

Thermal buckling analysis of SWBNNT on Winkler foundation by non local FSDT

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Abstract. In this work, the thermal buckling characteristics of zigzag single-walled boron nitride (SWBNNT) embedded in a one-parameter elastic medium modeled as Winkler-type foundation are investigated using a nonlocal first-order shear deformation theory (NFSDT). This model can take into account the small scale effect as well as the transverse shear deformation effects of nanotubes. A closed-form solution for nondimensional critical buckling temperature is obtained in this investigation. Further the effect of nonlocal parameter, Winkler elastic foundation modulus, the ratio of the length to the diameter, the transverse shear deformation and rotary inertia on the critical buckling temperature are being investigated and discussed. The results presented in this paper can provide useful guidance for the study and design of the next generation of nanodevices that make use of the thermal buckling properties of boron nitride nanotubes.

Keywords: boron nitride nanotube; critical buckling temperature; small scale effect; Winkler foundation

1. Introduction

Nanotubes have been modeled on carbon nanotubes from various materials with a lamellar structure similar to that of graphite. For example, nanotubes of MoS₂ and WS₂ could be synthesized in 1992 (Tenne *et al.* 1992, Margulis *et al.* 1993). Boron nitride nanotubes (BNNTs) are one of the most promising materials for nanotechnology due to the coupling characteristics of electromechanics field. This promising material was theoretically predicted in 1994 (Rubio *et al.* 1994, Blase *et al.* 1994) and carried out experimentally in 1995 (Chopra *et al.* 1995). These BNNTs have many of the superior CNTs properties, such as exceptional elastic properties (Goldberg *et al.* 2010, Moon and Hwang 2004, Pokropivny *et al.* 2008, Verma *et al.* 2007, Li and Chou 2006), high mechanical strength (Jeon and Mahan 2009, Ghassemi and Yassar 2010, Suryavanshi *et al.* 2004, Chopra and Zettl 1998), chemical inertness (Zhi *et al.* 2008) and structural stability (Ciofani *et al.* 2009), strong conduction thermal and piezoelectricity (Oh 2010). The growing interest that is brought to nanotubes of boron nitride is in particular due to the fact that, unlike carbon nanotubes, boron nitride nanotubes are large-gap semiconductors (of the order of 5.5 to 6 eV). Also, potential

applications of BNNTs include various materials reinforcements such as polymer, ceramic, and metal based composites, and key parts of nanomechanical systems. Recently, many researchers have investigated nanotubes based on non-local models, which have shown satisfying results compared to atomic models. Sudak (2003) studied infinitesimal column buckling of carbon nanotubes (CNTs), incorporating the van der Waals forces and small scale effect, and showed that the critical axial strain decreases compared with the results of classical beams. Lu *et al.* (2007) studied the propagation properties of waves and vibrations of single or multiple walled CNTs based on a non-local beam model.

Reddy (2007) developed non-local theories of Euler-Bernoulli, Timoshenko, Reddy and Levinson beams. Analytical solutions of bending, vibration and buckling are delivered from the non-local effect of bending, load and natural frequencies. Murmu and Pradhan (2009) used nonlocal elasticity and Timoshenko beam theory to investigate the stability response of single-walled carbon nanotubes (SWCNTs) embedded in an elastic medium. Zidour *et al.* (2012) carried out the thermal effect on vibration of zigzag single walled carbon nanotubes using nonlocal Timoshenko beam theory. Tounsi *et al.* (2013a) investigated the small scale effect on critical buckling temperature for DWCNTs based on Timoshenko beam theory. Adda Bedia *et al.* (2015) studied the thermal buckling characteristics of armchair single-walled carbon nanotube (SWCNT) embedded in a one-parameter elastic

