Effect of hall current in Transversely Isotropic magneto thermoelastic rotating medium with fractional order heat transfer due to normal force

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Abstract. This investigation is focused on the study of effect of hall current in transversely isotropic magneto thermoelastic homogeneous medium with fractional order heat transfer and rotation. As an application the bounding surface is subjected to normal force. The research becomes more interesting due to interaction of Hall current with the effect of rotation as it has found various applications. Laplace and Fourier transform is used for solving field equations. The analytical expressions of temperature, displacement components, stress components and current density components are computed in the transformed domain. The effects of hall current and fractional order parameter at different values are represented graphically.

Keywords: Laplace and Fourier Transform; thermoelastic; hall current; Transversely Isotropic; fractional order heat transfer

1. Introduction

When a material body is subjected to an external force or loads, it transmits mechanical waves. For example, if a sudden heat is applied in a solid body, it will create a mechanical wave through thermal expansion. It was observed that the interaction s between the mechanical and thermal fields occurred through the Lorentz forces, Ohm's law and the electric field created by the velocity of a material particle, moving in a magnetic field. Investigation of interaction between the strain and electromagnetic fields becomes new area of research, which is called magneto elasticity because of its effective aspects in various domains of science and technology, like damping of acoustic waves in a magnetic field, geophysics for understanding the effect of the Earth's magnetic field on seismic waves, electrical power engineering, development of a highly sensitive super conducting magnetometer, emissions at electromagnetic radiation from nuclear devices, optics and plasma physics.

Chen *et al.* (Chen and Gurtin 1968, Chen *et al.* 1968, 1969) formulated a two-temperature thermoelasticity of deformable bodies for the conduction of heat depending on two types of temperatures. Green and Naghdi (1991, 1992, and 1993) dealt with the linear and the nonlinear theories of thermoelastic body with and without energy dissipation. Three novel thermoelastic

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theories were proposed by them, based on entropy equality. Their theories are known as thermoelasticity type I theory, the thermoelasticity type II theory (i.e., thermoelasticity without energy dissipation) and the thermoelasticity type III theory (i.e., thermoelasticity with energy dissipation), "On linearization, type I becomes the classical heat equation whereas on linearization type-II as well as type-III theories gives finite speed of thermal wave propagation. Marin (1997a) had proved the Cesaro means of strain and kinetic energies of dipolar bodies with finite energy.

Ezzat *et al.* (2011) studied the linear theory of thermoelasticity using Fourier law of heat conduction with time-fractional order and three-phase lag and proved a uniqueness and reciprocity theorems. Bachher and Sarkar (2016) studied the theory of generalized thermoelasticity based on the heat conduction equation with the Caputo time-fractional derivative for magneto-thermoelastic response of a homogeneous isotropic two-dimensional rotating elastic half-space solid. Sheoran and Kundu (2016) gave a review and future prospects of fractional order generalized theory of thermoelasticity. Kumar *et al.* (2017a) investigated the Rayleigh waves in a homogeneous transversely isotropic magneto-thermoelastic in the presence of two temperature, hall current and rotation. Moreno-Navarro *et al.* (2018) proposed a fully coupled thermodynamic oriented transient finite element formulation for magnetic, electric, mechanic and thermal field's interactions.

Youssef and Abbas (2014) considered fractional order thermal conductivity as a linear function of temperature in the context of fractional order generalized thermoelasticity. Tripathi *et al.* (2017) studied the generalized thermoelasticity fractional order thermoelastic response due to a heat source that varies periodically with time with one relaxation time. Abbas (2018) studied the effect of fractional order 2-D GN-III model due to thermal shock for weak, normal and strong conductivity. Marin *et al.* (2017) studied the GN-thermoelastic theory for a dipolar body using mixed initial BVP and proved a result of Hölder's-type stability.

Kumar *et al.* (2017b) studied the thermomechanical interactions and effect of hall current and magnetic field in a homogeneous transversely isotropic thermoelastic rotating medium with two temperatures, due to thermomechanical sources by Green-Naghdi Theories of Type-II and Type-III. Ezzat *et al.* (2017) developed a unified mathematical fractional model of two-temperature phase-lag Green–Naghdi thermoelasticity theories based on two-temperature. Ezzat *et al.* (2012, 2015, 2016, 2017), presented a new mathematical models of two-temperature electro-thermo viscoelasticity theory in the perspective of heat conduction and provided applications of this model to different problems like concrete problems, a thermal shock problem and a problem for a half-space exposed to ramp-type heating respectively. Despite of this several researchers worked on different theory of thermoelasticity as Marin (1996, 2009, 2010), Marin and Baleanu (2016), Ezzat *et al.* (2012, 2015, 2016, 2017), Marin (1997b, 2008), Marin and Baleanu (2016), Marin and Stan (2013), Marin and Nicaise (2016), Marin and Ö chsner (2017), Othman and Marin (2017), Chauthale and Khobragade (2017), Marin (1998, 2009, 2010), Kumar *et al.* (2018), Marin *et al.* (2017), Lata and Kaur (2019a, b, c, d, e).

