Surface effects on scale-dependent vibration behavior of flexoelectric sandwich nanobeams

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Abstract. This paper infer the transient vibration of piezoelectric sandwich nanobeams, In present work, the flexoelectric effect on the mechanical properties of vibration piezoelectric sandwich nanobeam with different boundary conditions is investigated. According to the Nonlocal elasticity theory in nanostructures, the flexoelectricity is believed to be authentic for such size-dependent properties. The governing equations are derived by Hamilton's principle and boundary condition solved by Galerkin-based solution. This research develops a nonlocal flexoelectric sandwich nanobeam supported by Winkler-Pasternak foundation. The results of this work indicate that natural frequencies of a sandwich nanobeam increase by increasing the Winkler and Pasternak elastic constant. Also, increasing the nonlocal parameter at a constant length decreases the natural frequencies. By increasing the length to thickness ratio (L/h) of nanobeam, the nonlocal frequencies reduce.

Keywords: vibration; piezoelectric sandwich nanobeam; flexoelectricity; surface effect; nonlocal elasticity theory; thermal effect

1. Introduction

Smart materials have been used for a variety of smart applications and components of adaptive structures. One of the most significant forms of smart materials is their application, employing fibrous sensors or actuators. The intelligence of the fiber shaped memory alloys composites will make sense, weight savings, some control over directionality in actuation and resulting smart materials effects. Nanotechnology aided introduces structures and implement with good accuracy at nanoscale. Nanobeams are the core structures broadly used in many systems such as nano sensors and actuators for sensing and energy harvesting applications. According to the great potential of nano systems for increasing many applications, their mechanical behavior should be verified and well recognized before new designs can be proffered. The classical mechanic continuum theories assert to predict the response of structures up to a minimum size sub which they fail to provide accurate predictions. The nonlocal theories add a size parameter in the modeling of the continuum. This paper is deal with models developed according to the widely used nonlocal elasticity theory of Eringen (Eringen 1968, 1976, 2002, 2006, Eringen and Edelen 1972), Based on nonlocal beam theories, Xu (2006) verify the free vibrations of nanoto-micro scale beams, and the nonlocal effect becomes significant, especially for the high-order natural frequencies. Reddy (2007, 2010), Reddy and Pang (2008), and Reddy and El-Borgi (2014) linear and nonlinear average rotations to verify vibration characteristics of nanobeams. Investigate both surface and nonlocal elasticity, Lee and Chang (2010) and Elishakoff and Soret (2013) studied the coupled effects of nonlocal and surface effect on the vibration of nonlocal nanobeams using Gurtin-Murdoch model using EBT. Gheshlaghi and Hasheminejad (2012) developed an analytical model for predicting surface effects on the vibrations of piezoelectric non-local nanowires based on EBT. Presented clear statement for modal shapes and natural frequencies of TBT nanobeams into account the effects of length, shear deformation, and rotary inertia. Murmu and Pradhan (2009) showed the contributions of the nonlinearity and nonlocal effects on nonlinear vibration of nanobeams. Li et al. (2016) According to the surface elasticity theory developed by Gurtin and Murdoch (1975), the size-dependency of nanoscale structures due to the surface effects have been widely researched by the adjustment continuum models from static and dynamic properties (Wang and Wang 2011, Ebrahimi and Boreiry 2015, Ebrahimi et al. 2016a, Hosseini et al. 2016), Lately, a number of studies are administered to consolidate the surface effects in analysis of piezoelectric nanostructure. Yan and Jiang (2011) verified surface effects on vibration and of piezoelectric nanobeams with surface effects. Also, Yan and Jiang (2012) investigated the influence of surface piezoelasticity on the buckling behavior of piezoelectric nanofilms subjected to mechanical loadings. Yan and Jiang studied the influence of surface effects, including residual surface stress, surface elasticity and surface piezoelectricity, on the vibrational and buckling behavior of piezoelectric nanobeams by using the Euler-Bernoulli beam theory Ke

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and Wang (2012), A review on the Applications of shear thickening fluids has been presented by Stelson (2018) and Tian *et al.* (2018).

As a deficiency, the nonlocality of stress field is not considered in these papers. Recently, modeling of nanostructures by using the nonlocal elastic field theory of Eringen and Edelen (1972) has received wide importance. The prominence of nonlocal theory of elasticity has stimulated the researchers to investigate the behavior of the nanostructures much accurately Daneshmehr *et al.* 2015, Fernández-Sáez *et al.* 2016, Li *et al.* 2016. This theory includes a nonlocal stress which introduces a stiffness-softening influence on the nano-structures (Ebrahimi and Barati 2016a, b, Ebrahimi *et al.* 2016a).

Studied the electromechanical coupling behavior of a piezoelectric nanowire consolidating both surface and nonlocal effects. Also, Liu et al. (2013) investigated axial wave propagation of piezoelectric nanoplates considering surface and nonlocal effects. Liu et al. (2014) studied buckling and post-buckling behaviors of piezoelectric Timoshenko nanobeams under thermo-electro-mechanical loadings. Ke et al. (2015) reported vibration response of a nonlocal piezoelectric nanoplate considering various boundary conditions. Liu et al. (2015) presented large amplitude vibration of nonlocal piezoelectric nanoplates under electro-mechanical coupling. Asemi et al. (2015) researched the nanoscale mass detection using vibrating piezoelectric ultrathin films subjected to thermo-electromechanical loads. Ansari et al. (2016) Presented thermoelectrical vibrational analysis of post-buckled piezoelectric nanosize beams according to the nonlocal elasticity theory. Ebrahimi and Barati (2016c, d) investigated dynamic behavior of non-homogenous piezoelectric nanobeams under magnetic field. Wang et al. (2016a) investigated vibration response of piezoelectric circular nanoplates considering surface and nonlocal effects. A sandwich-plate model is developed for the vibration of piezoelectric nanofilms based on the Kirchhoff's assumption and the theory of the surface effect at the nanoscale. Viscoelastic vibration analysis of a sandwich nanoplate including a viscoelastic nanocore and two viscoelastic piezoelectric face sheets was investigated by Arefi and Zenkour (2016), Jamalpoor et al. (2016) studies free vibration and biaxial buckling of double-magnetoelectro-elastic nanoplate-systems (DMEENPS) subjected to initial external electric and magnetic potentials, using nonlocal plate theory. It is supposed that the two nanoplates are bonded with each other using a visco-Pasternak medium, and are also limited to the external elastic substrate. However, the flexoelectric effect on the vibration responses of piezoelectric sandwich nanobeams with different boundary conditions has not been reported thus far. Therefore, the objective of the present work is to investigate the influence of the flexoelectricity on the vibrating of piezoelectric nanobeams with different boundary conditions by using an EBT beam model.

