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Quantum computing using applied electric field to quantum dots

A. Meighan*, A. Rostami and K. Abbasian

School of Engineering Emerging-Technologies, University of Tabriz, Tabriz 51666, Iran

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Abstract. In recent years, spins of confined carriers in quantum dots are promising candidates for the logical units in quantum computers. In many concepts developed so far, the individual spin q-bits are being manipulated by magnetic fields, which is difficult to achieve. In the current research the recent developments of spin based quantum computing has been reviewed. Then, Single-hole spin in a molecular quantum dots with less energy and more speed has been electrically manipulated and the results have been compared with the magnetic manipulating of the spin.

Keywords: *g*-factor; electrical manipulation; spin manipulation; molecular quantum dot; magnetic field; g-tensor

1. Introduction

The importance of quantum computers is evidence and in years before theoretical potential and experimental challenges of quantum computers vastly clarified (Lloyd 1993). The building block of classical computers called bit and information is stored in bits, which take the discrete values 0 and 1. In contrast, the quantum computers are based on quantum-mechanical physic and the fabrication and algorithms that rely on each part of these computers are quantum-mechanical forms. In this case, the building blocks of the computers are called quantum bit or *q*-bit and information is stored in these q-bits. A *q*-bit can be in states labeled |0> and |1>, but it can also be in a superposition of these states, $a|0> + b|1>$, where a and b are complex numbers. The superposition of one and zero states is the power of quantum computers, and it differs from classical computers.

Spins of electron and holes in quantum dots are good candidates for the logical units in quantum computers (Loss and Divincenzo1998, Petta *et al*. 2005, Kroutvar *et al*. 2004). Nano-structures such as quantum dots can confine a single electron or hole in nanometer space. That is used in this research as the main advantage of quantum structures to electrically manipulating the spin.

2. Content

*Corresponding author, Msc., E-mail: arez.meighan@gmail.com

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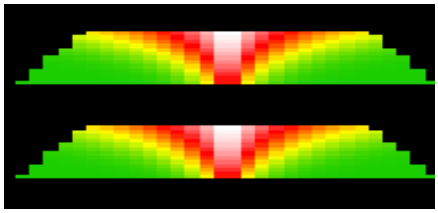


Fig. 1 Pyramid-shaped GaAs/InGaAs quantum dots with trump*et al*loy profile

2.1 Applying an electric field to change the spin precession axis

In many cases of spin based quantum computing, the individual spin *q*-bits are being manipulated by magnetic fields that in practice is hard to achieve. It is also difficult to address a special small point by applying a magnetic field (Koppens *et al.* 2006). These are the disadvantages of using magnetic fields to manipulate the spin. In contrast, it is important to note that since spin has a magnetic characteristic, so easily reacts with magnetic field.

A novel method to overcome the problems in magnetic fields is to address individual spin of electron or hole by a simple electrical gate (Salis *et al*. 2001, Pryor and Flatte 2006). Through the spin-orbit effect, the electron or hole *g*-tensor can be influenced by the shape of quantum dot and applied electric fields. These features enable the gating of a quantum dot and thereby the spin of a single electron or hole can be manipulated.

In heterostructures such as quantum wires, quantum wells and quantum dots, the electrical control of *g*-factor can be done by shifting the wave functions of electron or hole carriers between different material regions with different g-factors and by an applied electric bias (Jiang and Yablonovitch 2001). This electrical control and anisotropy of the *g*-tensor make it possible to control the Zeeman splitting and tuning the spin of electron or hole without time dependent magnetic fields (Salis *et al*. 2001) The *g*-factor of both electrons and holes have been considered extensively (Kato *et al*. 2003, Björk *et al*. 2005, Sheng and Babinski 2007, Andlauer *et al.* 2008). Self-assembled quantum dots have shown novel electrical *g*-factor engineering both experimentally and theoretically (Nakaoka *et al*. 2004, Mayer *et al.* 2006, Pingenot *et al*. 2008).

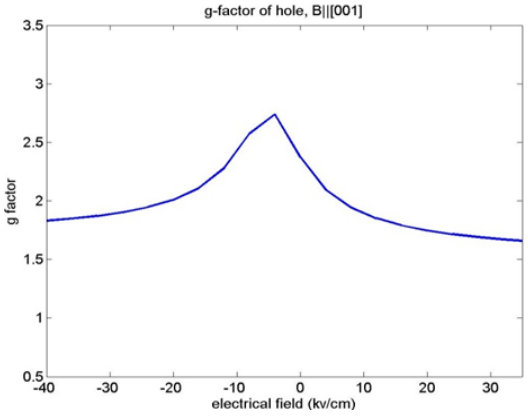
In this research, vertically stacked InAs/GaAs double dot structures (Fig. 1) are considered as benchmarks based on the experimental data (Krenner *et al*. 2005, Bracker *et al*. 2006). The quantum dot molecule itself is characterized by the dot separation d=1.5nm, the quantum dot height h=2nm, the dot width w=15nm, and the alloy profile within the individual dots that have identical size and composition.