From above study, it has been observed that numerous researches have been carried out in recent years on magneto-thermoelastic wave propagation in a non-rotating medium. It seems that slight attention has been given to the study of a rotating medium. Since most large bodies, like the earth, the moon, and other planets, have an angular velocity, it looks more realistic to study the propagation of magneto-thermoelastic waves in a rotating medium with Hall Effect. The fraction order derivatives are used to find viscoelasticity of such materials with a high precision.

In this paper, we have attempted to study the effect of hall current and fractional order heat transfer due to normal force in transversely isotropic magneto thermo elastic medium. The expressions of displacement components, conductive temperature and stresses components due to normal force are calculated in transformed domain by using the Laplace and Fourier transform. Numerical inversion technique is used to find the resulting quantities in the physical domain and effects of frequency at different values have been represented graphically.

2. Basic equations

Following Lata *et al.* (2016), the simplified Maxwell's linear equation of electrodynamics for a slowly moving and perfectly conducting elastic solid are

$$\operatorname{curl} \vec{h} = \vec{j} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t},\tag{1}$$

$$curl\,\vec{E} = -\,\mu_0 \frac{\partial\vec{h}}{\partial t},\tag{2}$$

$$\vec{E} = -\mu_0 \left(\frac{\partial \vec{u}}{\partial t} + \vec{H}_0 \right), \tag{3}$$

$$div\,\vec{h}=0.\tag{4}$$

Maxwell stress components are given by

$$t_{ij} = \mu_0 (H_i h_j + H_j h_i - H_k h_k \delta_{ij}).$$
(5)

The constitutive relations for a transversely isotropic thermoelastic medium (Dhaliwal and Sherief 1980) are given by

$$t_{ij} = C_{ijkl} e_{kl} - \beta_{ij} T.$$
(6)

Equation of motion as described by Schoenberg and Censor (1973) for a transversely isotropic thermoelastic medium rotating uniformly with an angular velocity $\boldsymbol{\Omega} = \boldsymbol{\Omega} \boldsymbol{n}$, where **n** is a unit vector representing the direction of axis of rotation and taking into account Lorentz force

$$t_{ij,i} + F_i = \rho\{\ddot{u}_i + (\Omega \times (\Omega \times \mathbf{u})_i + (2\Omega \times \dot{u})_i\},\tag{7}$$

where $F_i = \mu_0 (\vec{j} \times \vec{H}_0)$ are the components of Lorentz force, \vec{H}_0 is the external applied magnetic field intensity vector, \vec{j} is the current density vector, \vec{u} is the displacement vector, μ_0 and ε_0 are the magnetic and electric permeabilities respectively and t_{ij} the component of Maxwell stress tensor. The terms $\Omega \times (\Omega \times u)$ and $2\Omega \times \dot{u}$ are the additional centripetal acceleration due to the time-varying motion and Coriolis acceleration respectively.

The above equations are supplemented by generalized Ohm's law for media with finite conductivity and including the hall current effect

$$J = \frac{\sigma_0}{1+m^2} \left(E + \mu_0 \left(\dot{u} \times H - \frac{1}{en_e} J \times H_0 \right) \right)$$
(8)

The heat conduction equation following Ezzat et al. (2012)

$$K_{ij}\left(1+\frac{(\tau_t)^{\alpha}}{\alpha!}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\dot{T}_{,ji} + K_{ij}^*\left(1+\frac{(\tau_v)^{\alpha}}{\alpha!}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)T_{,ji}$$

$$= \left(1+\frac{(\tau_q)^{\alpha}}{\alpha!}\frac{\partial^{\alpha}}{\partial t^{\alpha}} + \frac{(\tau_q)^{2\alpha}}{2\alpha!}\frac{\partial^{2\alpha}}{\partial t^{2\alpha}}\right)\left[\rho C_E \ddot{T} + \beta_{ij}T_0\ddot{\mathbf{e}}_{ij} + bT_0\ddot{\phi}\right],$$
(9)

where $\beta_{ij} = C_{ijkl} \alpha_{ij}$,

=

$$e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}), \quad i, j = 1, 2, 3.$$
 (10)

 $\beta_{ij} = \beta_i \delta_{ij}$, $K_{ij} = K_i \delta_{ij}$, *i* is not summed.

