# On the bending and stability of nanowire using various HSDTs

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**Abstract.** In this article, various higher-order shear deformation theories (HSDTs) are developed for bending and buckling behaviors of nanowires including surface stress effects. The most important assumption used in different proposed beam theories is that the deflection consists of bending and shear components and thus the theories have the potential to be utilized for modeling of the surface stress influences on nanowires problems. Numerical results are illustrated to prove the difference between the response of the nanowires predicted by the classical and non-classical solutions which depends on the magnitudes of the surface elastic constants.

Keywords: surface effects; nanowires; bending; buckling

## 1. Introduction

The nanowires (NW)-based devices have found considerable range of applications in physics, engineering, and several other fields (Craighead 2000, Ekinci and Roukes 2005, He and Lilley 2008a, Jiang and Yan 2010, Liu *et al.* 2012, Wang and Feng 2009, Li *et al.* 2011, Chiu and Chen 2011a, Wang and Yang 2011, Eltaher *et al.* 2014). In physical applications, nanowires are often employed in advanced technological devices such as sensors, actuators, transistors, and resonators in nanoelectromechanical systems (NEMSs) (Craighead 2000, Ekinci and Roukes 2005). As is well known, conventional beam models failed to explain the size dependent mechanical response of nanostructrures. In the past few years, beam theories have been developed based on non-conventional continuum theories, such as the surface elasticity theory, strain gradient theory, and coupled stress theory to account for the size effect of 1D nanoscale structures (Al-Basyouni *et al.* 2015). Among these efforts, beam models based on the surface elasticity theory are attracting more and more attention due to their solid physical background (Wang and Feng 2009, Song and Huang 2009, Chiu and Chen 2011b, Ansari and Sahmani 2011, Mahmoud *et al.* 2012, Hosseini-Hashemi *et al.* 2013).

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Fig. 1 Simply supported-simply supported straight uniform beam with rectangular cross section and its coordinate system

Atoms near the surface and interface of a solid experience different local environment comparatively to those away from the surface because of the reduced coordination. Thus, the surface and interface of solids present different mechanical characteristics compared with the bulk material (Gurtin and Murdoch 1975, 1978, Dingreville *et al.* 2005). Gurtin and Murdoch (1975, 1978) and Gurtin *et al.* (1998) proposed a theoretical formulation to consider this surface/interface stress impact. This approach has been largely employed to investigate the mechanical response of nano defects, nano composites, and nanostructures (Sharma *et al.* 2003, Duan *et al.* 2005). He and Lim 2001). Recently, by employing the surface Cauchy-Born model, Park and Klein (2008) and Park (2008, 2009) examined the influences of the surface stress on the resonant frequencies of metallic/silicon NWs. He and Lilley (2008a, b) have considered the surface stress on all surfaces of the NWs and the effective Young's modulus of the NW was redefined. Yan and Jiang (2011) employed the Euler beam theory to investigate the buckling response of piezoelectric nanobeams with surface stress effect. Ansari and Sahmani (2011) adopted different beam theories for the buckling analysis of nanobeams with surface effect. Wang and Yang (2011) studied the buckling of nanobeams by considering the geometric nonlinearity.

In this work, various non-classical higher-order shear deformation beam theories are proposed to investigate the bending and axial buckling of a simply supported NWs including surface stress effect. Numerical results are presented to prove the significant effect of surface stress effects on the bending and buckling responses of NWs.

## 2. Formulation of the problem

Consider a beam of length L and rectangular cross-section of thickness h and width b. A coordinate system x, y, z is employed on the central axis of the beam, whereas the x axis is

Table 1	Shape	functions
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Model	f(z)	g(z)=1-f'(z)
TBT based on Reddy (1984)	$\frac{4z^3}{3h^2}$	$1 - \frac{4z^2}{h^2}$
SBT based on Touratier (1991)	$z - \frac{h}{\pi} \sin\left(\frac{\pi z}{h}\right)$	$\cos\!\left(\frac{\pi z}{h}\right)$
HBT based on Soldatos (1992)	$z - h \sinh\left(\frac{z}{h}\right) + z \cosh\left(\frac{1}{2}\right)$	$\cosh\left(\frac{z}{h}\right) - \cosh\left(\frac{1}{2}\right)$

considered along the length of the beam, the y axis in the width direction and the z axis is considered along the depth (height) direction. Also, the origin of the coordinate system is adopted at the left end of the beam (Fig. 1). The NW is subjected to transverse load q (point load or uniform load) and axial forces  $N_0$  at both ends.

