

Stability analysis of functionally graded heterogeneous piezoelectric nanobeams based on nonlocal elasticity theory

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Abstract. An analytical solution of the buckling governing equations of functionally graded piezoelectric (FGP) nanobeams obtained by using a developed third-order shear deformation theory is presented. Electro-mechanical properties of FGP nanobeam are supposed to change continuously in the thickness direction based on power-law model. To capture the small size effects, Eringen's nonlocal elasticity theory is adopted. Employing Hamilton's principle, the nonlocal governing equations of a FG nanobeams made of piezoelectric materials are obtained and they are solved using Navier-type analytical solution. Results are provided to show the effect of different external electric voltage, power-law index, nonlocal parameter and slenderness ratio on the buckling loads of the size-dependent FGP nanobeams. The accuracy of the present model is verified by comparing it with nonlocal Timoshenko FG beams. So, this study makes the first attempt for analyzing buckling behavior of higher order shear deformable FGP nanobeams.

Keywords: functionally graded piezoelectric nanobeam; buckling; nonlocal elasticity theory; third-order beam theory

1. Introduction

Functionally graded materials (FGMs) are well known as an alternative materials which are extensively applied in chemical, mechanical, electronic, civil, automotive and optical industries due to possessing supreme mechanical performance compared to classical composite materials. FGMs have microscopically heterogeneous structure and material properties changes with continuous composition gradation from one surface to another.

Nowadays, FGMs have gained its applicability in micro and nano electro-mechanical systems (MEMS/NEMS) which are composed of different structural elements including nanoscale beams and plates. Therefore, in the last decade nano scale structures achieved severe interest by researchers. The classical continuum theory can properly applied in the mechanical analysis of the macroscopic structures, but fails to capture the size effect on the mechanical behaviors on micro/nano structures. To consider the nanoscale influences the classical continuum theory must extended. So this can be achieved through the nonlocal elasticity theory proposed by Eringen

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which supposes that the stress state at a desired point is a function of the strain at all neighbor points of the body.

Many papers, dealing with static and dynamic behavior of FGM nanobeams, have been published recently. Among them, Eltaher *et al.* (2012, 2013a) presented a finite element analysis for free vibration of FG nanobeams using nonlocal Euler-Bernulli beam model. In another study they researched the static and stability behavior of FG nanobeams based on nonlocal continuum theory (Eltaher *et al.* 2013b). Also, based upon nonlocal Timoshenko and Euler-Bernulli beam models, Simsek and Yurtcu (2013) investigated bending and buckling behavior of size-dependent FG nanobeam using analytical method. Nonlinear free vibration of FG nanobeams within the framework of Euler-Bernulli beam model including the von Kármán geometric nonlinearity studied by Sharabiani and Yazdi (2013). Also, forced vibration analysis of FG nanobeams based on the nonlocal elasticity theory and using Navier method for various shear deformation theories studied by Uymaz (2013). Rahmani and Pedram (2014) analyzed the size effects on vibration of FG nanobeams based on nonlocal TBT. Nonlinear free vibration of FG nanobeams with fixed ends, i.e., simply supported-simply supported (SS) and simply supported-clamped (SC), using the nonlocal elasticity within the frame work of EBT with von Kármán type nonlinearity is studied by Nazemnezhad and Hosseini-Hashemi (2014). Also, recently Hosseini-Hashemi *et al.* (2014) investigated free vibration of FG nanobeams with consideration surface effects and piezoelectric field using nonlocal elasticity theory. Most recently Ebrahimi *et al.* (2015, 2016) examined the applicability of differential transformation method in investigations on vibrational characteristics of FG size-dependent nanobeams. In another work, Ebrahimi and Salari (2015a, b) presented a semi-analytical method for vibrational and buckling analysis of FG nanobeams considering the position of neutral axis. An exact solution for the nonlinear forced vibration of FG nanobeams in thermal environment based on surface elasticity theory is presented by Ansari *et al.* (2015). Recently, Rahmani and Jandaghian (2015) presented buckling analysis of FG nanobeams based on a nonlocal third-order shear deformation theory. Also, mechanical responses of FG nanoscale beams using a refined shear deformation beam theory examined by Zemri *et al.* (2015).

Moreover, piezoelectric materials are known as a type of smart structures which can create electricity when exposed to mechanical stresses. Also, they will work in reverse, producing a strain by the application of an electric field. Therefore, the piezoelectrics materials can be used in several fields including micro/nano electromechanical systems, resonators, mechanical and chemical sensors. Until now, several articles have investigated mechanical behavior of FGP beams. Mechanical behavior of a FGP cantilever beam exposed to various loadings is studied by Shi and Chen (2004). They characterized the piezoelectric beam by continuously graded properties for one elastic parameter and the material density. Also, static and vibrational responses of monomorph, bimorph, and multimorph actuators made of FGP materials under a combined thermal-electromechanical load based upon Timoshenko beam model studied by Yang and Xiang (2007). Doroushi *et al.* (2011) investigated the free and forced vibration characteristics of an FGPM beam subjected to thermo-electro-mechanical loads using the higher-order shear deformation beam theory. Kiani *et al.* (2011) analysed buckling behavior of FGM beams with or without surface-bonded piezoelectric layers subjected to both thermal loading and constant voltage. Komijani *et al.* (2013) studied free vibration of FGPM beams with rectangular cross sections under in-plane thermal and electrical excitations in pre/post-buckling regimes. Lezgy-Nazargah *et al.* (2013) suggested an efficient three-noded beam element model for static, free vibration and dynamic response of FGPM beams. Large amplitude free flexural vibration of shear deformable FGM beams with surface-bonded piezoelectric layers subjected to thermo-piezoelectric loadings with

random material properties presented by Shegokar and Lal (2014). Li *et al.* (2014) developed a size-dependent FGP beam model using the modified strain gradient theory and Timoshenko beam theory. Therefore it could be noted that the main deficiency of above-mentioned studies is that the small size effects is not considered in these works. To capture the size effect, recently a parametric study is performed to explore the influences of size-dependent shear deformation on static bending, buckling and free vibration behavior of microbeams based on modified couple stress classical and first shear deformation beam models by Dehrouyeh-Semnani and Nikkhah-Bahrami (2015). They mentioned that the effect of size-dependent shear deformation on mechanical responses of the microbeams has an ascending trend with respect to dimensionless material length scale parameter. Also, size-dependent dynamic stability analysis of microbeams actuated by piezoelectric voltage based on strain gradient theory investigated by Sahmani and Bahrami (2015). Therefore, it is clear that a work to study vibrational responses of FGP nanobeams using a parabolic shear deformation beam theory is not yet published. It can be seen that most of recent works for studying influences of piezoelectric materials on mechanical behavior of FG nanobeams have done based on Euler-Bernoulli (EBT) and Timoshenko beam (TBT) theories. It is well known that Euler-Bernoulli beam model fails to capture the influences of shear deformations and hence the buckling loads and natural frequencies of nanobeams are overestimated. Timoshenko beam model has the potential to capture the influences of shear deformations, but a shear correction factor is required to perfect demonstration of the deformation strain energy. Several higher-order shear deformation theories which are needless of shear correction factors are introduced such as the parabolic shear deformation theory proposed by Reddy (2007), the generalized beam theory proposed by Aydogdu (2009), sinusoidal shear deformation theory of Touratier (1991) and hyperbolic shear deformation presented by Soldatos (1992).

