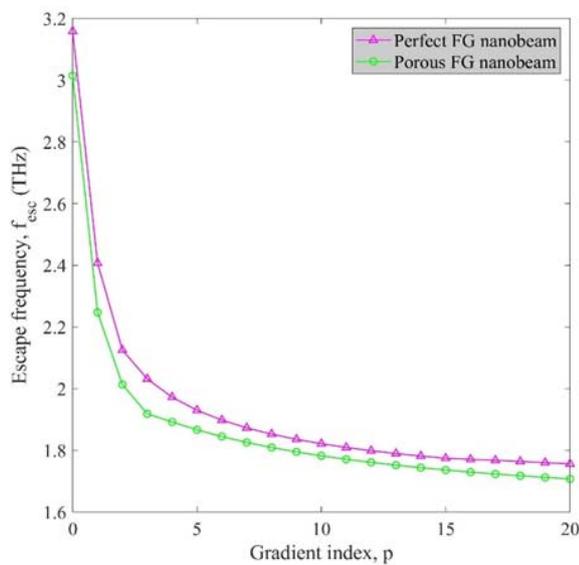
influence of nonlocal coefficient can be sensed more in high wave numbers. In other words, by changing the nonlocal parameter from zero to 0.5 nm, the percentage of changes of phase velocity for $\beta = 20$ (1/nm) is more than $\beta = 10$ (1/nm) followed by $\beta = 5$ (1/nm).

Finally, Fig. 6 is drawn to illustrate the variation of the wave dispersion responses of both porous and non-porous FG nanobeams against the gradient index. It can be seen that the escape frequency decreases continuously as the gradient index grows for both symmetric and uniform types of porosity distribution. The reason of the appeared decrease is that the elastic characteristics of the nanobeam will be lessened as the gradient index becomes greater. Hence, it is clear to observe a stiffness decrease which leads to seeing lower escape frequencies.

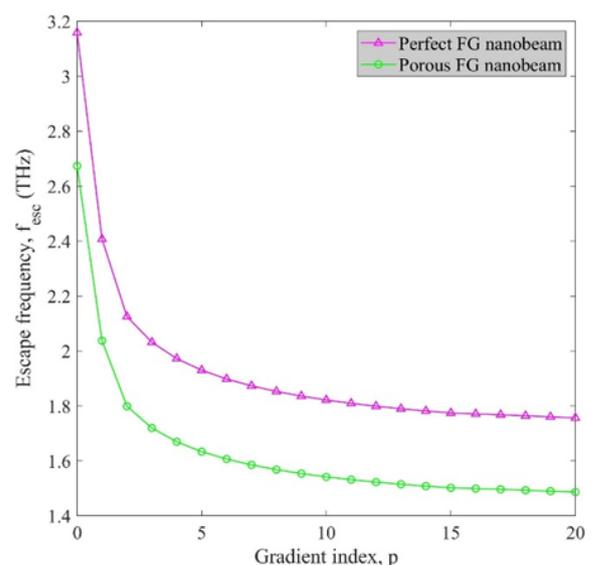
5. Conclusions

Herein, a two-step porosity based homogenization technique is presented and its application is shown in the wave propagation problem of a FG nanobeam. Incorporating Hamilton's variational principle, nonlocal elasticity and Euler-Bernoulli beam theory, the governing equations are achieved. Now, the most important results are reviewed:

- Wave dispersion responses of FG nanobeams decrease as gradient index increased.
- Porous materials cannot endure as enormous frequencies as perfect ones tolerate.
- The wave propagation responses become smaller in nonzero amounts of nonlocal coefficient.
- The mechanical responses of porous FG nanobeams can be more affected by porosities in uniform distribution.



(a) Symmetric porosity distribution



(b) Uniform porosity distribution

Fig. 6 Variation of escape frequency versus gradient index for both perfect and porous materials with respect to the influence of porosity distribution ($\mu = 0.5$ nm)

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