To the best of our knowledge, the vibration analyses of sandwich nanobeam considering elastic foundations with flexoelectric actuators have not received enough attentions so far. Motivated by these considerations, the aim of present work is to investigate vibrations of sandwich beam considering smart materials, surface effects, and elastic foundations. The purpose of this work is assessing vibration analysis of nano-structure multilayered nano beam using nonlocal elasticity theory. The study presented here aims to obtain the exact solutions for vibration of flexoelectric sandwich nanobeams employing the theory of nonlocal elasticity in detail. This paper deals with vibration behavior of flexoelectric sandwich nanobeams supported by Winkler-Pasternak elastic foundation. The governing equations are derived by energy method and boundary condition solved by Galerkin-based solution. we develops a nonlocal flexoelectric sandwich nanobeam including surface effect for vibration analysis of piezoelectric nanobeams supported by Winkler elastic constant and Pasternak elastic constant. Results show that natural frequencies of a sandwich nanobeam increase by increasing the Winkler elastic and Pasternak elastic constants while.

2. Material of piezoelectric nanobeams

In present work Assume a sandwich nanobeam made of PZT-5H piezoelectric material with a rectangular cross section, as shown in Fig. 1. A piezoelectric simply support nanobeam with L, b and t denoting its length, width and thickness.

2.1 Nonlocal elasticity theory for the piezoelectric materials with flexoelectric effect

Based on the nonlocal elasticity model (Eringen and Edelen 1972) which contains broad range interplays between points in a continuum solid, the stress state at a point inside a body is introduced as a function of the strains of all neighbor points. The influence of flexoelectricity due to the elastic polarization Pi induced by strain gradient, and the elastic stress created by electric field gradient, can be expressed by

$$\sigma_{ij} - (e_0 a)^2 \nabla^2 \sigma_{ij} = C_{ijkl} \varepsilon_{kl} - e_{kij} E_k + f_{klij} \frac{\partial E_k}{\partial x_l}$$
(1a)

$$P_{i} - (e_{0} a)^{2} \nabla^{2} P_{i} = \varepsilon_{0} \chi_{ij} E_{j} + e_{ikl} \varepsilon_{kl} + f_{ijkl} \frac{\partial \varepsilon_{kl}}{\partial x_{j}}$$
(1b)

where σ_{ij} , ε_{ij} , E_k denote the stress, strain and electric field components, respectively; C_{ijkl} , e_{kij} and k_{ik} are elastic, piezoelectric and dielectric constant, respectively. Also, χ_{ij} is the relative dielectric susceptibility and f_{ijkl} is the flexoelectric coefficient. Also, e_0a is nonlocal parameter which is introduced to describe the size-dependency of nanostructures. The effect of flexoelectricity is involved using the following expression of the electric enthalpy energy density was As follows

$$H = -\frac{1}{2}a_{kl}E_kE_l + \frac{1}{2}c_{ijkl}\varepsilon_{ij}\varepsilon_{kl} - e_{kij}E_k\varepsilon_{ij} - \frac{1}{2}f_{klij}(E_k\frac{\partial\varepsilon_{ij}}{\partial x_l} - \varepsilon_{ij}\frac{\partial E_k}{\partial x_l})$$
(2)



Fig. 1 Geometry and coordinates of flexoelectric sandwich nanobeam

Finally, the constitutive relations incorporating nonlocal and flexoelectricity effects can be expressed by

$$(1 - (e_0 a)^2 \nabla^2) \sigma_{ij} = \frac{\partial H}{\partial \varepsilon_{ij}} = C_{ijkl} \varepsilon_{kl} - e_{kij} E_k + \frac{f_{klij}}{2} \frac{\partial E_k}{\partial x_l} \quad (3a)$$

$$(1 - (e_0 a)^2 \nabla^2) \tau_{ijl} = \frac{\partial H}{\partial (\partial \varepsilon_{ij} / \partial x_l)} = -f_{ijkl} E_k$$
(3b)

$$(1 - (e_0 a)^2 \nabla^2) D_i = -\frac{\partial H}{\partial E_i} = a_{ij} E_j + e_{ikl} \varepsilon_{kl} + \frac{f_{ijkl}}{2} \frac{\partial \varepsilon_{kl}}{\partial x_j} \qquad (3c)$$

$$(1 - (e_0 a)^2 \nabla^2) Q_{ij} = \frac{\partial H}{\partial (\partial E_i / \partial x_j)} = -\frac{f_{ijkl}}{2} \varepsilon_{kl}$$
(3d)

in which τ_{ijl} denotes the moment stress tensor due to the converse flexoelectric effect, D_i is the electric displacement vector and Q_{ij} denotes the electric quadruple density due to flexoelectricity, respectively. The size-dependent phenomena in piezoelectric nanostructures due to flexoelectricity involved in Eq. (3) is reported in analysis of nanowires, nanobeams and nanoplates. Taking into account the surface effects, i.e., the residual surface stress, the surface elasticity, and the surface piezoelectricity, the surface internal energy density Us can be defined by the surface strain and the surface polarization as

$$U_{s} = \Gamma_{\alpha\beta}\varepsilon^{s}_{\alpha\beta} - \frac{1}{2}a^{s}_{\gamma\kappa}E^{s}_{\gamma}E^{s}_{\kappa} + \frac{1}{2}c^{s}_{\alpha\beta\gamma\kappa}\varepsilon^{s}_{\alpha\beta}\varepsilon^{s}_{\gamma\kappa} - e^{s}_{\kappa\alpha\beta}E^{s}_{\kappa}\varepsilon^{s}_{\alpha\beta} \qquad (4)$$

in which $\Gamma_{\alpha\beta}$ denotes the surface residual stress tensor, $a_{\gamma\kappa}^s$ and $c_{\alpha\beta\gamma\kappa}^s$ denote the surface permittivity and surface elastic constants. Also, $e_{\kappa\alpha\beta}^s$ and E_{κ}^s are the surface piezoelectric tensor and surface electric field. Finally, the nonlocal surface constitutive relations can be written as

$$(1 - (e_0 a)^2 \nabla^2) \sigma^s_{\alpha\beta} = \frac{\partial U_s}{\partial \varepsilon_{\alpha\beta}} = \Gamma_{\alpha\beta} + c^s_{\alpha\beta\gamma\kappa} \varepsilon^s_{\gamma\kappa} - e^s_{\kappa\alpha\beta} E^s_{\kappa} \quad (5a)$$