This paper aims to develop a higher order beam model for the buckling analysis of simply supported FG piezoelectric nanobeams. The electro-mechanical material properties of the beam are supposed to be graded in the thickness direction according to the power law distribution. The small size effect is captured using Eringen's nonlocal elasticity theory. Nonlocal governing equations for the buckling of a higher order FG nanobeam have been derived via Hamilton's principle. The most beneficial feature of the present theory is that it accounts for a parabolic variation of the transverse shear strains across the thickness direction and verifies the zero traction boundary conditions on the top and bottom surfaces of the beam without using shear correction factors. Governing equations are solved using Navier type method and various numerical examples are provided investigating the influences of external electric voltage, power-law index, slenderness ratio and nonlocal parameter on buckling behavior of FG piezoelectric nanobeams.

2. Theoretical formulations

2.1 The material properties of FGP nanobeams

Assume a FG nanobeam composed of PZT-4 and PZT-5H piezoelectric materials exposed to an electric potential $\Phi(x, z, t)$, with length L and uniform thickness h , as shown in Fig. 1. The effective material properties of the FGPM nanobeam are supposed to change continuously in the z -axis direction (thickness direction) based on the power-law model. So, the effective electro mechanical material properties of the FGP beam, P , can be stated in the following form (Komijani *et al.* 2013, 2014)

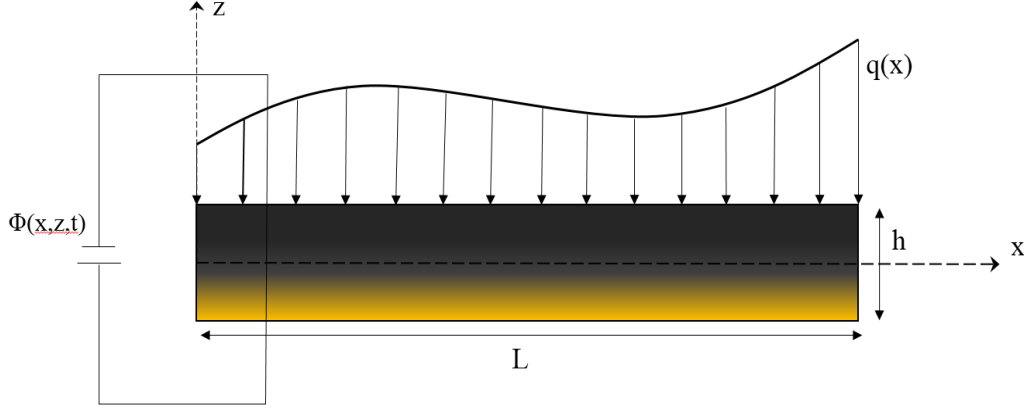


Fig. 1 Configuration of a functionally graded piezoelectric nanobeam

$$P(z) = (P_2 - P_1) \left(\frac{z}{h} + \frac{1}{2} \right)^p + P_1. \quad (1)$$

where P_1 and P_2 denote the material properties of the bottom and top surfaces, respectively, p is power-law exponent which is non-negative and estimates the material distribution through the thickness of the nanobeam and z is the distance from the mid-plane of the graded piezoelectric beam. It must be noted that, the top surface at $z = +h/2$ of FGP nanobeam is assumed PZT-4 rich, whereas the bottom surface ($z = -h/2$) is PZT-5H rich.

2.2 Nonlocal elasticity theory for the piezoelectric materials

Contrary to the constitutive equation of classical elasticity theory, Eringen's nonlocal theory (Eringen and Edelen 1972, Eringen 1972, 1983) notes that the stress state at a point inside a body is regarded to be function of strains of all points in the neighbor regions. For a nonlocal homogeneous piezoelectric solid the basic equations with zero body force may be defined as

$$\begin{aligned} \sigma_{ij}(x) &= \iiint_V \varrho(|x - x'|, \tau) [C_{ijkl} \varepsilon_{kl}(x') - e_{kij} E_k(x')] dv(x'), \\ D_i(x) &= \iiint_V \varrho(|x - x'|, \tau) [e_{ikl} \varepsilon_{kl}(x') + \kappa_{ik} E_k(x')] dv(x'), \end{aligned} \quad (2)$$

where σ_{ij} , ε_{ij} , D_i and E_i denote the stress, strain, electric displacement and electric field components, respectively; C_{ijkl} , e_{kij} and κ_{ik} are elastic, piezoelectric and dielectric constants, respectively; $\varrho(|x - x'|, \tau)$ is the nonlocal kernel function and $|x - x'|$ is the Euclidean distance. $\tau = e_0 a / l$ is defined as scale coefficient, where e_0 is a material constant which is determined experimentally or approximated by matching the dispersion curves of plane waves with those of atomic lattice dynamics; and a and l are the internal and external characteristic length of the nanostructures, respectively. Finally it is possible to represent the integral constitutive relations given by Eq. (2) in an equivalent differential form as

$$\begin{aligned}\sigma_{ij} - (e_0 a)^2 \nabla^2 \sigma_{ij} &= C_{ijkl} \varepsilon_{kl} - e_{kij} E_k, \\ D_i - (e_0 a)^2 \nabla^2 D_i &= e_{ikl} \varepsilon_{kl} + \kappa_{ik} E_k,\end{aligned}\quad (3)$$

where ∇^2 is the Laplacian operator and $e_0 a$ is the nonlocal parameter revealing the size influence on the response of nanostructures.

2.3 Nonlocal FG piezoelectric nanobeam model

Based on parabolic third-order beam theory, the displacement field at any point of the beam is supposed to be in the form

$$\begin{aligned}u_x(x, z) &= u(x) + z\psi(x) - \alpha z^3 \left(\psi + \frac{\partial w}{\partial x} \right), \\ u_z(x, z) &= w(x),\end{aligned}\quad (4)$$

where u and w are displacement components in the mid-plane along the coordinates x and z , respectively, while ψ denotes the total bending rotation of the cross-section.

To satisfy Maxwell's equation in the quasi-static approximation, the distribution of electric potential along the thickness direction is supposed to change as a combination of a cosine and linear variation as follows

$$\Phi(x, z, t) = -\cos(\xi z) \phi(x, t) + \frac{2z}{h} V, \quad (5)$$

where $\xi = \pi/h$. Also, V is the initial external electric voltage applied to the FGP nanobeam; and $\phi(x, t)$ is the spatial function of the electric potential in the x -direction. Considering strain-displacement relationships on the basis of parabolic beam theory, the non-zero strains can be stated as

$$\varepsilon_{xx} = \varepsilon_{xx}^{(0)} + z\varepsilon_{xx}^{(1)} + z^3\varepsilon_{xx}^{(3)}, \quad \gamma_{xz} = \gamma_{xz}^{(0)} + z^2\gamma_{xz}^{(2)}. \quad (6)$$

where

$$\begin{aligned}\varepsilon_{xx}^{(0)} &= \frac{\partial u}{\partial x}, \quad \varepsilon_{xx}^{(1)} = \frac{\partial \psi}{\partial x}, \quad \varepsilon_{xx}^{(3)} = -\alpha \left(\frac{\partial \psi}{\partial x} + \frac{\partial^2 w}{\partial x^2} \right), \\ \gamma_{xz}^{(0)} &= \psi + \frac{\partial w}{\partial x}, \quad \gamma_{xz}^{(2)} = -\beta \left(\psi + \frac{\partial w}{\partial x} \right), \quad \beta = \frac{4}{h^2}.\end{aligned}\quad (7)$$