### 2.1 Kinematics

Based on the same formulation proposed by Berrabah *et al* (2013) and Bourada *et al* (2015) where the transverse displacement is partitioned with two components (the bending part  $w_b$  and the shear part  $w_s$ ), the axial displacement, u, and the transverse displacement of any point of the beam, w, are given as

$$u(x,z) = -z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x}$$
(1a)

$$w(x, z) = w_b(x) + w_s(x)$$
 (1b)

The shape functions f(z) are chosen to satisfy the stress-free boundary conditions on the top and bottom surfaces of the beam, thus a shear correction factor is not needed. The displacement fields of the third-order beam theory (TBT) based on Reddy (1984), sinusoidal beam theory (SBT) based on Touratier (1991) and hyperbolic beam theory (HBT) based on Soldatos (1992) can be determined from Eq. (1) by employing different shape functions f(z) given in Table 1.

The non-zero strains associated with the displacements in Eq. (1) are

$$\varepsilon_x = -z \frac{\partial^2 w_b}{\partial x^2} - f(z) \frac{\partial^2 w_s}{\partial x^2} \text{ and } \gamma_{xz} = g(z) \frac{\partial w_s}{\partial x}$$
 (2)

where

$$g(z) = 1 - f'(z)$$
 and  $f'(z) = \frac{df(z)}{dz}$  (3)

### 2.2 Surface elasticity model for nanowires and constitutive relations

Surface impacts on the mechanical response of nanostructures can be investigated by

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considering surface energy and/or surface stresses. The resulting in-plane forces lead to surface stresses which can be derived by employing surface constitutive equations as

$$\sigma_x^s = (2\mu^s + \lambda^s)\varepsilon_x + \tau^s \text{ and } \tau_{xz}^s = \tau^s \frac{\partial w}{\partial x}$$
(4)

The superscript *s* is employed to represent the quantities corresponding to the surface.

The stress component  $\sigma_z$  is small comparatively to the  $\tau_{xz}$  for the classical beam theories and consequently it is supposed that  $\sigma_z=0$ . However this assumption does not respect the surface conditions considered in the Gurtin-Murdoch model. To solve this problem, it is supposed that the stress component  $\sigma_z$  changes linearly within the beam thickness and satisfies the balance conditions on the top and bottom surfaces (Lu *et al.* 2006). According to this assumption,  $\sigma_z$  can be determined as

$$\sigma_{z} = \frac{\frac{\partial \tau_{xz}}{\partial x}\Big|_{\text{at top}} + \frac{\partial \tau_{xz}}{\partial x}\Big|_{\text{at bottom}}}{2} + \frac{\frac{\partial \tau_{xz}}{\partial x}\Big|_{\text{at top}} + \frac{\partial \tau_{xz}}{\partial x}\Big|_{\text{at bottom}}}{h} z$$
(5)

Based on equations (2) together with equations (4), the components of surface stress for the present beam theories can be obtained in the following form

$$\sigma_{xx}^{s} = \left(2\mu^{s} + \lambda^{s}\right) \left(-z \frac{\partial^{2} w_{b}}{\partial x^{2}} - f(z) \frac{\partial^{2} w_{s}}{\partial x^{2}}\right) + \tau^{s}$$
(6a)

$$\tau_{xz}^{s} = \tau^{s} \left( \frac{\partial w_{b}}{\partial x} + \frac{\partial w_{s}}{\partial x} \right)$$
(6b)

The non-zero components of stress for the bulk ( $\sigma_x^b$  and  $\tau_{xz}^b$ ) of the beam can be determined as

$$\sigma_x^b = E\varepsilon_x + v\sigma_z = E\left(-z\frac{\partial^2 w_b}{\partial x^2} - f(z)\frac{\partial^2 w_s}{\partial x^2}\right) + \frac{2zv\tau^s}{h}\left(\frac{\partial^2 w_b}{\partial x^2} + \frac{\partial^2 w_s}{\partial x^2}\right)$$
(7a)

$$\tau_{xz}^{b} = G\gamma_{xz} = Gg(z)\frac{\partial w_{s}}{\partial x}$$
(7b)

The superscript b is employed to represent the quantities corresponding to the bulk.

