Buckling analysis of nanoplate-type temperature-dependent heterogeneous materials

Behrouz Karami^{*1} and Sara Karami²

¹ Department of Mechanical Engineering, Marvdasht Branch, Islamic Azad University, Marvdasht, Iran
² Department of Geology, Shiraz Branch, Islamic Azad University, Shiraz, Iran

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Abstract. This paper develops a four-unknown refined plate theory and the Galerkin method to investigate the size-dependent stability behavior of functionally graded material (FGM) under the thermal environment and the FGM having temperature-dependent material properties. In the current study two scale coefficients are considered to examine buckling behavior much accurately. Reuss micromechanical scheme is utilized to estimate the material properties of inhomogeneous nano-size plates. Governing differential equations, classical and non-classical boundary conditions are obtained by utilizing Hamiltonian principles. The results showed the high importance of considering temperature-dependent material properties for buckling analysis. Different influencing parametric on the buckling is studied which may help in design guidelines of such complex structures.

Keywords: temperature-dependent heterogeneous materials; refined plate theory; nonlocal strain gradient theory; thermal environment; Galerkin method

1. Introduction

In the current century, the use of materials that have the ability to work in environments with a high temperature is increasingly felt. These environments can be the spacecraft, aircraft and plasma coatings for fusion reactors etc. In order to overcome this need, composite materials which material properties graded in different directions such as thickness were presented (Shahsavari *et al.* 2018e, Kar and Panda 2015a, b, Chamkha *et al.* 2018, Shahsavari *et al.* 2018c, Nayebi *et al.* 2015, Damadam *et al.* 2018, Bensaid and Kerboua 2017).

These days due to the wonderful applications of materials at Nano-scale, these materials have been widely used in different nano-electro-mechanical systems (NEMSs). So the predictions of these structures is felt more and more. As you know classical continuum theories cannot predictions the size-dependent behavior of nanostructure systems. Therefore, to overcome this problem different sizedependent theories were presented (i.e., nonlocal theory (Karami et al. 2018a, f, Shahsavari et al. 2018a, 2017, Apuzzo et al. 2017, Barretta et al. 2018b, Romano and Barretta 2017, Bensaid and Guenanou 2017), modified couple stress theory (Ghayesh et al. 2013a, b, 2014, Farokhi et al. 2013, Farokhi and Ghayesh 2015, Ghayesh and Farokhi 2015), strain gradient theory (Karami and Janghorban 2016, Nami and Janghorban 2014b), and recently combinations of nonlocal and strain gradient theory so-called nonlocal strain gradient theory (Karami et al. 2017, 2018d, 2019a). Eringen nonlocal model considers size-dependent behavior of nanostructure systems using nonlocality effects. In the past decades, Eringen nonlocal theory has been widely used to model the different nanostructures (Tounsi et al. 2013, Zenkour 2016, Aydogdu and Arda 2016, Heydari and Shariati 2018, Eltaher et al. 2016, Bensaid 2017, Barzoki et al. 2015, Taghizadeh et al. 2015, Aissani et al. 2015, Tufekci et al. 2016, Karami et al. 2019b, 2018l, Shahsavari and Janghorban 2017, Nami et al. 2015), but due to the fact that the mentioned model only considers nonlocality mechanism of size-dependency, this theory cannot properly address the size-dependent behavior of nanostructures. Hence, Askes and Aifantis (2009) presented a complete model in which size-dependent behavior of nanotubes systems capture by two different approaches of size-dependency namely strain gradient sizedependency and nonlocality considering two small scale parameters. After that Lim et al. (2015) developed a more comprehensive model of this theory in which the authors showed that this theory is more accurate than the Eringen model considering both stiffness softening and hardening mechanisms. This claim proved using experimental tests by matching the results for the wave frequency of nano-size beams. Recently on the basis of nonlocal strain gradient model different analyzes have been done by various investigators (Houari et al. 2018, Arani et al. 2017, Sahmani et al. 2018, Ghayesh and Farajpour 2018, Malikan et al. 2018, El-Borgi et al. 2018, She et al. 2018b, d, Shafiei and She 2018, Karami et al. 2018b, c, e, h, i, j, Shahsavari et al. 2018b, Barretta and de Sciarra 2018, Barretta et al. 2018a, Bensaid et al. 2018a). Nami and Janghorban (2014a) studied the forced vibrations of micro/nano-plates via nonlocal strain gradient theory where the small scale effects

^{*}Corresponding author, Ph.D. Student, E-mail: behrouz.karami@miau.ac.ir

were considered separately. Using nonlocal strain gradient theory wave characteristics of nano-size beams made of FGMs was presented by (Li et al. 2015). Forced vibration analysis of nanoplates with uniform and graded porosities was studied by (Barati 2017) based on nonlocal strain gradient theory. (Karami et al. 2018k) investigated the wave propagation of temperature-dependent FG nanoplates based on nonlocal strain gradient theory. Effect of thickness on the mechanics of nanobeams was presented by (Li et al. 2018) using nonlocal strain gradient theory. Wave propagation, vibration and bending behavior of porous nanotubes were investigated by She et al. (2018a, c, 2019) via nonlocal strain gradient theory. Shahsavari et al. (2018d) presented shear buckling analysis of single layer graphene sheets based on the different nonlocal strain gradient theories for the first time.