To calculate the *g*-factor that in this structure is considered as a tensor, we need a constant magnetic field along the principal axes of the quantum dots, and an electric filed with a magnitude varying between -40kv/cm to 40kv/cm in Z direction.

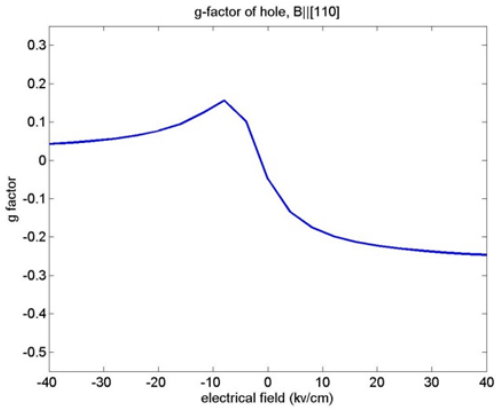
2.1.1 Calculation of the electrically controllable g-factor

In the Pyramid-shaped GaAs/InGaAs quantum dots, the Schrodinger equation with total Hamiltonian (Eq. (1)) is solved and energies with special consideration of the whole structure has been calculated (Andlauer *et al*. 2009)

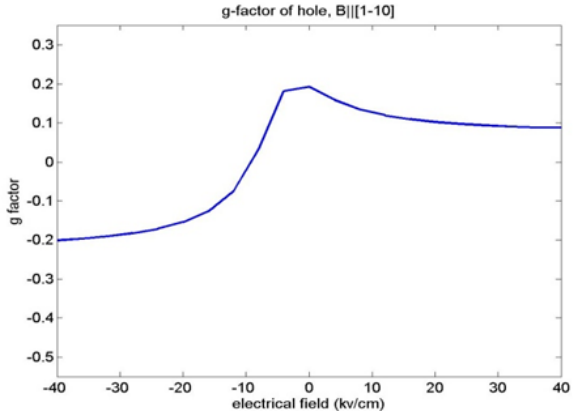
$$H = H_{k.p}^{8x8}(x, x', B) + \frac{g_0 \mu_B}{2} \hat{S}^{8x8} B + ex.F \tag{1}$$



(a) *g*-factor when magnetic field is parallel to [0 0 1]



(b) *g*-factor when magnetic field is parallel to [110]



(c) *g*-factor when magnetic field is parallel to [1 -1 0]

Fig. 2 Ground state *g*-factor of the hole in molecular quantum dots. It is shown that the sensitivity of ground state *g*-factor respect to variation of the electric field is high

By applying the magnetic field parallel to the three principal axes of the quantum dots ([001], [110], [1 -1 0]) the g-factor has been calculated as following

$$g = \frac{E_n \uparrow - E_n \downarrow}{\mu_B B} \tag{2}$$

Then the *g*-tensor is determined by

$$g_h = \begin{pmatrix} \frac{g_h^{[110]} + g_h^{[1\bar{1}0]}}{2} & \frac{g_h^{[110]} - g_h^{[1\bar{1}0]}}{2} & 0 \\ \frac{g_h^{[110]} - g_h^{[1\bar{1}0]}}{2} & \frac{g_h^{[110]} + g_h^{[1\bar{1}0]}}{2} & 0 \\ 0 & 0 & g_h^{[001]} \end{pmatrix} \tag{3}$$

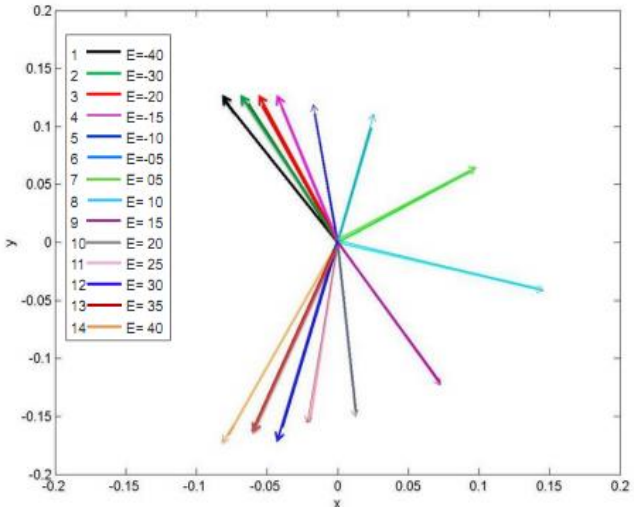


Fig. 3 The electric field dependency of the spin precession axis in the *x-y* plane. This figure shows the direction of the spin precession axis in *x-y* plane when the electric field has been increased gradually from −40kv/cm (first vector) to +40kv/cm (14th vector)