Here C_{ijkl} are elastic parameters and having symmetry due to homogeneous transversely isotropic medium. The basis of these symmetries of C_{ijkl} is due to

- (1) The stress tensor is symmetric, which is only possible if $(C_{ijkl} = C_{iikl})$
- (2) If a strain energy density exists for the material, the elastic stiffness tensor must satisfy $C_{ijkl} = C_{klij}$
- (3) From stress tensor and elastic stiffness tensor symmetries infer $(C_{ijkl} = C_{ijlk})$ and $C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk}$,

 β_{ij} is the thermal elastic coupling tensor, T is the absolute temperature, T_0 is the reference temperature, φ is the conductive temperature, t_{ij} are the components of stress tensor, e_{ij} are the components of strain tensor, u_i are the displacement components, ρ is the density, C_E is the specific heat, K_{ij} is the materialistic constant, a_{ij} are the two temperature parameters, α_{ij} is the coefficient of linear thermal expansion, τ_0 is the relaxation time, which is the time required to maintain steady state heat conduction in an element of volume of an elastic body when sudden temperature gradient is imposed on that volume element, δ_{ij} is the Kronecker delta, Ω is the angular velocity of the solid, τ_t is the phase lag of heat flux, τ_v is the phase lag of temperature gradient, τ_q is the phase lag of thermal displacement, α is the fractional order derivative, ϕ is the volume fraction field and b is the measure of diffusion effect.

3. Formulation and solution of the problem

We consider a perfectly conducting homogeneous transversely isotropic magnetothermoelastic medium without two temperature and which is rotating uniformly with an angular velocity Ω , in the context of three-phase-lag fractional model of generalized thermoelasticity initially at a uniform temperature T_0 , permeated by an initial magnetic field $\vec{H}_0 = (0, H_0, 0)$ acting along y-axis. The rectangular Cartesian co-ordinate system (x, y, z) having origin on the surface (z = 0) with z-axis pointing vertically downwards into the medium is introduced. The surface of the half-space is subjected to normal force acting at z = 0. For two dimensional problem in xz-plane, we take

$$\boldsymbol{u}=(u,0,w)$$

In addition, we consider that

$$\boldsymbol{E}=0,\boldsymbol{\Omega}=(0,\Omega,0).$$

From the generalized Ohm's law

$$J_2 = 0.$$
 (11)

The current density components J_1 and J_3 using (8) are given as

$$J_1 = \frac{\sigma_0 \mu_0 H_0}{1 + m^2} \left(m \frac{\partial u}{\partial t} - \frac{\partial w}{\partial t} \right), \tag{12}$$

$$J_3 = \frac{\sigma_0 \mu_0 H_0}{1 + m^2} \left(\frac{\partial u}{\partial t} + m \frac{\partial w}{\partial t} \right).$$
(13)

Now using the proper transformation on Eqs. (7)-(9) following Slaughter (2002) are as under

$$C_{11}\frac{\partial^{2}u}{\partial x^{2}} + C_{13}\frac{\partial^{2}w}{\partial x\partial z} + C_{44}\left(\frac{\partial^{2}u}{\partial z^{2}} + \frac{\partial^{2}w}{\partial x\partial z}\right) - \beta_{1}\frac{\partial}{\partial x}T - \mu_{0}J_{3}H_{0}$$

$$= \rho\left(\frac{\partial^{2}u}{\partial t^{2}} - \Omega^{2}u + 2\Omega\frac{\partial w}{\partial t}\right),$$

$$(C_{13} + C_{44})\frac{\partial^{2}u}{\partial x\partial z} + C_{44}\frac{\partial^{2}w}{\partial x^{2}} + C_{33}\frac{\partial^{2}w}{\partial z^{2}} - \beta_{3}\frac{\partial}{\partial z}T - \mu_{0}J_{1}H_{0}$$

$$= \rho\left(\frac{\partial^{2}w}{\partial t^{2}} - \Omega^{2}w - 2\Omega\frac{\partial u}{\partial t}\right),$$

