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Forced vibration of nanorods using nonlocal elasticity

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Abstract. Present study interests with the longitudinal forced vibration of nanorods. The nonlocal elasticity theory of Eringen is used in modeling of nanorods. Uniform, linear and sinusoidal axial loads are considered. Dynamic displacements are obtained for nanorods with different geometrical properties, boundary conditions and nonlocal parameters. The nonlocal effect increases dynamic displacement and frequency when compared with local elasticity theory. Present results can be useful for modeling of the axial nanomotors and nanoelectromechanical systems.

Keywords: nonlocal theory; nanorod model; forced vibration; dynamic displacement; mode shape

1. Introduction

Carbon nanotubes (CNTs) have been used in nanoscale engineering applications such as oscillator, sensor, nanomotor and fiber in nanocomposites, after the discovery by Iijima (1991). It is necessary to understand their mechanical behaviors properly.

Discrete and continuum models are used in order to understand statics and dynamics of CNTs. Discrete models are based on interactions in atomic lattice structure and Molecular Dynamic (MD) simulation is one of the discrete model approaches. Very expensive and time consuming computer softwares are using for MD simulation process. Modeling of CNTs which have longer or bigger radius takes much more time according to smaller CNTs. Because of the disadvantages of MD simulations, continuum models can also use in modeling. But, classical continuum mechanic approach couldn't be applied in nanoscale because of the size effect. It is well known from the previous studies that mechanics of nano-structures is size dependent and classical continuum mechanics does not include the size effect. Due to this fact, the nonlocal elasticity models have been considered in order to overcome this drawback. The nonlocal elasticity theory was apparently first proposed by Eringen (1983, 2007). He assumed that the stress at a point is a functional of the strain field at every point of the continuum. With this assumption, Eringen combined both discrete and continuum models into one theory that can be used at nano and macro scales.

Firstly Peddieson, Buchanan *et al*. (2003) developed a nonlocal Euler-Bernoulli beam model with using nonlocal elasticity theory. Sudak (2003) studied column buckling of multiwalled carbon nanotubes (MWCNTs) which were modeled based on nonlocal continuum mechanics.

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Wang and Wang (2007), Duan, Wang *et al*. (2007), Ece and Aydogdu (2007), Wang and Liew (2007), Demir, Civalek *et al*. (2010) used the nonlocal elasticity theory with various beam theories for static and dynamic analysis of CNTs. Reddy (2007) reconstituted various beam theories according to nonlocal differential constitutive relations of Eringen. Aydogdu (2009a) proposed an inclusive nonlocal beam theory which includes previous beam theories. Murmu and Pradhan (2009) developed an elastic beam model for analyzing the thermal vibration of SWCNT. Free vibration of SWCNT and nanobeams embedded in an elastic medium were investigated by Kiani (2010), Ansari, Gholami *et al*. (2011), respectively. Aydogdu and Arda (2014) studied the torsional vibration of DWCNTs with assuming the van der Waals interaction in circumferential direction. Kiani (2014) proposed nonlocal discrete and continuous models for dynamic analysis of SWCNTs.

Wang and Hu (2005) used continuum mechanics and MD simulation for flexural wave propagation in SWCNTs. Wang (2005), Wang and Varadan (2006), Hu, Liew *et al*. (2008) interested with wave propagation in CNTs. Narendar and Gopalakrishnan (2010) studied the nonlocal effect on ultrasonic wave characteristics of nanorods. Other studies about wave propagation in CNTs were carried out by Wang (2005), Narendar (2011)**,** Aydogdu (2012a, 2014) and Arda and Aydogdu (2016). Arash and Wang (2012) reviewed recent studies about the nonlocal continuum models.

Small scale effect on axial vibration of CNTs is investigated by Aydogdu (2009b), Danesh, Farajpour *et al*. (2012). Karaoglu and Aydogdu (2010), Kiani (2014), Şimşek (2010, 2011) interested with the forced vibration of the CNTs. A physically-based nonlocal model is formulated by Huang (2012) in order to study influences of the nonlocal long-range interactions on the longitudinal vibration of nanorod. The free longitudinal vibration of tapered nanowires is studied by using nonlocal continuum theory (Kiani 2010). Vibration of coupled nanorod system is investigated by (Murmu and Adhikari 2010). The axial wave propagation properties of a coupled nanorod system are investigated by using nonlocal elasticity theory (Narendar and Gopalakrishnan 2011). Free vibration of stocky ensembles of vertically aligned single-walled carbon nanotubes is investigated via nonlocal discrete and continuous modeling (Kiani 2014).

Kiani (2014) proposed nonlocal discrete and continuous models to explain free vibration of two- dimensional (2D) ensembles of single-walled carbon nanotubes (SWCNTs) in bending. Using a novel nonlocal integro-differential model accounting for the surface energy effect, free longitudinal vibration of functionally graded tapered nanorods embedded in an elastic matrix is investigated by Kiani (2016). Frequency analysis of elastically supported nanorods is performed via a novel nonlocal integro-differential surface energy-based model (Kiani 2016).

It should be stresses that the carbon nanotubes are used as atomic force microscope (AFM) probe tip. During scanning material samples may act as attached spring to probe and this may change dynamic characteristics of the AFM. Also similar behavior can be observed in biosensor applications. Some other possible applications of CNTs in such industries like nanoelectromechanical, pharmaceutical, nano-bearing or nano-gearbox and resonators are investigated by researchers. Axially tunable carbon nanotube resonators using co-integrated micro actuators are investigated by Truax, Lee *et al*. (2014). In this application axial dynamics of nanotubes due to external excitations is important.

According to author's limited knowledge, the axial vibration of nanorod under different external loads has not been considered, yet. The main goal of the present study is to fill the literature gap. In the present work, forced axial vibration of nanorods is studied using the nonlocal elasticity theory. The effect of various parameters like external force type, length of nanorod and nonlocal parameter are investigated in detail.

2. Analysis

A nanorod with diameter d and length L is considered. According to Hamilton's Principle (Reddy 2007, Aydogdu 2012b), the equation of motion in the axial direction can be found as

$$EA\frac{\partial^2 u}{\partial x^2} + f_x = \rho\frac{\partial^2 u}{\partial t^2} \tag{1}$$

Where E is the Young modulus, A is the cross sectional area of nanotube, u is the axial displacement, f_x is the axial force ρ is the mass density per unit length and t is the time. If the nanotube is inserted in an elastic matrix its effect can be considered as a part of f_x. For an embedded nanotube in axial vibration we get $f_x=ku$. In this relation k is the stiffness of the elastic medium.