Table 1 Flexoelectric properties of PZT-5H sandwich nanobeam

PZT-5H
102
31
35.5
17.05
1.76×10 ⁻⁸
10-7
102
3.3
2.13
-3.8×10 ⁻⁸

$$(1 - (e_0 a)^2 \nabla^2) D_{\gamma}^s = -\frac{\partial U_s}{\partial E_{\gamma}^s} = a_{\gamma\kappa}^s E_{\kappa}^s + e_{\gamma\alpha\beta}^s \mathcal{E}_{\alpha\beta}^s \qquad (5b)$$

where $\sigma_{\alpha\beta}^{s}$ and D_{γ}^{s} are the surface Cauchy stress and surface electric displacement.

2.2 Theoretical formulation

Here, the classical beam theory is employed for modeling of a piezoelectric sandwich nanobeam with surface, nonlocal and flexoelectric effects. The displacement field at any point of the nanobeam can be written as

$$u_1(x, y, z) = u - z \frac{\partial w}{\partial x}$$
(6a)

$$u_3(x, y, z) = w \tag{6b}$$

where u is displacement of the mid-surface and w is the bending displacement. Based on the Euler–Bernoulli beam theory, can be defined as

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$$\varepsilon_{xx} = \frac{\partial u_1}{\partial x} = \frac{\partial u}{\partial x} - z \frac{\partial^2 w}{\partial x^2},$$
 (7a)

$$\eta_{xxz} = \frac{\partial \varepsilon_{xx}}{\partial z} = -\frac{\partial^2 w}{\partial x^2}.$$
 (7b)

Through extended Hamilton's principle, the governing equations can be derived as follows

$$\int_{0}^{t} \delta(\Pi_{s} - \Pi_{\kappa} + \Pi_{w}) dt = 0$$
(8)

$$\Pi s_1 = \int_V (\sigma_{xx} \,\delta \varepsilon_{xx} + \tau_{xxz} \,\delta \eta_{xxz}) \mathrm{d}\mathbf{v} + \int_s (\sigma_{xx}^s \,\delta \varepsilon_{xx}) \mathrm{d}s \ (9a)$$

$$\Pi s_2 = \int_V (\sigma_{xx} \,\delta \varepsilon_{xx} + \tau_{xxz} \,\delta \eta_{xxz}) \mathrm{d}v + \int_s (\sigma_{xx}^s \,\delta \varepsilon_{xx}) \mathrm{d}s \tag{9b}$$

where Π_S and Π_W are strain energy and external forces work, respectively and Π_K is kinetic energy. The strain energy can be written as

$$\Pi s_{1} = \int_{0}^{l} [(N_{SS} + N_{XX}^{S}) \frac{\partial \delta u}{\delta x} - (M_{XX} + M_{XX}^{S}) \frac{\partial^{2} \delta w}{\partial x^{2}} + p_{xxz} \frac{\partial^{2} \delta w}{\partial x^{2}}] dx$$
(10a)

$$\Pi s_{2} = \int_{0}^{l} [(N_{SS} + N_{XX}^{S}) \frac{\partial \delta u}{\delta x} - (M_{XX} + M_{XX}^{S}) \frac{\partial^{2} \delta w}{\partial x^{2}} + p_{xxz} \frac{\partial^{2} \delta w}{\partial x^{2}}] dx$$
(10b)

in which the variables introduced in arriving at the last expression are defined as follows

$$(N_{xx}, M_{xx}) = \int_{A} (1, z) \sigma_{xx} \, dA,$$
 (11a)

$$P_{xxz} = \int_{A} \tau_{xxz} \, dA. \tag{11b}$$

The work done by applied forces can be written in the form

$$\delta\Pi_{w1} = \int_{0}^{l} (-N^{0} \frac{\partial w}{\partial x} \frac{\partial \delta w}{\delta x} - k_{w} \delta w$$

$$+k_{p} \frac{\partial w}{\partial x} \frac{\partial \delta w}{\partial x} dx + f_{13} (\delta w_{1} - \delta w_{2})$$

$$\delta\Pi_{w2} = \int_{0}^{l} (-N^{0} \frac{\partial w}{\partial x} \frac{\partial \delta w}{\delta x} - k_{w} \delta w$$

$$+k_{p} \frac{\partial w}{\partial x} \frac{\partial \delta w}{\partial x} dx + f_{13} (\delta w_{2} - \delta w_{1})$$
(12a)
(12b)

where N^0 is axial load and k_w , k_p are elastic foundation parameters. The first variational of the virtual kinetic energy of present beam model can be written in the form as

$$\delta \Pi_{k1} = \int_0^l I_0(\frac{\partial u}{\partial t}\frac{\partial \delta u}{\partial t} + \frac{\partial w}{\partial t}\frac{\partial \delta w}{\delta t})$$
(13a)

$$-I_{1}\left(\frac{\partial u}{\partial t}\frac{\partial \delta w}{\partial x \partial t} + \frac{\partial w}{\partial x \partial t}\frac{\partial \delta u}{\delta x}\right) + I_{2}\left(\frac{\partial w}{\partial x \partial t}\frac{\partial \delta w}{\partial x \partial t} + \frac{\partial w}{\partial y \partial t}\frac{\partial \delta w}{\partial y \partial t}\right)]dx$$
(13a)

$$\delta\Pi_{k2} = \int_{0}^{l} I_{0} \left(\frac{\partial u}{\partial t} \frac{\partial \delta u}{\partial t} + \frac{\partial w}{\partial t} \frac{\partial \delta w}{\delta t} \right) - I_{1} \left(\frac{\partial u}{\partial t} \frac{\partial \delta w}{\partial x \partial t} + \frac{\partial w}{\partial x \partial t} \frac{\partial \delta u}{\delta x} \right) + I_{2} \left(\frac{\partial w}{\partial x \partial t} \frac{\partial \delta w}{\partial x \partial t} + \frac{\partial w}{\partial y \partial t} \frac{\partial \delta w}{\partial y \partial t} \right) dx$$
(13b)

in which the mass inertias are defined as

$$I_{0a}, I_{1a}, I_{2a} = b \int_{-h/2}^{0} (1, z, z^2) \rho dz$$
(14)

$$I_{0b}, I_{1b}, I_{2b} = b \int_0^{h/2} (1, z, z^2) \rho dz$$
(15)