Among the different plate theories (i.e., classical plate theory (CPT), first-order shear deformation plate theory (FSDT), higher-order shear deformation refined plate theory (HSDT), etc. (Bensaid et al. 2017, 2018b), the authors in the current investigation decided to use a refined model of plate theories. This is due to this fact that the refined model does not require to use of any shear correction coefficients. Therefore, this theory has been widely considered to analyze the behavior of advanced composite material plates (Kant and Swaminathan 2001, Wu et al. 2008). Transient analysis of carbon nanotubes reinforced nanocomposite plates were presented by (Phung-Van et al. 2018) using nonlocal strain gradient theory considering thermal effects. Wave characteristics of nanosize plates made of FGMs were reported by (Karami et al. 2018g) via a nonlocal strain gradient refined model. In addition, (Ebrahimi et al. 2016) studied the wave dispersion of a nonlocal strain gradient FGM nnao-size plates.

It is clear that there are many studies have been reported to study the stability, dynamic and static response of nanoplates and nanobeams made of FGMs, but with the best knowledge of authors, it is the first time that thermally affected buckling of a nanoplates made of FGMs is studied in which the material properties are temperature- dependent.

In the current work, a nonlocal strain gradient refined plate model is presented to examine the size-dependent buckling response of nanoplates made of FGMs in the thermal environment. Reuss micromechanical model is utilized to estimate the temperature-dependent material properties where the properties are varying via the thickness direction. After owning governing equations and boundary



Fig. 1 Geometry of rectangular FGM plate with uniform thickness in the rectangular Cartesian coordinates

a

conditions via Hamiltonian principles, a semi-numerical solution is applied to find the critical buckling force of the nanoplate. Furthermore, the influence of material variation, temperature changes, and scale parameters on the stability response of such structures is presented afterward in a parametric investigation.

2. Theoretical formulation

2.1 Temperature-dependent material properties of FG elastic nanoplates

An FG nano-size plate made of a composition of ceramics and metals is assumed (see Fig. 1). The basic assumption for material properties is that, the properties for FGMs are varying via thickness direction. The produces estimation of the Young modulus and Poisson ratio are respectively as

$$E(z) = \frac{E_c E_m}{E_c (1 - V_f(z)) + E_m (1 - V_f(z))}$$
(1)

$$v(z) = \frac{V_c V_m}{V_c (1 - V_f (z)) + V_m (1 - V_f (z))}$$
(2)

$$\alpha(z) = \frac{\alpha_c \alpha_m}{\alpha_c (1 - V_f(z)) + \alpha_m (1 - V_f(z))}$$
(3)

where

$$V_{f}(z) = V_{m} + (V_{c} - V_{m})(\frac{2z+h}{2h})^{n}$$
(4)

in which *n* is the power law material index parameter and the subscripts *m* and *c* are the metallic and ceramic phases, respectively. It is clear that when n=0 the plate is fully ceramic, while $n = \rightarrow \infty$ means that the plate is fully metal. The variation of Young modulus with respect to different power-law material parameter index is illustrated in Fig. 2.

2.2 Nonlocal strain gradient elasticity

The classical continuum model cannot predict the sizedependent behavior of nanostructure systems. In this way (Askes and Aifantis 2009) presented a model incorporating



Fig. 2 The variation of Young modulus as a function of power-law index parameter

the two different approaches of size-dependency to investigate the nanostructure systems. This model includes two small scale parameters as

$$\sigma_{ij} = \sigma_{ij}^{(0)} - \nabla \partial \sigma_{ij}^{(1)} \tag{5}$$

where $\sigma_{ij}^{(0)}$ and $\sigma_{ij}^{(1)}$ are respectively related to strain ε_{ij} and strain gradient $\nabla \varepsilon_{ii}$, and are given as

$$\sigma_{ij}^{(0)} = \int_{0}^{L} C_{ijkl} \alpha_0(x, x', e_0 a) \varepsilon_{kl}'(x') dx'$$
(6)

$$\sigma_{ij}^{(1)} = l^2 \int_{0}^{L} C_{ijkl} \alpha_1(x, x', e_1 a) \nabla \varepsilon'_{kl}(x') dx'$$
(7)

herein C_{ijkl} , *ea* and *l* are the elastic constants, the effect of nonlocal stress field and the strain gradient length scale parameter (Karami and Janghorban 2016).

The general constitutive relation for nonlocal strain gradient elasticity can be expressed as

$$(1 - \mu \nabla^2) \sigma_{ij} = C_{ijkl} \left[(1 - \lambda \nabla^2) \varepsilon_{kl} \right]$$
(8)

in which $\lambda = l^2$ and $\mu = (ea)^2$. The equivalent form of Eq. (7) is presented as

$$\mathcal{L}_{\mu}\sigma_{ij} = C_{ijkl}\mathcal{L}_{l}\varepsilon_{kl} \tag{9}$$

where the linear operators are defined as

$$L_{\mu} = (1 - \mu \nabla^2), L_{l} = (1 - \lambda \nabla^2)$$
 (10)

It is obvious that by ignoring the strain gradient parameter in Eq. (8), the equation is reduced to the Eringen nonlocal theory. Besides, the gradient equations of motion without considering nonlocal effects can be obtained by omitting nonlocal parameter in Eq. (8). Further, the classical theory which was used for macro plates will be achieved by neglecting small-scale parameters.

