Effect of non-uniform temperature distributions on nonlocal vibration and buckling of inhomogeneous size-dependent beams

Farzad Ebrahimi ** and Erfan Salari

*Department of Mechanical Engineering, Faculty of Engineering,
Imam Khomeini International University, Qazvin, Iran*

Abstract. In the present investigation, thermal buckling and free vibration characteristics of functionally graded (FG) Timoshenko nanobeams subjected to nonlinear thermal loading are carried out by presenting a Navier type solution. The thermal load is assumed to be nonlinear distribution through the thickness of FG nanobeam. Thermo-mechanical properties of FG nanobeam are supposed to vary smoothly and continuously throughout the thickness based on power-law model and the material properties are assumed to be temperature-dependent. Eringen's nonlocal elasticity theory is exploited to describe the size dependency of nanobeam. Using Hamilton's principle, the nonlocal equations of motion together with corresponding boundary conditions based on Timoshenko beam theory are obtained for the thermal buckling and vibration analysis of graded nanobeams including size effect. Moreover, in following a parametric study is accompanied to examine the effects of the several parameters such as nonlocal parameter, thermal effect, power law index and aspect ratio on the critical buckling temperatures and natural frequencies of the size-dependent FG nanobeams in detail. According to the numerical results, it is revealed that the proposed modeling can provide accurate frequency results of the FG nanobeams as compared some cases in the literature. Also, it is found that the small scale effects and nonlinear thermal loading have a significant effect on thermal stability and vibration characteristics of FG nanobeams.

Keywords: thermal buckling; Timoshenko beam theory; vibration; functionally graded nanobeam; nonlocal elasticity theory

1. Introduction

Functionally graded material (FGM) is a composite material varying its microstructure from one material to another with a specific gradient, resulting in corresponding changes in the effective material properties (including elasticity modulus, thermal expansion coefficient and thermal conductivity) of the material (Ebrahimi and Rastgoo 2018). The FG materials can be designed to produce an optimum distribution of component materials for specific function and applications. Typically, FG materials are made of a mixture of two materials, mainly ceramic and metal phases, to achieve a composition with a certain functionality. In comparison with traditional composites, FGMs possess various advantages, for instance, higher fracture toughness, enhanced thermal

---

∗Corresponding author, Ph.D., Professor, E-mail: febrahimy@eng.ikiu.ac.ir

resistance, minimization or elimination of stress concentration, and increased bonding strength along the interface of two dissimilar materials. During the past decade, beams made of FGMs have found wide applications as structural elements in modern industries such as aeronautics/ astronautics manufacturing industry, mechanical engineering and engine combustion chamber, nuclear engineering and reactors. Motivated by these engineering applications, ceramic-metal FGMs have also attracted intensive research interests. Sofiyev and Kuruoglu (2015) studied buckling analysis of non-homogeneous orthotropic truncated conical shells subjected to combined loading of axial compression and external pressure. The superposition and Galerkin methods are used to achieve the expressions for critical loads of non-homogeneous orthotropic truncated conical shells with simply supported boundary conditions. Nanobeams are one of the basic components in micro/nano electromechanical systems (MEMS/NEMS), biomedical sensors and atomic force microscopy (AFM). Therefore understanding the mechanical and physical properties of nanostructures is necessary for its practical applications. For instance, Alizada *et al*. (2012) presented stress analysis of a substrate coated by nanomaterials with a vacancy under uniform extension load. They assumed that vacancies do not vary along the width of the substrate.

   Nanoscale engineering materials have attracted great interest in modern science and technology after the invention of carbon nanotubes (CNTs) by Iijima (1991). They have significant mechanical, thermal and electrical performances that are superior to the conventional structural materials. In recent years, nanobeams and CNTs hold a wide variety of potential applications (Zhang *et al*. 2004) such as sensors, actuators, transistors, probes, and resonators in NEMSs. For example, in MEMS/NEMS; nanostructures have been used in many areas including communications, machinery, information technology and biotechnology technologies. The classical continuum theory is quite efficient in the mechanical analysis of the macroscopic structures, but its applicability to the identification of the size effect on the mechanical behaviors on micro- or nano-scale structures is questionable. This limitation of the classical continuum theory is partly due to the fact that the classical continuum theory does not admit the size dependence in the elastic solutions of inclusions and inhomogeneities. However the classical continuum models need to be extended to consider the nanoscale effects and this can be achieved through the nonlocal elasticity theory proposed by Eringen (1983) which consider the size-dependent effect. According to this theory, the stress state at a reference point is considered as a function of strain states of all points in the body. This nonlocal theory is proved to be in accordance with atomic model of lattice dynamics and with experimental observations on phonon dispersion. Moreover, in recent years the application of nonlocal elasticity theory, in micro and nanomaterials has received a considerable attention within the nanotechnology community. Wang and Liew (2007) carried out the static analysis of micro- and nano-structures based on nonlocal continuum mechanics using Euler-Bernoulli beam theory and Timoshenko beam theory. Aydogdu (2009) proposed a generalized nonlocal beam theory to study bending, buckling, and free vibration of nanobeams based on Eringen model using different beam theories

   To improve the performance of composite structures, the development of the FG materials are being accelerated to optimize some certain functional properties of structures by tailoring the material architecture at nano/micro scale. The rapid developments of MEMS and NEMS make the FG materials possible to be applied in nano/micro scaled systems such as thin films in the form of shape memory alloys, atomic force microscopes (AFMs), micro sensors, micro piezoactuator and nano-motors to achieve high sensitivity and desired performance. In such applications, size effects or small scale effects play major role which should be considered to study the mechanical behaviors of such small scale structures. FG nanobeams are one of the most important

nanostructures which are commonly used as components in MEMS, NEMS and AFMS with the order of microns or sub-microns, and their properties are closely related to their microstructures. Nevertheless, the possible applications rely on a good understanding of the vibration and thermal stability characteristics of FG nanobeams at small-scale. Therefore, establishing an accurate model of FG nanobeams is very important for successful NEMS design.

Recently, FGMs find increasing applications in micro- and nano-scale structures such as thin films in the form of shape memory alloys, atomic force microscopes, micro sensors, micro piezoactuator and nano-motors (Fu *et al*. 2003). Moreover, with the development of the material technology, FGMs have also been employed in MEMS/NEMS. In all of these applications, the size effect plays major role which should be considered to study the mechanical behaviors of such small scale structures. Beams are the core structures widely used in MEMS, NEMS and AFMS with the order of microns or sub-microns, and their properties are closely related to their microstructures. On the other hands, FG nanobeams are important structural elements and hence, because of high sensitivity of MEMS/ NEMS to external stimulations, understanding mechanical properties and vibration behavior of them are of significant importance to the design and manufacture of FG MEMS/NEMS. Furthermore, different fabrication processes of nanoscale functionally graded material have been focused by the several researchers. The design and fabrication of stepwise functionally graded synthetic nanocomposites using combined powder stacking and compression molding techniques were studied by Bafekrpour *et al*. (2012). They concluded that the electrical and thermal properties of nanocomposites could be manipulated by changing the gradient patterns. Therefore, establishing an accurate model of FG nanobeams is a key issue for successful NEMS design. The free vibration analysis of FG microbeams was presented by Ansari *et al*. (2011) based on the strain gradient Timoshenko beam theory. They also concluded that the value of gradient index plays an important role in the vibrational response of the FG microbeams of lower slenderness ratios. Ebrahimi and Salari (2015) also studied the thermo-electrical buckling of the piezoelectric nanobeams subjected to in-plane thermal loads and applied electric voltage. Recently, Eltaher *et al*. (2012) presented a finite element formulation for free vibration analysis of FG nanobeams based on nonlocal Euler beam theory. Using nonlocal Timoshenko and Euler–Bernoulli beam theory, Simsek and Yurtcu (2013) investigated bending and buckling of FG nanobeam by analytical method. More recently, vibration behaviour of simply supported Timoshenko FG nanobeams were investigated by Rahmani and Pedram (2014). Material properties are assumed to be temperature independent in these works and the thermal environment effects were not considered. Furthermore, the common use of FGMs in high temperature environment leads to considerable changes in material properties. For example, Young's modulus usually decreases when temperature increases in FGMs. To predict the behavior of FGMs under high temperature more accurately, it is necessary to consider the temperature dependency on material properties.