And by the following relations the spin precession axis is calculated

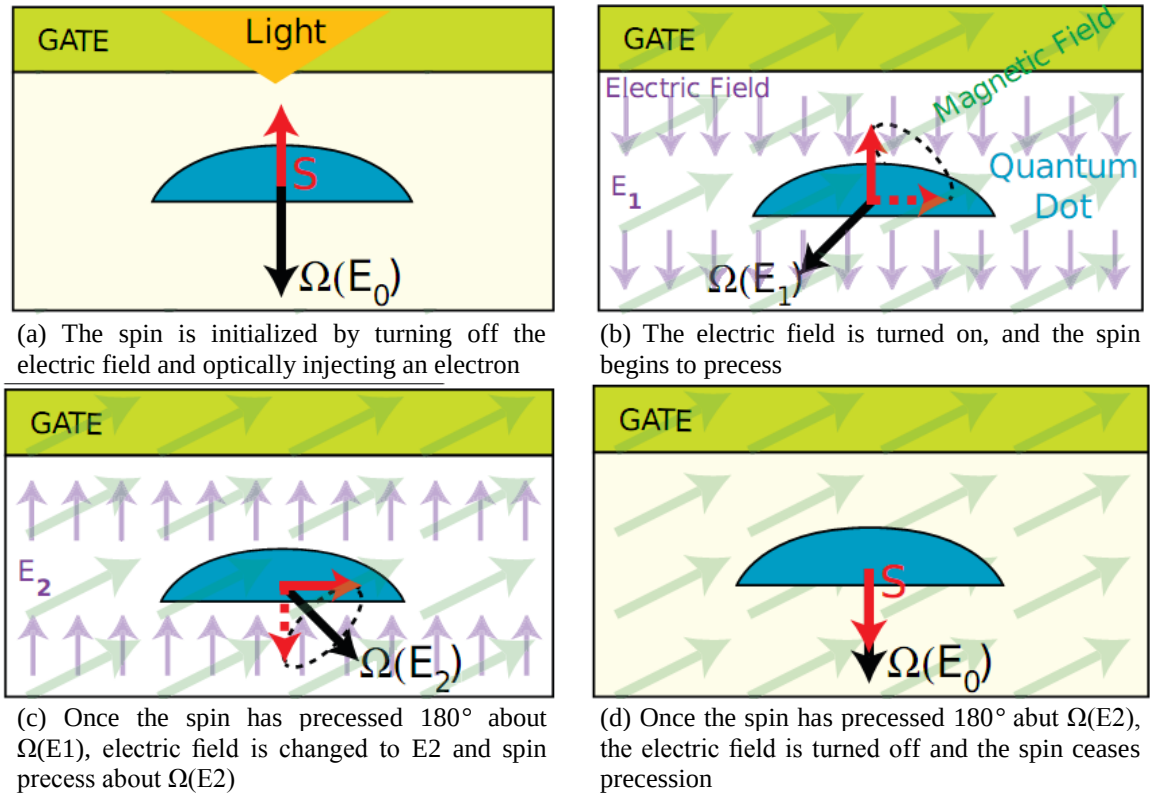
$$H_h = \sigma.\Omega_h \tag{4}$$

$$\Omega_h = \frac{\mu_B}{2} g.B \tag{5}$$

Where Ω_h is the spin precession axis and $g_h^{[110]}$, $g_h^{[001]}$ and $g_h^{[1\text{-}10]}$ are electric field dependent *g*-factors in the direction of principal axes of the quantum dots. Figs. 2(a)-(c) show the variations of the *g*-factor respect to the electric filed. We used Eq. (5) to calculate the spin precession axis and because it depends on the g-factor and the g-factor depends on the electric field (Fig. 2), the spin precession axis will change respect to the variations of the electric field .As shown in Fig. 3, when the electric field has been increased gradually from −40kv/cm to +40kv/cm the direction of the spin precession axis has changed about 270° in the *x-y* plane.