2.3 Refined higher order plate theory

For isotropic FGMs, the nonlocal strain gradient constitutive relations can be written as

$$L_{\mu} \begin{cases} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xz} \\ \tau_{yz} \\ \tau_{xy} \end{cases} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66} \end{bmatrix} L_{\eta} \begin{cases} \varepsilon_{x} \\ \varepsilon_{y} \\ \gamma_{xz} \\ \gamma_{yz} \\ \gamma_{xy} \end{cases}$$
(11)

herein (σ_x , σ_y , σ_z , τ_{yz} , τ_{xxy}) and (ε_x , ε_y , ε_z , γ_{yz} , γ_{xy}) are the stress and strain components, respectively. Elastic constants of FGM layer can be defined as

$$Q_{11} = Q_{22} = \frac{E(z)}{(1-v^2)}, Q_{12} = vQ_{11}$$
 (12)

$$Q_{44} = Q_{55} = Q_{66} = \frac{E(z)}{2(1+\nu)}$$
(12)

Based on refined plate model proposed by Shimpi and Patel (2006), the basic assumption for the displacement field of the plate can be described as

$$u(x, y, z, t) = u_0(x, y, t) - z \frac{\partial w_b}{\partial x} - f(z) \frac{\partial w_s}{\partial x}$$
(13)

$$v(x, y, z, t) = v_0(x, y, t) - z \frac{\partial w_b}{\partial y} - f(z) \frac{\partial w_s}{\partial y}$$
(14)

$$w(x, y, z, t) = w_{b}(x, y, t) + w_{s}(x, y, t)$$
(15)

where u_0 and v_0 are displacement of mid-plane along *x*, *y*-axis and w_b , w_s are the bending and shear components of transverse displacement of a point on the mid-plane of the plate and t is the time. The shape function of transverse shear deformation is considered as Shimpi and Patel (2006)

$$f(z) = -\frac{z}{4} + \frac{5z^3}{3h^2}$$
(16)

The nonzero strain-displacment relastions are given as

$$\begin{cases} \varepsilon_{x} \\ \varepsilon_{y} \\ \gamma_{xy} \end{cases} = \begin{cases} \varepsilon_{x}^{0} \\ \varepsilon_{y}^{0} \\ \gamma_{xy}^{0} \end{cases} + z \begin{cases} K_{x}^{b} \\ K_{y}^{b} \\ K_{xy}^{b} \end{cases} + f \begin{cases} K_{x}^{s} \\ K_{y}^{s} \\ K_{xy}^{s} \end{cases}, \qquad (17)$$
$$\begin{cases} \gamma_{yz} \\ \gamma_{xz}^{s} \end{cases} = g \begin{cases} \gamma_{yz}^{s} \\ \gamma_{xz}^{s} \end{cases}, \qquad g = 1 - \frac{\partial f}{\partial z}$$

in which

$$\begin{cases} \mathcal{E}_{x}^{0} \\ \mathcal{E}_{y}^{0} \\ \gamma_{xy}^{0} \end{cases} = \begin{cases} \frac{\partial u_{0}}{\partial x} \\ \frac{\partial v_{0}}{\partial y} \\ \frac{\partial u_{0}}{\partial y} + \frac{\partial v_{0}}{\partial x} \end{cases}, \begin{cases} k_{x}^{b} \\ k_{y}^{b} \\ k_{xy}^{b} \end{cases} = \begin{cases} -\frac{\partial^{2} w_{b}}{\partial x^{2}} \\ -\frac{\partial^{2} w_{b}}{\partial y^{2}} \\ -2\frac{\partial^{2} w_{b}}{\partial x \partial y} \end{cases}, \\ \begin{cases} k_{x}^{s} \\ k_{y}^{s} \\ k_{xy}^{s} \end{cases} = \begin{cases} -\frac{\partial^{2} w_{s}}{\partial x^{2}} \\ -\frac{\partial^{2} w_{s}}{\partial y^{2}} \\ -2\frac{\partial^{2} w_{s}}{\partial x \partial y} \end{cases}, \begin{cases} \gamma_{xz}^{s} \\ \gamma_{yz}^{s} \end{cases} = \begin{cases} \frac{\partial w_{s}}{\partial x} \\ \frac{\partial w_{s}}{\partial y} \\ \frac{\partial w_{s}}{\partial y} \end{cases} \end{cases}$$
(18)

2.4 Governing equations

The equations of motion for the buckling of the FG nanoplate can be derived from Hamilton's principle as follows

$$\int_0^t \delta(U+V)dt = 0 \tag{19}$$

where U and V are respectively strain energy and work done by external (applied) forces. The first variation of strain energy can be concluded as

$$\delta U = \int_{V} \left[\sigma_x \, \delta \varepsilon_x + \sigma_y \, \delta \varepsilon_y + \tau_{yz} \, \varepsilon_{yz} + \tau_{xz} \, \varepsilon_{xz} + \tau_{xy} \, \varepsilon_{xy} \right] dV$$

$$= \int_{0}^{L} \left[N_x \, \delta \varepsilon_x^0 + N_y \, \delta \varepsilon_y^0 + N_{xy} \, \delta \gamma_{xy}^0 + M_x^b \, \delta k_x^b + M_y^b \, \delta k_y^b \right]$$

$$+ M_{xy}^b \, \delta k_{xy}^b + M_x^s \, \delta k_x^s + M_y^s \, \delta k_y^s + M_{xy}^s \, \delta k_{xy}^s$$

$$+ Q_{yz}^s \, \gamma_{yz}^0 + Q_{xz}^s \, \gamma_{xz}^0 \right] dx = 0$$

(20)

The first variation of work done by applied forces can be stated as

$$\delta V = \int_{0}^{L} \left[N_{x}^{0} \frac{\partial (w_{b} + w_{s})}{\partial x} \frac{\partial \delta (w_{b} + w_{s})}{\partial x} + N_{y}^{0} \frac{\partial (w_{b} + w_{s})}{\partial y} \frac{\partial \delta (w_{b} + w_{s})}{\partial y} + 2\delta N_{xy}^{0} \frac{\partial (w_{b} + w_{s})}{\partial x} \frac{\delta (w_{b} + w_{s})}{\partial y} + N^{T} \left(\frac{\partial w}{\partial x} \frac{\partial \delta w}{\partial x} + \frac{\partial w}{\partial y} \frac{\partial \delta w}{\partial y} \right) \right] = 0$$

$$(21)$$

where N_x^0 , N_y^0 and N_{xy}^0 are in-plane applied loads (buckling loads); the external force N^T according to the changes of temperature is expressed as

$$N^{T} = \int_{-\frac{h}{2}}^{\frac{h}{2}} E(z) \alpha(z) (\Delta T) dz$$
 (22)

Note: The variation of temperature field will be introduced in section 3.