The following equations are obtained by inserting Eqs. (10a), (10b) and (12a), (12b), (13a), (13b) in Eq. (8) when the coefficients of δu , δw are equal to zero

$$\frac{\partial (N_{xx} + N_{xx}^s)}{\partial x} = 0 \tag{16a}$$

$$\frac{\partial (N_{xx} + N_{xx}^s)}{\partial x} = 0 \tag{16b}$$

$$\frac{\partial}{\partial x^2} (M_{xx} + M_{xx}^s) + \frac{\partial^2 P_{xxz}}{\partial x^2}$$

$$+ (2b\sigma_0 - bN^0) \nabla^2 w - bk_w w + bk_p \nabla^2 w = 0$$
(16c)

$$\frac{\partial}{\partial x^2} \left(M_{xx} + M_{xx}^s \right) + \frac{\partial^2 P_{xxz}}{\partial x^2} + (2b\sigma_0 - bN^0) \nabla^2 w \quad (16d) - bk_w w + bk_p \nabla^2 w = 0$$

Where N_{xx} is the critical load of buckling.

$$u = 0, or (N_{xx} + N^s_{xx})n_x = 0$$
 (17)

$$w = 0$$
, or $n_x \left(\frac{\partial (M_{xx} + M_{xx}^s)}{\partial x} + \frac{\partial P_{xxz}}{\partial x} - N^0 \frac{\partial w}{\partial x} + k_p \frac{\partial w}{\partial x}\right) = 0$ (18a)

$$\frac{\partial w}{\partial x} = 0, \text{ or } (M_{xx} + M^s_{xx})n_x = 0$$
 (18b)

For a piezoelectric sandwich nanobeam with the flexoelectric effect, the nonlocal constitutive relations for the bulk may be written as

$$\sigma_{xx} - (e_0 a)^2 \nabla^2 \sigma_{xx} = c_{11} \varepsilon_{xx} + e_{31} \frac{\partial \varphi}{\partial z} - \frac{f_{31}}{2} \frac{\partial^2 \varphi}{\partial z^2} \quad (19a)$$

$$\tau_{xxz} - (e_0 a)^2 \nabla^2 \tau_{xxz} = + \frac{f_{31}}{2} \frac{\partial \varphi}{\partial z}$$
(19b)

$$D_{z} - (e_{0}a)^{2}\nabla^{2}D_{z} = e_{31}\varepsilon_{xx} - k_{33}\frac{\partial\varphi}{\partial z} + \frac{f_{31}}{2}\eta_{xxz} \quad (19c)$$

$$Q_{zz} - (e_0 a)^2 \nabla^2 Q_{zz} = -\frac{f_{31}}{2} \varepsilon_{xx}$$
 (19d)

where φ is the electrostatic potential and $E_z = -\frac{\partial \varphi}{\partial z}$. Also, the nonlocal constitutive relations for the surface layer can be expressed by

$$\sigma_{xx}^{s} - (e_0 a)^2 \nabla^2 \sigma_{xx}^{s} = \sigma_{xx}^{0} + c_{11}^{s} \varepsilon_{xx} + e_{31}^{s} \frac{\partial \varphi}{\partial z}$$
(20)

Under the open circuit condition, the electric displacement on the surface is zero. Therefore, one can obtain the electric field an electric field gradient as

$$E_{z} = -(\frac{e_{31}}{k_{33}}\frac{\partial u}{\partial x}) + (z\frac{e_{31}}{k_{33}} + \frac{f_{31}}{k_{33}})(\frac{\partial^{2}w}{\partial x^{2}})$$
(21)

Finally, the electric field gradient can be written as

$$E_{z,z} = \frac{e_{31}}{k_{33}} \frac{\partial^2 w}{\partial x^2}$$
(22)

Using Eqs. (20) and (21) the nonlocal constitutive relations for the bulk and surface can be expressed by the following for

$$\sigma_{xx1} - (e_0 a) \nabla^2 \sigma_{xx1} = \left(c_{11} + \frac{e_{31}^2}{k_{33}} \right) \frac{\partial u}{\partial x} - \left(c_{11} + \frac{e_{31}^2}{k_{33}} \right) z \frac{\partial w^2}{\partial x^2} - \left(\frac{e_{31} f_{31}}{2k_{33}} \right) \frac{\partial^2 w}{\partial x^2}$$
(23)

$$\sigma_{xx2} - (e_0 a) \nabla^2 \sigma_{xx2} = \left(c_{11} + \frac{e_{31}^2}{k_{33}} \right) \frac{\partial u}{\partial x} - \left(c_{11} + \frac{e_{31}^2}{k_{33}} \right) z \frac{\partial w^2}{\partial x^2} - \left(\frac{e_{31} f_{31}}{2k_{33}} \right) \frac{\partial^2 w}{\partial x^2}$$
(24)

$$\tau_{xxz\,1} - (e_0 a) V^2 \tau_{xxz\,1} = \left(\frac{e_{31\,f_{31}}}{2k_{33}}\right) \frac{\partial u}{\partial x} - \left(\frac{e_{31\,f_{31}}}{2k_{33}}\right) z \frac{\partial^2 w}{\partial x^2} - \frac{f_{31}}{2k_{33}} \frac{\partial^2 w}{\partial x^2}$$
(25)

$$\tau_{xxz\,2} - (e_0 a) \nabla^2 \tau_{xxz\,2} = \left(\frac{e_{31\,f_{31}}}{2k_{33}}\right) \frac{\partial u}{\partial x} - \left(\frac{e_{31\,f_{31}}}{2k_{33}}\right) z \frac{\partial^2 w}{\partial x^2} - \frac{f_{31}}{2k_{33}} \frac{\partial^2 w}{\partial x^2}$$
(26)

$$\sigma_{xx1}^{s} - (e_{0}a)\nabla^{2}\sigma_{xx1}^{s}$$

$$= \sigma_{xx}^{0} - \left(c_{11}^{s} + \frac{e_{31e_{31}}^{s}}{k_{33}}\right)\frac{\partial u}{\partial x}$$

$$- \left(c_{11}^{s} + \frac{e_{31e_{31}}^{s}}{k_{33}}\right)z\frac{\partial^{2}w}{\partial x^{2}} - \left(\frac{e_{31f_{31}}^{s}}{k_{33}}\frac{\partial^{2}w}{\partial x^{2}}\right)$$
(27)

$$\sigma_{xx2}^{s} - (e_{0}a)\nabla^{2}\sigma_{xx2}^{s}$$

$$= \sigma_{xx}^{0} - \left(c_{11}^{s} + \frac{e_{31e_{31}}^{s}}{k_{33}}\right)\frac{\partial u}{\partial x}$$

$$- \left(c_{11}^{s} + \frac{e_{31e_{31}}^{s}}{k_{33}}\right)z\frac{\partial^{2}w}{\partial x^{2}} - \left(\frac{e_{31f_{31}}^{s}}{k_{33}}\frac{\partial^{2}w}{\partial x^{2}}\right)$$
(28)