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medium are investigated using a new nonlocal first-order shear deformation theory (NFSDT). Semmah *et al.* (2014) analyzed the thermal buckling properties of zigzag single-walled carbon nanotubes using Timoshenko beam theory. Semmah *et al.* (2015) studied the effect of the chirality on critical buckling temperature of zigzag single-walled carbon nanotubes using higher-order variation of transverse shear strain. The particular structure of BNNTs and their extraordinary properties motivated some researchers to study BNNTs using nonlocal methods. Ghorbanpour Arani *et al.* (2012) used nonlocal piezoelectricity theory to investigate nonlinear vibration of embedded SWBNNTs with zigzag atomic structure. Hady Elmerabet *et al.* (2017) estimated the critical buckling temperature of SWBNNTs using a new first-order shear deformation beam theory. Kumar (2018) investigated the mechanical vibration of double-walled carbon nanotubes with inter-tube Van der Waals forces. Bensaid *et al.* (2018) employed a nonlocal strain gradient theory to study the dynamic response of higher order shear-deformable nanobeams resting on elastic foundation. Ebrahimi and Mahmoodi (2018) analyzed the vibration of carbon nanotubes with multiple cracks in thermal environment. Ebrahimi and Haghi (2018) presented elastic wave dispersion modelling within rotating functionally graded nanobeams in thermal environment. Selmi and Bisharat (2018) investigated the Free vibration of functionally graded SWNT reinforced aluminum alloy beam. Bensattalah *et al.* (2018) determined critical buckling loads of carbon nanotube embedded in Kerr's medium. Hajmohammad *et al.* (2018) used a layerwise theory for buckling analysis of truncated conical shells reinforced by CNTs and carbon fibers integrated with piezoelectric layers in hygrothermal environment. Karami *et al.* (2018a) studied the thermal buckling of smart porous functionally graded nanobeam rested on Kerr foundation. Eltaher *et al.* (2018) proposed a modified porosity model in analysis of functionally graded porous nanobeams. Mehar *et al.* (2018) presented a finite-element solution to nonlocal elasticity and scale effect on frequency behavior of shear deformable nanoplate structure. Faleh *et al.* (2018) studied vibrations of porous functionally graded nanoshells. Bouadi *et al.* (2018) presented a new nonlocal HSDT to study the stability of single layer graphene sheet. Zemri *et al.* (2015) employed refined nonlocal shear deformation theory beam theory for mechanical response of functionally graded nanoscale beam. Yazid *et al.* (2018) proposed a novel nonlocal refined plate theory for stability response of orthotropic single-layer graphene sheet resting on elastic medium. Other works on nanostructures can be found in literature (Ahouel *et al.* 2016, Bounouara *et al.* 2016, Karami *et al.* 2017, 2018b, c, d, e, 2019, Bellifa *et al.* 2017a, Khetir *et al.* 2017, Cherif *et al.* 2018, Bakhadda *et al.* 2018, Akbaş 2018, Ebrahimi *et al.* 2019, Karami and Karami 2019, Ebrahimi and Salari 2018).

In this work the critical buckling temperature of zigzag SWBNNT embedded in elastic medium modeled as Winkler-type foundation is estimated using a new first-order shear deformation beam theory. The influence of the scale parameter, the Winkler modulus parameter, and the transverse shear deformation of zigzag SWBNNT are taken into account. It is hoped that this work will help researchers

and engineers using BNNTs to strengthen nanocomposite materials and polymers.

2. Theoretical formulations

2.1 Basic assumptions

The displacement field of the proposed theory is chosen based on the following assumptions (Bouremana *et al.* 2013):

The displacements are small in comparison with the nanobeam thickness and, therefore, strains involved are infinitesimal.