Based on above relations, the governing equations are obtained by inserting Eqs. (20) and (21) in Eq. (19) when the coefficients of δu_0 , δv_0 , δw_b , and δw_s are equal to zero

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0$$
(23)

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{y}}{\partial y} = 0$$
(24)

$$\frac{\partial^2 M_x^b}{\partial x^2} + 2 \frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + N^T \nabla^2 (w_b + w_s) \cdot + N_x^0 \frac{\partial^2 (w_b + w_s)}{\partial x^2} + N_y^0 \frac{\partial^2 (w_b + w_s)}{\partial y^2} + 2N_{xy}^0 \frac{\partial^2 (w_b + w_s)}{\partial x \partial y} = 0$$
(25)

$$\frac{\partial^2 M_x^s}{\partial x^2} + 2 \frac{\partial^2 M_{xy}^s}{\partial x \partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial Q_{xz}^s}{\partial x} + \frac{\partial Q_{yz}^s}{\partial y}$$
(26)

$$+N^{T}\nabla^{2}(w_{b}+w_{s})+N^{0}_{x}\frac{\partial^{2}(w_{b}+w_{s})}{\partial x^{2}}$$

+ $N^{0}_{y}\frac{\partial^{2}(w_{b}+w_{s})}{\partial y^{2}}+2N^{0}_{xy}\frac{\partial^{2}(w_{b}+w_{s})}{\partial x\partial y}=0$ (26)

herein N, M, and Q denote the stress resultants and are given as follows

$$\begin{cases} \{N\}\\ \{M^{b}\}\\ \{M^{s}\} \end{cases} = \begin{bmatrix} [A] & [B] & [B^{s}]\\ [B] & [D] & [D^{s}]\\ [B^{s}] & [D^{s}] & [H^{s}] \end{cases} \begin{cases} \varepsilon^{0}\\ k^{b}\\ k^{s} \end{cases}, \begin{cases} \{Q_{xz}\}\\ \{Q_{yz}\} \end{cases} = \begin{bmatrix} A_{44}^{s} & 0\\ 0 & A_{55}^{s} \end{bmatrix} \begin{bmatrix} \gamma_{xz}^{s}\\ \gamma_{Yz}^{s} \end{cases}$$
(27)

in which

$$(A, B, B^{s}, D, D^{s}, H^{s}) = \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{ij}(z, T)(1, z, f, z^{2}, zf, f^{2})dz$$

$$(i, i = 1, 2, 4, 5, 6)$$
(28)

and

$$A_{44}^{s} = A_{55}^{s} = \int_{-\frac{h}{2}}^{\frac{h}{2}} Q_{44}(z,T) g^{2} dz$$
(29)

The classical and non-classical boundary conditions can be observed in the derivation process when using the integrations by parts. Thus, we obtain classical boundary conditions at x = 0 or a and y = 0 or b as (Barati 2018)

Specify
$$w_{b}$$
 or $\left(\frac{\partial M_{x}^{b}}{\partial x} + \frac{\partial M_{xy}^{b}}{\partial y}\right)n_{x} + \left(\frac{\partial M_{y}^{b}}{\partial y} + \frac{\partial M_{xy}^{b}}{\partial x}\right)n_{y} = 0$
Specify w_{s} or $\left(\frac{\partial M_{x}^{s}}{\partial x} + \frac{\partial M_{xy}^{s}}{\partial y} + Q_{xz}\right)n_{x} + \left(\frac{\partial M_{y}^{s}}{\partial y} + \frac{\partial M_{xy}^{s}}{\partial x} + Q_{yz}\right)n_{y} = 0$ (30)
Specify $\frac{\partial w_{b}}{\partial n}$ or $M_{x}^{b}n_{x}^{2} + n_{x}n_{y}M_{xy}^{b} + M_{y}^{b}n_{y}^{2} = 0$

in which $\frac{\partial O}{\partial n} = n_x \frac{\partial O}{\partial x} + n_y \frac{\partial O}{\partial y}$; n_x and n_y denote the x and

y-components of the unit normal vector on the nanoplate boundaries, respectively and the non-classical boundary conditions are

Specify
$$\frac{\partial^2 w_b}{\partial x^2}$$
 or $M_x^b = 0$, Specify $\frac{\partial^2 w_b}{\partial y^2}$ or $M_y^b = 0$
Specify $\frac{\partial^2 w_s}{\partial x^2}$ or $M_x^s = 0$, Specify $\frac{\partial^2 w_s}{\partial y^2}$ or $M_y^s = 0$
(31)