It can be evaluated from the literature survey that there is no study on the thermal buckling and vibration of FG Timoshenko nanobeams under nonlinear temperature rise via nonlocal elasticity theory. In fact, it is crucial to consider simultaneously nonlinear temperature rise and nonlocal effects for more accurate thermal buckling and vibration analysis and design of temperature dependent inhomogeneous nanobeams.

In view of the above, the aim of the present article is to develop a inhomogeneous nanobeam under nonlinear temperature distribution for thermal buckling and vibration analysis of Timoshenko nanobeams within the framework of nonlocal elasticity theory. An analytical method called Navier solution is employed for vibration and thermal buckling analysis of size-dependent

FG nanobeams. It is assumed that material properties of the beam, vary continuously through the beam thickness according to power-law form and are temperature dependent. Nonlocal Timoshenko beam model and Eringen's nonlocal elasticity theory are employed. Governing equations and boundary conditions for the free vibration of a nonlocal FG beam have been derived via Hamilton's principle. These equations are solved using Navier type method and numerical solutions are obtained. The detailed mathematical derivations are presented while the emphasis is placed on investigating the effect of several parameters such as thermal effects, constituent volume fractions, mode number, aspect ratio and small scale on critical buckling temperature and vibration characteristics of FG nanobeams. Comparisons with analytical solutions and the results from the existing literature are provided for two-constituents metal–ceramic nanobeams and the good agreement between the results of this article and those available in literature validated the presented approach. Numerical results are presented to serve as benchmarks for the application and the design of nanoelectronic and nano-drive devices, nano-oscillators, and nanosensors, in which nanobeams act as basic elements. They can also be useful as valuable sources for validating other approaches and approximate methods.

2. Theory and formulation

2.1 Nonlocal power-law FG nanobeam equations

Consider a FG nanobeam of length L, width b and uniform thickness h in the unstressed reference configuration. The coordinate system for FG nanobeam is shown in Fig. 1. The nanobeam is made of elastic and isotropic functionally graded material with properties varying smoothly in the z thickness direction only. The effective material properties of the FG nanobeam such as Young's modulus E_f, shear modulus G_f and mass density ρ_f are assumed to vary continuously in the thickness direction (z-axis direction) according to a power function of the volume fractions of the constituents. According to the rule of mixture, the effective material properties, P_f, can be expressed as Şimşek and Yurtcu (2013)

$$P_f = P_c V_c + P_m V_m \tag{1}$$

where P_m, P_c, V_m and V_c are the material properties and the volume fractions of the metal and the ceramic constituents related by

$$V_c + V_m = 1 \tag{2a}$$

The volume fraction of the ceramic constituent of the beam is assumed to be given by

$$V_c = (\frac{z}{h} + \frac{1}{2})^P \tag{2b}$$

here p is the non-negative variable parameter (power-law index) which determines the material distribution through the thickness of the beam and z is the distance from the mid-plane of the FG nanobeam.

The FG nanobeam becomes a fully ceramic beam when p is set to be zero. Therefore, from Eqs.

Table 1 Temperature dependent coefficients of Young's modulus, thermal expansion coefficient, mass density and Poisson's ratio for Si_3N_4 and SUS304 (Tang *et al.* 2018)

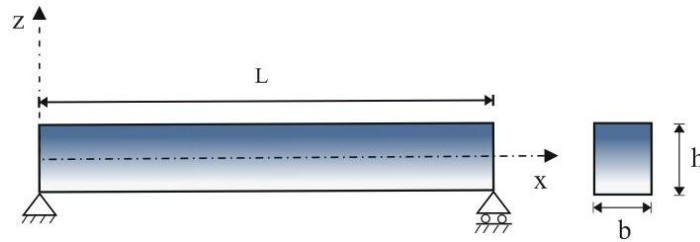| Material	Properties	P_0	P_{-1}	P_1	P_2	P_3
Si_3N_4	E (Pa)	348.43e+9	0	-3.070e-4	2.160e-7	-8.946e-11
	α (K^{-1})	5.8723e-6	0	9.095e-4	0	0
	ρ (Kg/m^3)	2370	0	0	0	0
	κ (W/mK)	13.723	0	-1.032e-3	5.466e-7	-7.876e-11
	v	0.24	0	0	0	0
SUS304	E (Pa)	201.04e+9	0	3.079e-4	-6.534e-7	0
	α (K^{-1})	12.330e-6	0	8.086e-4	0	0
	ρ (Kg/m^3)	8166	0	0	0	0
	κ (W/mK)	15.379	0	-1.264e-3	2.092e-6	-7.223e-10
	v	0.3262	0	-2.002e-4	3.797e-7	0

Fig. 1 Geometry and coordinates of Timoshenko FG nanobeam

(1)-(2), the effective material properties of the FG nanobeam such as Young's modulus (E), mass density (ρ), thermal expansion (α), thermal conductivity (κ) and Poisson's ratio (v) can be expressed as follows

$$E(z) = \left(E_c - E_m\right)\left(\frac{z}{h} + \frac{1}{2}\right)^p + E_m, \qquad \rho(z) = \left(\rho_c - \rho_m\right)\left(\frac{z}{h} + \frac{1}{2}\right)^p + \rho_m$$

$$\alpha(z) = \left(\alpha_c - \alpha_m\right)\left(\frac{z}{h} + \frac{1}{2}\right)^p + \alpha_m, \qquad \kappa(z) = \left(\kappa_c - \kappa_m\right)\left(\frac{z}{h} + \frac{1}{2}\right)^p + \kappa_m$$

$$(3)$$

$$v(z) = \left(v_c - v_m\right)\left(\frac{z}{h} + \frac{1}{2}\right)^p + v_m \tag{3}$$

To predict the behavior of FGMs under high temperature more accurately, it is necessary to consider the temperature dependency on material properties. Temperature dependency of the FGM constituents is frequently expressed based on Touloukian formula which captures the higher order dependencies. The temperature-dependent case, on the other hand, represents the conditions where properties are calculated at current temperature based on the Touloukian model described by Eq. (4). The nonlinear equation of thermo-elastic material properties in function of temperature $T(K)$

can be expressed as

$$P = P_0(P_{-1}T^{-1} + 1 + P_1 T + P_2 T^2 + P_3 T^3)$$ (4)

where P_0, P_{-1}, P_1, P_2 and P_3 are the temperature dependent coefficients which can be seen in the table of materials properties (Table 1) for Si_3N_4 and SUS304. The bottom surface ($z = -h/2$) of FG nanobeam is pure metal (SUS304), whereas the top surface ($z = h/2$) is pure ceramics (Si_3N_4).