The transverse displacement w includes two components of bending w_b , and shear w_s . These components are functions of coordinate x only (Bouremana *et al.* 2013, Al-Basyouni *et al.* 2015, Boudierba *et al.* 2016, Bellifa *et al.* 2016, Youcef *et al.* 2018)

$$w(x, z) = w_b(x) + w_s(x) \quad (1)$$

- (i) The transverse normal stress σ_z is negligible in comparison with in-plane stresses σ_x .
- (ii) The displacement u in x -direction given by the classical beam theory.

$$u = -z \frac{\partial w_b}{\partial x} \quad (2)$$

2.2 Kinematics

Based on the assumptions made in the preceding section, the displacement field can be obtained using Eqs. (1)-(2) as

$$u(x, z, t) = -z \frac{\partial w_b}{\partial x} \quad (3a)$$

$$w(x, z, t) = w_b(x, t) + w_s(x, t) \quad (3b)$$

The strains associated with the displacements in Eq. (3) are

$$\varepsilon_x = -z \frac{\partial^2 w_b}{\partial x^2} \quad \text{and} \quad \gamma_{xz} = \frac{\partial w_s}{\partial x} \quad (4)$$

2.3 Constitutive relations

Response of materials at the nanoscale is different from those of their bulk counterparts. Nonlocal elasticity is first considered by Eringen (1983). He assumed that the stress at a reference point is a functional of the strain field at every point of the continuum. Eringen (1983) proposed a differential form of the nonlocal constitutive relation as (Belkhorissat *et al.* 2015, Larbi Chaht *et al.* 2015, Bouafia *et al.* 2017, Besseghier *et al.* 2017, Mouffoki *et al.* 2017, Mokhtar *et al.* 2018, Kadari *et al.* 2018)

$$\sigma_x - (e_0 a)^2 \frac{d^2 \sigma_x}{dx^2} = E \varepsilon_x \quad (5a)$$

$$\tau_{xz} - (e_0 a)^2 \frac{d^2 \tau_{xz}}{dx^2} = G \gamma_{xz} \quad (5b)$$

where σ_x , τ_{xz} , E and G are the axial stress, the shear stress, the elastic modulus and shear modulus of the nanobeam, respectively; e_0a is the nonlocal parameter, e_0 is a constant appropriate to each material and a is an internal characteristic length. So far, there is no rigorous study made on estimating the value of the nonlocal parameter. It is suggested that the value of nonlocal parameter can be determined by conducting a comparison of dispersion curves from the nonlocal continuum mechanics and molecular dynamics simulation (Arash and Ansari 2010, Wang 2005, Wang and Wang 2007).

2.4 Equations of motion

Hamilton's principle is used herein to derive the equations of motion. The principle can be stated in analytical form as (Ahmed 2014, Belabed *et al.* 2014, Attia *et al.* 2015, Yahia *et al.* 2015, Mahi *et al.* 2015, Bourada *et al.* 2015, Boukhari *et al.* 2016, Bennoun *et al.* 2016, Houari *et al.* 2016, Hachemi *et al.* 2017, Zidi *et al.* 2017, Zine *et al.* 2018, Fourn *et al.* 2018, Kaci *et al.* 2018, Bourada *et al.* 2019, Tlidji *et al.* 2019, Khiloun *et al.* 2019)

$$\delta \int_0^T (U + V - K) dt = 0 \quad (6)$$

where δU is the virtual variation of the strain energy; δV is the virtual variation of the potential energy; and δK is the virtual variation of the kinetic energy. The variation of the strain energy of the beam can be stated as

$$\begin{aligned} \delta U &= \int_0^L \int_A (\sigma_x \delta \varepsilon_x + \tau_{xz} \delta \gamma_{xz}) dA dx \\ &= \int_0^L \left(-M_b \frac{d^2 \delta w_b}{dx^2} + Q \frac{d \delta w_s}{dx} \right) dx \end{aligned} \quad (7)$$

where M_b and Q are the stress resultants defined as

$$M_b = \int_A z \sigma_x dA \quad \text{and} \quad Q = \int_A \tau_{xz} dA \quad (8)$$

The variation of the potential energy by the applied loads can be written as

$$\delta V = - \int_0^L q \delta(w_b + w_s) dx - \int_0^L N_0 \frac{d(w_b + w_s)}{dx} \frac{d \delta(w_b + w_s)}{dx} dx \quad (9)$$

where q and N_0 are the transverse and axial loads, respectively.