2.5 Equations of motion

Integrating Eq. (11) over the plate's cross-section area, the nonlocal strain gradient refined FG nanoplates relations can be obtained as follows

$$L_{\eta}\left\{A_{11}\frac{\partial^{2}u_{0}}{\partial x^{2}}+A_{66}\frac{\partial^{2}u_{0}}{\partial y^{2}}+(A_{12}+A_{66})\frac{\partial^{2}v_{0}}{\partial x\,\partial y}-B_{11}\frac{\partial^{3}w_{b}}{\partial x^{3}}\right\}$$
(32)

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$$+ L_{\eta} \left\{ -(B_{12} + 2B_{66}) \frac{\partial^3 w_b}{\partial x \, \partial y^2} - B_{11}^s \frac{\partial^3 w_s}{\partial x^3} - (B_{12}^s + 2B_{66}^s) \frac{\partial^3 w_s}{\partial x \, \partial y^2} \right\} = 0$$
(32)

$$L_{1}\left\{A_{66}\frac{\partial^{2}v_{0}}{\partial x^{2}} + A_{22}\frac{\partial^{2}v_{0}}{\partial y^{2}} + (A_{12} + A_{66})\frac{\partial^{2}u_{0}}{\partial x\partial y} - B_{22}\frac{\partial^{3}w_{b}}{\partial y^{3}}\right\}$$

+
$$L_{1}\left\{-(B_{12} + 2B_{66})\frac{\partial^{3}w_{b}}{\partial x^{2}\partial y} - B_{22}^{s}\frac{\partial^{3}w_{s}}{\partial x^{3}} - (B_{12}^{s} + 2B_{66}^{s})\frac{\partial^{3}w_{s}}{\partial x^{2}\partial y}\right\} = 0$$
(33)

$$L_{l}\left\{B_{11}\frac{\partial^{3}u_{0}}{\partial x^{3}} + (B_{12} + 2B_{66})\frac{\partial^{3}u_{0}}{\partial x \partial y^{2}} + (B_{12} + 2B_{66})\frac{\partial^{3}v_{0}}{\partial x^{2} \partial y}\right\}$$
$$+L_{l}\left\{+B_{22}\frac{\partial^{3}v_{0}}{\partial y^{3}} - D_{11}\frac{\partial^{4}w_{b}}{\partial x^{4}} - 2(D_{12} + 2D_{66})\frac{\partial^{4}w_{b}}{\partial x^{2} \partial y^{2}}\right\}$$
$$+L_{l}\left\{-D_{22}\frac{\partial^{4}w_{b}}{\partial x^{4}} - D_{11}^{s}\frac{\partial^{4}w_{s}}{\partial x^{4}} - 2(D_{12}^{s} + 2D_{66})\frac{\partial^{4}w_{s}}{\partial x^{2} \partial y^{2}} - D_{22}^{s}\frac{\partial^{4}w_{s}}{\partial y^{4}}\right\}$$
(34)
$$+L_{\mu}\left\{N^{T}\nabla^{2}(w_{b} + w_{s})N_{x}^{0}\frac{\partial^{2}(w_{b} + w_{s})}{\partial x^{2}} + N_{y}^{0}\frac{\partial^{2}(w_{b} + w_{s})}{\partial x^{2}}\right\}$$
$$+L_{\mu}\left\{+2N_{xy}^{0}\frac{\partial^{2}(w_{b} + w_{s})}{\partial x \partial y}\right\} = 0$$

$$L_{l} \left\{ B_{11}^{s} \frac{\partial^{3} u_{0}}{\partial x^{3}} + (B_{12}^{s} + 2B_{66}^{s}) \frac{\partial^{3} u_{0}}{\partial x \partial y^{2}} + (B_{12}^{s} + 2B_{66}^{s}) \frac{\partial^{3} v_{0}}{\partial x^{2} \partial y} \right\}$$

$$+ L_{l} \left\{ + B_{22} \frac{\partial^{3} v_{0}}{\partial y^{3}} - D_{11}^{s} \frac{\partial^{4} w_{b}}{\partial x^{4}} A_{55}^{s} \frac{\partial^{2} w_{s}}{\partial x^{2}} + A_{44}^{s} \frac{\partial^{2} w_{s}}{\partial y^{2}} \right\}$$

$$+ L_{l} \left\{ -2(D_{12}^{s} + 2D_{66}^{s}) \frac{\partial^{4} w_{b}}{\partial x^{2} \partial y^{2}} - D_{22}^{s} \frac{\partial^{4} w_{b}}{\partial x^{4}} - H_{11}^{s} \frac{\partial^{4} w_{s}}{\partial x^{4}} \right\}$$

$$+ L_{l} \left\{ -2(H_{12}^{s} + 2H_{66}^{s}) \frac{\partial^{4} w_{s}}{\partial x^{2} \partial y^{2}} - H_{22}^{s} \frac{\partial^{4} w_{s}}{\partial y^{4}} \right\}$$

$$+ L_{\mu} \left\{ N^{T} \nabla^{2} (w_{b} + w_{s}) + N_{x}^{0} \frac{\partial^{2} (w_{b} + w_{s})}{\partial x^{2}} + N_{y}^{0} \frac{\partial^{2} (w_{b} + w_{s})}{\partial x^{2}} \right\}$$

$$+ L_{\mu} \left\{ 2N_{xy}^{0} \frac{\partial^{2} (w_{b} + w_{s})}{\partial x \partial y} \right\} = 0$$

$$(35)$$

3. Temperature field

Temperature dependent material properties can be expressed as following form (Touloukian and Ho 1970, Bensaid and Bekhadda 2018)

$$\mathbf{P} = \mathbf{P}_0 \left(\mathbf{P}_{-1} T^{-1} + \mathbf{P}_1 T + \mathbf{P}_2 T^2 + \mathbf{P}_3 T^3 + 1 \right)$$
(36)

in which P_0 , P_{-1} , P_1 , P_2 and P_3 are the temperaturedependent coefficients, which are needed to be uniquely determined for a specified material (see Ref. Reddy and Chin 1998) and temperature-dependent material properties of SUS304 and Si₃N₄ can be seen in the Table 1.