Therefore, by integrating Eqs. (23)-(28) over the beam's cross-section area, the force and moment stress resultants can be rewritten in the following form

$$N_{XX1} - (e_0 a)^2 \nabla^2 N_{XX1} = A_{11} \frac{\partial u}{\partial x} - B_{11} \frac{\partial^2 y}{\partial x^2}$$
(29)

$$N_{XX2} - (e_0 a)^2 \nabla^2 N_{XX2} = A_{22} \frac{\partial u}{\partial x} - B_{22} \frac{\partial^2 y}{\partial x^2}$$
(30)

$$M_{XX1} - (e0_a)^2 \nabla^2 M_{XX1} = -C_{11} \frac{\partial^2 y}{\partial x^2}$$
(31)

$$P_{XXZ1} - (e_0)^2 \nabla^2 p_{XXZ1} = B_{22} \frac{\partial u}{\partial x} - D_{11} \frac{\partial^2 w}{\partial x^2}$$
(32)

$$P_{XXZ2} - (e_0)^2 \nabla^2 p_{xxZ2} = B_{22} \frac{\partial u}{\partial x} - D_{11} \frac{\partial^2 w}{\partial x^2}$$
(33)

and the cross sectional rigidities are defined as

$$A_{11} = \left(c_{11} + \frac{e_{31}^2}{k_{33}}\right)\frac{bh}{2}, \qquad B_{11} = \left(\frac{e_{31}f_{31}}{2k_{33}}\right)\frac{bh}{2} \qquad (34)$$

$$c_{11} = \left(c_{11} + \frac{e_{31}^2}{k_{33}}\right) \frac{bh}{24}, \qquad D_{11} = \left(\frac{f_{31}^2}{2k_{33}}\right) \frac{bh}{2}$$
(35)

$$A_{22} = \left(c_{11} + \frac{e_{31}^2}{k_{33}}\right)\frac{bh}{2}, \qquad B_{22} = \left(\frac{e_{31}f_{31}}{2k_{33}}\right)\frac{bh}{2} \qquad (36)$$

$$c_{22} = \left(c_{11} + \frac{e_{31}^2}{k_{33}}\right)\frac{bh}{24}, \qquad D_{22} = \left(\frac{f_{31}^2}{2k_{33}}\right)\frac{bh}{2} \qquad (37)$$

And the force and moment stress resultants due to surface piezoelectricity may be expressed as

$$N_{xx}^{s} - (e_0 a)^2 \nabla^2 N_{xx}^{s} = A_{11}^{s} \frac{\partial u}{\partial x} - B_{11}^{s} \frac{\partial^2 w}{\partial x^2}$$
(38)

$$M_{xx}^{s} - (e_0 a)^2 \nabla^2 M_{xx}^{s} = F_{11}^{s} \frac{\partial u}{\partial x} - C_{11}^{s} \frac{\partial^2 w}{\partial x^2}$$
(39)

In which

$$A_{11}^{S} = \left(C_{11}^{S} + \frac{e_{31}e_{31}^{S}}{k_{33}}\right)h,$$

$$B_{11}^{S} = \left(C_{11}^{S} + \frac{e_{31}e_{31}^{S}}{k_{33}}\right)\frac{bh^{2}}{4} + \frac{f_{31}e_{31}^{S}}{k_{33}}h$$
(40)

$$F_{11}^{s} = \left(C_{11}^{s} + \frac{e_{31}e_{31}^{s}}{k_{33}}\right)\frac{bh^{2}}{4},$$

$$C_{11}^{s} = \left(C_{11}^{s} + \frac{e_{31}e_{31}^{s}}{k_{33}}\right)\frac{h^{3}}{12} + \frac{f_{31}e_{31}^{s}}{k_{33}}\frac{bh^{2}}{4}$$
(41)

$$A_{22}^{S} = \left(C_{11}^{S} + \frac{e_{31}e_{31}^{S}}{k_{33}}\right)h,$$

$$S = \left(C_{11}^{S} + \frac{e_{31}e_{31}^{S}}{k_{33}}\right)bh^{2} + \frac{f_{31}e_{31}^{S}}{k_{33}}h$$
(42)

$$B_{22}^{S} = \left(C_{11}^{S} + \frac{e_{31}e_{31}}{k_{33}}\right)\frac{bh^{2}}{4} + \frac{f_{31}e_{31}}{k_{33}}h$$

$$F_{22}^{s} = \left(C_{11}^{s} + \frac{e_{31}e_{31}^{s}}{k_{33}}\right)\frac{bh^{2}}{4},$$

$$C_{22}^{s} = \left(C_{11}^{s} + \frac{e_{31}e_{31}^{s}}{k_{33}}\right)\frac{h^{3}}{12} + \frac{f_{31}e_{31}^{s}}{k_{33}}\frac{bh^{2}}{4}$$
(43)