Substituting the expressions for δU and δV from Eqs. (8), (9) and (10) into Eq. (7) and integrating by parts, and collecting the coefficients of δw_b , and δw_s , the following equations of motion of the proposed beam theory are obtained

$$\delta w_b : \frac{d^2 M_b}{dx^2} + q - N_0 \frac{d^2(w_b + w_s)}{dx^2} = 0 \quad (10a)$$

$$\delta w_s : \frac{dQ}{dx} + q - N_0 \frac{d^2(w_b + w_s)}{dx^2} = 0 \quad (10b)$$

when the shear deformation effect is neglected ($w_s = 0$), the equilibrium equations in Eq. (11) recover those derived from the Euler-Bernoulli beam theory.

By substituting Eq. (6) into Eq. (11) and the subsequent results into Eq. (9), the stress resultants are obtained as

$$M_b - (e_0a)^2 \frac{d^2 M_b}{dx^2} = -D \frac{d^2 w_b}{dx^2} \quad (11a)$$

$$Q - (e_0a)^2 \frac{d^2 Q}{dx^2} = A_s \frac{dw_s}{dx} \quad (11b)$$

where

$$D = \int_A z^2 E dA, \quad A_s = K_s \int_A G dA \quad (12)$$

By substituting Eq. (11) into Eq. (12), the nonlocal equations of motion can be expressed in terms of displacements (w_b , w_s) as

$$\begin{aligned} -D \frac{d^4 w_b}{dx^4} + \left(K_f (w_b + w_s) - (e_0a)^2 K_f \frac{d^2(w_b + w_s)}{dx^2} \right) \\ - N_0 \left(\frac{d^2(w_b + w_s)}{dx^2} - (e_0a)^2 \frac{d^4(w_b + w_s)}{dx^4} \right) = 0 \end{aligned} \quad (13a)$$

$$\begin{aligned} A_s \frac{d^2 w_s}{dx^2} + \left(K_f (w_b + w_s) - (e_0a)^2 K_f \frac{d^2(w_b + w_s)}{dx^2} \right) \\ - N_0 \left(\frac{d^2 w_b}{dx^2} - (e_0a)^2 \frac{d^4(w_b + w_s)}{dx^4} \right) = 0 \end{aligned} \quad (13b)$$

The equations of motion of local beam theory can be obtained from Eq. (14) by setting the scale parameter e_0a equal to zero.

3. Analytical solution of simply supported nanobeam

In this study, analytical solutions are given for the hinged boundary condition case, the solution of these eqs for a simply supported borone nitride nanotube can be expressed as follows

$$w_b = w_s = M_b = M_s = 0 \quad \text{at} \quad x = 0, L \quad (14)$$

The following displacement field satisfies boundary conditions and governing equations.

$$\begin{Bmatrix} w_b \\ w_s \end{Bmatrix} = \sum_{n=1}^{\infty} \begin{Bmatrix} W_{bn} \sin(\alpha x) e^{i\omega t} \\ W_{sn} \sin(\alpha x) e^{i\omega t} \end{Bmatrix} \quad (15)$$

where W_{bn} , and W_{sn} are arbitrary parameters to be determined, ω is the eigenfrequency associated with m the

igenmode, and $\alpha = m\pi/L$.

Substituting the expansions of w_b and w_s from Eqs. (16) into Eq. (14), the closed-form solutions can be obtained from the following equations

$$\begin{pmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{pmatrix} - \lambda(K_f + N_0\alpha^2) \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{Bmatrix} w_{bn} \\ w_{sn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (16)$$

where

$$S_{11} = D\alpha^4, \quad S_{12} = 0, \quad S_{22} = A_s\alpha^2, \quad \lambda = 1 + (e_0a)^2\alpha^2 \quad (17)$$

3.1 Buckling

The buckling load is obtained from Eq. (15).

$$S_{11} = D\alpha^4, \quad S_{12} = 0, \quad S_{22} = A_s\alpha^2, \quad \lambda = 1 + (e_0a)^2\alpha^2 \quad (18)$$

On the basis of the theory of thermal elasticity mechanics, the axial force N_0 can be written as

$$N_0 = \frac{EA\beta}{(1-2\nu)}T \quad (19)$$

where β is the coefficient of thermal expansion in the direction of x -axis, and ν is Poisson's ratio, respectively. T presents the change in temperature.