In the present study, three different pattern of temperature field variations are used as follows.

3.1 Uniform temperature

The uniform case temperature rise can be defined by Li et al. (2009)

$$T(z) = T_0 + \Delta T(z)$$
(37)

Often the initial temperature T_0 is the temperature of the surface with pure metal T_m and equals the room temperature (i.e., $T_0 = T_m = 300$ K), and $\Delta T(z) = T_c - T_m$ where T_c denotes the temperature of the surface with pure ceramic.

3.2 Nonlinear temperature

In this subsection with basic assumptions in which temperature distribution is only along the thickness direction the plate. In this non-linear case, one-dimensional steady-state heat conduction equation is applied for the temperature field in the thickness direction and can be given by Li *et al.* (2009)

$$-\frac{d}{dz}\left(\kappa(z)\frac{dT}{dz}\right) = 0 \tag{38}$$

By utilizing polynomials series Eq. (38) solve as following form

Material	Properties	P_0	P.1	P_1	P_2	P ₃
SUS304	E (GPa)	201.4	0	-3.079×10 ⁻⁴	-6.534×10 ⁻⁷	0
	α (K ⁻¹)	12.330×10 ⁻⁶	0	8.086×10 ⁻⁶	0	0
	κ (W/mk)	15.379	0	-1.264×10 ⁻³	2.092×10 ⁻⁶	-7.223×10 ⁻¹⁰
	ρ (Kg/m ³)	8166	0	0	0	0
	v	0.3262	0	-2.002×10^{-4}	3.797×10 ⁻⁷	0
Si ₃ N ₄	E (GPa)	348.43	0	-3.070×10 ⁻⁴	2.160×10 ⁻⁷	-8.964×10 ⁻¹¹
	α (K ⁻¹)	5.8723×10 ⁻⁶	0	9.095×10 ⁻⁶	0	0
	κ (W/mk)	13.723	0	-1.032×10 ⁻³	5.466×10 ⁻⁷	-7.876×10 ⁻¹¹
	ρ (Kg/m ³)	2370	0	0	0	0
	ν	0.2400	0	0	0	0

Table 1 Temperature-dependent coefficients for SUS304 and Si₃N₄ (Reddy and Chin 1998)

$$T(z) = T_{\rm m} + (T_{\rm c} - T_{\rm m})\eta(z)$$
⁽³⁹⁾

where

$$\eta(z) = \frac{1}{C} \left[\left(\frac{2z+h}{2h} \right) - \frac{\kappa_{\rm cm}}{(N+1)\kappa_{\rm m}} \left(\frac{2z+h}{2h} \right)^{N+1} + \frac{\kappa_{\rm cm}^2}{(2N+1)\kappa_{\rm m}^2} \left(\frac{2z+h}{2h} \right)^{2N+1} - \frac{\kappa_{\rm cm}^3}{(3N+1)\kappa_{\rm m}^3} \left(\frac{2z+h}{2h} \right)^{3N+1} + \frac{\kappa_{\rm cm}^4}{(4N+1)\kappa_{\rm m}^2} \left(\frac{2z+h}{2h} \right)^{4N+1} - \frac{\kappa_{\rm cm}^5}{(5N+1)\kappa_{\rm m}^2} \left(\frac{2z+h}{2h} \right)^{5N+1} \right]$$
(40)

$$C = 1 - \frac{\kappa_{\rm cm}}{(N+1)\kappa_{\rm m}} + \frac{\kappa_{\rm cm}^2}{(2N+1)\kappa_{\rm m}^2} - \frac{\kappa_{\rm cm}^3}{(3N+1)\kappa_{\rm m}^3} + \frac{\kappa_{\rm cm}^4}{(4N+1)\kappa_{\rm m}^4} - \frac{\kappa_{\rm cm}^5}{(5N+1)\kappa_{\rm m}^5}$$
(41)

here $\kappa_{cm} = \kappa_c - \kappa_m$ in which κ_c and κ_m respectively represent thermal conductivity of the bottom and top surfaces.

3.3 Sinusoidal temperature

In the case of sinusoidal temperature rise, the temperature rise can be defined as follow (Gupta and Talha 2017)

$$T(z) = T_{\rm m} + (T_{\rm c} - T_{\rm m})\eta(z)$$
(42)

with $T\left(\frac{h}{2}\right) = T_c$, $T\left(-\frac{h}{2}\right) = T_m$.