2.1 Equations of motion of nanorod using nonlocal elasticity

In order to account size dependence in the elastic behavior of continuum, the nonlocal constitute relation can be defined as

$$(1-\mu\nabla^2)\tau_{kl} = \lambda\varepsilon_{rr}\delta_{kl} + 2G\varepsilon_{kl} \tag{2}$$

where τ_{kl} and ε_{kl} are the nonlocal stress and strain tensor, λ and G are the Lame constants, $\mu=(e_0a)^2$ is called the nonlocal parameter, a is an internal characteristic length and e_0 is Eringen constant. Eringen determined this parameter as 0.39 for longitudinal wave results according to atomic lattice model (Eringen 1983, Eringen 2007). Aydogdu (2012a) has showed that e_0 is material and length dependent for axial wave propagation. For one dimensional case, Eq. (2) can be written as

$$\left(1-\mu\frac{\partial^2}{\partial x^2}\right)\tau_{xx} = E\varepsilon \tag{3}$$

If Eq. (3) is integrated respect to cross sectional area A of the nanorod ($N_x = \int_A \tau_{xx}\,dA$), Eq. (4) will be obtained

$$N_x - \mu\frac{\partial^2 N_x}{\partial x^2} = {N_x}^L \tag{4}$$

where N_x and ${N_x}^L$ denote axial force per unit length according to nonlocal and local elasticity respectively. Using Eqs. (1), (3) and (4) the following equation of motion for the forced longitudinal vibration of nanorod can be obtained

$$EA\frac{\partial^2 u}{\partial x^2} + f(x,t) - \mu\frac{\partial^2 f(x,t)}{\partial x^2} = \left(1-\mu\frac{\partial^2}{\partial x^2}\right)\rho\frac{\partial^2 u}{\partial t^2} \tag{5}$$

Eq. (8) is the nonlocal rod model for the forced longitudinal vibration of nanorods. If the nonlocal parameter is assumed as equal to zero (μ=0), Eq. (5) is turned into the classical elasticity equation.

2.2 Free vibration case

In order to use results of the free vibration, this case is briefly explained below (Aydogdu

2009b, Karaoglu and Aydogdu 2010, Aydogdu 2012b). Free vibration case occurs when the external axial force is assumed zero ($f(x,t)=0$).

$$EA\frac{\partial^2 u}{\partial x^2} = \left(1 - \mu\frac{\partial^2}{\partial x^2}\right)\rho\frac{\partial^2 u}{\partial t^2} \tag{6}$$

If Eq. (6) is rearranged with assumption of dimensionless nanotube length $\left(\bar{x} = \frac{x}{L}\right)$ and harmonic vibration $\left(u(\bar{x},t) = U(\bar{x})\,sin\,\omega t\right)$

$$\frac{\partial^2 U}{\partial \bar{x}^2} + \beta^2 U = 0 \tag{7}$$

where

$$\beta^2 = \frac{\Omega^2}{1-\frac{\mu}{L^2}\Omega^2} \quad , \quad \Omega^2 = \frac{\rho\omega^2 L^2}{EA} \tag{8}$$

Here, Ω is the non-dimensional frequency parameter (NDFP). Clamped and free boundary cases are taking into account in this study with following conditions

$$\text{Clamped (C): } u = 0 \tag{9}$$

$$\text{Free (F): } N = EA\frac{\partial u}{\partial x} + \mu\rho\frac{\partial^3 u}{\partial x\partial t^2} - \mu\frac{\partial f}{\partial x} = 0 \tag{10}$$

Free vibration results for Clamped-Clamped (C-C) and Clamped-Free (C-F) boundary cases can be seen at Table 1. It is important to note that, classical theory (μ=0) frequency results increase with increasing mode number, but nonlocal theory (μ=1) results approach a constant value in both boundary conditions. Small-scale effect can be seen with this example, clearly.

2.3 Forced vibration case

It is assumed that a harmonic force is applied to the nanorod as given below

$$f(\bar{x},t) = F(\bar{x})\,sin\,\omega t \tag{11}$$

Table 1 Free vibration frequencies of nanorod in C-C and C-F boundary conditions (L=5 nm)

	Clamped-Clamped		Clamped-Free	
Mode Number	μ=0	μ=1	μ=0	μ=1
1	3.14	2.66	1.57	1.49
2	6.28	3.91	4.71	3.42
3	9.42	4.41	7.85	4.21
4	12.56	4.64	10.99	4.55
5	15.70	4.76	14.13	4.71
6	18.84	4.83	17.27	4.80
7	21.99	4.87	20.42	4.85
8	25.13	4.90	23.56	4.89
9	28.27	4.92	26.70	4.91
10	31.41	4.93	29.84	4.93

where $F(\bar{x})$ gives variation of the axial force in the space. Inserting Eqs. (7) and (11) in Eq. (5) gives

$$\frac{\partial^2 U}{\partial \bar{x}^2} + \beta^2 U = -\frac{Q}{\alpha} F + \frac{\mu}{L^2} \frac{Q}{\alpha} \frac{\partial^2 F}{\partial \bar{x}^2} \tag{12}$$

where

$$Q = \frac{F_0 L^2}{EA} \quad , \quad \alpha = 1 - \frac{\mu}{L^2} \Omega^2 \tag{13}$$

where F_0 is the amplitude of the axial external force. In the present study three different axial loads are investigated. Namely: uniform, linear and sinusoidal loads.