$$(15)$$

$$K_{1}\left(1+\frac{(\tau_{t})^{\alpha}}{\alpha!}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\frac{\partial^{2}\dot{T}}{\partial x^{2}}+K_{3}\left(1+\frac{(\tau_{t})^{\alpha}}{\alpha!}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\frac{\partial^{2}\dot{T}}{\partial z^{2}} +K_{1}^{*}\left(1+\frac{(\tau_{v})^{\alpha}}{\alpha!}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\frac{\partial^{2}T}{\partial z^{2}}+K_{3}^{*}\left(1+\frac{(\tau_{v})^{\alpha}}{\alpha!}\frac{\partial^{\alpha}}{\partial t^{\alpha}}\right)\frac{\partial^{2}T}{\partial z^{2}}$$

$$=\left(1+\frac{(\tau_{q})^{\alpha}}{\alpha!}\frac{\partial^{\alpha}}{\partial t^{\alpha}}+\frac{(\tau_{q})^{2\alpha}}{2\alpha!}\frac{\partial^{2\alpha}}{\partial t^{2\alpha}}\right)\left[\rho C_{E}\ddot{T}+T_{0}\left\{\beta_{1}\frac{\partial\ddot{u}}{\partial x}+\beta_{1}\frac{\partial\ddot{w}}{\partial z}\right\}\right],$$
(16)

and

$$t_{xx} = C_{11}e_{xx} + C_{13}e_{xz} - \beta_1 T, \qquad (17)$$

$$t_{zz} = C_{13}e_{xx} + C_{33}e_{zz} - \beta_3 T, \qquad (18)$$

$$t_{xz} = 2C_{44}e_{xz},\tag{19}$$

where

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$$\beta_1 = (C_{11} + C_{12})\alpha_1 + C_{13}\alpha_{3,}$$
$$\beta_3 = 2C_{13}\alpha_1 + C_{33}\alpha_{3,}$$

To facilitate the solution, below mentioned dimensionless quantities are used

$$\begin{aligned} x' &= \frac{x}{L}, & u' &= \frac{\rho c_1^2}{L\beta_1 T_0} u, & t' &= \frac{c_1}{L} t, \\ w' &= \frac{\rho c_1^2}{L\beta_1 T_0} w, & T' &= \frac{T}{T_0}, & t'_{xx} &= \frac{t_{xx}}{\beta_1 T_0}, & t'_{zz} &= \frac{t_{zz}}{\beta_1 T_0}, \\ t'_{xz} &= \frac{t_{xz}}{\beta_1 T_0}, & z' &= \frac{z}{L}, \\ \Omega' &= \frac{L}{C_1} \Omega, & \tau'_T &= \frac{C_1}{L} \tau_T, & \tau'_v &= \frac{C_1}{L} \tau_v, & \tau'_q &= \frac{C_1}{L} \tau_q. \end{aligned}$$
(20)

Making use of (20) in Eqs. (14)-(16), after suppressing the primes, yield

$$\frac{\partial^2 u}{\partial x^2} + \delta_1 \frac{\partial^2 w}{\partial x \partial z} + \delta_2 \frac{\partial^2 u}{\partial z^2} - \frac{\partial T}{\partial x} = \frac{M}{1 + m^2} \left[\frac{\partial u}{\partial t} + m \frac{\partial w}{\partial t} \right] + \frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega \frac{\partial w}{\partial t}, \tag{21}$$

$$\delta_1 \frac{\partial^2 u}{\partial x \partial z} + \delta_2 \frac{\partial^2 w}{\partial x^2} + \delta_3 \frac{\partial^2 w}{\partial z^2} - \frac{\beta_3}{\beta_1} \frac{\partial T}{\partial z} = -\frac{M}{1+m^2} \left[m \frac{\partial u}{\partial t} - \frac{\partial w}{\partial t} \right] + \frac{\partial^2 w}{\partial t^2} - \Omega^2 w - 2\Omega \frac{\partial u}{\partial t}, \quad (22)$$

$$\left(1 + \frac{C_1(\tau_t)^{\alpha}}{\alpha! L} \frac{\partial^{\alpha+1}}{\partial t^{\alpha+1}}\right) \left(K_1 \frac{\partial^2 T}{\partial x^2} + K_3 \frac{\partial^2 T}{\partial z^2}\right)_1 + \left(1 + \frac{(\tau_v)^{\alpha}}{\alpha!} \frac{\partial^{\alpha}}{\partial t^{\alpha}}\right) \left(K_1^* \frac{\partial^2 T}{\partial x^2} + K_3^* \frac{\partial^2 T}{\partial z^2}\right)$$