In this work, we consider a superposition between the quantities corresponding to the surface and the bulk and this summation is considered to facilitate only the mathematical formulation

$$\sigma_x = \sigma_x^b + \sigma_x^s \text{ and } \tau_{xz} = \tau_{xz}^b + \tau_{xz}^s \tag{8}$$

#### 2.3 Governing equations

The minimum total potential energy principle, is employed here to obtain the governing equations (Reddy 2002, Draiche *et al.* 2014).

$$\partial \Pi = \delta \left( U_{\text{int}} - W_{ext} \right) = 0 \tag{9}$$

where  $\prod$  is the total potential energy.  $\delta U_{int}$  is the virtual variation of the strain energy; and  $\delta W_{ext}$  is the variation of work done by external forces. The first variation of the strain energy is given as:

$$\delta U_{\text{int}} = \int_{0}^{L} \int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_x \delta \varepsilon_x + \tau_{xz} \delta \gamma_{xz}) dz dx$$

$$= \int_{0}^{L} \left( -M_b \frac{d^2 \delta w_b}{dx^2} - M_s \frac{d^2 \delta w_s}{dx^2} + Q \frac{d \delta w_s}{dx} \right) dx$$
(10)
are the stress resultants defined as

where  $M_b$ ,  $M_s$  and Q are the stress resultants defined as

$$(M_b, M_s) = \int_A (z, f) \,\sigma_x dA \text{ and } Q = \int_A g \,\tau_{xz} dA \tag{11}$$

The first variation of the work done by the axial compressive force is given by

$$\delta V = \int_{0}^{L} q \delta w dx + \int_{0}^{L} N_0 \frac{d w}{dx} \frac{d \delta w}{dx} dx$$
(12)

where q and  $N_0$  are the transverse and axial loads, respectively.

Substituting Eqs. (10) and (12) into Eq. (9) and carrying out the integration by parts, the equations of motion of the proposed beam theory are determined as follows

$$\delta w_b : \frac{d^2 M_b}{dx^2} - N_0 \frac{d^2 (w_b + w_s)}{dx^2} + q = 0$$
(13a)

$$\delta w_s: \ \frac{d^2 M_s}{dx^2} + \frac{dQ}{dx} - N_0 \frac{d^2 (w_b + w_s)}{dx^2} + q = 0$$
(13b)

By substituting Eqs. (6) and (7) into Eq. (8), and the subsequent results into Eq. (11), the constitutive equations for the stress resultants are obtained as

$$M_{b} = \left[\frac{2I\nu\tau^{s}}{h} - D_{11} - \left(2\mu^{s} + \lambda^{s}\left(\frac{h^{3}}{6} + \frac{Ah}{2}\right)\right]\frac{\partial^{2}w_{b}}{\partial x^{2}} + \left[\frac{2I\nu\tau^{s}}{h} - D_{11}^{s} - \left(2\mu^{s} + \lambda^{s}\right)I_{p4}\right]\frac{\partial^{2}w_{s}}{\partial x^{2}}$$
(14a)

$$M_{s} = \left(\frac{2I_{1}\nu\tau^{s}}{h} - D_{11}^{s} - \left(2\mu^{s} + \lambda^{s}\right)I_{p4}\right)\frac{\partial^{2}w_{b}}{\partial x^{2}} + \left[\frac{2I_{1}\nu\tau^{s}}{h} - H_{11}^{s} - \left(2\mu^{s} + \lambda^{s}\right)I_{p5}\right]\frac{\partial^{2}w_{s}}{\partial x^{2}} + I_{p3}\tau^{s} \quad (14b)$$

$$Q = \left[ A_{55}^{s} + \mu^{s} J_{p1} + \frac{1}{2} \tau^{s} \left( J_{p3} - J_{p2} \right) \right] \frac{\partial w_{s}}{\partial x}$$
(14c)

where

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$$(D_{11}, D_{11}^s, H_{11}^s) = \int_A E(z^2, z f(z), f^2(z)) dA$$
 and  $A_{55}^s = \int_A G[g(z)]^2 dA$  (15a)

$$I = \int_{A} z^{2} dA = \frac{bh^{3}}{12}; \quad I_{1} = \int_{A} zf(z) dA;$$
(15b)

$$J_{p1} = 2 \int_{-h/2}^{h/2} [g(z)]^2 dz \quad ; \quad J_{p2} = \int_{-h/2}^{h/2} g(z) dz \; ; \\ J_{p3} = \int_{-h/2}^{h/2} f'(z)g(z) dz \; ; \qquad (15c)$$

$$I_{p3} = \int_{S} f(z) ds \; ; \; I_{p4} = \int_{S} z f(z) ds \; ; \; I_{p5} = \int_{S} [f(z)]^2 ds$$
(15d)