2.3.1 $F(\bar{x}) = F_0$ uniform distributed load

The general solution of Eq. (12) for uniform axial load F_0 can be defined in the following form.

$$U(\bar{x}) = A \sin \beta \bar{x} + B \cos \beta \bar{x} - \frac{Q}{\alpha \beta^2} \tag{14}$$

where A and B are the undermined coefficients. For C-C boundary case following equation is obtained

$$U(\bar{x}) = \frac{Q}{\alpha \beta^2} \left[\left(\frac{(1 - \cos \beta)}{\sin \beta} \sin \beta \bar{x} \right) + \cos \beta \bar{x} - 1 \right] \tag{15}$$

and for C-F boundary case following dynamic displacement equation is obtained

$$U(\bar{x}) = \frac{Q}{\alpha \beta^2} [(\tan \beta \sin \beta \bar{x}) + \cos \beta \bar{x} - 1] \tag{16}$$

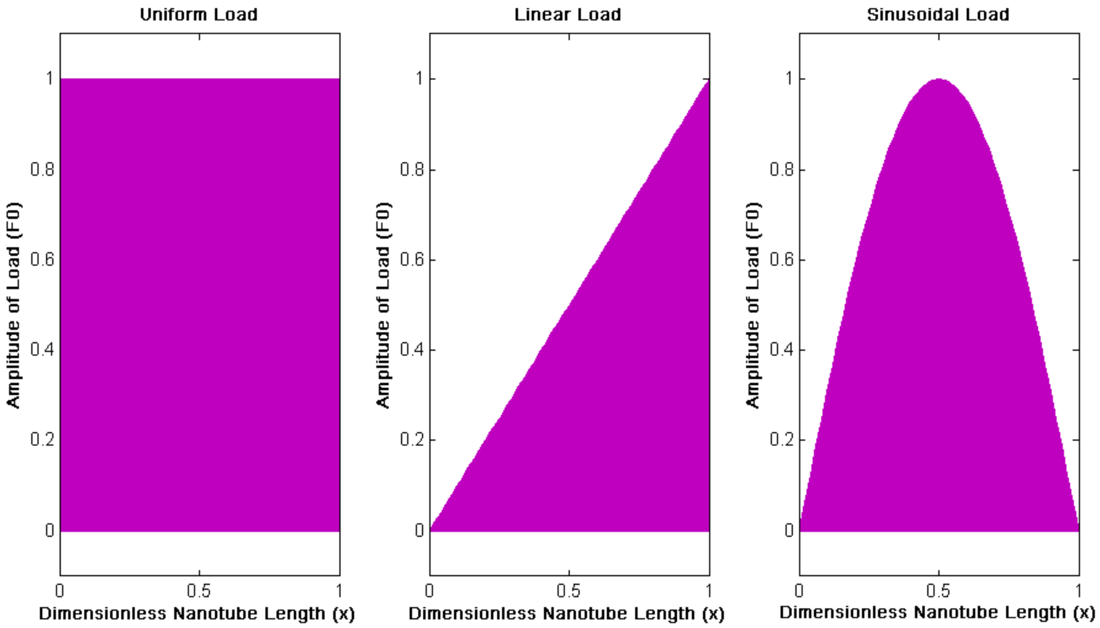

Fig. 1 Uniform, linear and sinusoidal loading in the axial direction

2.3.2. $F(\bar{x}) = F_0\bar{x}$ linear varying load

In the linearly varying load case the general solution of Eq. (12) can be written as

$$U(\bar{x}) = A\sin\beta\bar{x} + B\cos\beta\bar{x} - \frac{Q}{\alpha\beta^2}\bar{x} \tag{17}$$

where *A* and *B* are undetermined coefficients. For the C-C and C-F boundary conditions dynamic displacement are obtained in the following forms

$$U(\bar{x}) = \frac{Q}{\alpha\beta^2}\left[\frac{\sin\beta\bar{x}}{\sin\beta} - \bar{x}\right] \qquad (C-C) \tag{18}$$

$$U(\bar{x}) = \frac{Q}{\alpha\beta^2}\left[\left(\left(1 + \frac{\mu}{L^2}\beta^2\right)\frac{\sin\beta\bar{x}}{\beta\cos\beta}\right) - \bar{x}\right] \qquad (C-F) \tag{19}$$

2.3.3 $F(\bar{x}) = F_0\, sin(m\pi\bar{x})$ sinusoidal varying load

In the sinusoidal varying load case the general solution of Eq. (12) can be written as

$$U(\bar{x}) = A\sin\beta\bar{x} + B\cos\beta\bar{x} - K_1\sin(m\pi\bar{x}) \tag{20}$$

where *A* and *B* are undetermined coefficients, m is the mode number of wave and K_1 is defined as

$$K_1 = \frac{Q}{\alpha(\beta^2 - m^2\pi^2)}\left[1 + 1 + \frac{\mu}{L^2}m^2\pi^2\right] \tag{21}$$

For the C-C and C-F boundary conditions dynamic displacement are obtained in the following form

$$U(\bar{x}) = K_1\left[\frac{\sin m\pi}{\sin\beta}\sin\beta\bar{x} - \sin(m\pi\bar{x})\right] \qquad (C-C) \tag{22}$$

$$U(\bar{x}) = K_2\sin\beta\bar{x} - K_1\sin(m\pi\bar{x}) \qquad (C-F) \tag{23}$$

where

$$K_2 = m\pi\frac{\cos m\pi}{\beta\cos\beta}\left[\frac{Q}{\alpha}\frac{\mu}{L^2} + K_1\right] \tag{24}$$

3. Numerical results and discussion

The dimensionless dynamic displacements of axially vibrating nanorods are given for different geometrical, material and nonlocal parameter, in this section. All dynamic displacements are obtained at the midpoint of the nanorods ($\bar{x} = 0.5$). The dimensionless dynamic displacements are defined as

$$\underline{U} = U\frac{EA}{F_0L^2} \tag{25}$$

Figs. 2-3 show the variation of dynamic displacement of nanorod under uniform axial harmonic load for C-C and C-F boundary condition. Results are obtained for different nanotube lengths and nonlocal parameters. Dynamic displacements increase with increasing nanotube length (L). The difference between the local and nonlocal dynamic displacement are greater for shorter nanotube length values for both C-C and C-F boundary conditions. Moreover the difference is more pronounced for C-C boundary conditions. This is due to higher natural frequency for C-C