4. Solution procedure

In this subsection to satisfy the equations of motion, using Galerkin's method following series are presented (Barati 2018)

$$u = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} U_{mn} \frac{\partial X_m(x)}{\partial x} Y_n(y)$$
(43)

$$v = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} V_{mn} X_m(x) \frac{\partial Y_n(y)}{\partial y}$$
(44)

$$w_{b} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{bmn} X_{m}(x) Y_{n}(y)$$
(45)

$$w_{s} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{smn} X_{m}(x) Y_{n}(y)$$
(46)

where $(U_{mn}, V_{mn}, W_{bmn}, W_{smn})$ are the unknown coefficients and the functions X_m and Y_n satisfy the boundary conditions. The classical and non-classical boundary condition based on the present plate model are (Barati 2018)

$$w_{b} = w_{s} = 0$$

$$\frac{\partial^{2} w_{b}}{\partial x^{2}} = \frac{\partial^{2} w_{s}}{\partial x^{2}} = \frac{\partial^{2} w_{b}}{\partial y^{2}} = \frac{\partial^{2} w_{s}}{\partial y^{2}} = 0$$
(47)

$$\frac{\partial^4 w_b}{\partial x^4} = \frac{\partial^4 w_s}{\partial x^4} = \frac{\partial^4 w_b}{\partial y^4} = \frac{\partial^4 w_s}{\partial y^4} = 0$$
(47)

By substituting Eqs. (43)-(46) into equations of motion (Eqs. (32)-(35)), one can write these four equations in matrix format to find the critical buckling force. It is important to note that to investigate the stability behavior of mentioned nanostructure following relegation is considered:

$$N_{x}^{0} = \xi_{1}P_{1}, N_{y}^{0} = \xi_{2}P_{1}, N_{xy}^{0} = 0$$
(48)

"where P_1 is the force per unit length; ζ_1 and ζ_2 are changed according to different loading conditions." In present investigation biaxial compression load is used to investigate the size-dependent buckling response of nanopltes made of functionally graded materials. Note that the following parameters are introduced to consider the small-scale effects on the results in the next section

$$\eta_1 = \mu/a, \ \eta_2 = \lambda/a \tag{49}$$

5. Numerical results and discussions

Some of researchers have been presented the importance of temperature-dependent material properties to investigate the vibration of plate type structures (Shahrjerdi *et al.* 2011, Huang and Shen 2004). Therefore, the authors of the present paper decided to study the stability response of nano-size plates made of FGMs having temperaturedependent properties due to the lack of any study on the stability analysis of such nanostructures. Reuss micromechanical scheme is considered to estimate the material properties where. the influence of thermal environments is captured. For the geometrical parameters, it is assumed that the length of the nanoplate is a=10 nm, and the thickness of the plate is variable.

Firstly, to show the accuracy of present model the results for free vibration of rectangular FG plate is compared with those of (Shahrjerdi *et al.* 2011) and (Huang and Shen 2004) and results tabulated in Table 2. The effect of thermal environment is also validated with mentioned studies. Good agreement can be seen between different mathematical model.

The variations of size-dependent non-dimensional critical buckling loads as a function of uniform, nonlinear and sinusoidal temperature fields in simply-supported nanoplates made of FGMs are illustrated in Figs. 3-5 when $\eta_1 = \eta_2 = 0.1$. the geometrical conditions are a = b = 10 nm and a/h = 10. The non-dimensional critical buckling load parameter is defined as $\Omega = Ra^2/h^3 E_m$. The bottom surface temperature is equal to room temperature and heat applied from the top surface.

It is concluded that non-dimensional critical buckling loads are reduced with increasing temperature. This is due to the changes in Young's modulus by temperature variations. In a similar area of temperature variation, the greatest impact of temperature conditions is obtained for the plate under uniform temperature variation condition followed by nonlinear, and sinusoidal respectively. Further

		$T_t = 300$	$T_t = 400$		$T_{t} = 600$	
п	Model		Temperature- dependent	Temperature- independent	Temperature- dependent	Temperature- independent
Ceramic	SSDT ^a	12.506	12.175	12.248	11.461	11.716
	TSDT ^b	12.495	13.397	12.382	11.984	12.213
	Present	12.5317	12.2728	12.4001	11.7429	12.0785
<i>n</i> = 0.5	SSDT ^a	8.652	8.361	8.405	7.708	7.887
	TSDT ^b	8.675	8.615	8.641	8.269	8.425
	Present	8.6313	8.4603	8.5205	8.1133	8.2688
<i>n</i> = 1	SSDT ^a	7.584	7.306	7.342	6.674	6.834
	TSDT ^b	7.555	7.474	7.514	7.171	7.305
	Present	7.5657	7.4180	7.4591	7.1190	7.2242
<i>n</i> = 2	SSDT ^a	6.811	6.545	6.575	5.929	6.077
	TSDT ^b	6.777	6.693	6.728	6.398	6.523
	Present	6.7897	6.6570	6.6856	6.3893	6.4625
Metal	SSDT ^a	5.410	5.161	5.178	4.526	4.682
	TSDT ^b	5.405	5.311	5.335	4.971	5.104
	Present	5.4210	5.3116	5.3116	5.0857	5.0857

Table 2 Non-dimensional natural frequency parameter of simply supported (Si₃N₄/SUS304) FG plate in thermal environments (nonlinear temperature distribution)

*a: Shahrjerdi et al. 2011; b: Huang and Shen 2004



Fig. 3 The non-dimensional critical buckling load versus uniform temperature field for simply supported FG nanoplates (a = b = 10 nm, a/h = 20, $\eta_1 = \eta_2 = 0.1$)

more, from Fig. 3, it is observed that the buckling load of the nanoplate decreases with the rises of temperature until it approaches the critical buckling temperature. This is due to the reduction in total stiffness of the plate since geometrical stiffness decreases when the temperature rises. Buckling load reaches to zero at the critical temperature point. The increase in temperature yields in higher buckling load after the branching point. Moreover, it is seen from Figs. 4 and 5 that the branching point of the nanoplate is postponed by changing temperate variation model.