By substituting Eq. (14) into Eq. (13), the governing equations can be expressed in terms of displacements  $(w_b, w_s)$  as

$$\begin{bmatrix}
\frac{2I\nu\tau^{s}}{h} - D_{II} - \left(2\mu^{s} + \lambda^{s}\left(\frac{h^{3}}{6} + \frac{Ah}{2}\right)\right]\frac{\partial^{4}w_{b}}{\partial x^{4}} + \left[\frac{2I\nu\tau^{s}}{h} - D_{II}^{s} - \left(2\mu^{s} + \lambda^{s}\right)I_{p4}\right]\frac{\partial^{4}w_{s}}{\partial x^{4}} + q + \left(H - N_{0}\left(\frac{\partial^{2}w_{b}}{\partial x^{2}} + \frac{\partial^{2}w_{s}}{\partial x^{2}}\right)\right) = 0$$
(16a)

$$\left[\frac{2I_{I}\nu\tau^{s}}{h} - D_{II}^{s} - (2\mu^{s} + \lambda^{s})I_{p4}\right]\frac{\partial^{4}w_{b}}{\partial x^{4}} + \left[\frac{2I_{I}\nu\tau^{s}}{h} - H_{II}^{s} - (2\mu^{s} + \lambda^{s})I_{p5}\right]\frac{\partial^{4}w_{s}}{\partial x^{4}} + \left[A_{55}^{s} + \mu^{s}J_{p1} + \frac{1}{2}\tau^{s}(J_{p3} - J_{p2})\right]\frac{\partial^{2}w_{s}}{\partial x^{2}} + q + (H - N_{0})\left(\frac{\partial^{2}w_{b}}{\partial x^{2}} + \frac{\partial^{2}w_{s}}{\partial x^{2}}\right) = 0$$
(16b)

where *H* is the constant parameter which is determined by the residual surface tension  $\tau^x$  (generally assumed as a positive number) and the shape of cross section. For rectangular beam cross sections, the surface elasticity tension is expressed by

$$H = 2b\tau^s \tag{17}$$

## 3. Closed-form solution for simply supported nanowires

A simply supported beam with length L subjected to transverse load q and axial load  $N_0$  is considered here. The following expansions of displacements  $(w_b, w_b)$  are chosen to satisfy the simply supported boundary conditions of beam

$$w_b = \sum_{n=1}^{\infty} W_{bn} \sin(\alpha x)$$
(18a)

$$w_s = \sum_{n=1}^{\infty} W_{sn} \sin(\alpha x)$$
(18b)

where  $W_{bn}$ , and  $W_{sn}$  are arbitrary parameters to be determined, and  $\alpha = n\pi/L$ . The transverse load q is also expanded in the Fourier sine series as

$$q(x) = \sum_{n=1}^{\infty} Q_n \sin \alpha x, \quad Q_n = \frac{2}{L} \int_0^L q(x) \sin(\alpha x) dx$$
(19)

The Fourier coefficients  $Q_n$  associated with some typical loads are given

$$Q_n = q_0, \ n = 1$$
 for sinusoidal load, (20a)

$$Q_n = \frac{4q_0}{n\pi}, \ n = 1,3,5....$$
 for uniform load, (20b)

Substituting the expansions of  $w_b$ ,  $w_s$ , and q from Eqs. (19) and (20) into Eq. (18), the closed-form solutions can be obtained from the following equations

$$\begin{pmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{bmatrix} - N_0 \alpha^2 \begin{bmatrix} I & I \\ I & I \end{bmatrix} \end{pmatrix} \begin{pmatrix} W_{bn} \\ W_{sn} \end{pmatrix} = \begin{pmatrix} Q_n \\ Q_n \end{pmatrix}$$
(21)

where

$$S_{11} = \alpha^4 \left( H + \left( \frac{2I\nu\tau^s}{h} - D_{11} - \left( 2\mu^s + \lambda^s \left( \frac{h^3}{6} + \frac{Ah}{2} \right) \right) \right)$$
(22a)

$$S_{12} = \alpha^4 \left( H + \left( \frac{2I\nu\tau^s}{h} - D_{11}^s - \left( 2\mu^s + \lambda^s \right) I_{p4} \right) \right)$$
(22b)

$$S_{22} = \alpha^{4} \left( H + \left( \frac{2I_{1}v\tau^{s}}{h} - H_{11}^{s} - (2\mu^{s} + \lambda^{s})I_{p5} \right) \right) - \alpha^{2} \left( A_{55}^{s} + \mu^{s}J_{p1} + \frac{1}{2}\tau^{s} \left( J_{p3} - J_{p2} \right) \right)$$
(22c)