2.2 Kinematic relations

The equations of motion is derived based on the Timoshenko beam theory according to which the displacement field at any point of the beam can be written as

$$u_x(x,z,t) = u(x,t) + z\,\varphi(x,t)$$ (5a)

$$u_z(x,z,t) = w(x,t)$$ (5b)

where t is time, φ is the total bending rotation of the cross-section, u and w are displacement components of the mid-plane along x and z directions, respectively. Therefore, according to the Timoshenko beam theory, the nonzero strains are obtained as

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + z\,\frac{\partial \varphi}{\partial x}$$ (6)

$$\gamma_{xz} = \frac{\partial w}{\partial x} + \varphi$$ (7)

where ε_{xx} and γ_{xy} are the normal strain and shear strain, respectively. Based on the Hamilton's principle, which states that the motion of an elastic structure during the time interval $t_1 < t < t_2$ is such that the time integral of the total dynamics potential is extremum

$$\int_0^t \delta(U - T + V)\,dt = 0$$ (8)

here U is strain energy, T is kinetic energy and V is work done by external forces. The virtual strain energy can be calculated as

$$\delta U = \int_v \sigma_{ij}\,\delta\varepsilon_{ij}\,dV = \int_v (\sigma_{xx}\,\delta\varepsilon_{xx} + \sigma_{xz}\,\delta\gamma_{xz})\,dV$$ (9)

Substituting Eqs. (6) and (7) into Eq. (9) yields

$$\delta U = \int_0^L (N\,(\delta\frac{\partial u}{\partial x}) + M\,(\delta\frac{\partial \varphi}{\partial x}) + Q\,(\delta\frac{\partial w}{\partial x} + \delta\varphi))\,dx$$ (10)

In which N is the axial force, M is the bending moment and Q is the shear force. These stress resultants used in Eq. (10) are defined as

$$N = \int_A \sigma_{xx}\, dA, \quad M = \int_A \sigma_{xx} z\, dA, \quad Q = \int_A K_s \sigma_{xz}\, dA \tag{11}$$

The kinetic energy for Timoshenko beam can be written as

$$T = \frac{1}{2} \int_0^L \int_A \rho(z,T)\left(\left(\frac{\partial u_x}{\partial t}\right)^2 + \left(\frac{\partial u_z}{\partial t}\right)^2\right) dA\, dx \tag{12}$$

Also the virtual kinetic energy can be expressed as

$$\delta T = \int_0^L \left[I_0\left(\frac{\partial u}{\partial t}\frac{\partial \delta u}{\partial t} + \frac{\partial w}{\partial t}\frac{\partial \delta w}{\partial t}\right) + I_1\left(\frac{\partial \varphi}{\partial t}\frac{\partial \delta u}{\partial t} + \frac{\partial u}{\partial t}\frac{\partial \delta \varphi}{\partial t}\right) + I_2 \frac{\partial \varphi}{\partial t}\frac{\partial \delta \varphi}{\partial t} \right] dx \tag{13}$$

where (I_0, I_1, I_2) are the mass moment of inertias, defined as follows

$$(I_0, I_1, I_2) = \int_A \rho(z,T)(1, z, z^2)\, dA \tag{14}$$

For a typical FG nanobeam which has been in high temperature environment for a long period of time, it is assumed that the temperature can be distributed nonlinearly across its thickness. So that the case of nonlinear temperature rise is taken into consideration. Hence, the first variation of the work done corresponding to temperature change can be written in the form

$$\delta V = \int_0^L N^T \frac{\partial w}{\partial x}\frac{\partial \delta w}{\partial x}\, dx \tag{15}$$

where N^T is thermal resultant can be expressed as

$$N^T = \int_{-h/2}^{h/2} E(z,T)\alpha(z,T)(T - T_0)\, dz \tag{16}$$

where T_0 is the reference temperature. By Substituting Eqs. (10), (13) and (15) into Eq. (8) and setting the coefficients of δu, δw and $\delta \varphi$ to zero, the following Euler–Lagrange equation can be obtained

$$\frac{\partial N}{\partial x} = I_0 \frac{\partial^2 u}{\partial t^2} + I_1 \frac{\partial^2 \varphi}{\partial t^2} \tag{17a}$$

$$\frac{\partial Q}{\partial x} - N^T \frac{\partial^2 w}{\partial x^2} = I_0 \frac{\partial^2 w}{\partial t^2} \tag{17b}$$

$$\frac{\partial M}{\partial x} - Q = I_1 \frac{\partial^2 u}{\partial t^2} + I_2 \frac{\partial^2 \varphi}{\partial t^2} \tag{17c}$$

Under the following boundary conditions

$$N = 0 \quad \text{or} \quad u = 0 \quad \text{at} \quad x = 0 \quad \text{and} \quad x = L \tag{18a}$$

$$Q = 0 \quad \text{or} \quad w = 0 \quad \text{at} \quad x = 0 \quad \text{and} \quad x = L \tag{18b}$$

$$M = 0 \quad \text{or} \quad \varphi = 0 \quad \text{at} \quad x = 0 \quad \text{and} \quad x = L \tag{18c}$$

2.3 The nonlocal elasticity model for FG nanobeam

Based on Eringen nonlocal elasticity model (Eringen 1983), the stress at a reference point x in a body is considered as a function of strains of all points in the near region. This assumption is agreement with experimental observations of atomic theory and lattice dynamics in phonon scattering in which for a homogeneous and isotropic elastic solid the nonlocal stress-tensor components σ_{ij} at any point x in the body can be expressed as

$$\sigma_{ij}(x) = \int_{\Omega} \alpha(|x'-x|,\tau) t_{ij}(x') d\Omega(x') \tag{19}$$

where $t_{ij}(x')$ are the components of the classical local stress tensor at point x which are related to the components of the linear strain tensor ε_{kl} by the conventional constitutive relations for a Hookean material, i.e.

$$t_{ij} = C_{ijkl}\varepsilon_{kl} \tag{20}$$

The meaning of Eq. (19) is that the nonlocal stress at point x is the weighted average of the local stress of all points in the neighborhood of x, the size of which is related to the nonlocal kernel $\alpha(|x'-x|,\tau)$. Here $|x'-x|$ is the Euclidean distance and τ is a constant given by

$$\tau = \frac{e_0 a}{l} \tag{21}$$

which represents the ratio between a characteristic internal length, a (such as lattice parameter, C–C bond length and granular distance) and a characteristic external one, l (e.g., crack length, wavelength) trough an adjusting constant, e_0, dependent on each material. The magnitude of e_0 is determined experimentally or approximated by matching the dispersion curves of plane waves with those of atomic lattice dynamics. According to Eringen (1983), for a class of physically admissible kernel $\alpha(|x'-x|,\tau)$ it is possible to represent the integral constitutive relations given by Eq. (19) in an equivalent differential form as

$$(1-(e_0 a)^2 \nabla^2)\sigma_{kl} = t_{kl} \tag{22}$$

where ∇^2 is the Laplacian operator. Thus, the scale length $e_0 a$ takes into account the size effect on the response of nanostructures. For an elastic material in the one dimensional case, the nonlocal constitutive relations may be simplified as

$$\sigma_{xx} - (e_0 a)^2 \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E\varepsilon_{xx} \tag{23}$$

$$\sigma_{xz} - (e_0 a)^2 \frac{\partial^2 \sigma_{xz}}{\partial x^2} = G\gamma_{xz} \tag{24}$$

where σ and ε are the nonlocal stress and strain, respectively. E is the Young's modulus, $G = E / 2(1 + v)$ is the shear modulus. For Timoshenko nonlocal FG beam, Eqs. (23) and (24) can be rewritten as

$$\sigma_{xx} - \mu \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E(z)\varepsilon_{xx} \tag{25}$$

$$\sigma_{xz} - \mu \frac{\partial^2 \sigma_{xz}}{\partial x^2} = G(z)\gamma_{xz} \tag{26}$$

where $(\mu = (e_0 a)^2)$. Integrating Eqs. (25) and (26) over the beam's cross-section area, the force-strain and the moment-strain of the nonlocal Timoshenko FG beam theory can be obtained as follows

$$N - \mu \frac{\partial^2 N}{\partial x^2} = A_{xx} \frac{\partial u}{\partial x} + B_{xx} \frac{\partial \varphi}{\partial x} \tag{27}$$

$$M - \mu \frac{\partial^2 M}{\partial x^2} = B_{xx} \frac{\partial u}{\partial x} + D_{xx} \frac{\partial \varphi}{\partial x} \tag{28}$$

$$Q - \mu \frac{\partial^2 Q}{\partial x^2} = C_{xz} (\frac{\partial w}{\partial x} + \varphi) \tag{29}$$