$$= \left(1 + \frac{(\tau_q)^{\alpha}}{\alpha!} \frac{\partial^{\alpha}}{\partial t^{\alpha}} + \frac{(\tau_q)^{2\alpha}}{2\alpha!} \frac{\partial^{2\alpha}}{\partial t^{2\alpha}}\right) \left[\rho C_E \ddot{T} + \frac{\beta_1}{\rho} T_0 \left\{\beta_1 \frac{\partial \ddot{u}}{\partial x} + \beta_1 \frac{\partial \ddot{w}}{\partial z}\right\}\right],$$

$$(23)$$

where

$$\delta_1 = \frac{c_{13} + c_{44}}{c_{11}}, \qquad \delta_2 = \frac{c_{44}}{c_{11}}, \qquad \delta_3 = \frac{c_{33}}{c_{11}}, \qquad M = \left(\frac{L\sigma_0 \mu_0^2 H_0^2}{\rho C_1}\right).$$

We assume that the medium is initially at rest. The undisturbed state is maintained at reference temperature. Then we have the initial and regularity conditions which are given by

$$u(x, z, 0) = 0 = \dot{u}(x, z, 0)$$
$$w(x, z, 0) = 0 = \dot{w}(x, z, 0)$$
$$T(x, z, 0) = 0 = \dot{T}(x, z, 0)$$

For $z \ge 0$ & $-\infty \le x \le \infty$ u(x, z, t) = w(x, z, t) = T(x, z, t) = 0 for t > 0 when $z \to \infty$. Apply Laplace and Fourier transforms defined by

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$$\tilde{f}(x,z,s) = \int_{0}^{\infty} f(x,z,t)e^{-st}dt,$$
(24)

$$\hat{f}(\xi, z, s) = \int_{-\infty}^{\infty} \tilde{f}(x, z, s) e^{i\xi x} dx.$$
(25)

On Eqs. (17)-(19), we obtain a system of equations

$$\begin{bmatrix} -\xi^{2} - s^{2} + \delta_{2}D^{2} - \frac{Ms}{1 + m^{2}} + \Omega^{2} \end{bmatrix} \hat{u}(\xi, z, s) + \left[\delta_{1}Di\xi - \frac{mMs}{1 + m^{2}} - 2\Omega s \right] \hat{w}(\xi, z, s) + (-i\xi)\hat{T}(\xi, z, s) = 0,$$
(26)

$$\left[\delta_1 Di\xi + \frac{mMs}{1+m^2} + 2\Omega \right] \hat{u}(\xi, z, s) + \left[-\delta_2 \xi^2 + \delta_3 D^2 - s^2 + \Omega^2 - \frac{Ms}{1+m^2} \right] \hat{w}(\xi, z, s) - \frac{\beta_3}{\beta_1} D\hat{T}(\xi, z, s) = 0,$$

$$(27)$$

$$\frac{\beta_{1}^{2}T_{0}s^{2}i\xi}{\rho} \left[1 + \frac{\tau_{q}^{\alpha}s^{\alpha}}{\alpha!} + \frac{\tau_{q}^{2\alpha}s^{2\alpha}}{2\alpha!} \right] \hat{u}(\xi, z, s) + \frac{\beta_{1}\beta_{3}T_{0}s^{2}}{\rho} \left[1 + \frac{\tau_{q}^{\alpha}s^{\alpha}}{\alpha!} + \frac{\tau_{q}^{2\alpha}s^{2\alpha}}{2\alpha!} \right] D\hat{w}(\xi, z, s) + \left\{ \frac{\rho C_{E}C_{1}^{2}s^{2} \left[1 + \frac{\tau_{q}^{\alpha}s^{\alpha}}{\alpha!} + \frac{\tau_{q}^{2\alpha}s^{2\alpha}}{2\alpha!} \right] + K_{1}\xi^{2} \left[1 + \frac{C_{1}\tau_{T}^{\alpha}s^{\alpha+1}}{\alpha!L} \right] \\ + K_{1}^{*}\xi^{2} \left[1 + \frac{\tau_{v}^{\alpha}s^{\alpha}}{\alpha!L} \right] - \left[K_{3} \left[1 + \frac{C_{1}\tau_{T}^{\alpha}s^{\alpha+1}}{\alpha!L} \right] + K_{3}^{*} \left[1 + \frac{\tau_{v}^{\alpha}s^{\alpha}}{\alpha!L} \right] D^{2} \right] \right\} \hat{T}(\xi, z, s) = 0$$
(28)