# 3.1 Bending

The static deflection is obtained from Eq. (21) by setting  $N_0$  to zero

$$w(x) = \sum_{n=1}^{\infty} \left( \frac{Q_n}{S_{11} - S_{12}^2 / S_{22}} + \frac{Q_n}{S_{22} - S_{12}^2 / S_{11}} - \frac{2Q_n}{S_{11} S_{22} / S_{12} - S_{12}} \right) \sin \alpha x$$
(23)

# 3.2 Buckling

The buckling load is obtained from Eq. (21) by setting q to zero

$$N_0 = \frac{S_{11}S_{22} - S_{12}^2}{\alpha^2 (S_{11} + S_{22} - 2S_{12})}$$
(24)

Table 2 Comparison between maximum center deflections under uniform load of nanowires obtained with classical and non-classical solutions

	EBT		FBT		Present	t TBT	Presen	t SBT	Present HBT	
L/h	classical	non- classical	classical	non- classical	classical	non- classical	classical	non- classical	classical	non- classical
10	8.8127	2.7423	9.0276	2.7538	9.0276	2.7544	9.0273	2.7544	8.9890	2.7518
15	44.6145	7.2189	45.0980	7.2238	45.0980	7.2240	45.0973	7.2240	45.0110	7.2229
20	141.0039	13.6198	141.8635	13.6216	141.8635	13.6217	141.8622	13.6217	141.7089	13.6213
25	344.2479	21.8829	345.5910	21.8835	345.5909	21.8835	345.5890	21.8835	345.3494	21.8834
30	713.8325	31.9907	715.7665	31.9909	715.7664	31.9909	715.7636	31.9909	715.4186	31.9909
35	1322.4628	43.9385	1325.0952	43.9386	1325.0952	43.9386	1325.0913	43.9386	1324.6216	43.9386
40	2256.0632	57.7251	2259.5014	57.7251	2259.5013	57.7251	2259.4964	57.7251	2258.8829	57.7251
45	3613.7770	73.3501	3618.1285	73.3501	3618.1284	73.3501	3618.1221	73.3501	3617.3456	73.3501
50	5507.9667	90.8133	5513.3390	90.8133	5513.3389	90.8133	5513.3311	90.8133	5512.3725	90.8133

Table 3 Comparison between maximum center deflections under point load of nanowires obtained with classical and non-classical solutions

	EBT		FBT		Present TBT		Present SBT		Present HBT	
L/h	classical	non- classical	classical	non- classical	classical	non- classical	classical	non- classical	classical	non- classical
10	1.3743	0.4220	1.4062	0.4229	1.4062	0.4230	1.4061	0.4230	1.4004	0.4228
15	4.6382	0.7335	4.6861	0.7334	4.6861	0.7334	4.6860	0.7334	4.6774	0.7334
20	10.9942	1.0303	11.0580	1.0300	11.0580	1.0300	11.0580	1.0300	11.0466	1.0300
25	21.4731	1.3173	21.5529	1.3171	21.5529	1.3171	21.5528	1.3171	21.5385	1.3172
30	37.1055	1.5992	37.2013	1.5991	37.2013	1.5991	37.2011	1.5991	37.1840	1.5991
35	58.9222	1.8782	59.0339	1.8781	59.0339	1.8781	59.0337	1.8781	59.0138	1.8781
40	87.9539	2.1555	88.0815	2.1554	88.0815	2.1554	88.0813	2.1554	88.0585	2.1554
45	125.2312	2.4317	125.3748	2.4317	125.3748	2.4317	125.3746	2.4317	125.3489	2.4317
50	171.7849	2.7073	171.9444	2.7073	171.9444	2.7073	171.9442	2.7073	171.9157	2.7073

# 4. Numerical results and discussion

In this section, numerical results are provided for analytical solutions shown in the previous sections. The following material characteristics are used in computations as follows (Gurtin and Murdoch 1978):

$$E = 17.73 \ 10^{10} \ \text{N/m}^2$$
,  $\nu = 0.27$ ,  $\lambda^s = -8 \ \text{N/m}$ ,  $\mu^s = 2.5 \ \text{N/m}$ ,  $\tau^s = 1.7 \ \text{N/m}$ 

It is supposed that h=b=1 nm and L varies from L/h=10 to 50.