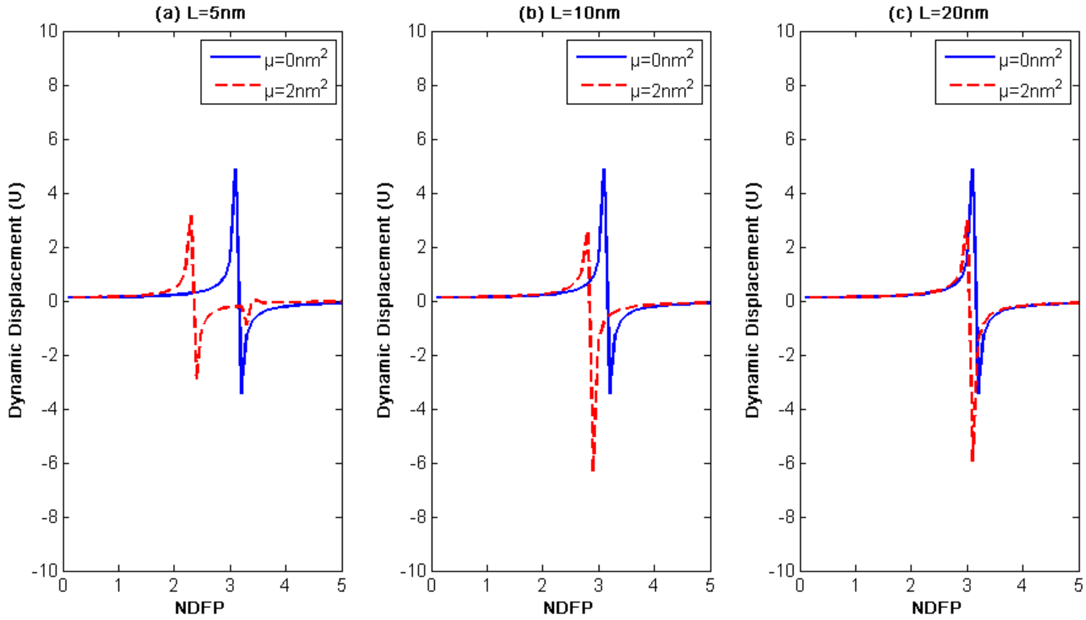

Fig. 2 Dynamic displacement of the nanorod under uniform axial load at C-C boundary condition

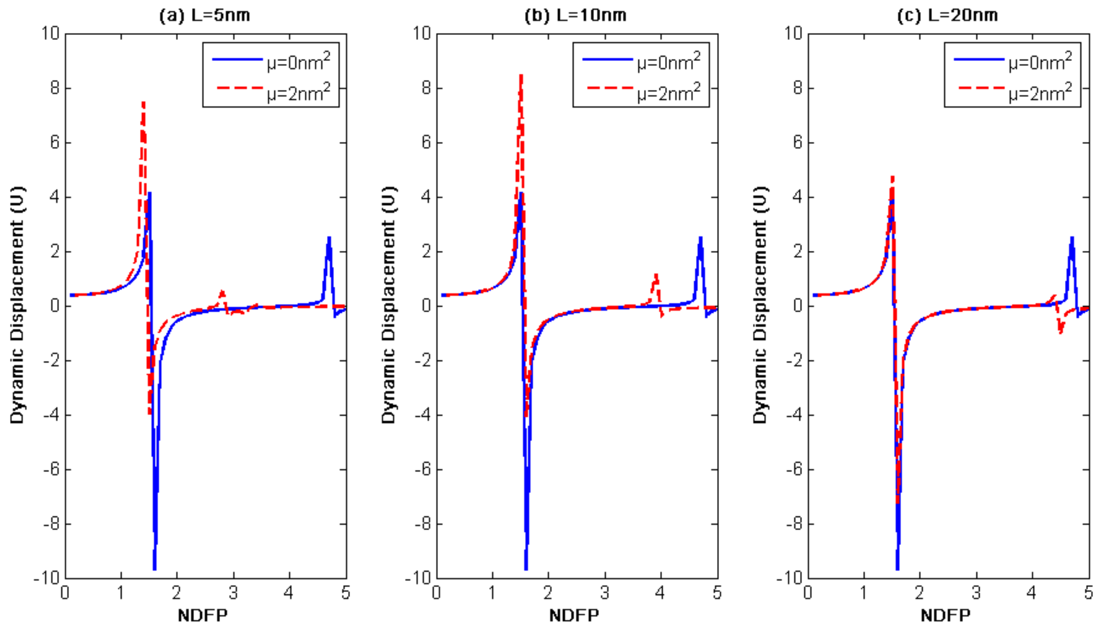

Fig. 3 Dynamic displacement of the nanorod under uniform axial load at C-F boundary condition

boundary conditions. Resonance frequencies are smaller for the nonlocal elasticity especially for shorter nanotube length values. Long range interactions are more pronounced for shorter nanotubes. This effect is vanished in longer nanotubes. The nonlocal effects are again more

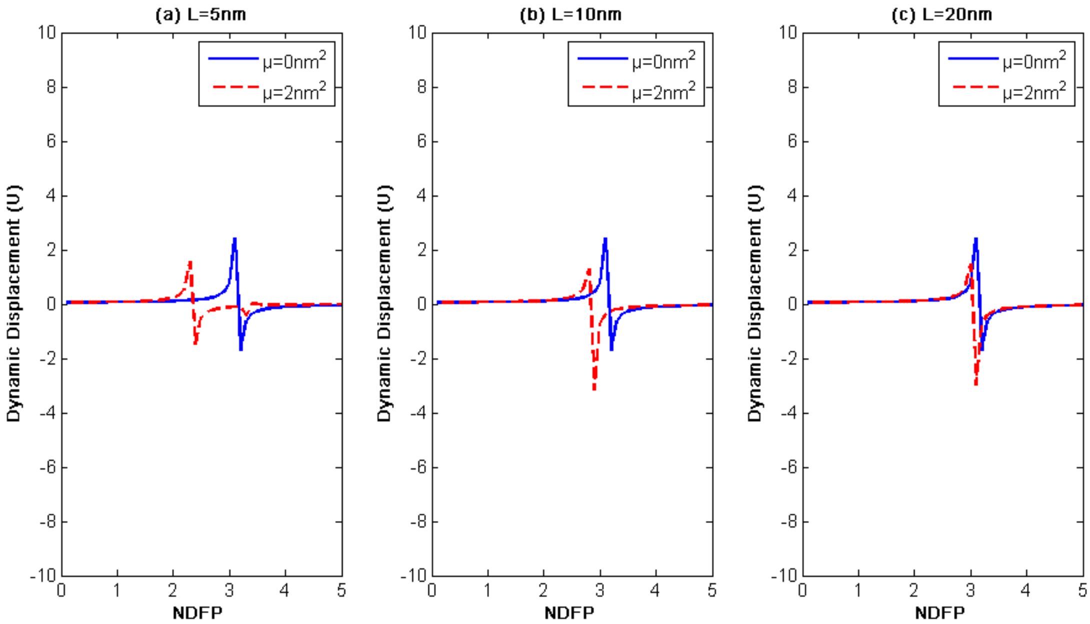

Fig. 4 Dynamic displacement of the nanorod under linear axial load at C-C boundary condition