The non trivial solution of (26)-(28) yields

$$AD^6 + BD^4 + CD^2 + E = 0, (29)$$

where

$$\begin{split} \mathbf{D} &= \frac{d}{dz}, \qquad \mathbf{A} = -\delta_2 \delta_3 \zeta_4 ,\\ B &= -\zeta_4 \zeta_9 \delta_2 + \delta_2 \delta_3 \zeta_3 - \zeta_5 \zeta_4 \delta_3 - \zeta_2 \zeta_{10} \delta_2 + \delta_1^2 \zeta_7^2 \zeta_4 ,\\ C &= \zeta_3 \zeta_9 \delta_2 - \zeta_9 \zeta_4 \zeta_5 + \zeta_5 \zeta_3 \delta_3 - \zeta_2 \zeta_{10} \zeta_5 + \zeta_8 \zeta_4 \zeta_6 - \delta_1^2 \zeta_7^2 \zeta_3 + \zeta_7 \zeta_1 \delta_1 \zeta_{10} - \delta_1 \zeta_7^2 \zeta_2 + \zeta_1 \zeta_7 \delta_3 \\ E &= \zeta_5 \zeta_9 \zeta_3 - \zeta_6 \zeta_8 \zeta_3 + \zeta_1 \zeta_9 \zeta_7 ,\\ \zeta_1 &= \frac{\beta_1^2 T_0 s^2 i \xi}{\rho} \bigg[1 + \frac{\tau_q^\alpha s^\alpha}{\alpha!} + \frac{\tau_q^{2\alpha} s^{2\alpha}}{2\alpha!} \bigg], \end{split}$$

$$\begin{split} \zeta_{2} &= \frac{\beta_{1}\beta_{3}T_{0}s^{2}}{\rho} \bigg[1 + \frac{\tau_{q}^{a}s^{a}}{\alpha!} + \frac{\tau_{q}^{2a}s^{2a}}{2\alpha!} \bigg], \\ \zeta_{3} &= \rho C_{E}C_{1}^{2}s^{2} \bigg[1 + \frac{\tau_{q}^{a}s^{a}}{\alpha!} + \frac{\tau_{q}^{2a}s^{2a}}{2\alpha!} \bigg] + K_{1}\xi^{2} \bigg[1 + \frac{c_{1}\tau_{T}^{a}s^{a+1}}{\alpha!L} \bigg] + K_{1}^{*}\xi^{2} \bigg[1 + \frac{\tau_{v}^{a}s^{a}}{\alpha!L} \bigg], \\ \zeta_{4} &= K_{3} \bigg[1 + \frac{c_{1}\tau_{T}^{a}s^{a+1}}{\alpha!L} \bigg] + K_{3}^{*} \bigg[1 + \frac{\tau_{v}^{a}s^{a}}{\alpha!L} \bigg], \\ \zeta_{5} &= -\xi^{2} - s^{2} - \frac{Ms}{1 + m^{2}}, \\ \zeta_{6} &= -\frac{mMs}{1 + m^{2}} - 2\Omega s, \\ \zeta_{7} &= i\xi, \\ \zeta_{8} &= \frac{mMs}{1 + m^{2}} + 2\Omega s, \\ \zeta_{9} &= -\xi^{2}\delta_{2} - s^{2} + \Omega^{2} - \frac{Ms}{1 + m^{2}}, \\ \zeta_{10} &= -\frac{\beta_{3}}{\beta_{1}}. \end{split}$$

The roots of the Eq. (29) are $\pm \lambda_i$, (i = 1, 2, 3), the solution of the Eqs. (26)-(28) satisfying the radiation condition that $\tilde{u}, \tilde{v}, \tilde{w} \to 0$ as $z \to \infty$ can be written as

$$\hat{u}(\xi, z, s) = \sum_{i=1}^{3} A_i e^{-\lambda_i z},$$
(30)

$$\widehat{w}(\xi, z, s) = \sum_{i=1}^{3} d_i A_i e^{-\lambda_i z}, \qquad (31)$$

$$\hat{T}(\xi, z, s) = \sum_{i=1}^{3} l_i A_i e^{-\lambda_i z},$$
(32)

where $A_{i,i} = 1, 2, 3$ being undetermined constants and d_i and l_i are given by