In which the cross-sectional rigidities are defined as follows

$$(A_{xx}, B_{xx}, D_{xx}) = \int_A E(z,T)(1, z, z^2) dA \tag{30}$$

$$C_{xz} = K_s \int_A G(z) \, dA \tag{31}$$

where $K_s = 5/6$ is the shear correction factor. The explicit relation of the nonlocal normal force can be derived by substituting for the second derivative of N from Eq. (17a) into Eq. (27) as follows

$$N = A_{xx} \frac{\partial u}{\partial x} + B_{xx} \frac{\partial \varphi}{\partial x} + \mu(I_0 \frac{\partial^3 u}{\partial x \partial t^2} + I_1 \frac{\partial^3 \varphi}{\partial x \partial t^2}) \tag{32}$$

Also the explicit relation of the nonlocal bending moment can be derived by substituting for the second derivative of M from Eq. (17c) into Eq. (28) as follows

$$M = B_{xx} \frac{\partial u}{\partial x} + D_{xx} \frac{\partial \varphi}{\partial x} + \mu(I_0 \frac{\partial^2 w}{\partial t^2} + I_1 \frac{\partial^3 u}{\partial x \partial t^2} + I_2 \frac{\partial^3 \varphi}{\partial x \partial t^2} + N^T \frac{\partial^2 w}{\partial x^2}) \tag{33}$$

By substituting for the second derivative of Q from Eq. (17b) into Eq. (29), the following expression for the nonlocal shear force is derived

$$Q = C_{xz} (\frac{\partial w}{\partial x} + \varphi) + \mu(I_0 \frac{\partial^3 w}{\partial x \partial t^2} + N^T \frac{\partial^3 w}{\partial x^3}) \tag{34}$$

The nonlocal governing equations of Timoshenko FG nanobeam in terms of the displacement can be drieved by substituting for N, M and Q from Eqs. (32)-(34), respectively, into Eq. (17) as follows

$$A_{xx}\frac{\partial^2 u}{\partial x^2}+B_{xx}\frac{\partial^2 \varphi}{\partial x^2}+\mu\left(I_0\frac{\partial^4 u}{\partial t^2 \partial x^2}+I_1\frac{\partial^4 \varphi}{\partial t^2 \partial x^2}\right)-I_0\frac{\partial^2 u}{\partial t^2}-I_1\frac{\partial^2 \varphi}{\partial t^2}=0 \tag{35a}$$

$$C_{xz}(\frac{\partial^2 w}{\partial x^2}+\frac{\partial \varphi}{\partial x})+\mu(N^T\frac{\partial^4 w}{\partial x^4}+I_0\frac{\partial^4 w}{\partial t^2 \partial x^2})-N^T\frac{\partial^2 w}{\partial x^2}-I_0\frac{\partial^2 w}{\partial t^2}=0 \tag{35b}$$

$$B_{xx}\frac{\partial^2 u}{\partial x^2}+D_{xx}\frac{\partial^2 \varphi}{\partial x^2}-C_{xz}(\frac{\partial w}{\partial x}+\varphi)+\mu(I_1\frac{\partial^4 u}{\partial t^2 \partial x^2}+I_2\frac{\partial^4 \varphi}{\partial t^2 \partial x^2})-I_1\frac{\partial^2 u}{\partial t^2}-I_2\frac{\partial^2 \varphi}{\partial t^2}=0 \tag{35c}$$

2.4 Non-uniform temperature distribution

In this study, nonlinear temperature rise across the thickness is assumed. The steady-state one-dimensional heat conduction equation with the known temperature boundary conditions on bottom and top surfaces of the FG nanobeam can be obtained by solving the following equation (Zhang 2013)

$$-\frac{d}{dz}\left(\kappa(z,T)\frac{dT}{dz}\right)=0 \qquad T\left(\frac{h}{2}\right)=T_c, \ \ T\left(-\frac{h}{2}\right)=T_m \tag{36}$$

Solving this equation via polynomial series and taking the sufficient terms to assure the convergence, yields the temperature distribution across the nanobeam thickness as

$$T=T_m+\Delta T\left[\sum_{i=0}^{\infty}\frac{(-1)^i}{pi+1}\left(\frac{\kappa_c-\kappa_m}{\kappa_m}\right)^i\right]^{-1}\sum_{i=0}^{\infty}\left[\frac{(-1)^i}{pi+1}\left(\frac{1}{2}+\frac{z}{h}\right)^{pi+1}\left(\frac{\kappa_c-\kappa_m}{\kappa_m}\right)^i\right] \tag{37}$$

where $\Delta T=T_c-T_m$.

3. Solution procedures

Here, based on the Navier type method, analytical solutions of the governing equations for free vibration and thermal buckling of a simply supported FG nanobeam is presented. The displacement functions are expressed as product of undetermined coefficients and known trigonometric functions to satisfy the governing equations and the conditions at $x = 0$, L. The following displacement fields are assumed to be of the form

$$u(x,t)=\sum_{n=1}^{\infty}U_n\cos(\frac{n\pi}{L}x)e^{i\omega_n t} \tag{38}$$

$$w(x,t)=\sum_{n=1}^{\infty}W_n\sin(\frac{n\pi}{L}x)e^{i\omega_n t} \tag{39}$$

$$\varphi(x,t) = \sum_{n=1}^{\infty} \varphi_n \cos\left(\frac{n\pi}{L}x\right)e^{i\omega_n t} \tag{40}$$

where (U_n, W_n, φ_n) are the unknown Fourier coefficients to be determined for each n value. Boundary conditions for simply supported beam are as Eq. (41)

$$u(0) = 0 \;,\; \frac{\partial u}{\partial x}(L) = 0$$
$$w(0) = w(L) = 0 \;,\; \frac{\partial \varphi}{\partial x}(0) = \frac{\partial \varphi}{\partial x}(L) = 0 \tag{41}$$

Substituting Eqs. (38)-(40) into Eqs. (35a)-(35c) respectively, leads to Eqs. (42)-(44)

$$\left(-A_{xx}\left(\frac{n\pi}{L}\right)^2 + I_0\left(1+\mu\left(\frac{n\pi}{L}\right)^2\right)\omega_n^2\right)U_n + \left(-B_{xx}\left(\frac{n\pi}{L}\right)^2 + I_1\left(1+\mu\left(\frac{n\pi}{L}\right)^2\right)\omega_n^2\right)\phi_n = 0 \tag{42}$$

$$\left(-C_{xz}\left(\frac{n\pi}{L}\right)^2 + I_0\left(1+\mu\left(\frac{n\pi}{L}\right)^2\right)\omega_n^2 + N^T\left(\frac{n\pi}{L}\right)^2\left(1+\mu\left(\frac{n\pi}{L}\right)^2\right)\right)W_n - C_{xz}\left(\frac{n\pi}{L}\right)\phi_n = 0 \tag{43}$$

$$\left(-B_{xx}\left(\frac{n\pi}{L}\right)^2 + I_1\left(1+\mu\left(\frac{n\pi}{L}\right)^2\right)\omega_n^2\right)U_n$$
$$+\left(-D_{xx}\left(\frac{n\pi}{L}\right)^2 - C_{xz} + I_2\left(1+\mu\left(\frac{n\pi}{L}\right)^2\right)\omega_n^2\right)\phi_n - C_{xz}\left(\frac{n\pi}{L}\right)W_n = 0 \tag{44}$$

By setting the determinant of the coefficient matrix of the above equations, the analytical solutions can be obtained from the following equations

$$\left\{([K] + \Delta T[K_T]) - \omega^2[M]\right\}\begin{Bmatrix} U_n \\ W_n \\ \phi_n \end{Bmatrix} = 0 \tag{45}$$

where $[K]$ and $[K_T]$ are stiffness matrix and the coefficient matrix of temperature change, respectively, and $[M]$ is the mass matrix. By setting this polynomial to zero, we can find natural frequencies ω_n and critical buckling temperature ΔT_{cr}.