Present investigation tries to provide a benchmark results for nanoplates made of FGMs in which material properties is depend on temperature. So, as a benchmark results Tables 3-4 present buckling response of FG nanoplates under nonlinear temperate variation condition



Fig. 4 The non-dimensional critical buckling load versus nonlinear temperature field for simply supported FG nanoplates (a = b = 10 nm, a/h = 20, $\eta_1 = \eta_2 = 0.1$)



Fig. 5 The non-dimensional critical buckling load versus sinusoidal temperature field for simply supported FG nanoplates (a = b = 10 nm, a/h = 20, $\eta_1 = \eta_2 = 0.1$)

Table 3 Non-dimensional critical buckling load of simply supported SUS304/Si₃N₄ FG square plate under biaxial compression load in thermal environments and for different values of strain gradient length scale parameter (a = 10, a/h = 10, $\eta_2 = 0$)

		$T_{b} = 300 (K)$					
n	η_1	$T_t = 300 (K)$	$T_t = 400 (K)$		$T_t = 600 (K)$		
			Temperature- dependent	Temperature- independent	Temperature- dependent	Temperature- independent	
Ceramic	0	1.7112	1.6540	1.6202	1.5167	1.4336	
	0.1	1.4291	1.3907	1.3623	1.2987	1.2275	
	0.2	0.9562	0.9495	0.9301	0.9333	0.8822	
	0	1.4371	1.3907	1.3623	1.2797	1.2096	
0.5	0.1	1.2002	1.1712	1.1473	1.1004	1.0400	
	0.2	0.8030	0.8032	0.7868	0.7998	0.7560	
1	0	1.3585	1.3133	1.2865	1.2070	1.1409	
	0.1	1.1345	1.1068	1.0842	1.0400	0.9830	
	0.2	0.7591	0.7606	0.7451	0.7601	0.7185	
2	0	1.2996	1.2541	1.2285	1.1491	1.0861	
	0.1	1.0854	1.0578	1.0362	0.9924	0.9380	
	0.2	0.7262	0.7286	0.7137	0.7298	0.6898	
Metal	0	1.1033	1.0543	1.0327	0.9433	0.8916	
	0.1	0.9214	0.8923	0.8741	0.8247	0.7795	
	0.2	0.6165	0.6209	0.6082	0.6259	0.5916	

Table 4Non-dimensional critical buckling load of simply supported SUS304/Si₃N₄ FG square plate under biaxial compression load in thermal environments and for different values of strain gradient length scale parameter (a = 10, a/h = 10, $\eta_2 = 0$)

		T _b = 300 (K)					
n	η_1	$T_t = 300 (K)$	$T_t = 400 (K)$		$T_t = 600 (K)$		
			Temperature- dependent	Temperature- independent	Temperature- dependent	Temperature- independent	
Ceramic	0	1.7112	1.6540	1.6202	1.5167	1.4336	
	0.1	2.0490	1.9917	1.9511	1.8544	1.728	
	0.2	3.0623	3.0051	2.9637	2.8678	2.7106	
0.5	0	1.4371	1.3907	1.3623	1.2797	1.2096	
	0.1	1.7207	1.6769	1.6426	1.5702	1.4842	
	0.2	2.5717	2.5354	2.4836	2.4418	2.3079	
1	0	1.3585	1.3133	1.2865	1.2070	1.1409	
	0.1	1.6266	1.5846	1.5522	1.4836	1.4023	
	0.2	2.4311	2.3982	2.3492	2.3133	2.1865	
2	0	1.2996	1.2541	1.2285	1.1491	1.0861	
	0.1	1.5562	1.5141	1.4832	1.4151	1.3358	
	0.2	2.3258	2.2941	2.2472	2.1326	2.0920	
Metal	0	1.1033	1.0543	1.0327	0.9433	0.8916	
	0.1	1.3211	1.2766	1.2505	1.1737	1.1094	
	0.2	1.9744	1.9436	1.9039	1.8650	1.7627	

when (a = 10, a/h = 10). Non-dimensional critical buckling loads $\tilde{\Omega} = Ra^2/h^3E_c$ of FG nanoplates with respect to nonlocality are tabulated in Table 3 by ignoring strain gradient length scale parameter while Table 4 shows the buckling behavior of nanoplates considering strain gradient size-dependency by omitting nonlocal parameter.

Results are obtained for different power-law index parameters. To see the effect of the power index on the buckling response, the same values of the thermal load is used. It is concluded that the result for nanoplates is in between those for pure material nanoplates. This is due to the fact that Young's modulus increases from pure metal to pure ceramic. Also, the non-dimensional critical buckling load decrease by increasing the temperature difference between the top and bottom surfaces for the same value of the power-law index parameter. Furthermore, the nonlocal parameter has decreasing effects on the results, while the strain gradient length scale parameter has increasing effects on the results of nanoplate. This phenomenon obtained in different power-lave indexes and temperature values. The difference between temperature-dependent and independent FG plates is less significant.

6. Conclusions

For the first time buckling analysis of size-dependent of temperature-dependent rectangular plates made functionally graded materials (FGMs) is studied. Material properties of FGMs are varied along thickness direction and obtained based on Reuss micromechanical model. A refined nonlocal strain gradient plate model is presented to model the nano-size plate. Hamiltonian principles are used to obtain the governing equations and boundary conditions where the Galerkin method is adopted to solve the buckling problem. Influence of thermal environment with respect to three different distribution of temperatures namely uniform, nonlinear and sinusoidal is also investigated. The results for vibration analysis of FGM plates are validated and good agreement achieved. As a result, the non-dimensional critical buckling load decreases as temperature change increases in all types of temperature fields. Moreover, the impacts of small scale parameters on critical buckling load are discussed, which concluded the parameters play a significant role in the critical buckling loads of nano-size plates made of FGM.

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