$$d_{i} = \frac{\delta_{2}\zeta_{4}\lambda_{i}^{4} + (-\zeta_{5}\zeta_{4} + \delta_{2}\zeta_{3})\lambda_{i}^{2} + \zeta_{1}\zeta_{7} + \zeta_{5}\zeta_{3}}{-\delta_{3}\zeta_{4}\lambda_{i}^{4} + (\delta_{3}\zeta_{3} - \zeta_{2}\zeta_{10} - \zeta_{4}\zeta_{9})\lambda_{i}^{2} + \zeta_{3}\zeta_{6}},$$

$$l_{i} = \frac{\delta_{2}\delta_{3}\lambda_{i}^{4} + (\delta_{2}\zeta_{9} + \zeta_{5}\delta_{3} - \delta_{1}^{2}\zeta_{7}^{2})\lambda_{i}^{2} + \zeta_{5}\zeta_{9} - \zeta_{6}\zeta_{8}}{-\delta_{3}\zeta_{4}\lambda_{i}^{4} + (\delta_{3}\zeta_{3} - \zeta_{2}\zeta_{10} - \zeta_{4}\zeta_{9})\lambda_{i}^{2} + \zeta_{3}\zeta_{6}}.$$

4. Boundary conditions

On the half-space surface (z = 0) normal force is applied. The appropriate boundary conditions are

$$t_{zz}(x,z,t) = G(t)\delta(x), \tag{33}$$

$$t_{xz}(x,z,t) = 0,$$
 (34)

$$T(x, z, t) = 0,$$
 (35)

where $\delta(x)$ is a dirac delta function of x and G(t) is a function defined as

$$G(t) = \begin{cases} 0; t \le 0, \\ T_1 \frac{t}{t_0}; 0 \le t \le t_0, \\ T_1; t > t_0, \end{cases}$$

where t_0 indicates the length of time to rise the heat and T_1 is a constant.

Applying the Laplace and Fourier transform to both sides of (33)

$$\hat{t}_{zz}(\xi, 0, s) = \bar{G}(s),$$

where

$$\bar{G}(s) = T_1 \frac{(1 - e^{-st_0})}{t_0 s^2}.$$

Applying the Laplace and Fourier transform defined by (24) and (25) on the boundary conditions (33)-(35), using (18)-(19) and with the help of Eqs. (30)-(32), we obtain the components of displacement, normal stress, tangential stress, conductive temperature and current density components

$$\hat{u}(\xi, z, s) = T_1 \frac{(1 - e^{-st_0})}{\Lambda t_0 s^2} \left(N_1 e^{-\lambda_1 z} + N_2 e^{-\lambda_2 z} + N_3 e^{-\lambda_3 z} \right),$$
(36)

$$\widehat{w}(\xi, z, s) = T_1 \frac{(1 - e^{-st_0})}{\Lambda t_0 s^2} \left(d_1 N_1 e^{-\lambda_1 z} + d_2 N_2 e^{-\lambda_2 z} + d_3 N_3 e^{-\lambda_3 z} \right), \tag{37}$$

$$\hat{T}(\xi, z, s) = T_1 \frac{(1 - e^{-st_0})}{\Lambda t_0 s^2} \left(l_1 N_1 e^{-\lambda_1 z} + l_2 N_2 e^{-\lambda_2 z} + l_3 N_3 e^{-\lambda_3 z} \right),$$
(38)

$$\hat{t}_{xz}(\xi, z, s) = T_1 \frac{(1 - e^{-st_0})}{\Lambda t_0 s^2} \left(\Lambda_{21} N_1 e^{-\lambda_1 z} + \Lambda_{22} N_2 e^{-\lambda_2 z} + \Lambda_{23} N_3 e^{-\lambda_3 z} \right), \tag{39}$$

$$\hat{t}_{zz}(\xi, z, s) = T_1 \frac{(1 - e^{-st_0})}{\Lambda t_0 s^2} \left(\Lambda_{11} N_1 e^{-\lambda_1 z} + \Lambda_{12} N_2 e^{-\lambda_2 z} + \Lambda_{13} N_3 e^{-\lambda_3 z} \right), \tag{40}$$