Then, the critical temperature with the nonlocal continuum theory can be derived as

$$T_{cr} = \frac{(1-2\nu)I\alpha^2K_sG}{\beta\lambda(EI\alpha^2 + K_sGA)} - K_f \frac{(1-2\nu)}{EA\beta\alpha^2} \quad (20)$$

the non-dimensional critical temperature can be expressed as the following form

$$\bar{T}_{cr} = \frac{L^2\alpha^2K_sGA}{\lambda(EI\alpha^2 + K_sGA)}(1-2\nu) - \bar{K}_f \frac{1}{L^2\alpha^2}(1-2\nu) \quad (21)$$

For the sake of simplicity the following dimensionless variable is introduced for Winkler foundation parameter

$$\bar{K}_f = K_f \frac{L^4}{EI} \quad (22)$$

4. Validity and applicability of continuum beam model for CNTs

Applicability of continuum beam theory for carbon nanotubes (CNTs) is discussed by several authors (Wang and Hu 2005, Harik 2001, 2002). The ranges of applicability for the continuum beam theory in the mechanics of CNTs and nanorods were reported by Wang and Hu (2005) and Harik (2001, 2002).

Recently, Tounsi *et al.* (2013b) and Adda Bedia *et al.* (2015) presented numerical results for critical buckling strains obtained from the continuum mechanics theory (using the nonlocal Timoshenko beam theory (NTBT) and new nonlocal first-order shear deformation theory

(NFSDT), respectively) which are compared with those obtained from MD simulations, and the Sanders shell theory (SST) (Silvestre *et al.* 2011). It can be seen that both the present nonlocal first-order shear deformation theory and the conventional nonlocal Timoshenko model give identical results. Since the MD simulations referenced herein consider the CNTs with fixed ends, also the NFSDT and NTBT are developed with fully clamped boundary conditions. In addition, CNT (5, 5) is studied with a diameter $d = 6.71\text{\AA}$ and CNT (7, 7) with a diameter $d = 9.40\text{\AA}$, for different lengths. Both nanotubes are modeled using a thickness $t = 0.66\text{\AA}$, Young's modulus $E = 5.5\text{ TPa}$, and Poisson's ratio $\nu = 0.19$ (Yakobson *et al.* 1996). The lengths of CNTs used in the following table are extracted from the work done by Silvestre *et al.* (2011). The results from MD simulations, the present NFSDT, NTBT and SST are compared in Table 1. It is seen that the critical buckling strains are in good agreement when compared to the results obtained from MD simulations as well as Sanders shell theory (SST). Based on the MD simulation results, the value of nonlocal constant is determined for CNTs based on an averaging process. The best match between MD simulations and nonlocal formulations is achieved for a nonlocal constant value of $e_0a = 0.54\text{ nm}$ for CNT (5, 5) and $e_0a = 1.05\text{ nm}$ for CNT (7, 7) with good accuracy (the error is less than 10%).

5. Numerical results and discussion

In this section, numerical computations for the thermal buckling characteristics of embedded zigzag SWBNNTs are carried out. The dimensions and characteristics employed in numerical results for the SWBNNTs with zigzag structure are considered as follows (GhorbanpourArani *et al.* 2012, Hadj Elmerabet *et al.* 2017): the wall thickness $h = 0.075\text{ mm}$, mean radius $r = 0.313\text{ nm}$, Poisson's ratio $\nu = 0.34$, elastic modulus $E = 1.8\text{ TPa}$, and the values of thermal expansion is $\alpha x = 1.2 \times 10^{-6}$. To show the influences of the transverse shear deformation, the critical buckling temperature of the zigzag SWBNNT by the present nonlocal theory NFSDT to the nonlocal Euler-Bernoulli beam model with different values of the Length to diameter ratios (L/d) is presented in Fig. 1. The mode number (m) and the nonlocal parameter (e_0a) nm are considered.