4. Numerical results and discussions

This section is dedicated to discuss about the effects of nonlocal parameter as well as the power law index, temperature change and thickness ratios on the thermal buckling and vibration behavior of FG nanobeam based on the nonlocal elasticity theory. For this purpose, the FG nanobeam considered here consist of Steel (SUS304) and Silicon nitride (Si_3N_4) with thermo-mechanical material properties listed in Table 1. The bottom surface of the beam is pure Steel, whereas the top surface of the beam is pure Silicon nitride. The beam geometry has the following dimensions: *L*

(length) = 10 nm, b (width) = 1 nm and h (thickness) = varied. It is assumed that the temperature increase in metal surface to reference themperature T_0 of the FG nanobeam is $T_m - T_0 = 5K$ (Kiani and Eslami 2013). Relation described in Eq. (46) are performed in order to calculate the non-dimensional natural frequencies

$$\hat{\omega} = \omega L^2 \sqrt{\rho_c A / EI_c} \qquad (46)$$

where $I = bh^3/12$ is the moment of inertia of the cross section of the beam. To evaluate accuracy of the natural frequencies predicted by the present method, the non-dimensional natural frequencies of simply supported FG nanobeam with various nonlocal parameters previously analyzed by Navier method are reexamined. Table 2 compares the results of the present study and the results presented by Rahmani and Pedram (2014) which has been obtained by analytical method for FG nanobeam with different nonlocal parameters (varying from 0 to 5). The reliability of the presented method and procedure for FG nanobeam may be concluded from Table 2; where the results are in an excellent agreement as values of non-dimensional fundamental frequency are consistent with presented analytical solution.

After making certain about the credibility of the numerical results of current study, the effects of different parameters such as aspect ratio, nonlocality parameter and gradient index on the thermal buckling of FG nanobeam are investigated. In Table 3 critical buckling temperature of the simply supported FG nanobeams are presented for various values of the gradient index ($p = 0, 0.2, 0.5, 1, 2, 5$), nonlocal parameters ($\mu = 0, 1, 2, 3, 4$) and three different values of aspect ratio ($L/h = 40, 50, 60$) based on Navier solution method. It is indicated that increasing of nonlocal parameter leads to lower critical buckling temperature difference. In other words, by increasing the influence of small scale effect, the stiffness of FG nanobeams decreases. It can be also observed that the critical buckling temperature difference of FG nanobeams decreases by increasing the value of L/h ratio and this behavior is the same for all values of power index. Moreover, an increase in the nonlocal parameter leads to the decrease of the buckling temperature and it is more tangible for the nanobeams with lower values of L/h ratio. Also, it can be seen that an increase in the power law index lead to lower critical temperature by decrease the stiffness of nanobeam.

In order to highlight the effect of the nonlocal parameter and aspect ratio on the thermal buckling behavior of the FG nanobeams, the critical buckling temperature against the L/h ratio with the assumption of $p = 0.5, 2$, is illustrated in Fig. 2. This figure indicates that the decrease of

Table 2 Comparison of the nondimensional fundamental frequency for a S-S FG nanobeam with various gradient indexes when $L/h = 50$

μ	$p = 0$		$p = 0.2$		$p = 1$		$p = 5$	
	Rahmani and Pedram (2014)	Present study	Rahmani and Pedram (2014)	Present study	Rahmani and Pedram (2014)	Present study	Rahmani and Pedram (2014)	Present study
0	9.8631	9.86315733	8.6895	8.68954599	6.9917	6.99174004	5.9389	5.93894397
1	9.4097	9.40973040	8.2901	8.29007206	6.6703	6.67031728	5.6659	5.66592012
2	9.0136	9.01358936	7.9411	7.94106762	6.3895	6.38950303	5.4274	5.42739007
3	8.6636	8.66360601	7.6327	7.63272858	6.1414	6.14140878	5.2166	5.21665314
4	8.3515	8.35146095	7.3577	7.35772548	5.9201	5.92013713	5.0287	5.02869994
5	8.0708	8.07079327	7.1104	7.11045428	5.7212	5.72117899	4.8597	4.85970034

Table 3 Material graduation and aspect ratio effect on the critical buckling temperature ΔT_{cr} [K] of a S-S FG nanobeam with different nonlocality parameters

| μ	L/h	Gradient index					
		0	0.2	0.5	1	2	5
	40	127.3340	111.4220	98.1295	87.4235	78.9567	70.9984
0	50	77.9423	67.5578	58.9611	52.0963	46.7232	41.7252
	60	51.0893	43.7102	37.6665	32.8897	29.1981	25.8089
	40	114.9980	100.4660	88.3464	78.5996	70.9053	63.6861
1	50	70.0424	60.5421	52.6965	46.4459	41.5674	37.0425
	60	45.6016	38.8368	33.3148	28.9647	25.6165	22.5559
	40	104.6950	91.3161	80.1761	71.2304	64.1812	57.5792
2	50	63.4449	54.6830	47.4646	41.7270	37.2614	33.1317
	60	41.0186	34.7668	29.6805	25.6867	22.6254	19.8392
	40	95.9606	83.5598	73.2501	64.9835	58.4811	52.4023
3	50	57.8521	49.7163	43.0296	37.7267	33.6113	29.8165
	60	37.1336	31.3167	26.5997	22.9079	20.0898	17.5362
	40	88.4628	76.9011	67.3042	59.6207	53.5877	47.9581
4	50	53.0508	45.4524	39.2222	34.2926	30.4777	26.9705
	60	33.7984	28.3548	23.9549	20.5224	17.9130	15.5592
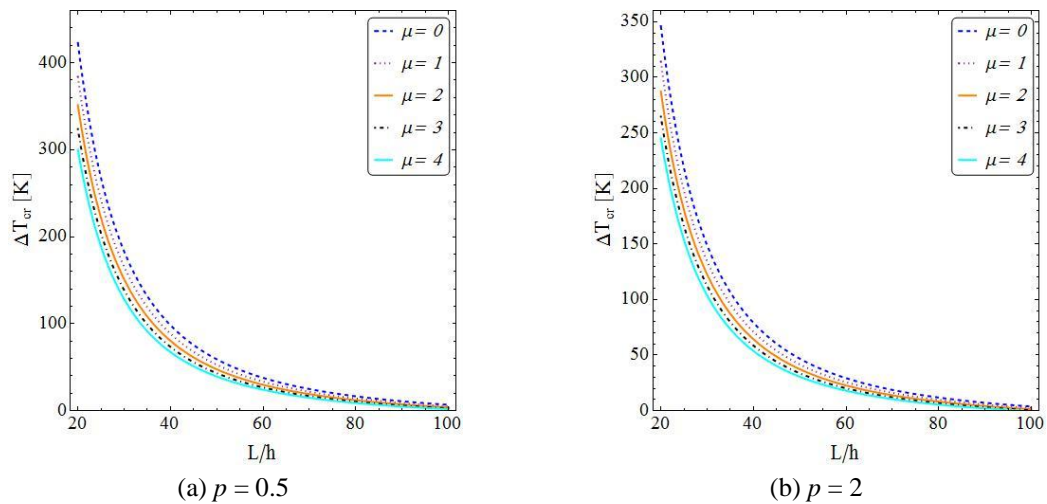
Fig. 2 The variation of the critical buckling temperature of S-S FG nanobeam with aspect ratios and nonlocality parameters for different material graduations

the critical buckling temperature resulting from the increase of the nonlocal parameter occurs in all the L/h ratios so that at the lower L/h ratios, the rate of variations is higher. It is also obvious that the effect of the nonlocal parameter becomes more pronounced for lower L/h ratios. This illustrates the size-dependence of the nanoscale beams which plays an important role in the nonlocal theory.