Tables 2 and 3 show, respectively, the maximum deflections of a simply supported nanowire subjected to uniform load and point load by using the classical and non-classical theories. The obtained results are compared with those computed independently for the first time based on the Euler-Bernoulli beam theory (EBT), and First beam theory (FBT) for a wide range of thickness ratio. It can be seen that the results of present theories are in excellent agreement with those



Fig. 2 Variation of maximum center deflections with the aspect ratio corresponding to different values  $\tau^s$  of with the assumption of  $2\mu^s + \lambda^s = 0$ 



Fig. 3 Variation of maximum center deflections with the aspect ratio corresponding to different magnitudes of  $2\mu^s + \lambda^s$  with the assumption of  $\tau^s = 0$ 

predicted by FBT for all values of thickness ratio L/h. The TBT, SBT, and HBT provide solutions which are almost the same for all values of thickness ratio L/h, whereas the EBT underestimates deflections. The difference between EBT and shear deformation theories (i.e., TBT, SBT, HBT and FBT) is negligible for slender nanowires and considerable for deep nanowires. It can be proved from the results that by introducing the surface stress impacts, the deflections corresponding to all values of aspect ratio decrease which shows the fact that with consideration of the surface stress effects, the stiffness of nanowire will be increased.

Fig. 2 shows the influence of value of  $\tau^s$  on the transverse deflection of nanowires. The value of

 $(2\mu^s + \lambda^s)$  is taken zero, and the variation of the transverse deflection with the span-to-depth ratio (L/h) of nanowire is shown corresponding to various magnitude of  $\tau^s$  by using various higher beam theories (i.e., TBT, SBT and HBT). It is seen that the overall bending stiffness of nanowire tends to increase as the value of  $\tau^s$  increases.

Fig. 3 presents the variation of the transverse deflection of nanowires versus the span-to-depth ratio (L/h) of nanowire for three different conditions. It is seen that by taking  $\tau^s$  zero, the positive value of  $2\mu^s + \lambda^s$  makes nanowire stiffer. However, the non-positive value of  $2\mu^s + \lambda^s$  diminishes the stiffness of the nanowire.



Fig. 4 Variation of critical buckling load with the aspect ratio corresponding to different values of magnitudes of  $\tau^s$  with the assumption of  $2\mu^s + \lambda^s = 0$ 

Table 4 Critical buckling loads corresponding to the first mode obtained with classical and non-classical solutions (nN)

L/h	Ref <sup>(a)</sup>		Present TBT		Pres	ent SBT	Present HBT	
	classical	non-classical	classical	non-classical	classical	non-classical	classical	non-classical
10	1.4226	4.6272	1.4226	4.6272	1.4226	4.6272	1.4226	4.6272
15	0.6410	3.9518	0.6410	3.9518	0.6410	3.9518	0.6410	3.9518
20	0.3623	3.7117	0.3623	3.7117	0.3623	3.7117	0.3623	3.7117
25	0.2324	3.5998	0.2324	3.5998	0.2324	3.5998	0.2324	3.5998
30	0.1616	3.5389	0.1616	3.5389	0.1616	3.5389	0.1616	3.5389
35	0.1188	3.5021	0.1188	3.5021	0.1188	3.5021	0.1188	3.5021
40	0.0910	3.4782	0.0910	3.4782	0.0910	3.4782	0.0910	3.4782
45	0.0719	3.4618	0.0719	3.4618	0.0719	3.4618	0.0719	3.4618
50	0.0583	3.4501	0.0583	3.4501	0.0583	3.4501	0.0583	3.4501

<sup>(a)</sup> Taken from Ref (Ansari and Sahmani 2011)

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In order to demonstrate the validity of the present formulation in the case of buckling analysis of nanowires, comparative studies are presented in Tables 4-6. The obtained results based on TBT, SBT and HBT are compared with those of Ansari and Sahmani (2011). Excellent agreement can be observed for different values of span-to-depth ratio (L/h). It can be seen that the results from the classical theories due to ignoring the surface stress effect are highly underestimate when comparing with those from the non-classical theories. This effect is more pronounced for the lower mode numbers. The obtained results confirm again that this effect is important and makes nanowire stiffer.