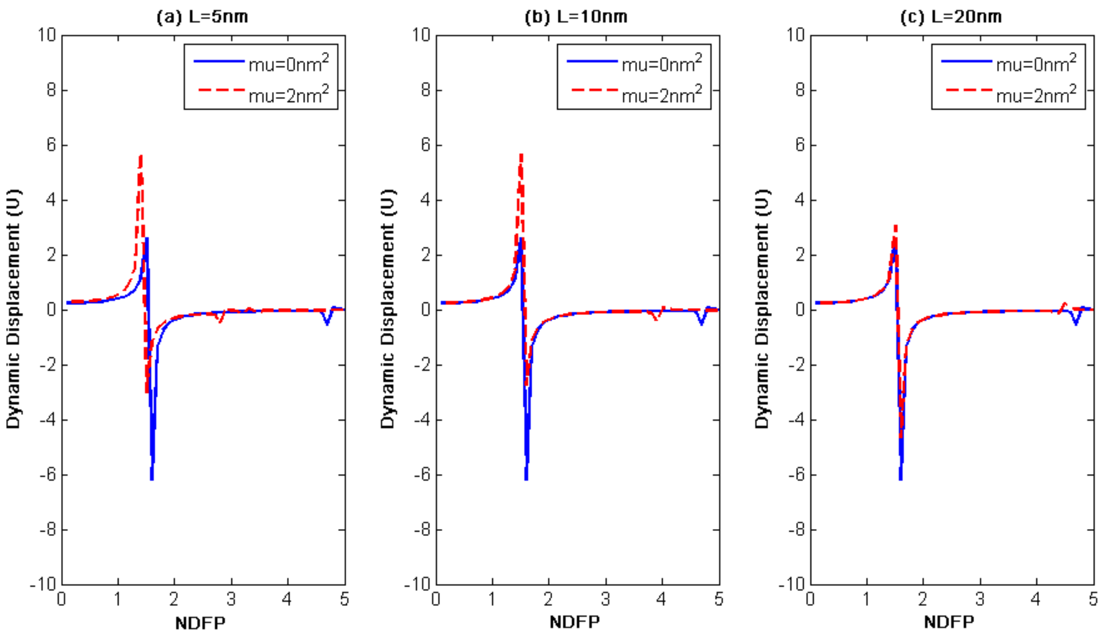

Fig. 5 Dynamic displacement of the nanorod under linear axial load at C-F boundary condition

pronounced for C-C boundary condition then C-F boundary condition.

Figs. 4-5 depict the variation of dynamic displacement of nanorod under linearly varying load. Dynamic displacements are smaller for linear load when compared with uniform load. The effect

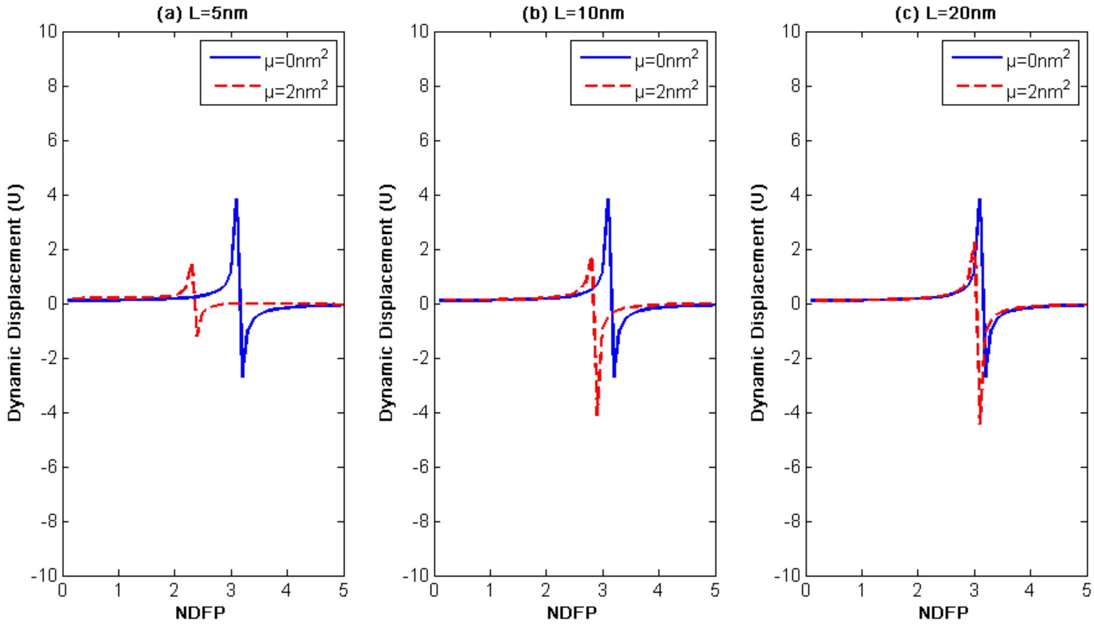

Fig. 6 Dynamic displacement of the nanorod under sinusoidal axial load at C-C boundary condition