2.2 Manipulation of the spin precession axis

By the use of full Bloch-sphere method (Pingenot *et al*. 2008), control of a single spin requires the ability to switch between two orthogonal spin precession axes (Fig. 4). Flipping a spin requires positioning the spin at 45° from the spin precession axis or equivalently, positioning the spin precession axis at 45° from the spin. The spin then precesses around the precession axis until it reaches to the position of 45° from the spin precession axis on the other side. The spin precession axis is then switched off, and the orthogonal spin precession axis is switched on. Then the spin will precess around the new axis (starting out 45° from that axis) until it is 45° from the axis on the other side. The spin is now “down” and full spin switching is done.



(a) The spin is initialized by turning off the electric field and optically injecting an electron

(b) The electric field is turned on, and the spin begins to precess

(c) Once the spin has precessed 180° about Ω(E1), electric field is changed to E2 and spin precess about Ω(E2)

(d) Once the spin has precessed 180° abut Ω(E2), the electric field is turned off and the spin ceases precession

Fig. 4 Full Bloch-sphere manipulation of the spin (Pingenot *et al.* 2008)

2.2.1 Calculating device performance with the hole spins

To calculate device performance with the hole spins, first we should find the spin precession time-t_1 around one axis ($\vec{\Omega}_1 = \vec{\Omega}(E_1)$). This is done by constructing the spin precession vector at 45° on each side of the spin precession axes. Without loss of generality, the *Z*-axis can be defined parallel to $\vec{\Omega}_1$. In the *Z*-basis

$$|+Z\rangle = |\uparrow\rangle \tag{6}$$

$$|+X\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \tag{7}$$

$$|-X\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle) \tag{8}$$

The first state that is 45° from *Z*-axis defined as

$$k\left[|\uparrow\rangle + \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)\right] \tag{9}$$

And the second state that is 225° from *Z*-axis defined as

$$k\left[\left|\uparrow\right\rangle+\frac{1}{\sqrt{2}}(\left|\uparrow\right\rangle-\left|\downarrow\right\rangle)\right] \tag{10}$$

Where, *k* is normalization constant and is determined by

$$k=k'=\frac{\sqrt{2}}{\sqrt{4+2\sqrt{2}}} \tag{11}$$

And the states then are

$$\left|\psi_0\right\rangle=\frac{1}{\sqrt{4+2\sqrt{2}}}\left[(\sqrt{2}+1)\left|\uparrow\right\rangle+\left|\downarrow\right\rangle\right]\pi \tag{12}$$

$$\left|\psi_1\right\rangle=\frac{1}{\sqrt{4+2\sqrt{2}}}\left[(\sqrt{2}+1)\left|\uparrow\right\rangle-\left|\downarrow\right\rangle\right]\sqrt{a^2+b^2} \tag{13}$$

The initial state $|\psi_0\rangle$ will then precess until it overlaps completely with the final state, $|\psi_1\rangle$. That is

$$\left|\left\langle\psi_1\left|e^{-i\frac{\Omega t}{2\hbar}\sigma_x}\right|\psi_0\right\rangle\right|^2=1 \tag{14}$$

This equation is solved when

$$\frac{\Omega t}{2\hbar}=\frac{\pi}{2} \tag{15}$$

And therefore the time required to flip a spin 180° about the *z* axis is

$$t=\frac{\pi\hbar}{\Omega} \tag{16}$$

Because of the complexity, we numerically find the optimum electric field values corresponding to these to orthogonal axes. We used Matlab software to numerically solve the minimum time, and corresponding electric fields and the results are

$$E_1=-4KV/cm,\quad E_1=+4KV/cm \tag{17}$$

In these electric fields we found the spin switching time about 19 picoseconds that is about ten times faster than last reports (Pingenot *et al.* 2008).

3. Conclusions

In this research we considered a Pyramid-shaped GaAs/InGaAs quantum dots with trump*et al*loy profile. In this molecular quantum dot *g*-factor is very sensitive to the electric field and by changing the electric field the spin precession axis can be rotated about 270° in the *x*-*y* plane. The full control of a single spin requires the ability to switch between two orthogonal spin precession axes and we found that this structure has all of the mentioned features. In addition we numerically calculated the amount of energy and found that in these quantum dots the amount of energy to flip the spin from up to down is less than last reports. Also we calculated the switching time that is about ten times faster than the previous works. This allows a full spin switching of a single hole in a quantum dot molecule by a gate voltage that is much easier than applying a variable magnetic field.

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