Table 1 Comparison between critical buckling strains of CNT (5, 5) and CNT (7, 7) obtained from MD simulations, Sanders shell theory (SST), nonlocal Timoshenko theory (NTBT) and the present new nonlocal first shear deformation theory (NFSDT)

L (Å)	d (Å)	MD	SST	NTBT	NFSDT
16.09	6.71	0.08146	0.08729	0.08216	0.08216
21.04	6.71	0.07528	0.08288	0.07460	0.07460
28.46	6.71	0.06992	0.07858	0.06302	0.06302
28.29	9.40	0.06514	0.06582	0.06542	0.06542
40.59	9.40	0.04991	0.05885	0.05763	0.05763
52.88	9.40	0.04710	0.05600	0.04962	0.04962

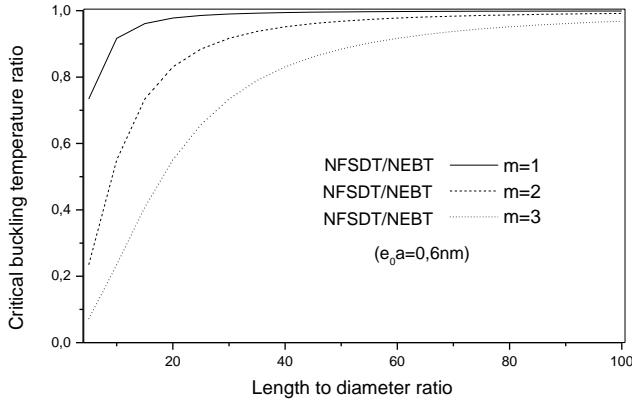


Fig. 1 Ratio of the critical buckling temperature by NFSDT to the nonlocal EBT and the Length to diameter ratio (L/d) for different mode numbers (m)

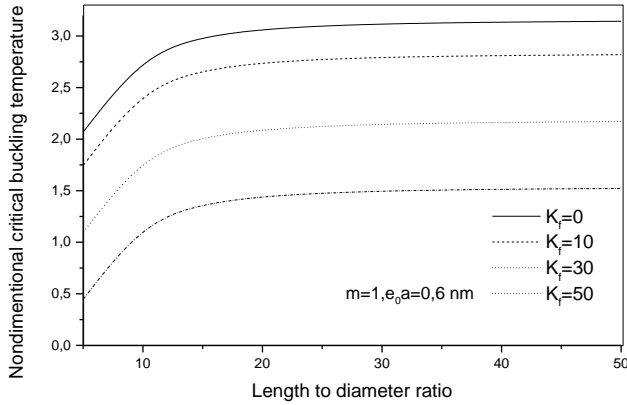


Fig. 2 Relation between the nondimensional critical buckling temperature and the Length to diameter ratio (L/d) with different values of the Winkler foundation parameters $\overline{K_f}$

From Fig. 1, it can be observed that for different mode numbers, all of the ratios are smaller than 1.0. It means that because of the effects of the transverse shear deformation, the critical buckling temperature of the nonlocal NFSDT is lower than that of the nonlocal Euler-Bernoulli beam model. This phenomenon is more obvious for higher mode numbers and smaller slenderness ratios. It means that the effects of the transverse shear deformation should be considered and the nonlocal NFSDT is more accurate for short boron nitride nanotube.