Table 4 Temperature and material graduation effect on first three dimensionless frequency of a S-S FG
nanobeam with different nonlocality parameters ($L/h = 20$)

μ	$\hat{\omega}_i$	$\Delta T = 10[K]$				$\Delta T = 30[K]$				$\Delta T = 60[K]$			
		Gradient index				Gradient index				Gradient index			
		0	0.2	1	5	0	0.2	1	5	0	0.2	1	5
	1	9.6481	7.7976	5.7716	4.6905	9.4632	7.6281	5.6135	4.5334	9.1788	7.3665	5.3678	4.2869
0	2	38.6707	31.3183	23.2774	18.9846	38.4886	31.1520	23.1233	18.8321	38.2137	30.9008	22.8902	18.6010
	3	85.5836	69.3213	51.5487	42.0461	85.4002	69.1540	51.3939	41.8928	85.1245	68.9024	51.1608	41.6618
	1	9.1873	7.4229	5.4910	4.4600	8.9929	7.2446	5.3246	4.2944	8.6932	6.9687	5.0648	4.0334
1	2	32.6830	26.4612	19.6563	16.0230	32.4673	26.2641	19.4735	15.8420	32.1410	25.9657	19.1961	15.5666
	3	62.1630	50.3359	37.4093	30.4970	61.9103	50.1053	37.1957	30.2853	61.5294	49.7575	36.8730	29.9649
	1	8.7839	7.0948	5.2451	4.2580	8.5804	6.9081	5.0707	4.0842	8.2658	6.6181	4.7972	3.8087
2	2	28.7999	23.3105	17.3059	14.0997	28.5548	23.0865	17.0980	13.8937	28.1833	22.7463	16.7814	13.5789
	3	51.1661	41.4188	30.7643	25.0663	50.8588	41.1382	30.5042	24.8083	50.3944	40.7138	30.1098	24.4162
	1	8.4269	6.8043	5.0273	4.0788	8.2146	6.6094	4.8450	3.8971	7.8853	6.3057	4.5580	3.6074
3	2	26.0190	21.0533	15.6212	12.7204	25.7475	20.8050	15.3906	12.4917	25.3348	20.4270	15.0381	12.1405
	3	44.4503	35.9715	26.7025	21.7451	44.0963	35.6481	26.4025	21.4471	43.5599	35.1574	25.9458	20.9923
	1	8.1079	6.5446	4.8325	3.9186	7.8870	6.3417	4.6426	3.7290	7.5434	6.0246	4.3422	3.4251
4	2	23.9000	19.3329	14.3363	11.6679	23.6042	19.0623	14.0847	11.4181	23.1533	18.6489	13.6986	11.0329
	3	39.8026	32.2004	23.8890	19.4432	39.4069	31.8388	23.5531	19.1094	38.8057	31.2884	23.0401	18.5975

In order to investigate the vibration characteristics of the FG nanobeam under nonlinear temperature rise, the first three non-dimensional fundamental frequencies of simply-supported FG nanobeam is presented in Table 4, which figures out the effect of nonlocal parameter (varying from 0 to 4 (nm)^2), gradient index (varying from 0 to 5) and three different values of linear temperature changes ($\Delta T = 10, 30, 60$) for $L/h = 20$ on the natural frequency characteristics of FG nanobeam.

First of all, when the two parameters vanish ($\mu = 0$ and $p = 0$) the classical isotropic beam theory is rendered. Furthermore, the effects of temperature change, nonlocal parameter and gradient indexes on the dimensionless frequencies are presented in this table. Also, there is no rigorous study made on estimating the value of small scale to simulate mechanical behaviour of functionally graded micro/nanobeams. Hence all researchers who worked on size-dependent mechanical behaviour of FG nanobeams based on the nonlocal elasticity method investigated the effect of small scale parameter on mechanical behaviour of FG nanobeams by changing the value of the small scale parameter. The nonlocal parameter, $\mu = (e_0 a)^2$, is experimentally obtained for various materials. Therefore, in the present study, a conservative estimate of the nonlocal parameter is in the range of 0-4 (nm)^2 (Rahmani and Pedram 2014).

As seen in Table 4, the natural frequencies decreases for increasing values of the power index from 0 up to 5. This is due to the fact that for large values of power law index, the material properties of the nanobeam become similar to the material properties of a metal with higher thermal expansion coefficients in comparison to the ceramic. Thus, one could easily control the frequencies and critical buckling temperature of the FG nanobeams by tuning the power index. Also, it is revealed that the major effect of power law index is for $p < 5$. Whilst the variations of

natural frequencies with respect to p for $p > 5$ are not significant when compared to $p < 5$. However, the increasing of nonlocal parameter causes the decreasing in fundamental frequency, at a constant material graduation index. In addition, it is seen that the first three dimensionless natural frequencies decrease by increasing temperature change and it can be stated that temperature change has a significant effect on the dimensionless natural frequencies, especially for lower mode numbers.

Variations of the first three dimensionless natural frequencies of the simply supported FG nanobeams with respect to temperature changes for different values of gradient indexes and nonlocal parameters are depicted in Figs. 3-5, respectively. It is seen from the figures that the
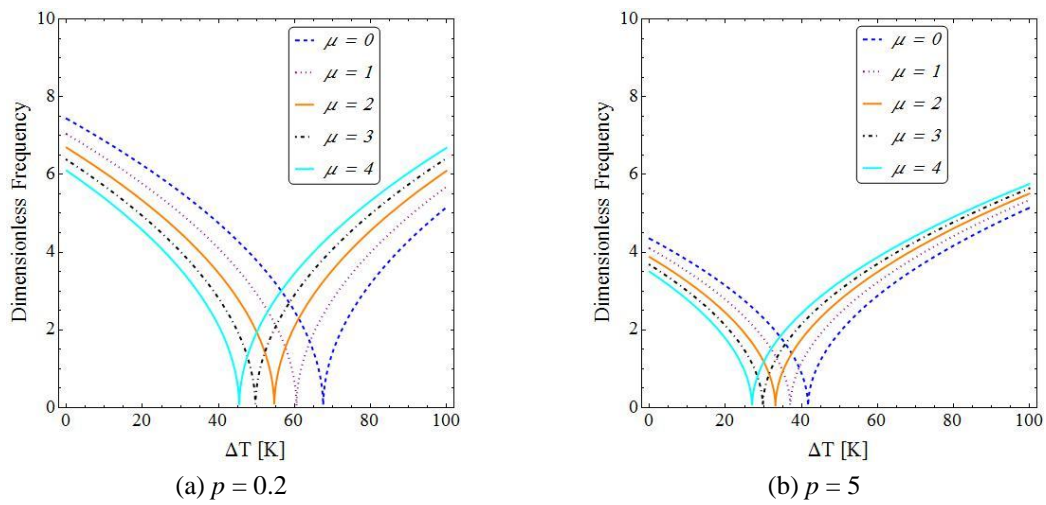

(a) $p = 0.2$ (b) $p = 5$

Fig. 3 Variations of the first dimensionless natural frequency of the S-S FG nanobeam with respect to temperature change for different values of nonlocal parameters and gradient indexes ($L/h = 50$)
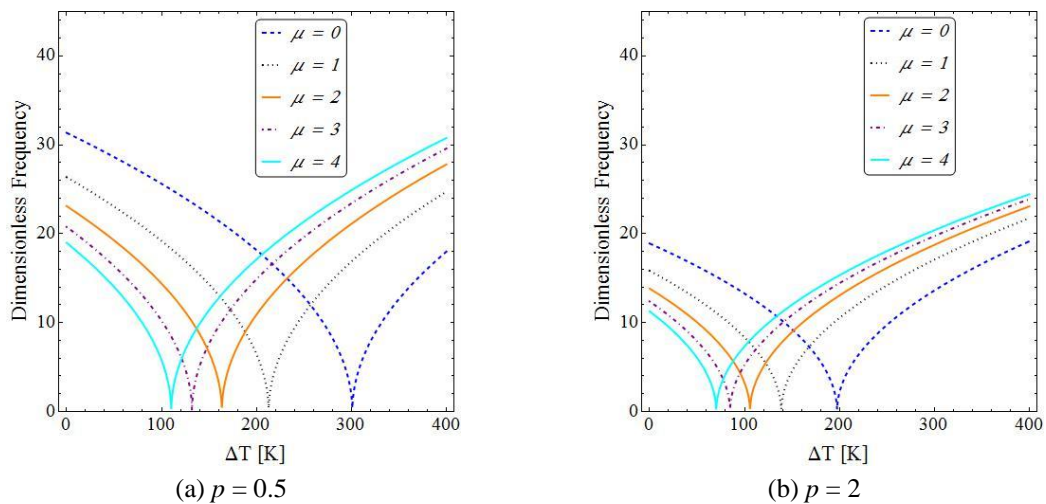