According to the defined electric potential in Eq. (5), the non-zero components of electric field (E_x, E_z) can be obtained as

$$E_x = -\frac{\partial \Phi}{\partial x} = \cos(\xi z) \frac{\partial \phi}{\partial x}, \quad E_z = -\frac{\partial \Phi}{\partial z} = -\xi \sin(\xi z) \phi - \frac{2}{h} V. \quad (8)$$

The principle of virtual work can be stated in the following form to obtain the governing equations of equilibrium

$$\int_0^t \delta(\Pi_S + \Pi_W) dt = 0, \quad (9)$$

where Π_S is the total strain energy and Π_W is the work done by external applied forces. The first variation of strain energy Π_S can be calculated as:

$$\delta\Pi_S = \int_0^L \int_{-h/2}^{h/2} (\sigma_{xx} \delta\varepsilon_{xx} + \sigma_{xz} \delta\gamma_{xz} - D_x \delta E_x - D_z \delta E_z) dz dx. \quad (10)$$

Inserting Eqs. (6) and (8) into Eq. (10) yields

$$\begin{aligned} \delta\Pi_S = & \int_0^L \left(N\delta\varepsilon_{xx}^{(0)} + M\delta\varepsilon_{xx}^{(1)} + P\delta\varepsilon_{xx}^{(3)} + Q\delta\gamma_{xz}^{(0)} + R\delta\gamma_{xz}^{(2)} \right) dx \\ & + \int_0^L \int_{-h/2}^{h/2} \left[-D_x \cos(\xi z) \frac{\partial\delta\phi}{\partial x} + D_z \xi \sin(\xi z) \delta\phi \right] dz dx. \end{aligned} \quad (11)$$

in which N , M and Q are the axial force, bending moment and shear force resultants, respectively. Relations between the stress resultants and stress components used in Eq. (11) are defined as

$$\begin{aligned} \{N, M, P\} &= \iint_A \sigma_{xx} \{1, z, z^3\} dA, \\ \{Q, R\} &= \iint_A \sigma_{xz} \{1, z^2\} dz. \end{aligned} \quad (12)$$

The first variation of the work done due to electric voltage, $\delta\Pi_W$, can be written in the form:

$$\begin{aligned} \delta\Pi_W = & \int_0^L \left\{ \left[(N_E + N_B) \frac{\partial w}{\partial x} \frac{\partial}{\partial x} + \alpha P \frac{\partial^2}{\partial x^2} + q \right] \delta w + f \delta u \right. \\ & \left. - N \delta\varepsilon_{xx}^{(0)} - \bar{M} \frac{\partial\delta\psi}{\partial x} - \bar{Q} \delta\gamma_{xz}^{(0)} \right\} dx, \end{aligned} \quad (13a)$$

in which $\bar{M} = M - \alpha P$, $\bar{Q} = Q - \beta R$ and $q(x)$ and $f(x)$ denote the transverse and axial distributed loads and N_B is the buckling load and N_E is normal force due to applied electric voltage V which is defined as

$$N_E = - \int_{-h/2}^{h/2} e_{31} \frac{2}{h} V dz. \quad (13b)$$

For a FGPM nanobeam exposed to electro-mechanical loading in the one dimensional case, the nonlocal constitutive relations (3) may be rewritten as

$$\sigma_{xx} - (e_0 a)^2 \frac{\partial^2 \sigma_{xx}}{\partial x^2} = c_{11} \varepsilon_{xx} - e_{31} E_z, \quad (14)$$

$$\sigma_{xz} - (e_0 a)^2 \frac{\partial^2 \sigma_{xz}}{\partial x^2} = c_{55} \gamma_{xz} - e_{15} E_x, \quad (15)$$

$$D_x - (e_0 a)^2 \frac{\partial^2 D_x}{\partial x^2} = e_{15} \gamma_{xz} + \kappa_{11} E_x, \quad (16)$$

$$D_z - (e_0 a)^2 \frac{\partial^2 D_z}{\partial x^2} = e_{31} \varepsilon_{xx} + \kappa_{33} E_z. \quad (17)$$

Inserting Eqs. (11) and (13) in Eq. (9) and integrating by parts, and gathering the coefficients of δu , δw , $\delta \psi$ and $\delta \phi$, the following governing equations are obtained

$$\frac{\partial N}{\partial x} + f = 0, \quad (18)$$

$$\frac{\partial \bar{M}}{\partial x} - \bar{Q} = 0, \quad (19)$$

$$\frac{\partial \bar{Q}}{\partial x} - (N_E + N_B) \frac{\partial^2 w}{\partial x^2} + \alpha \frac{\partial^2 P}{\partial x^2} + q = 0, \quad (20)$$

$$\int_{-h/2}^{h/2} \left[\cos(\xi z) \frac{\partial D_x}{\partial x} + \xi \sin(\xi z) D_z \right] dz = 0. \quad (21)$$

Also, the boundary conditions are

$$\text{Specify } u \text{ or } N \quad (22a)$$

$$\text{Specify } \psi \text{ or } M \quad (22b)$$

$$\text{Specify } w \text{ or } \bar{Q} - (N_E + N_B) \frac{\partial w}{\partial x} + \alpha \frac{\partial P}{\partial x} \quad (22c)$$

$$\text{Specify } \phi \text{ or } \int_{-h/2}^{h/2} [\cos(\xi z) D_x] dz \quad (22d)$$

By integrating Eqs. (14)-(17) over the beam's cross-section area, the force-strain and the moment-strain of the nonlocal third-order FGP beam theory can be obtained as follows

$$N - \mu \frac{\partial^2 N}{\partial x^2} = A_{xx} \frac{\partial u}{\partial x} + (B_{xx} - \alpha E_{xx}) \frac{\partial \psi}{\partial x} - \alpha E_{xx} \frac{\partial^2 w}{\partial x^2} + \bar{A}_{31}^e \phi - N_E, \quad (23)$$

$$M - \mu \frac{\partial^2 M}{\partial x^2} = B_{xx} \frac{\partial u}{\partial x} + (D_{xx} - \alpha F_{xx}) \frac{\partial \psi}{\partial x} - \alpha F_{xx} \frac{\partial^2 w}{\partial x^2} + \bar{B}_{31}^e \phi, \quad (24)$$

$$P - \mu \frac{\partial^2 P}{\partial x^2} = E_{xx} \frac{\partial u}{\partial x} + (F_{xx} - \alpha H_{xx}) \frac{\partial \psi}{\partial x} - \alpha H_{xx} \frac{\partial^2 w}{\partial x^2} + \bar{E}_{31}^e \phi, \quad (25)$$

$$Q - \mu \frac{\partial^2 Q}{\partial x^2} = (A_{xz} - \beta D_{xz}) \left(\frac{\partial w}{\partial x} + \psi \right) - \hat{A}_{15}^e \frac{\partial \phi}{\partial x}, \quad (26)$$

$$R - \mu \frac{\partial^2 R}{\partial x^2} = (D_{xz} - \beta F_{xz}) \left(\frac{\partial w}{\partial x} + \psi \right) - \hat{D}_{15}^e \frac{\partial \phi}{\partial x}, \quad (27)$$