Fig. 2 shows the variation of the critical buckling temperature of zigzag SWBNNT with aspect ratios L/d for various Winkler modulus parameters. Four different values of Winkler modulus parameter are considered for the study, viz. $K_f = 0, 10, 30$ and 50 . In this present computation, a constant value of nonlocal parameter ($e_0a = 0.6\text{ nm}$) and the mode number ($m = 1$) are used for the proposed model. From the figure, it is seen that as the aspect ratios (L/d) increase, the critical buckling temperature increases until taken as a constant value for higher values of L/d . Thus, for a slender SWBNNT, the effect of shear deformation is less compared to short SWBNNT. The influences of both the mode number (m) and the Winkler modulus parameter K_f ,

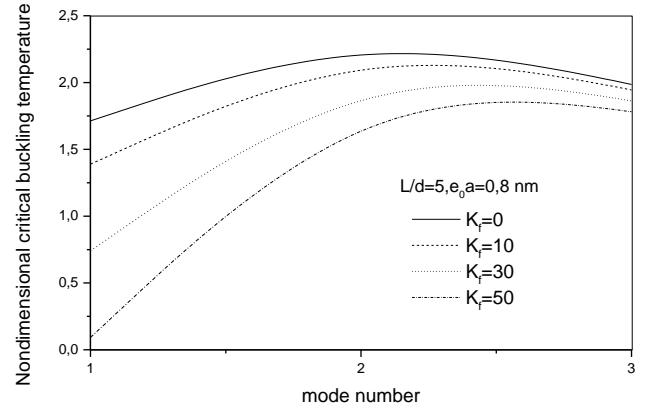


Fig. 3 Relation between the nondimensional critical buckling temperature and the mode number with different values of the Winkler foundation parameters $\overline{K_f}$

on the non-dimensional critical buckling temperature are shown in Fig. 3.

The nonlocal parameter is $= 0.8\text{ nm}$. From Fig. 3, it is apparent that when the mode number is less than 3, the effect of the elastic medium is obvious. This influence becomes insignificant when the mode number is larger than 3. Moreover, the non-dimensional critical buckling temperatures for all of four values of the Winkler modulus parameter are elevated with the mode number increasing. This implies that the elastic medium has significant influence on the non-dimensional critical buckling temperature for lower mode numbers, and should be considered in the case where NNBTs are used as reinforcement for polymers or in similar applications. The relation between the nondimensional critical buckling temperature and the axial mode number as well as the nonlocal parameter is illustrated in Fig. 4.

The most notable feature is that the effect of the nonlocal parameter (e_0a) on the critical buckling temperature is relatively weak for small mode numbers. However, the difference becomes obvious with the mode number increasing.

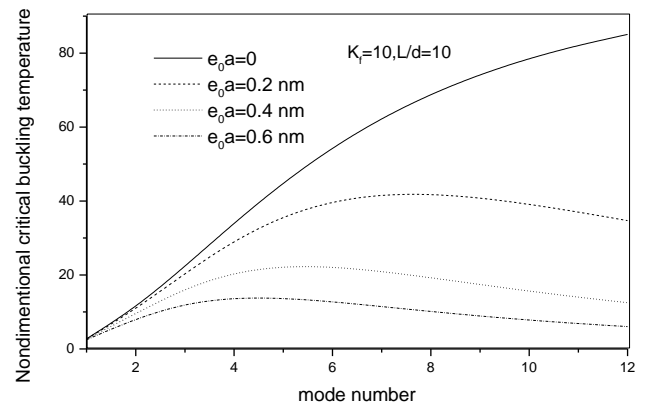


Fig. 4 Relation between the nondimensional critical buckling temperature and the mode number with different values of the scale coefficients (e_0a)

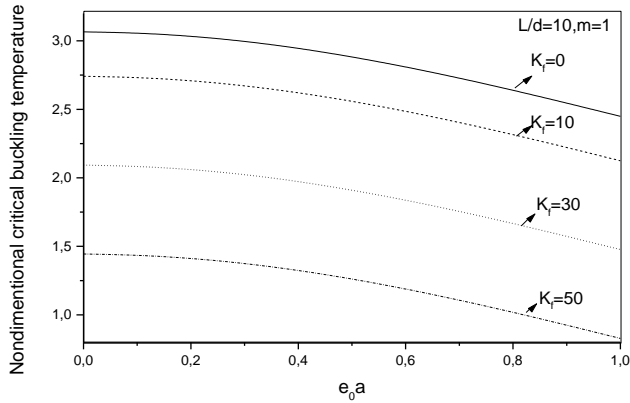


Fig. 5 Relation between the nondimensional critical buckling temperature and the scale coefficient (e_0a) with different values of the Winkler foundation parameters \bar{K}_f