(a) $p = 0.5$ (b) $p = 2$

Fig. 4 Variations of the second dimensionless natural frequency of the S-S FG nanobeam with respect to temperature change for different values of nonlocal parameters and gradient indexes ($L/h = 50$)
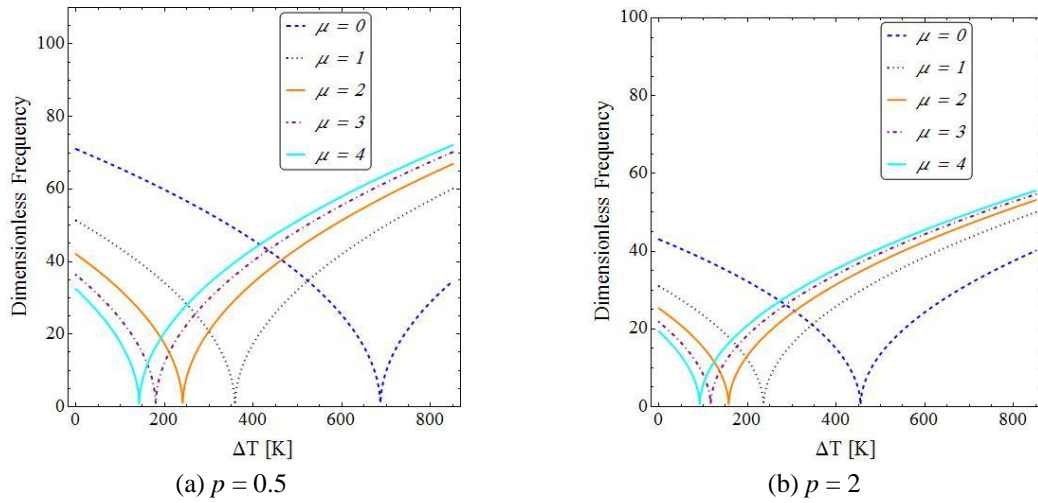
(a) $p = 0.5$ (b) $p = 2$

Fig. 5 Variations of the third dimensionless natural frequency of the S-S FG nanobeam with respect to temperature change for different values of nonlocal parameters and gradient indexes ($L/h = 50$)
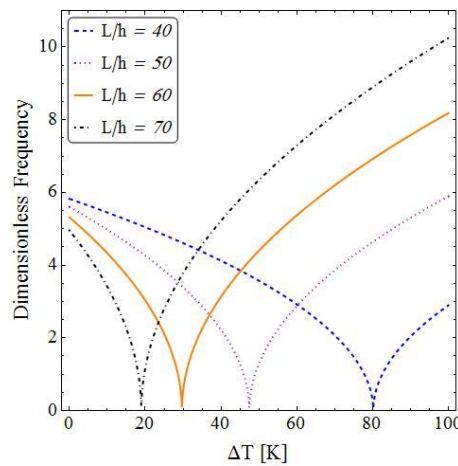

Fig. 6 Variations of the first dimensionless natural frequency of the S-S FG nanobeam with respect to temperature change for different values of aspect ratios ($p = 0.5$, $\mu = 2$)

fundamental frequency of FG nanobeam decreases with the increase of temperature until it approaches to the critical buckling temperature. This is due to the reduction in total stiffness of the beam, since geometrical stiffness decreases when temperature rises. Frequency reaches to zero at the critical temperature point. Also, the increase in temperature yields in higher frequency after the branching point. One important observation within the range of temperature before the critical temperature, it is seen that the FG nanobeams with lower value of gradient index (higher percentage of ceramic phase) usually provide larger values of the frequency results. However, this behavior is opposite in the range of temperature beyond the critical temperature. It is also observable that the branching point of the FG nanobeam is postponed by consideration of the lower gradient indexes due to the fact that the lower gradient indexes result in the increase of

stiffness of the structure. It can be also seen that regardless of the values of power index, the natural frequencies are considerably reduced by increasing the nonlocal parameter. This implies that the nonlocal effects make the stiffness of the FG nanobeam diminish and so, the small scale impact must be accounted.

Depicted in Fig. 6 is the influences of *L/h* ratio and temperature changes on dimensionless natural frequency of FG nanobeams at $p = 0.5$, $\mu = 2$. As expected, the increase in aspect ratio results in reduction of natural frequency of temperature-dependent FG nanobeams. Also, an increase in the temperature change leads to the decrease of the dimensionless natural frequency.

5. Conclusions

In the present research, the study of thermal buckling and vibrational behavior of the temperature-dependent FG nanobeams subjected to nonlinear temperature distribution through the thickness direction was carried out based on the nonlocal elasticity theory accounting for the small scale effects. The power law distribution is assumed for the variation of the material properties in the nanobeam thickness. Thermo-mechanical properties of the FG nanobeams are assumed to be functions of both temperature and thickness. Eringen's theory of nonlocal elasticity together with Timoshenko beam theory is used to model the nanobeam. The governing differential equations and related boundary conditions in thermal environment are derived by implementing Hamilton's principle. Accuracy of the results is examined using available data in the literature. After performing comparison studies, parametric studies are done to investigate the influences of nonlocal parameter, gradient index, mode number, nonlinear temperature rise and aspect ratio on the critical buckling temperature and natural frequencies of FG nanobeams.

It is concluded that various factors such as nonlocal parameter, gradient index, temperature-dependent material properties, thermal environment and aspect ratio play important roles in buckling and vibration behavior of FG nanobeams. It is illustrated that presence of nonlocality leads to reduction in natural frequency and buckling temperature. This behavior was more tangible for the nanobeams with lower values of *L/h* ratio. Moreover it is revealed that critical buckling temperature decreases with the increase in aspFFPect ratio and this behavior is the same for all values of power index. It is also observed that the fundamental frequency decreases with the increase in temperature and tends to the minimum point closing to zero at the critical temperature. This decrease in frequency with thermal load is attributed to the fact that the thermally induced compressive stress weakens the beam stiffness. However, after the critical temperature region, the fundamental frequency increases with the increment of temperature. Also, it is concluded that an increase in the power law index lead to lower critical buckling temperature and natural frequencies by decrease the stiffness of nanobeam.

References

Alizada, A.N., Sofiyev, A.H. and Kuruoglu, N. (2012), "Stress analysis of a substrate coated by nanomaterials with vacancies subjected to uniform extension load", *Acta Mechanica*, **223**(7), 1371-1383.

Ansari, R. and Norouzzadeh, A. (2016), "Nonlocal and surface effects on the buckling behavior of functionally graded nanoplates: An isogeometric analysis", *Phys. E: Low-dimens. Syst. Nanostruct.*, **84**, 84-97.

Ansari, R., Gholami, R. and Sahmani, S. (2011), "Free vibration analysis of size-dependent functionally

graded microbeams based on the strain gradient Timoshenko beam theory", *Compos. Struct.*, **94**(1), 221-228.

Aydogdu, M. (2009), "A general nonlocal beam theory: its application to nanobeam bending, buckling and vibration", *Phys. E*: *Low-dimens. Syst. Nanostruct.*, **41**(9), 1651-1655.

Bafekrpour, E., Simon, G.P., Habsuda, J., Naebe, M., Yang, C. and Fox, B. (2012), "Fabrication and characterization of functionally graded synthetic graphite/phenolic nanocomposites", *Mater. Sci. Eng.: A*, **545**, 123-131.