$$\int_{-h/2}^{h/2} \left(D_x - \mu \frac{\partial^2 D_x}{\partial x^2} \right) \cos(\xi z) dz = (\hat{A}_{15}^e - \beta \hat{D}_{15}^e) \left(\frac{\partial w}{\partial x} + \psi \right) + \check{A}_{11}^k \frac{\partial \phi}{\partial x}, \quad (28)$$

$$\int_{-h/2}^{h/2} \left(D_z - \mu \frac{\partial^2 D_z}{\partial x^2} \right) \xi \sin(\xi z) dz = \bar{A}_{31}^e \frac{\partial u}{\partial x} + (\bar{B}_{31}^e - \alpha \bar{E}_{31}^e) \frac{\partial \psi}{\partial x} - \alpha E_{31}^e \frac{\partial^2 w}{\partial x^2} - \check{A}_{33}^k \phi, \quad (29)$$

where $\mu = (e_0 a)^2$ and all quantities used in the above equations are defined as

$$\{A_{xx}, B_{xx}, D_{xx}, E_{xx}, F_{xx}, H_{xx}\} = \int_{-h/2}^{h/2} c_{11} \{1, z, z^2, z^3, z^4, z^6\} dz, \quad (30)$$

$$\{A_{xz}, D_{xz}, F_{xz}\} = \int_{-h/2}^{h/2} c_{55} \{1, z^2, z^4\} dz. \quad (31)$$

$$\{\bar{A}_{31}^e, \bar{B}_{31}^e, \bar{E}_{31}^e\} = \int_{-h/2}^{h/2} e_{31} \{1, z, z^3\} \xi \sin(\xi z) dz, \quad (32)$$

$$\{\hat{A}_{15}^e, \hat{D}_{15}^e\} = \int_{-h/2}^{h/2} e_{15} \{1, z^2\} \cos(\xi z) dz \quad (33)$$

$$\{\check{A}_{11}^k, \check{A}_{33}^k\} = \int_{-h/2}^{h/2} \{\kappa_{11} \cos^2(\xi z), \kappa_{33} \xi^2 \sin^2(\xi z)\} dz. \quad (34)$$

The explicit relation of the nonlocal normal force can be derived by substituting for the second derivative of N from Eq. (19) into Eq. (23) as follows

$$N = A_{xx} \frac{\partial u}{\partial x} + K_{xx} \frac{\partial \psi}{\partial x} - \alpha E_{xx} \frac{\partial^2 w}{\partial x^2} + \bar{A}_{31}^e \phi - N_E - \mu \frac{\partial f}{\partial x}. \quad (35)$$

Omitting \bar{Q} from Eqs. (20) and (21), we obtain the following equation

$$\frac{\partial^2 \bar{M}}{\partial x^2} = (N_E + N_B) \frac{\partial^2 w}{\partial x^2} - \alpha \frac{\partial^2 P}{\partial x^2} - q. \quad (36)$$

Also the explicit relation of the nonlocal bending moment can be derived by substituting for the second derivative of \bar{M} from the above equation into Eq. (24) and using Eq. (25) as follows

$$\bar{M} = K_{xx} \frac{\partial u}{\partial x} + \bar{I}_{xx} \frac{\partial \psi}{\partial x} - \alpha J_{xx} \frac{\partial^2 w}{\partial x^2} + (\bar{B}_{31}^e - \alpha \bar{E}_{31}^e) \phi + \mu \left[(N_E + N_B) \frac{\partial^2 w}{\partial x^2} - \alpha \frac{\partial^2 P}{\partial x^2} - q \right], \quad (37)$$

where

$$K_{xx} = B_{xx} - \alpha E_{xx}, \quad I_{xx} = D_{xx} - \alpha F_{xx}, \quad J_{xx} = F_{xx} - \alpha H_{xx}, \quad \bar{I}_{xx} = I_{xx} - \alpha J_{xx}. \quad (38)$$

By substituting for the second derivative of \bar{Q} from Eq. (21) into Eq. (26) and using Eq. (27), the following expression for the nonlocal shear force is derived

$$\bar{Q} = A_{xz}^* \left(\frac{\partial w}{\partial x} + \psi \right) - (\hat{A}_{15}^e - \beta \hat{D}_{15}^e) \frac{\partial \phi}{\partial x} + \mu \left[(N_E + N_B) \frac{\partial^3 w}{\partial x^3} - \alpha \frac{\partial^3 P}{\partial x^3} - \frac{\partial q}{\partial x} \right], \quad (39)$$

where

$$A_{xz}^* = \bar{A}_{xz} - \beta \bar{D}_{xz}, \quad \bar{A}_{xz} = A_{xz} - \beta D_{xz}, \quad \bar{D}_{xz} = D_{xz} - \beta F_{xz}. \quad (40)$$

In addition, the second derivative of the identity of Eq. (25) may be written as

$$\alpha \frac{\partial^2}{\partial x^2} \left(P - \mu \frac{\partial^2 P}{\partial x^2} \right) = \alpha E_{xx} \frac{\partial^3 u}{\partial x^3} + \alpha J_{xx} \frac{\partial^3 \psi}{\partial x^3} - \alpha^2 H_{xx} \frac{\partial^4 w}{\partial x^4} + \alpha \bar{E}_{31}^e \frac{\partial^2 \phi}{\partial x^2}. \quad (41)$$

For a higher order FGP nanobeam by substituting for N , \bar{M} and \bar{Q} from Eqs. (35), (37) and (39), respectively, and using Eq. (41) in Eq. (21), the nonlocal governing equations can be obtained as below

$$A_{xx} \frac{\partial^2 u}{\partial x^2} + K_{xx} \frac{\partial^2 \psi}{\partial x^2} - \alpha E_{xx} \frac{\partial^3 w}{\partial x^3} + \bar{A}_{31}^e \frac{\partial \phi}{\partial x} + f - \mu \frac{\partial^2 f}{\partial x^2} = 0, \quad (42)$$

$$K_{xx} \frac{\partial^2 u}{\partial x^2} + \bar{I}_{xx} \frac{\partial^2 \psi}{\partial x^2} - \alpha J_{xx} \frac{\partial^3 w}{\partial x^3} - A_{xz}^* \left(\frac{\partial w}{\partial x} + \psi \right) + \bar{A}_{15}^e \frac{\partial \phi}{\partial x} = 0, \quad (43)$$

$$\begin{aligned} & A_{xz}^* \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial \psi}{\partial x} \right) + \alpha E_{xx} \frac{\partial^3 u}{\partial x^3} + \alpha J_{xx} \frac{\partial^3 \psi}{\partial x^3} - \alpha^2 H_{xx} \frac{\partial^4 w}{\partial x^4} \\ & - (N_E + N_B) \frac{\partial^2 w}{\partial x^2} + q - \tilde{A}_{15}^e \frac{\partial^2 \phi}{\partial x^2} + \mu \left[(N_E + N_B) \frac{\partial^4 w}{\partial x^4} - \frac{\partial^2 q}{\partial x^2} \right] = 0, \end{aligned} \quad (44)$$

where

$$\tilde{A}_{15}^e = \hat{A}_{15}^e - \alpha E_{31}^e - \beta \hat{D}_{15}^e, \quad \bar{\bar{A}}_{15}^e = \hat{A}_{15}^e + \bar{B}_{31}^e - \alpha \bar{E}_{31}^e - \beta \hat{D}_{15}^e. \quad (45)$$