The nonlocal governing equations of a piezoelectric sandwich nanobeam with surface and flexoelectric effects in terms of the displacement can be derived by substituting Eqs. (34)-(37), into Eqs. (14), (15) as follows

$$(A_{11} + A_{11}^{S})\frac{\partial^2 u}{\partial x^2} - (B_{11} + B_{11}^{S})\frac{\partial^3 w}{\partial x^3} = 0$$
(44)

$$(A_{22} + A_{22}^{S})\frac{\partial^{2} u}{\partial x^{2}} - (B_{22} + B_{22}^{S})\frac{\partial^{3} w}{\partial x^{3}} = 0$$
(45)

$$(B_{11} + F_{11}^{S})\frac{\partial^{3}u}{\partial x^{3}} - (C_{11} + C_{11}^{S} + D_{11})\frac{\partial^{4}W}{\partial x^{4}} + 2b\sigma_{0}(\frac{\partial^{2}w}{\partial x^{2}}) - (e_{0}a)^{2}2b\sigma_{0}\left(\frac{\partial^{2}}{\partial x^{2}}\right)\left(\frac{\partial^{2}w}{\partial x^{2}}\right) -bN^{0}\left(\frac{\partial^{2}w}{\partial x^{2}}\right) + (e_{0})^{2}bN^{0}\left(\frac{\partial^{2}}{\partial x}\right)\left(\frac{\partial^{2}w}{\partial x^{2}}\right) - bk_{w} + (e_{0}a)^{2}bk_{w}\left(\frac{\partial^{2}w}{\partial x^{2}}\right) + k_{p}\left(\frac{\partial^{2}w}{\partial x^{2}}\right) - (e_{0}a)^{2}bk_{p}\left(\frac{\partial^{2}}{\partial x^{2}}\right)\left(\frac{\partial^{2}w}{\partial x^{2}}\right) = 0$$

$$a^{3}w \qquad a^{4}W$$

$$(B_{22} + F_{22}^{S})\frac{\partial^{2}u}{\partial x^{3}} - (C_{22} + C_{22}^{S} + D_{22})\frac{\partial^{2}w}{\partial x^{4}} + 2b\sigma_{0}(\frac{\partial^{2}w}{\partial x^{2}}) - (e_{0}a)^{2}2b\sigma_{0}\left(\frac{\partial^{2}}{\partial x^{2}}\right)\left(\frac{\partial^{2}w}{\partial x^{2}}\right) - bN^{0}\left(\frac{\partial^{2}w}{\partial x^{2}}\right) + (e_{0})^{2}bN^{0}\left(\frac{\partial^{2}}{\partial x}\right)\left(\frac{\partial^{2}w}{\partial x^{2}}\right) - bk_{w}$$
(47)
$$+ (e_{0}a)^{2}bk_{w}\left(\frac{\partial^{2}w}{\partial x^{2}}\right) + k_{p}\left(\frac{\partial^{2}w}{\partial x^{2}}\right) - (e_{0}a)^{2}bk_{p}\left(\frac{\partial^{2}}{\partial x^{2}}\right)\left(\frac{\partial^{2}w}{\partial x^{2}}\right) = 0$$

2.3 Solution procedure

Here, an analytical solution of the governing equations for vibration of a flexoelectric nanobeam having simply-supported (S) and clamped (C) boundary conditions is

presented which they are given as:

• Simply-supported (S):

$$w = N_{xx} = M_{xx} = 0$$
 $x = 0, L$ (48)

• Clamped (C):

$$u = w = 0 \qquad x = 0, L \tag{49}$$

To satisfy above-mentioned boundary conditions, the displacement quantities are presented in the following form

$$u = \sum_{n=1}^{\infty} U_m \frac{\partial X_m(x)}{\partial x} e^{i\omega_n t}$$
(50)

$$w = \sum_{m=1}^{\infty} W_m X_m(x) e^{i\omega_n t}$$
(51)

where (U_m, W_m) are the unknown coefficients. Inserting Eqs. (50)-(51) into Eqs. (44)-(47) respectively, leads to

$$\begin{cases} k_{1,1} & k_{1,2} & k_{1,3} \\ k_{2,1} & k_{2,2} & k_{2,3} \\ k_{3,1} & k_{3,2} & k_{3,3} \end{cases} - \overline{\varpi}^2_n \begin{bmatrix} m_{1,1} & m_{1,2} & m_{1,3} \\ m_{2,1} & m_{2,2} & m_{2,3} \\ m_{3,1} & m_{3,2} & m_{3,3} \end{bmatrix} \} = 0 (52)$$

$$K_{1,1} = (A_{11} + A_{11}^S)K_{12},$$

$$K_{1,2} = (B_{11} + F_{11}^S)K_{13},$$

$$k_{1,3} = 0$$

$$K_{2,1} = -(B_{11} + B_{11}^S)K_{12},$$

$$K_{2,2} = -(C_{11} + C_{11}^S + D_{11})K_{13} - b(N^0 - 2\sigma_0)(K_{11})$$

$$+ (e_0a)^2b(N^0 - 2\sigma_0)(K_{13}) - bk_wk_1$$

$$+ (e_0a)^2bk_w(k_{11}) + bk_p(k_{11})$$

$$- (e_0a)^2bk_p(k_{13}) - (C_{22} + C_{22}^S + D_{22})K_{31}$$

$$- b(N^0 - 2\sigma_0)(K_{22})$$

$$+ (e_0a)^2b(N^0 - 2\sigma_0)(K_{31})$$

$$- bk_wk_2 + (e_0a)^2bk_w(k_{22}) + bk_p(k_{22})$$

$$- (e_0a)^2bk_p(k_{31}) = 0$$

$$K_{2,3} = (B_{22} + F_{22}^S)K_{31},$$

$$K_{3,3} = (A_{11} + A_{11}^S)K_{21}$$

$$m_{1,1} = +I_{0a}k_6 - (e_0a)^2I_{0a}K_{12},$$

$$m_{1,2} = 0,$$

$$m_{1,3} = 0,$$

$$m_{2,2} = I_{0a}k_1 + I_{0b}k_2 - I_{2a}k_{11} - I_{2b}k_{22}$$

$$- (e_0a)^2I_{0a}k_{11} - (e_0a)^2I_{0b}k_{22}$$

$$+ (e_0a)^2I_{2a}k_{13} + (e_0a)^2I_{2b}(k_{13}) = 0$$

$$m_{2,3} = 0,$$

$$m_{3,3} = I_{0b}k_6 - (e_0a)^2I_{0b}K_{12}$$

$$k_{1} = \int_{0}^{l} (X_{m}X_{m}) dx$$

$$k_{2} = \int_{0}^{l} (X_{m}X_{m}) dx$$

$$k_{6} = \int_{0}^{l} (X_{m}^{''}X_{m}) dx$$

$$k_{11} = \int_{0}^{l} (X_{m}^{'''}X_{m}) dx$$

$$k_{22} = \int_{0}^{l} (X_{m}^{'''}X_{m}) dx$$

$$k_{12} = \int_{0}^{l} (X_{m}^{''''}X_{m}) dx$$

$$k_{21} = \int_{0}^{l} (X_{m}^{''''}X_{m}) dx$$

$$k_{13} = \int_{0}^{l} (X_{m}^{''''}X_{m}) dx$$

$$k_{31} = \int_{0}^{l} (X_{m}^{'''''}X_{m}) dx$$

By finding determinant of the coefficients of above matrix and setting it to zero, we can find natural frequencies. The function X_m for different boundary conditions is defined by