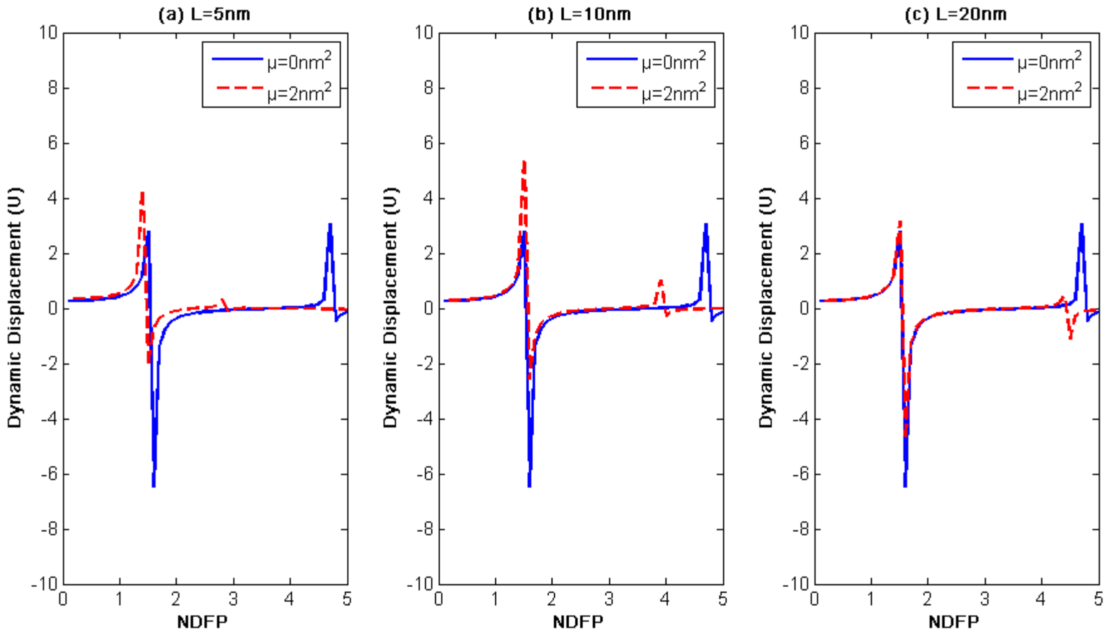

Fig. 7 Dynamic displacement of the nanorod under sinusoidal axial load at C-F boundary condition

of the nonlocal elasticity is greater for higher modes. This is due to increasing long range interactions with decreasing wave length in the continuum.

Change of dynamic displacement with sinusoidal load can be seen on Figs. 6-7. Nonlocal effect is more obvious in C-C boundary condition rather than the C-F boundary condition case. Also nonlocal effect decreases with increasing nanotube length, but in higher modes, it may become more pronounced. Nonlocal effect can decrease the NDFP for higher modes, but not the dynamic

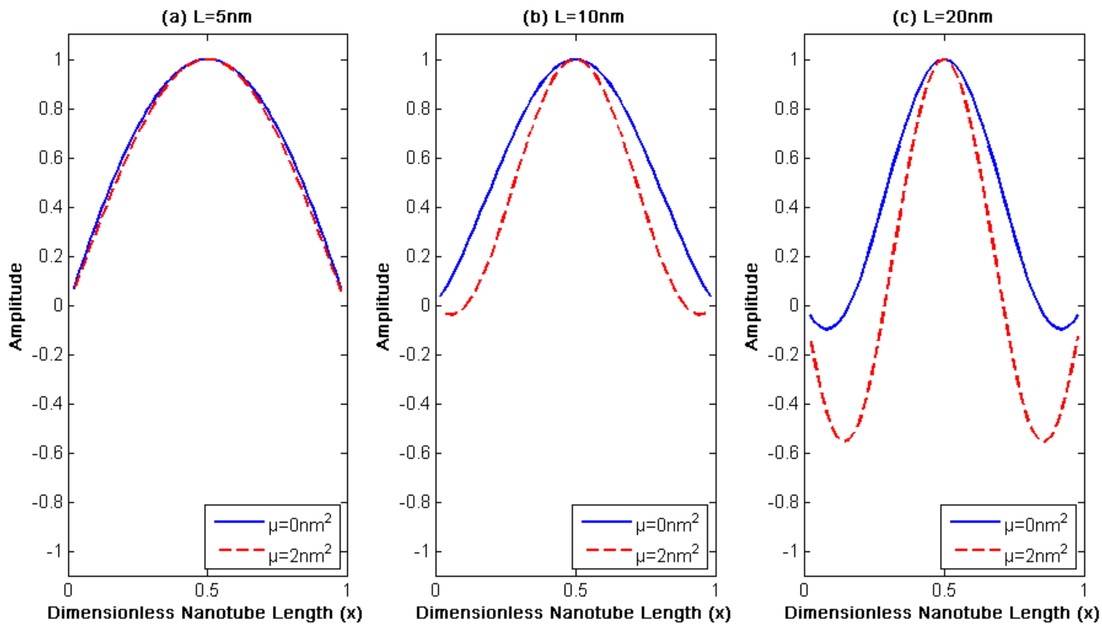

Fig. 8 Mode shapes uniform axial load at C-C boundary condition (a) Ω=2.5 (b) Ω=5 (c) Ω=7.5