It must be cited that inserting Eq. (22) into Eqs. (28) and (29) does not provide an explicit

expressions for D_x and D_z . To overcome this problem, Eq. (23) can be re-expressed in terms of u , w , ψ and ϕ by using Eqs. (28) and (29) as

$$\bar{A}_{31}^e \frac{\partial u}{\partial x} + \bar{A}_{15}^e \frac{\partial^2 w}{\partial x^2} + \bar{A}_{15}^e \frac{\partial \psi}{\partial x} + \check{A}_{11}^k \frac{\partial^2 \phi}{\partial x^2} - \check{A}_{33}^k \phi = 0. \quad (46)$$

3. Solution procedure

Here, on the basis the Navier method, an analytical solution of the governing equations for free vibration of a simply supported FGP nanobeam is presented. To satisfy governing equations of motion and the simply supported boundary condition, the displacement variables are adopted to be of the form

$$\begin{pmatrix} u(x) \\ \psi(x) \\ w(x) \\ \phi(x) \end{pmatrix} = \sum_{n=1}^{\infty} \begin{pmatrix} U_n \cos(\zeta x) \\ \Psi_n \cos(\zeta x) \\ W_n \sin(\zeta x) \\ \Phi_n \sin(\zeta x) \end{pmatrix}, \quad (47)$$

where $\zeta = n\pi/L$, and U_n , W_n , Ψ_n and Φ_n are the unknown Fourier coefficients to be determined for each n value. The boundary conditions for simply-supported FGP beam can be identified as

$$\begin{aligned} \left. \frac{\partial u}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial u}{\partial x} \right|_{x=L} = 0, \quad w(0, t) = w(L, t) = 0, \\ \left. \frac{\partial \psi}{\partial x} \right|_{x=0} = \left. \frac{\partial \psi}{\partial x} \right|_{x=L} = 0, \quad \phi(0, t) = \phi(L, t) = 0. \end{aligned} \quad (48)$$

Inserting Eqs. (47) into Eqs. (42), (43), (44) and (46), yields

$$[K]\{\Delta\} = \{0\}, \quad (49)$$

where $\{\Delta\} = \{U_n, \Psi_n, W_n, \Phi_n\}^T$ and $[K]$ is the stiffness matrix. The coefficients of the symmetric stiffness matrix $k_{ij} = k_{ji}$ are given by

$$\begin{aligned} k_{11} &= \zeta^2 A_{xx}, \quad k_{12} = \zeta^2 K_{xx}, \quad k_{13} = \alpha \zeta^3 E_{xx}, \quad k_{14} = -\zeta \bar{A}_{31}^e, \\ k_{22} &= A_{xz}^* + \zeta^2 \bar{I}_{xx}, \quad k_{23} = \zeta (A_{xz}^* - \alpha \zeta^2 J_{xx}), \quad k_{24} = -\zeta \bar{A}_{15}^e, \\ k_{33} &= \zeta^2 (A_{xz}^* + \alpha^2 \zeta^2 H_{xx}) - (1 + \mu \zeta^2) \zeta^2 (N_E + N_B), \\ k_{34} &= -\zeta^2 \bar{A}_{15}^e, \quad k_{44} = -\zeta^2 \check{A}_{11}^k - \check{A}_{33}^k. \end{aligned} \quad (50)$$

By setting determinant of the coefficient matrix of Eq. (49) to zero, we can find buckling loads of the FGP nanobeam exposed to electrical loading.

4. Results and discussion

In this section, some numerical examples are presented to show the influence of electric field, material composition, nonlocal parameter and slenderness ratio on the electro-mechanical buckling behavior of higher-order shear deformable FGP nanobeams. So, the nonlocal FGP beam composed of PZT-4 and PZT-5H, with electro-mechanical material properties listed in Table 1, is supposed. The beam geometry has the following dimensions: L (length) = 10 nm and h (thickness) = varied.

Table 1 Electro-mechanical coefficients of material properties for PZT-4 and PZT-5H (Doroushi *et al.* 2011)

Properties	PZT-4	PZT-5H
c_{11} (GPa)	81.3	60.6
c_{55} (GPa)	25.6	23.0
e_{31} (Cm ⁻²)	-10.0	-16.604
e_{15} (C/m ²)	40.3248	44.9046
κ_{11} (C ² m ⁻² N ⁻¹)	0.6712e-8	1.5027e-8
κ_{33} (C ² m ⁻² N ⁻¹)	1.0275e-8	2.554e-8

Table 2 Comparison of the non-dimensional buckling load for a S-S FG nanobeam with various power-law index ($L/h = 20$)

p	$\mu = 1$			$\mu = 2$		
	EBT (Eltaher <i>et al.</i> 2013b)	RBT (Rahmani and Jandaghian 2015)	Present	EBT (Eltaher <i>et al.</i> 2013b)	RBT (Rahmani and Jandaghian 2015)	Present
0	8.9843	8.9258	8.925759	8.2431	8.1900	8.190046
0.1	10.1431	9.7778	9.777865	9.2356	8.9719	8.971916
0.2	10.2614	10.3898	10.389845	9.7741	9.5334	9.533453
0.5	11.6760	11.4944	11.494448	10.6585	10.5470	10.547009
1	12.4581	12.3709	12.370918	12.0652	11.3512	11.351234
2	13.1254	13.1748	13.174885	12.4757	12.0889	12.088934
5	13.5711	14.2363	14.236343	13.2140	13.0629	13.062900
p	$\mu = 3$			$\mu = 4$		
	EBT (Eltaher <i>et al.</i> 2013b)	RBT (Rahmani and Jandaghian 2015)	Present	EBT (Eltaher <i>et al.</i> 2013b)	RBT (Rahmani and Jandaghian 2015)	Present
0	7.6149	7.5663	7.566381	7.0765	7.0309	7.030978
0.1	8.5786	8.2887	8.288712	8.0416	7.7021	7.702196
0.2	9.3545	8.8074	8.807489	8.3176	8.1842	8.184264
0.5	9.8093	9.7438	9.743863	9.0585	9.0543	9.054379
1	10.9776	10.4869	10.486847	9.9816	9.7447	9.744790
2	11.7415	11.1683	11.168372	10.4649	10.3781	10.378089
5	12.2786	12.0682	12.068171	11.5231	11.2142	11.214218

Also, the following relation is described to calculate the non-dimensional buckling loads

$$N_{Bcr} = \frac{N_B L^2}{(C_{11} I)_{PZT-4}}, \quad (51)$$

where $I = bh^3/12$ is the moment of inertia of the cross section of the nanobeam and $A = bh$. Table 2 compares dimensionless buckling loads of the present theory with those of nonlocal FGM Euler and Timoshenko beams, because there are no available numerical results for the buckling loads of FGP nanobeams based on the nonlocal elasticity theory. For comparison study, the material selection is performed as follows: $E_m = 70$ GPa, $\nu_m = 0.3$ for Steel and $E_c = 390$ GPa, $\nu_c = 0.24$ for Alumina.