S-S:

$$X_{m}(x) = \sin(\lambda_{m}x)$$
(55)

$$\lambda_{m} = \frac{m\pi}{L}$$
(55)

$$X_{m}(x) = \sin(\lambda_{m}x) - \sinh(\lambda_{m}x) -\xi_{m}(\cos(\lambda_{m}x) - \cosh(\lambda_{m}x))$$
(56)

$$\xi_{m} = \frac{\sin(\lambda_{m}x) - \sinh(\lambda_{m}x)}{\cos(\lambda_{m}x) - \cosh(\lambda_{m}x)}$$
(56)

$$\lambda_{1} = 3.961, \quad \lambda_{2} = 6.867,$$
(56)

$$\lambda_{3} = 9.991, \quad \lambda_{4} = 14.118,$$
(56)

$$\lambda_{m \ge 5} = \frac{(m + 0.5)\pi}{L}$$

3. Numerical results and discussions

Comparison is performed with those of a piezoelectric nanobeam presented by Yan and Jiang (2011) In Fig. 2. The frequency ratio (ω/ω^0) is presented as a function of sandwich nanobeam thickness. Also, ω^0 is the natural frequency of piezoelectric sandwich nanobeam without surface effect. The results are in an excellent agreement with those of Yan and Jiang (2011) for a simply-supported nanobeam. Also, for better presentation of the results the following dimensionless quantity is adopted

$$\overline{\omega} = \omega \frac{L^2}{h} \sqrt{\frac{\rho}{c_{11}}}, \quad K_w = k_w \frac{L^4}{D}, \quad K_p = k_p \frac{L^2}{D}, \quad D = \frac{1}{12} c_{11} h^3, \quad \mu = \frac{(e_0 a)}{L}$$
(57)

which is expressed for three cases of deflection such as

$$K_{w} = K_{w} \frac{L^{2}}{D}, \qquad K_{P} = K_{P} \frac{L^{2}}{D},$$

$$D = \frac{1}{12} C_{11} h^{3}, \qquad \mu = \frac{e_{0} a}{L}$$
(58)

Fig. 3 determines the surface and flexoelectric effects on the variation of natural frequencies of piezoelectric sandwich nanobeams with respect to thickness for S-S boundary conditions at $\mu = 0.1$. In this figure, NL refers to nonlocal piezoelectric sandwich nanobeam without surface and flexoelectric effects. NL-Flexoelectric refers to a nonlocal flexoelectric nanobeam without surface effect. Also, NL-SE denotes a nonlocal piezoelectric sandwich nanobeam without flexoelectric effect. It is observable from this figure that neglecting the surface effect leads to lower natural frequencies. In fact, inclusion of surface effect enhances the stiffness of flexoelectric nanobeams and natural frequencies increase. It is found that flexoelectricity effect leads to higher natural frequencies, especially at smaller values of nanobeam thickness. Therefore, the maximum natural frequencies are observed for NL-SE-Flexoelectric nanobeam, while nonlocal (NL) piezoelectric nanobeam has the minimum buckling load. For the nonlocal (NL) piezoelectric nanobeams, natural frequencies are not dependent on the value of nanobeam thickness. But, when the flexoelectric effect is involved, natural frequencies reduce as the value of thickness rises. So, flexoelectricity has an important size effect on vibration behavior of piezoelectric nanobeams. It can be concluded that surface and flexoelectric effects are important at lower thicknesses. In other words, effects surface elasticity and flexoelectricity are negligible at large thicknesses.

Examination of flexoelectric and nonlocal effects on vibration behavior of flexoelectric sandwich nanobeams under S-S boundary condition when L/h = 10 is presented in Fig. 4. It is observable from this figure that neglecting the flexoelectric effect leads to lower natural frequencies at a fixed nonlocal parameter. It is also found that the nonlocal flexoelectric nanobeam has lower natural frequencies compared with local flexoelectric sandwich nanobeam ($\mu = 0 \text{ nm}^2$), regardless of the type of boundary conditions. So,



Fig. 2 Comparison of frequency ratio of S-S piezoelectric nanobeams (L/h = 20)



Fig. 3 Surface and flexoelectricity effects on vibration frequency of nonlocal S-S piezoelectric sandwich nanobeams with respect to h/l ($\mu = 0.1$, $K_w = K_p = 0$)

Table 2 Surface and flexoelectricity effects on vibration
frequency of nonlocal S-S piezoelectric sandwich
nanobeams with respect to h/l
$(\mu = 0.1, K_w = K_p = 0)$

	NL without flexoelectricity	NL-SE without flexoelectricity	NL-SE with flexoelectricity
h/L	Di	псу	
0.05	3.312	6.778	7.304
0.1	3.312	4.215	4.463
0.15	3.312	3.283	3.481
0.2	3.312	3.156	3.218
0.25	3.312	3.132	3.156

inclusion of nonlocal stress field parameter reduces the natural frequencies of a flexoelectric sandwich nanobeam. Such observation is neglected in all previous analyzes on flexoelectric sandwich nanobeams. So, by ignoring the effect of nonlocality in analysis of flexoelectric nanobeams, the obtained results are overestimated. Hence, it can be concluded that the vibration behavior of flexoelectric sandwich nanobeams is sensitive to the nonlocal parameter. The maximum and minimum natural frequencies of flexoelectric nanobeam are obtained for S-S boundary condition. In fact, stronger supports at ends make the flexoelectric nanobeam stiffer and natural frequencies rise.