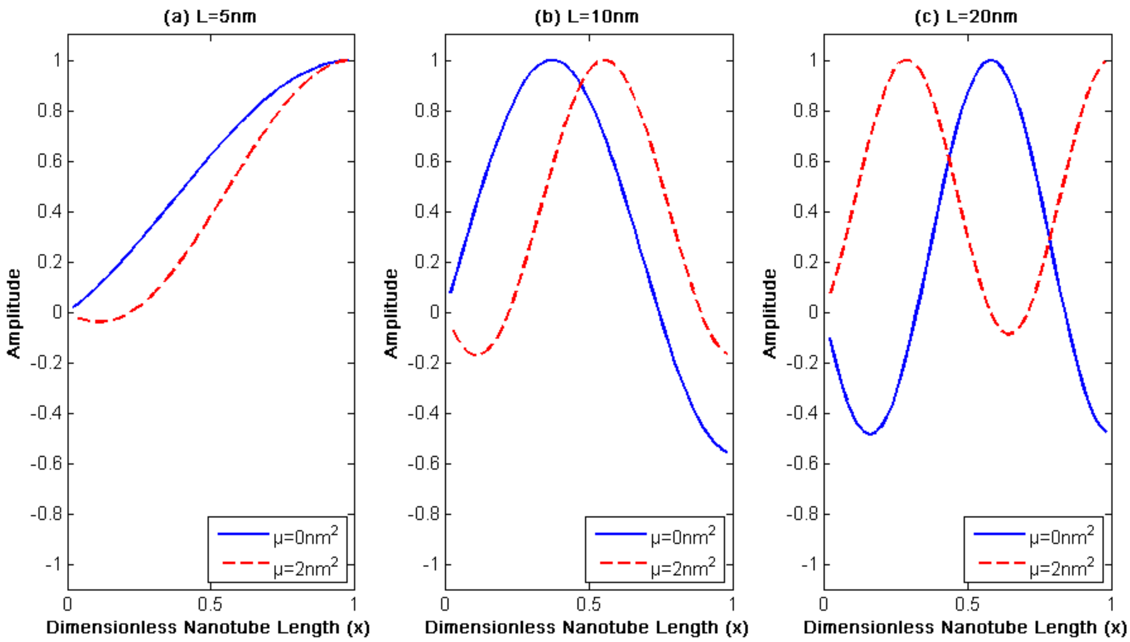

Fig. 9 Mode shapes uniform axial load at C-F boundary condition (a) Ω=2.5 (b) Ω=5 (c) Ω=7.5

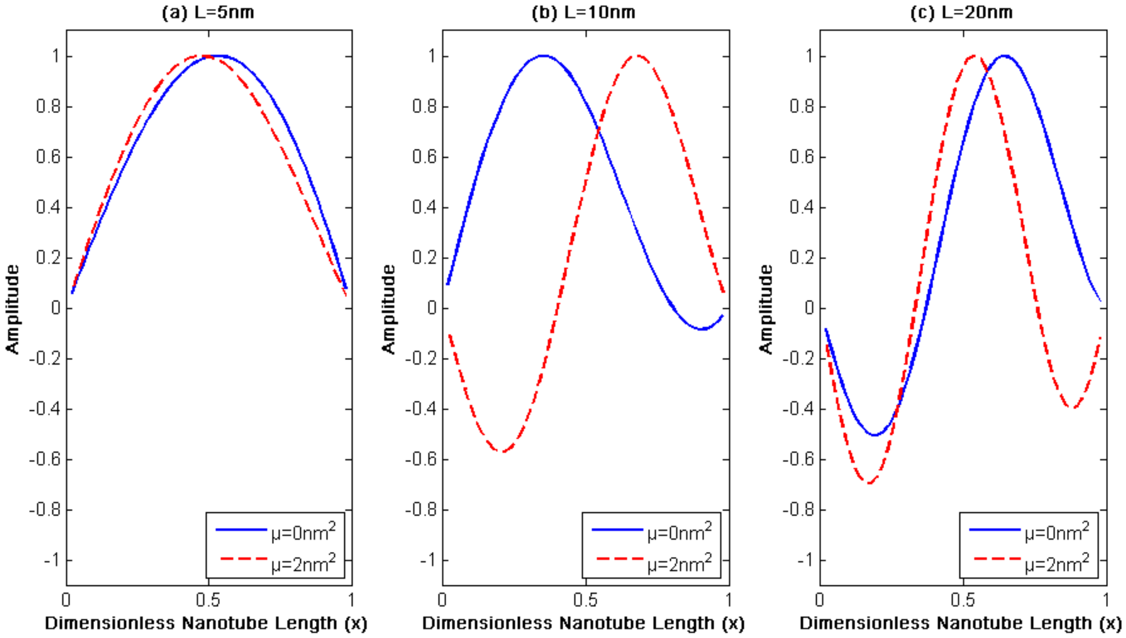

Fig. 10 Mode shapes linear axial load at C-C boundary condition (a) Ω=2.5 (b) Ω=5 (c) Ω=7.5

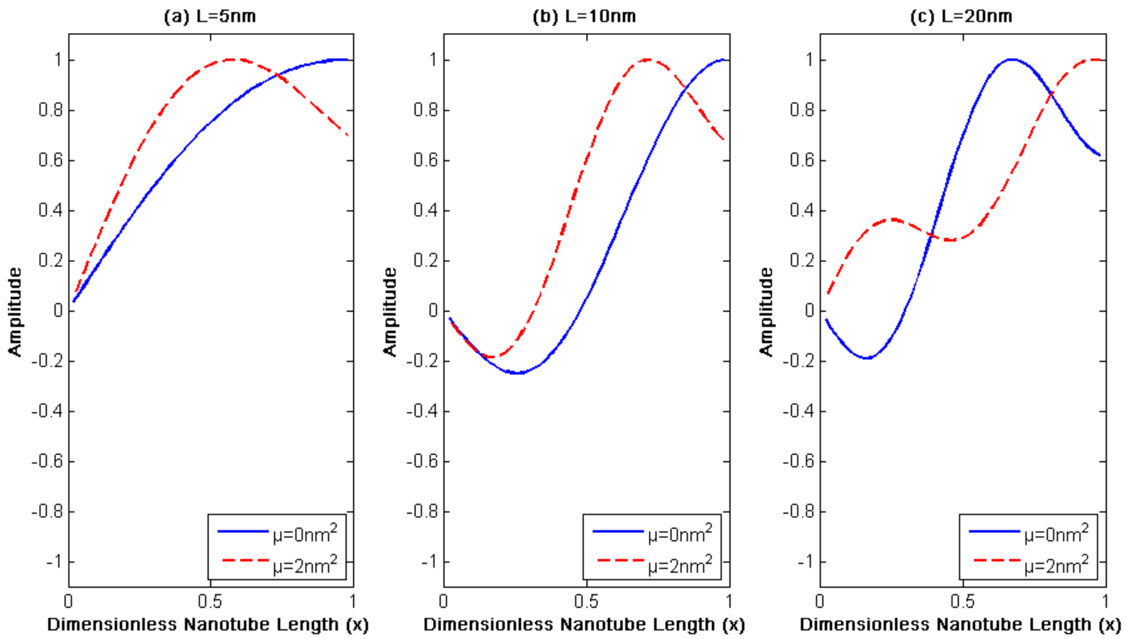

Fig. 11 Mode shapes linear axial load at C-F boundary condition (a) Ω=2.5 (b) Ω=5 (c) Ω=7.5

displacement. In all loading cases, dynamic displacements are always highest at fundamental frequencies.