In Tables 3-6, the effects of some parameters such as nonlocal scale parameter (μ), electric

Table 3 Influence of external electric voltage and material composition on the non-dimensional buckling load of a S-S FGP nanobeam ($\mu = 0$)

L/h	Voltage (V)	Gradient index (p)						
		0	0.2	0.5	1	2	5	10
20	-0.25	11.6038	11.1412	10.7834	10.5247	10.3484	10.1632	10.0004
	0	11.0134	10.4858	10.0630	9.73938	9.49802	9.24789	9.05552
	+0.25	10.4230	9.83041	9.34262	8.95402	8.64768	8.33256	8.11066
25	-0.25	12.1739	11.7735	11.4775	11.2805	11.1658	11.0423	10.9074
	0	11.0207	10.4935	10.0705	9.74657	9.50495	9.25456	9.06191
	+0.25	9.86759	9.21343	8.66351	8.21266	7.84413	7.46681	7.21648
30	-0.25	13.0174	12.7096	12.5059	12.4011	12.3787	12.3474	12.2543
	0	11.0247	10.4977	10.0746	9.75050	9.50875	9.25820	9.06541
	+0.25	9.03212	8.28575	7.64333	7.09992	6.63884	6.16898	5.87649

Table 4 Influence of external electric voltage and material composition on the non-dimensional buckling load of a S-S FGP nanobeam ($\mu = 1$ (mn)²)

L/h	Voltage (V)	Gradient index (p)						
		0	0.2	0.5	1	2	5	10
20	-0.25	10.6145	10.1993	9.87940	9.64984	9.49516	9.33248	9.18693
	0	10.0241	9.54386	9.15903	8.86449	8.64481	8.41715	8.24206
	+0.25	9.43366	8.88847	8.43866	8.07913	7.79447	7.50182	7.29720
25	-0.25	11.1839	10.8309	10.5728	10.4049	10.3119	10.2110	10.0933
	0	10.0307	9.55085	9.16585	8.87103	8.65112	8.42322	8.24788
	+0.25	8.87759	8.27079	7.75887	7.33713	6.99030	6.63547	6.40244
30	-0.25	12.0270	11.7666	11.6009	11.5252	11.5245	11.5158	11.4400
	0	10.0344	9.55468	9.16959	8.87461	8.65457	8.42654	8.25106
	+0.25	8.04177	7.34274	6.73833	6.22403	5.78467	5.33731	5.06215

Table 5 Influence of external electric voltage and material composition on the non-dimensional buckling load of a S-S FGP nanobeam ($\mu = 2 \text{ (mn)}^2$).

L/h	Voltage (V)	Gradient index (p)						
		0	0.2	0.5	1	2	5	10
20	-0.25	9.78823	9.41259	9.12446	8.91918	8.7826	8.63869	8.50757
	0	9.19782	8.7572	8.40409	8.13382	7.93226	7.72336	7.56270
	+0.25	8.60742	8.10181	7.68371	7.34847	7.08192	6.80803	6.61784
25	-0.25	10.3571	10.0437	9.81733	9.67373	9.59887	9.51667	9.41348
	0	9.20394	8.76362	8.41035	8.13983	7.93805	7.72893	7.56804
	+0.25	8.05080	7.48356	7.00337	6.60593	6.27722	5.94118	5.72260
30	-0.25	11.1999	10.9791	10.8450	10.7937	10.8111	10.8212	10.7599
	0	9.20730	8.76713	8.41378	8.14311	7.94121	7.73197	7.57096
	+0.25	7.21468	6.55519	5.98252	5.49253	5.07131	4.64275	4.38204

Table 6 Influence of external electric voltage and material composition on the non-dimensional buckling load of a S-S FGP nanobeam ($\mu = 2 \text{ (mn)}^2$).

L/h	Voltage (V)	Gradient index (p)						
		0	0.2	0.5	1	2	5	10
20	-0.25	9.08782	8.74574	8.48450	8.29980	8.17856	8.05056	7.93167
	0	8.49742	8.09035	7.76412	7.51444	7.32822	7.13523	6.98681
	+0.25	7.90701	7.43496	7.04375	6.72908	6.47788	6.21991	6.04195
25	-0.25	9.65620	9.37633	9.17689	9.05389	8.99439	8.92812	8.83718
	0	8.50307	8.09627	7.76991	7.51999	7.33357	7.14038	6.99174
	+0.25	7.34993	6.81621	6.36293	5.98608	5.67275	5.35263	5.14630
30	-0.25	10.4988	10.3115	10.2043	10.1736	10.2064	10.2324	10.1834
	0	8.50617	8.09952	7.77308	7.52302	7.33650	7.14319	6.99444
	+0.25	6.51355	5.88758	5.34182	4.87244	4.46659	4.05396	3.80552

voltage (V), gradient index (p) and slenderness ratio on the non-dimensional buckling load of the S-S FGP nanobeams are tabulated. The results of these tables indicate that nonlocal parameter has a considerable reducing influence on the non-dimensional buckling loads and weakens the nanobeam structure. Also, it must be mentioned that when the gradient index rises the non-dimensional buckling loads of FGP nanobeams reduce at a constant electric voltage and nonlocal parameter. Another conclusion is that external voltage values with positive sign yields lower buckling loads than electric voltages with negative sign.

Also, Table 7 present the buckling loads of a piezoelectric nanobeam for classical and Reddy beam theories and different nonlocal parameters when $p = 1$ and $V = 0$. One can see that buckling results according to Reddy beam model are smaller than that of classical beam model. This shows the importance of size-dependent shear deformation effect on piezoelectric nanostructures.

Fig. 2 shows the variations of the non-dimensional buckling load of FG piezoelectric nanobeams with the gradient index for various external voltages ($V = -0.5, -0.25, 0, +0.25$,

Table 7 Dimensionless buckling load according to classical and Reddy beam theories

μ	$L/h = 5$		$L/h = 10$	
	RBT	CBT	RBT	CBT
1	8.49316	8.7512	8.77446	8.9541
2	7.7931	8.0334	8.05121	8.2212
3	7.19966	7.3872	7.43812	7.6018