Influences of Winkler (K_w) and Pasternak (K_p) foundation parameters on natural frequencies of flexoelectric nanobeam with surface effect for different nonlocal parameters at L/h = 10 is presented in Figs. 5 and 6, respectively. It is found that presence of elastic medium has a significant effect on the vibration behavior flexoelectric nanobeams. In fact, elastic medium makes the flexoelectric nanobeam more rigid and natural frequencies increase at a constant nonlocal parameter. Moreover, the frequency results of embedded flexoelectric nanobeam depend on the value of nonlocal parameter. It is observed that increasing the value of nonlocal parameter leads to reduction in dimensionless natural frequencies of flexoelectric nanobeam at every magnitude of Winkler and



Fig. 4 Nonlocal and flexoelectricity effects on vibration frequency of nonlocal piezoelectric sandwich nanobeams for S-S and C-C boundary conditions $(L/h = 10, K_w = K_p = 0)$

Table 3 Nonlocal and flexoelectricity effects on vibration frequency of nonlocal piezoelectric sandwich nanobeams for various boundary conditions $(L/h = 10, K_w = K_p = 0)$

	-				
	S-S with flexoelectricity	S-S without flexoelectricity			
μ	Dimensionless frequency				
0.2	6.123	5.787			
0.4	5.913	5.589			
0.6	5.033	4.604			
0.8	4.283	3.795			
1	3.642	3.147			

Pasternak foundation parameters. This is due to stiffness reduction of flexoelectric sandwich nanobeam by considering the nonlocal stress field parameter.

Another investigation on the effect of elastic medium, surface elasticity and flexoelectricity on natural frequencies of flexoelectric sandwich nanobeams is presented in Figs. 7 and 8 at L/h = 10, $\mu = 0.1$. It is found that existence of elastic medium leads to larger natural frequencies. In fact, natural frequencies of piezoelectric sandwich nanobeam



Fig. 5 Nonlocal and Winkler foundation effects on vibration frequency of piezoelectric nanobeams $(L/h = 10, K_p = 0)$



Fig. 6 Nonlocal and Pasternak foundation effects on vibration frequency of piezoelectric nanobeams $(L/h = 10, K_w = 0)$

Table 4 Nonlocal and Winkler foundation effects on vibration frequency of piezoelectric nanobeams $(L/h = 10, K_p = 0)$

`	; P	,		
	<i>Kw</i> = 25	Kw = 50	<i>Kw</i> = 25	Kw = 50
	S-S	S-S	C-C	C-C
μ		Dimensionle	ss frequency	
0.2	6.72	7.34	12.67	12.83
0.4	5.72	6.65	11.53	11.72
0.6	5.17	5.67	9.32	9.12
0.8	5.01	5.38	9.15	8.24
1	4.78	5.01	8.78	7.95

Table 5 Nonlocal and Pasternak foundation effects on vibration frequency of piezoelectric nanobeams $(L/h = 10, K_p = 0)$

```	, p	- /		
	<i>Kw</i> = 25	Kw = 50	<i>Kw</i> = 25	Kw = 50
	S-S	S-S	C-C	C-C
μ		Dimensionle	ss frequency	
0.2	9.763	12.135	12.67	12.83
0.4	8.083	9.631	11.53	11.72
0.6	6.562	8.934	9.32	9.12
0.8	5.539	6.505	9.15	8.24
1	4.268	4.09	8.78	7.95

increase linearly with the rise of Winkler or Pasternak parameters. Also, it is found that Pasternak layer has more significant impact on natural frequencies of flexoelectric sandwich nanobeam than Winkler layer. In fact, Pasternak layer has a continuous interaction with the flexoelectric sandwich nanobeams. However, Winkler layer is modeled via infinite parallel springs and has a discontinuous interaction with the flexoelectric sandwich nanobeam. But, these observations are dependent on the surface and flexoelectric effects. Considering both surface and flexoelectric effects leads the largest natural frequencies at a constant elastic foundation parameters. Neglecting flexoelectric or surface effects leads to lower natural frequencies at fixed elastic foundation parameters.

Table 6 Surface and flexoelectric effects on vibration frequency of piezoelectric nanobeams with respect to Pasternak parameter ( $L/h = 10, \mu = 0.1$ )

	S-S without flexo- electricity	S-S with flexoelectricity, surface	C-C without flexo- electricity	C-C with flexoelectricity, surface
$K_w$	$K_w$ Dimensionless frequency (HZ)			
0	3.385	6.017	8.879	9.659
50	3.839	6.385	9.368	10.397
100	4.279	6.693	9.839	10.968
150	4.901	6.937	10.375	11.739
200	5.238	7.179	11.493	12.321



Fig. 7 Surface and flexoelectric effects on vibration frequency of piezoelectric nanobeams with respect to Winkler parameter ( $L/h = 10, \mu = 0.1$ )

## 4. Conclusions

This paper develops vibration characteristics of flexoelectric nanobeams are investigated based on nonlocal elasticity theory considering surface effects. Nonlocal flexoelectric sandwich nanobeam incorporating surface effect for vibration analysis of piezoelectric sandwich nanobeams supported by Winkler-Pasternak foundation. The model includes a nonlocal stress field parameter and a flexoelectric coefficient to capture the size effect. The governing differential equations and natural boundary edges were derived by exploiting the use of the Hamilton's principle. The solution of these equations is provided employing a Galerkin-based approach which has the potential to capture various boundary conditions. The evaluation of influences of nonlocality, flexoelectricity, surface, elastic foundation, boundary conditions and plate thickness is conducted for vibration behavior of flexoelectric nanobeams. Numerical results show that:

• Nonlocal flexoelectric sandwich nanobeams possess lower frequency results than local one attributed to the softening influence of nonlocal parameter on the beam rigidity.

Table 7 Surface and flexoelectric effects on vibration frequency of piezoelectric nanobeams with respect to Pasternak parameter ( $L/h = 10, \mu = 0.1$ )

	S-S without flexo-	S-S with flexoelectricity,	C-C without flexo-	C-C with flexoelectricity,
17	electricity	Surface		surface
Kp		Dimensionless	frequency (H	<i>L</i> )
0	5.938	6.469	11.089	12.112
50	6.398	6.907	11.369	12.759
100	6.831	7.340	11.840	13.080
150	7.294	8.03	12.094	13.709
200	7.930	8.909	12.348	14.006



Fig. 8 Surface and flexoelectric effects on vibration frequency of piezoelectric nanobeams with respect to Pasternak parameter ( $L/h = 10, \mu = 0.1$ )

- Flexoelectricity shows an increasing influence on the natural frequencies, especially at lower thicknesses.
- Effect of flexoelectricity depends on the nonlocality.
- The natural frequencies of flexoelectric nanobeams depend on the number of restraints at edges.
- That effect of Pasternak parameter on natural frequencies is more announced compared with Winkler parameter for every value of nonlocal parameter.

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