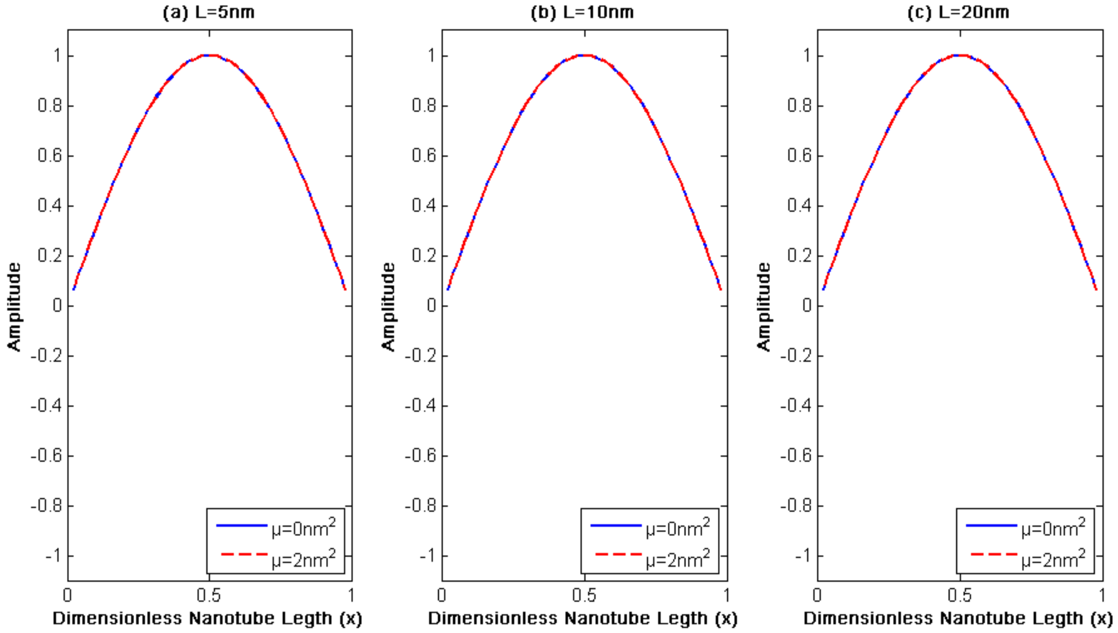

Fig. 12 Mode shapes of sinusoidal axial load at C-C boundary condition (a) Ω=2.5 (b) Ω=5 (c) Ω=7.5

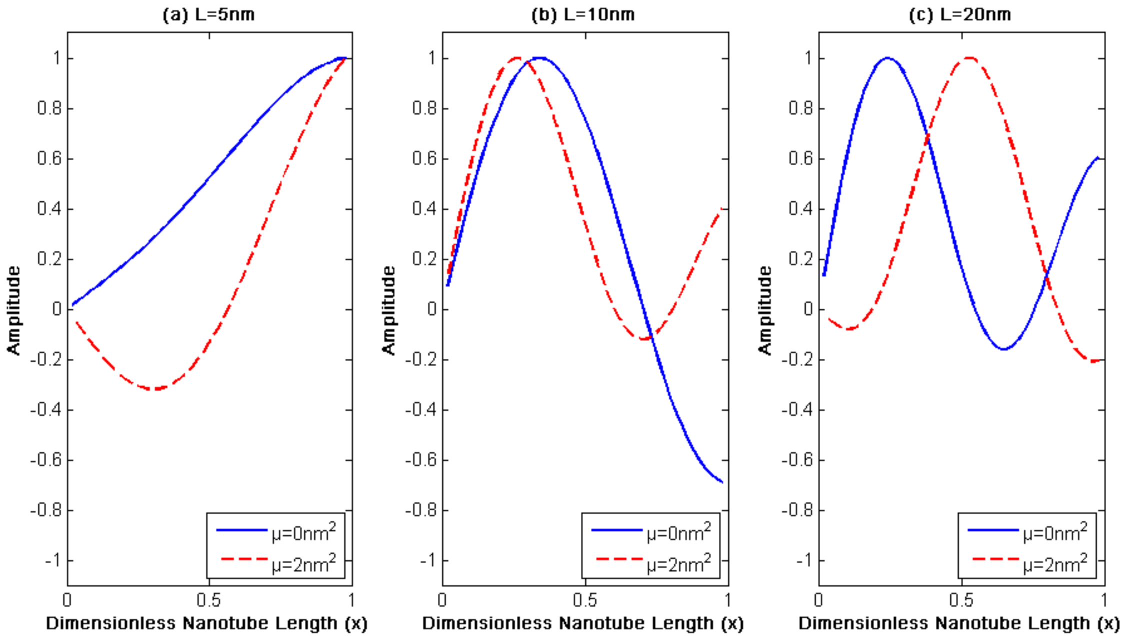

Fig. 13 Mode shapes of sinusoidal axial load at C-F boundary condition (a) Ω=2.5 (b) Ω=5 (c) Ω=7.5

Mode shapes are depicted as a dimensionless amplitude value that is a ratio between the local and nonlocal theory amplitude results. In Figs. 8-13, mode shapes of CNT at different NDFP values can be seen. The frequency of external load is chosen as Ω=2.5, 5 and 7.5. It can be seen form Table 1 that these external excitation frequencies are near to different frequencies in the local and nonlocal theories. In C-C boundary case, nonlocal and local theory results show similar mode shapes. Especially sinusoidal load case, mode shapes are identically similar for all mode numbers. Nonlocal effect can be seen in C-F boundary cases, more evidently.

4. Conclusions

In present study, nonlocal elastic nanorod model were developed with Eringen's Nonlocal Elasticity Theory. Small scale effect on the forced axial vibrations of nanorods is investigated with using the local and nonlocal rod models. Evident expressions are derived for dynamic displacements at clamped and free boundary conditions. Effects of parameters on forced vibration characteristics of nanorod are investigated. General results can be concluded as:

- Nonlocal effect is more pronounced in shorter nanotube lengths,
- Natural frequencies increase with increasing nonlocal parameter especially for shorter nanotube lengths.
- Nonlocal effect can change the mode shapes of CNT, particularly in C-F boundary case.
- Different mode shapes for nanorod are obtained with local and nonlocal theories

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