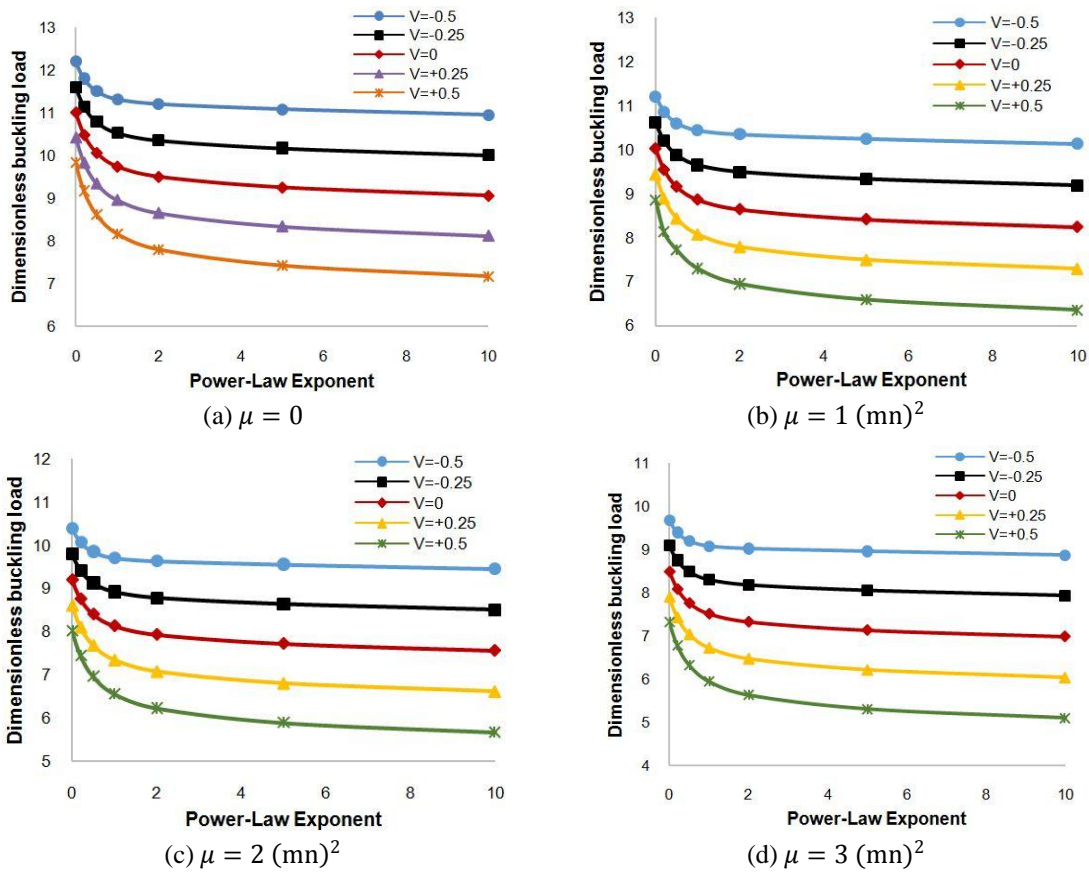


Fig. 2 Effect of external electric voltage on the dimensionless buckling load of the S-S FGP nanobeam with respect to gradient index for different values of nonlocal parameters ($L/h = 20$)

+0.5) and nonlocal parameters at $L/h = 20$. It can be seen from the figure that the non-dimensional buckling load reduce significantly for lower values of gradient indexes, but the higher values of gradient index will not affect the buckling load notably. Also, it should be cited that the dimensionless buckling load for positive voltages reduces with a higher slope compared to negative voltages which indicate importance of the sign of external electric voltage.

Fig. 3 demonstrates the variations of the non-dimensional buckling load with respect to external electric voltage for different nonlocal parameters and gradient indices at slenderness ratio

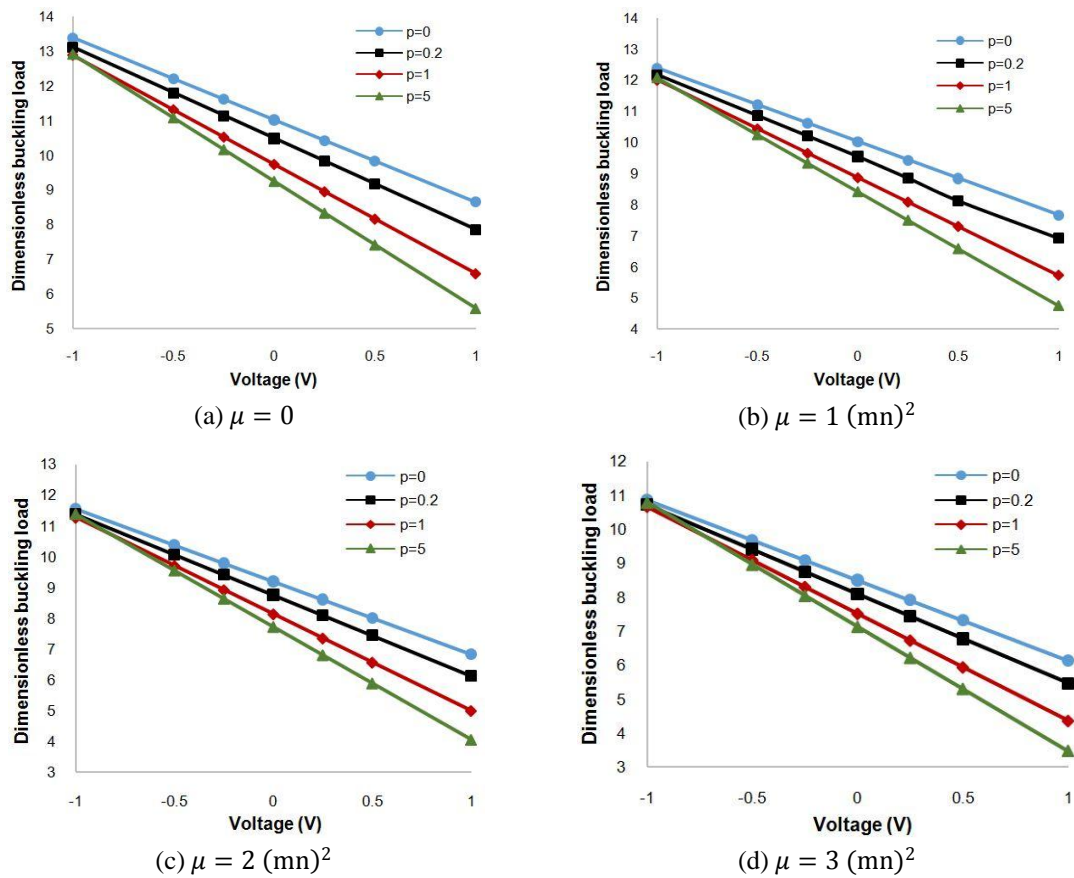


Fig. 3 The variation of dimensionless buckling load of the S-S FGP nanobeam with respect to external voltage for different values of nonlocal parameters and gradient indexes ($L/h = 20$)

$L/h = 20$. It is observable that when the voltage changes from $V = -1$ to $V = +1$ the dimensionless buckling load of FGP nanobeams approximately decreases linearly. Another observation is that at the negative voltages the obtained buckling loads for various gradient indexes are closer than those obtained for positive electric voltages. Therefore, as the positive voltage raises the difference between buckling loads for various values of gradient index at a constant nonlocal parameter increases.

The influences of electric voltage and nonlocal parameter on the dimensionless buckling loads of FGM piezoelectric nanobeams at $L/h = 20$ and various gradient indexes are plotted in Fig. 4. As a consequence, when the electric voltage rises from $V = -1$ to $V = +1$ the dimensionless buckling load reduce for all values of nonlocal parameter with a same manner. It is also seen from the figure that, as the external electric voltage increases the difference between the results of various nonlocal parameter remains invariant, so it can be concluded that nonlocal scale parameter influence is not dependent on the values of electric voltage.