Carbonari, R.C., Silva, E.C. and Paulino, G.H. (2009), "Multi-actuated functionally graded piezoelectric micro-tools design: A multiphysics topology optimization approach", *Int. J. Numer. Methods Eng.*, **77**(3), 301-336.

Daneshmehr, A., Rajabpoor, A. and Hadi, A. (2015), "Size dependent free vibration analysis of nanoplates made of functionally graded materials based on nonlocal elasticity theory with high order theories", *Int. J. Eng. Sci.*, **95**, 23-35.

Ebrahimi, F. and Barati, M.R. (2016), "Temperature distribution effects on buckling behavior of smart heterogeneous nanosize plates based on nonlocal four-variable refined plate theory", *Int. J. Smart Nano Mater.*, **7**(3), 119-143.

Ebrahimi, F. and Rastgoo, A. (2008), "Free vibration analysis of smart annular FGM plates integrated with piezoelectric layers", *Smart Mater. Struct.*, **17**(1), 015044.

Ebrahimi, F. and Salari, E. (2015), "Size-dependent thermo-electrical buckling analysis of functionally graded piezoelectric nanobeams", *Smart Mater. Struct.*, **24**(12), p.125007.

Ebrahimi, F., Rastgoo, A. and Kargarnovin, M.H. (2008), "Analytical investigation on axisymmetric free vibrations of moderately thick circular functionally graded plate integrated with piezoelectric layers", *J. Mech. Sci. Technol.*, **22**(6), 1058-1072.

Ebrahimi, F., Naei, M.H. and Rastgoo, A. (2009), "Geometrically nonlinear vibration analysis of piezoelectrically actuated FGM plate with an initial large deformation", *J. Mech. Sci. Technol.*, **23**(8), 2107-2124.

Ebrahimi, F., Ghasemi, F. and Salari, E. (2016), "Investigating thermal effects on vibration behavior of temperature-dependent compositionally graded Euler beams with porosities", *Meccanica*, **51**(1), 223-249.

Ebrahimi, F., Barati, M.R. and Haghi, P. (2017), "Thermal effects on wave propagation characteristics of rotating strain gradient temperature-dependent functionally graded nanoscale beams", *J. Thermal Stress.*, **40**(5), 535-547.

Eltaher, M.A., Emam, S.A. and Mahmoud, F.F. (2012), "Free vibration analysis of functionally graded size-dependent nanobeams", *Appl. Math. Computat.*, **218**(14), 7406-7420.

Eringen, A.C. (1983), "On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves", *J. Appl. Phys.*, **54**(9), 4703-4710.

Fu, Y., Du, H. and Zhang, S. (2003), "Functionally graded TiN/TiNi shape memory alloy films", *Mater. Lett.*, **57**(20), 2995-2999.

Iijima, S. (1991), "Helical microtubules of graphitic carbon", *Nature*, **354**(6348), p. 56.

Kiani, Y. and Eslami, M.R. (2013), "An exact solution for thermal buckling of annular FGM plates on an elastic medium", *Compos. Part B: Eng.*, **45**(1), 101-110.

Lee, Z., Ophus, C., Fischer, L.M., Nelson-Fitzpatrick, N., Westra, K.L., Evoy, S., Radmilovic, V., Dahmen, U. and Mitlin, D. (2006), "Metallic NEMS components fabricated from nanocomposite Al–Mo films", *Nanotechnology*, **17**(12), p. 3063.

Lü, C.F., Lim, C.W. and Chen, W.Q. (2009), "Size-dependent elastic behavior of FGM ultra-thin films based on generalized refined theory", *Int. J. Solids Struct.*, **46**(5), 1176-1185.

Lun, F.Y., Zhang, P., Gao, F.B. and Jia, H.G. (2006), "Design and fabrication of micro-optomechanical vibration sensor", *Microfabr. Technol.*, **120**(1), 61-64.

Lv, Z., Qiu, Z., Zhu, J., Zhu, B. and Yang, W. (2018), "Nonlinear free vibration analysis of defective FG nanobeams embedded in elastic medium", *Compos. Struct.*.

Mahmoudpour, E., Hosseini-Hashemi, S.H. and Faghidian, S.A. (2018), "Nonlinear vibration analysis of FG

nano-beams resting on elastic foundation in thermal environment using stress-driven nonlocal integral model", *Appl. Math. Model.*, **57**, 302-315.

Rahaeifard, M., Kahrobaiyan, M.H. and Ahmadian, M.T. (2009), "Sensitivity analysis of atomic force microscope cantilever made of functionally graded materials", *ASME 2009 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, January, pp. 539-544.

Rahmani, O. and Pedram, O. (2014), "Analysis and modeling the size effect on vibration of functionally graded nanobeams based on nonlocal Timoshenko beam theory", *Int. J. Eng. Sci.*, **77**, 55-70.

Şimşek, M. and Yurtcu, H.H. (2013), "Analytical solutions for bending and buckling of functionally graded nanobeams based on the nonlocal Timoshenko beam theory", *Compos. Struct.*, **97**, 378-386.

Sofiyev, A.H. and Kuruoglu, N. (2015), "Buckling of non-homogeneous orthotropic conical shells subjected to combined load", *Steel Compos. Struct., Int. J.*, **19**(1), 1-19.

Tang, Y.G., Liu, Y. and Zhao, D. (2018), "Effects of neutral surface deviation on nonlinear resonance of embedded temperature-dependent functionally graded nanobeams", *Compos. Struct.*, **184**, 969-979.

Thermophysical Properties Research Center (1967), Thermophysical properties of high temperature solid materials; Volume 1, Elements.-Pt. 1. Ed. Yeram Sarkis Touloukian, Macmillan.

Wang, Q. and Liew, K.M. (2007), "Application of nonlocal continuum mechanics to static analysis of micro-and nano-structures", *Phys. Lett. A*, **363**(3), 236-242.

Witvrouw, A. and Mehta, A. (2005), "The use of functionally graded poly-SiGe layers for MEMS applications", In: *Materials Science Forum*, Volume 492, pp. 255-260. Trans Tech Publications.

Zhang, D.G. (2013), "Nonlinear bending analysis of FGM beams based on physical neutral surface and high order shear deformation theory", *Compos. Struct.*, **100**, 121-126.

Zhang, Y.Q., Liu, G.R. and Wang, J.S. (2004), "Small-scale effects on buckling of multiwalled carbon nanotubes under axial compression", *Phys. Rev. B*, **70**(20), p. 205430.

JL

Nomenclature

A	area of the cross section
A_{xx}, B_{xx}, D_{xx}	cross-sectional rigidities
E	Young's modulus
h	thickness of the nanobeam
I	moment of inertia of the cross section
I_0, I_1, I_2	mass moment of inertias
L	length of the nanobeam
M	bending moment
N	axial force
N^T	thermal resultant
p	power-law index
P_C	material properties of the ceramic constituent
P_m	material properties of the metal constituent
$P_0, P_{-1}, P_1, P_2, P_3$	temperature dependent coefficients
T_0	initial temperature
ΔT	temperature change
u	axial displacement
U	strain energy
V	work done by external forces
w	transverse displacement
t	time
T	kinetic energy
ρ	mass density
α	thermal expansion coefficient
σ_{ij}	nonlocal stress-tensor components
ω	natural frequency
$\hat{\omega}$	non-dimensional natural frequency
μ	nonlocal parameter

Highlights

- Thermal buckling and free vibration analysis of Timoshenko nanobeams subjected to nonlinear temperature rise are studied.
- The material properties of the nanobeam vary continuously through the beam thickness according to power-law form and are temperature dependent based on the Touloukian model.
- Nonlocal equations are derived by using the Hamilton's principle and they are solved by applying an analytical solution.
- Effects of nonlocal parameter, nonlinear temperature distribution, geometrical characteristics and power law index on the critical buckling temperatures and natural frequencies of size-dependent FG nanobeams are discussed.

Graphical abstract