The effect of the values of  $\tau^s$  on the variation of the critical buckling load of nanowires is shown in Fig. 4. It can be observed that the increase of the value of  $\tau^s$  induces an increase in the overall bending stiffness of nanowire.

Table 5 Critical buckling loads corresponding to the second mode obtained with classical and non-classical solutions (nN)

L/h	Ref <sup>(a)</sup>		Present TBT		Pres	ent SBT	Present HBT	
	classical	non-classical	classical	non-classical	classical	non-classical	classical	non-classical
10	5.3019	8.0214	5.3019	8.0204	5.3027	8.0201	5.3019	8.0204
15	2.4819	5.5471	2.4819	5.5469	2.4820	5.5468	2.4819	5.5469
20	1.4226	4.6272	1.4226	4.6272	1.4226	4.6272	1.4226	4.6272
25	0.9185	4.1914	0.9185	4.1913	0.9185	4.1913	0.9185	4.1913
30	0.6410	3.9518	0.6410	3.9518	0.6410	3.9518	0.6410	3.9518
35	0.4723	3.8064	0.4723	3.8064	0.4723	3.8064	0.4723	3.8064
40	0.3623	3.7117	0.3623	3.7117	0.3623	3.7117	0.3623	3.7117
45	0.2866	3.6465	0.2866	3.6465	0.2866	3.6465	0.2866	3.6465
50	0.2324	3.5998	0.2324	3.5998	0.2324	3.5998	0.2324	3.5998

<sup>(a)</sup> Taken from Ref (Ansari and Sahmani, 2011)

Table 6 Critical buckling loads corresponding to the third mode obtained with classical and non-classical solutions (nN)

	R. An	sari et Al.	Present RBT		Pres	ent SBT	Present HBT	
L/h	classical	non-classical	classical	non-classical	classical	non-classical	classical	non- classical
10	10.7134	12.8760	10.7134	12.8700	10.7175	12.8690	10.7134	12.8703
15	5.3019	8.0214	5.3019	8.0204	5.3027	8.0200	5.3019	8.0204
20	3.1060	6.0915	3.1060	6.0912	3.1062	6.0910	3.1060	6.0912
25	2.0267	5.1512	2.0267	5.1511	2.0268	5.1511	2.0267	5.1511
30	1.4226	4.6272	1.4226	4.6272	1.4226	4.6272	1.4226	4.6272
35	1.0520	4.3066	1.0520	4.3066	1.0520	4.3066	1.0520	4.3066
40	0.8089	4.0967	0.8089	4.0966	0.8089	4.0966	0.8089	4.0966
45	0.6410	3.9518	0.6410	3.9518	0.6410	3.9518	0.6410	3.9518
50	0.5203	3.8478	0.5203	3.8478	0.5203	3.8477	0.5203	3.8478

<sup>(a)</sup>Taken from Ref (Ansari and Sahmani 2011)

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Fig. 5 Variation of critical buckling load with the aspect ratio to different magnitudes of  $2\mu^s + \lambda^s$  with the assumption of  $\tau^s = 0$ 

The variation of the critical buckling load of nanowires as function of the span-to-depth ratio (L/h) of nanowire for three different conditions, is illustrated in Fig. 5. It is seen that by taking  $\tau^s$  zero, the non-positive value of  $2\mu^s + \lambda^s$  makes the nanowire softer, while, for the positive value of  $2\mu^s + \lambda^s$  the nanowire becomes stiffer.

### 5. Conclusions

In this work, we have presented a framework of high-order surface stresses, based on various higher-order shear deformation beam theories, to investigate the bending and buckling response of NWs. Our results showed that the bending and buckling behaviors of nanowires are significantly affected by the surface stress impacts. Indeed, it is demonstrated that the inclusion of surface stress effect makes a nanowire stiffer, and hence, leads to a reduction of deflection and an increase of buckling load. The formulation lends itself particularly well to functionally graded structures (Bouderba *et al.* 2013, Ait Amar Meziane *et al.* 2014, Belabed *et al.* 2014, Hebali *et al.* 2014, Bousahla *et al.* 2014, Ait Yahia *et al.* 2015, Belkorissat *et al.* 2015, Hamidi *et al.* 2015, Larbi Chaht *et al.* 2015) and nanotubes (Besseghier *et al.* 2015, Tounsi *et al.* 2013), which will be considered in the near future.

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