The effect of slenderness ratio (L/h) on the dimensionless buckling load of FGP nanobeams with changing of gradient index at $\mu = 1 \text{ (nm)}^2$ is presented in Fig. 5. It is found that for negative electric voltages as the slenderness ratio increases the dimensionless buckling load

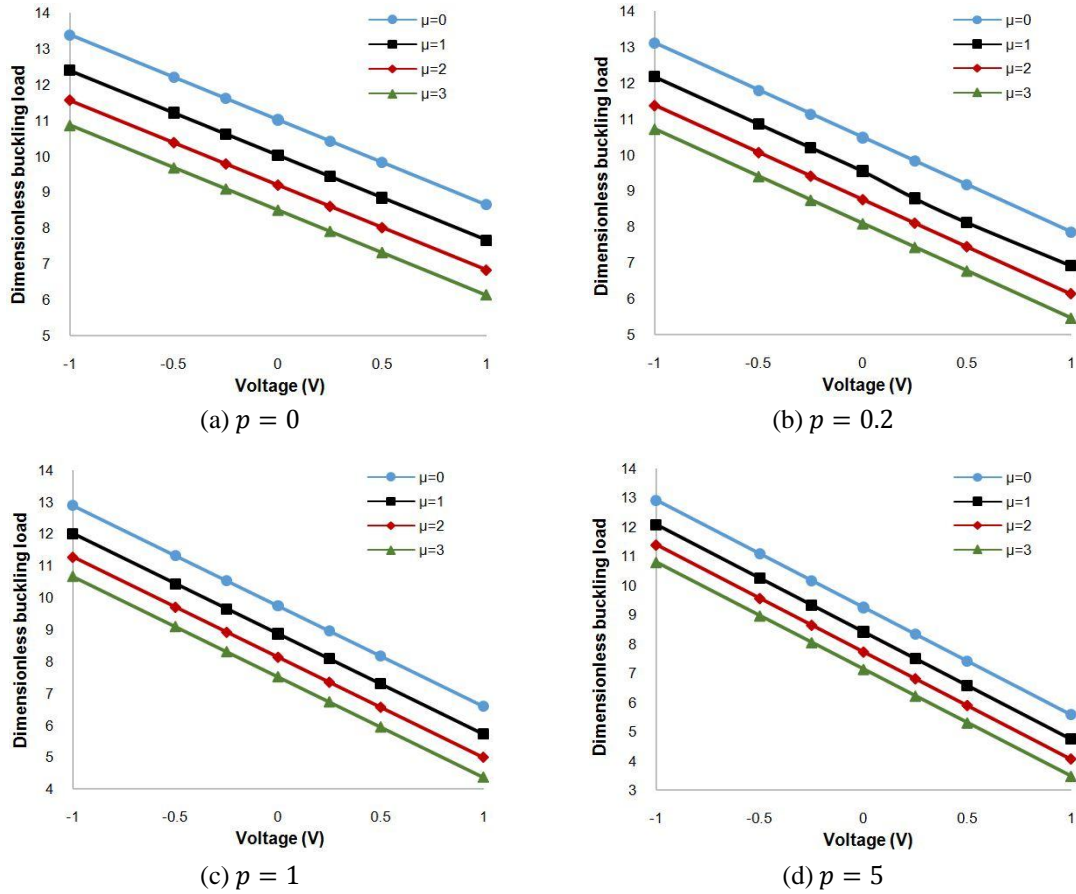


Fig. 4 The variation of dimensionless buckling load of the S-S FGP nanobeam with respect to external voltage for different values of nonlocal parameters and gradient indexes ($L/h = 20$)

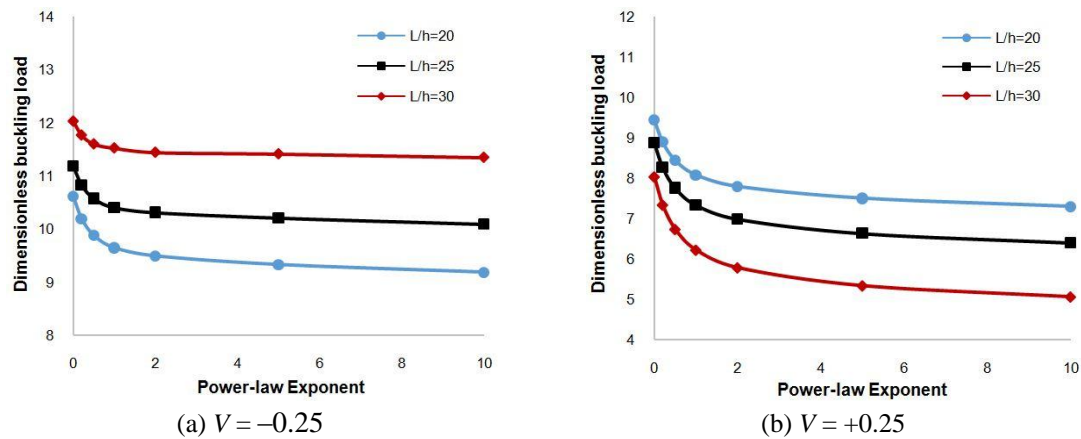


Fig. 5 Effect of slenderness ratio on the dimensionless buckling load of the S-S FGP nanobeam with respect to gradient index for different values of nonlocal parameters ($\mu = 1$ $(mn)^2$)

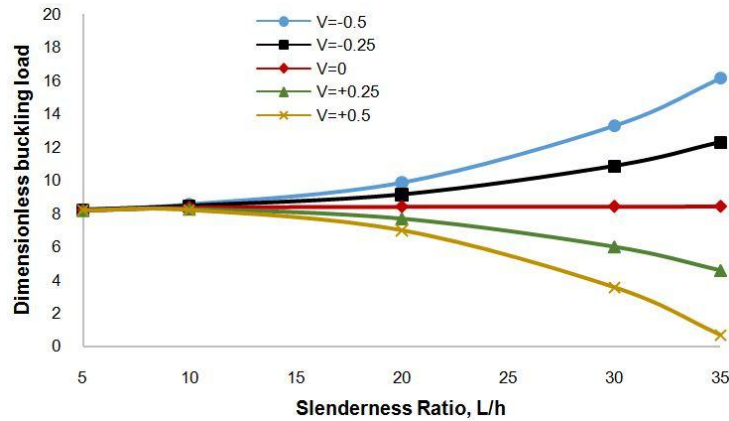


Fig. 6 The variation of dimensionless buckling load of the S-S FGP nanobeam with respect to slenderness ratio for different values of External

increases, whereas a reverse trend is observed for positive voltages. Also, it is deduced that for all values of slenderness ratio with an increase in gradient index the buckling load reduces. More specifically, for a positive voltage the buckling load changes is more sensible at the higher values of slenderness ratio. Fig. 6 illustrates the variations of the dimensionless buckling load of nonlocal piezoelectric FG beams versus slenderness ratio at gradient index $p = 0.5$ and nonlocal parameter $\mu = 2$. It must be mentioned that electric voltage has a significant effect on the non-dimensional buckling load especially for higher values of slenderness ratio. Therefore negative external voltages provide larger values of buckling loads while positive voltages provide smaller values for buckling loads. Another finding is that, when the external electric voltage is equal to zero ($V = 0$) the dimensionless buckling load variation is approximately independent of slenderness ratio.

5. Conclusions

The purpose of this article is to develop a nonlocal higher-order parabolic beam model for buckling analysis of piezoelectric FG nanobeams using nonlocal elasticity theory which captures the small size effects. Nonlocal governing equations are solved using Navier solution method. Electro-mechanical properties of the FGP nanobeams are supposed to be position dependent according to power-law model. The validity of the present model is investigated with comparison to some of the present in literature. The influences of various parameters such as external electric voltage, gradient index, nonlocal parameter and slenderness ratio on the buckling loads of nonlocal FGP beams are discussed. It is indicated that nonlocal parameter shows a reducing influence on stiffness of the nanobeam and buckling loads. Another important observation is that according to the sign of the electric voltage the buckling loads of FGP nanobeams experience both decreasing and increasing trends. Also, it is found that nonlocal scale parameter is not dependent on the values of electric voltage.

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