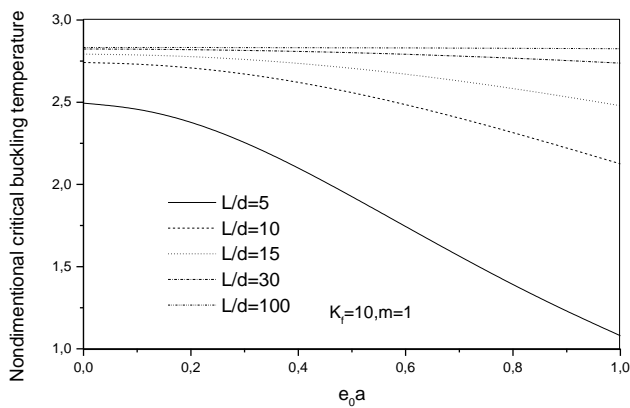


Fig. 6 Relation between the nondimensional critical buckling temperature and the scale coefficient (e_0a) with different values of the Length to diameter ratios (L/d)

Fig. 5 indicates the effect of the small scale on the critical buckling temperature of zigzag SWBNNT for various Winkler modulus parameters. As the nonlocal parameter (e_0a) increases, the critical buckling temperature decreases. Thus, it can be concluded that the classical elastic (i.e., the local) model, which does not consider the small-scale effects, will give a higher approximation for the critical buckling temperature. But the nonlocal continuum theory will present an accurate and reliable result. In addition, an interesting feature that can be deduced is that as the Winkler foundation parameter increases, the value of critical buckling temperature decreases irrespective of the nonlocal parameter. To show the influences of both aspect ratio and de nonlocal parameter, the critical buckling temperature of the zigzag SWBNNT is presented in Fig. 6.

It can be observed that the effect of aspect ratio increases the critical buckling temperature unlike that of the nonlocal parameter which decreases the critical buckling temperature.

6. Conclusions

In this paper, the thermal buckling characteristics of zigzag SWBNNTs, which are embedded in elastic medium, are predicted using a new nonlocal first-order shear deformation theory. The mathematical formulations include the nonlocal parameter effect, the temperature change. The effects of the scale coefficient, the ratio of the length to the diameter, the transverse shear deformation and the stiffness of the surrounding elastic medium of the thermal buckling properties are investigated. This work is expected to be useful in the design of the next generation of nanodevices that make use of the thermal buckling characteristics of SWBNNTs. An improvement of the present formulation will be considered in the future work to consider the shear deformation effect without using the shear correction factors (Bouderba *et al.* 2013, Tounsi *et al.* 2013b, Bousahla *et al.* 2014, 2016, Meziane *et al.* 2014, Hebali *et al.* 2014, Zidi *et al.* 2014, Hamidi *et al.* 2015, Beldjelili *et al.* 2016, Draiche *et al.* 2016, Abdelaziz *et al.* 2017, El-Haina *et al.* 2017, Fahsi *et al.* 2017, Chikh *et al.* 2017, Sekkal *et al.* 2017a, b, Menasria *et al.* 2017, Klouche *et al.* 2017, Benahmed *et al.* 2017, Benadouda *et al.* 2017, Bellifa *et al.* 2017b, Mehar and Panda 2018, 2019, Abualnour *et al.* 2018, Katariya *et al.* 2018, Mouli *et al.* 2018, Attia *et al.* 2018, Benchohra *et al.* 2018, Younsi *et al.* 2018, Belabed *et al.* 2018, Katariya and Panda 2018, 2019, Bourada *et al.* 2018, Bouhadra *et al.* 2018, Patle *et al.* 2018, Dash *et al.* 2018, Kar *et al.* 2019, Zaoui *et al.* 2019, Meksi *et al.* 2019).

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