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$$\hat{f}_1(\xi, z, s) = T_1 \frac{(1 - e^{-st_0})}{\Lambda t_0 s^2} \left(S_1 N_1 e^{-\lambda_1 z} + S_2 N_2 e^{-\lambda_2 z} + S_3 N_3 e^{-\lambda_3 z} \right), \tag{41}$$

$$\hat{f}_{3}(\xi, z, s) = T_{1} \frac{(1 - e^{-st_{0}})}{\Lambda t_{0}s^{2}} \left(R_{1}N_{1}e^{-\lambda_{1}z} + R_{2}N_{2}e^{-\lambda_{2}z} + R_{3}N_{3}e^{-\lambda_{3}z} \right), \tag{42}$$

where

$$\begin{split} \Lambda_{1j} &= C_{13}i\xi - C_{33}d_j\lambda_j - \beta_3l_i, \quad \Lambda_{2j} = -C_{44}\lambda_j + i\xi d_j, \\ \Lambda_{3j} &= A_jl_j, \quad & \Lambda = N_1\Lambda_{11} - N_2\Lambda_{12} + N_3\Lambda_{13}, \\ N_1 &= \Lambda_{22}\Lambda_{33} - \Lambda_{23}\Lambda_{32}, \quad & N_2 = \Lambda_{21}\Lambda_{33} - \Lambda_{23}\Lambda_{31}, \quad & N_3 = \Lambda_{21}\Lambda_{32} - \Lambda_{22}\Lambda_{31}, \\ S_j &= \frac{\sigma_0 H_0 \mu_0}{(1+m^2)}s(m-d_j), \quad & R_j = \frac{\sigma_0 H_0 \mu_0}{(1+m^2)}s(1+md_j). \end{split}$$

5. Inversion of the transformation

For obtaining the result in physical domain, invert the transforms in Eqs. (36)-(42) by inverting the Fourier transform using

$$\tilde{f}(x,z,s) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\xi x} \, \hat{f}(\xi,z,s) d\xi = \frac{1}{2\pi} \int_{-\infty}^{\infty} |\cos(\xi x)f_e - i\sin(\xi x)f_o| \, d\xi \tag{43}$$

Where f_e and f_o are respectively the odd and even parts of $\hat{f}(\xi, z, s)$. Following Honig and Hirdes (1984), the Laplace transform function $\tilde{f}(x, z, s)$ can be inverted to f(x, z, t) by

$$f(x, z, t) = \frac{1}{2\pi i} \int_{\nu - i\infty}^{\nu + i\infty} \tilde{f}(x, z, s) e^{-st} ds.$$
(44)

The last step is to calculate the integral in Eq. (43), "The method for evaluating this integral by using Romberg's integration with adaptive step size is described in Press *et al.* (1986).

6. Numerical results and discussion

In order to illustrate our theoretical results in the proceeding section and to show the effect of hall current parameter m and fractional order parameter α , we now present some numerical results. Following Dhaliwal and Sherief (1980), cobalt material has been taken for transversely isotropic thermoelastic material as

$$\begin{split} c_{11} &= 3.07 \times 10^{11} Nm^{-2}, \qquad c_{33} &= 3.581 \times 10^{11} Nm^{-2}, \qquad c_{13} &= 1.027 \times 10^{10} Nm^{-2}, \\ c_{44} &= 1.510 \times 10^{11} Nm^{-2}, \qquad \beta_1 &= 7.04 \times 10^6 Nm^{-2} deg^{-1}, \\ \beta_3 &= 6.90 \times 10^6 Nm^{-2} deg^{-1}, \qquad \rho &= 8.836 \times 10^3 Kgm^{-3}, \end{split}$$

$$\begin{split} & C_E = 4.27 \times 10^2 j K g^{-1} deg^{-1}, \\ & K_1 = 0.690 \times 10^2 W m^{-1} K deg^{-1}, \\ & T_0 = 298 \text{ K}, \\ & H_0 = 1 \text{Jm}^{-1} \text{nb}^{-1}, \\ & \tau_t = 0.4 \text{s}, \\ & \tau_v = 0.5 \text{s}, \\ & \tau_q = 0.6 \text{s}, \\ & K_1^* = 0.02, \\ & K_3^* = 0.04. \end{split}$$

Using the above values, the graphical representations of normal displacement, induced magnetic effect, normal stress and conductive temperature for transversely isotropic thermoelastic medium have been investigated for normal force/ thermal source and uniformly distributed force/source.

A comparison of the dimensionless form of the field variables displacement components, tangential stress t_{zx} , temperature *T*, current density components J_1 and J_3 for a transversely isotropic medium with hall current effect and fractional order parameter is demonstrated graphically as

- (i) The black line with square symbol relates to hall current for m = 0.0 and $\alpha = 0.5$,
- (ii) The red line with circle symbol relates to hall current for m = 0.3 and $\alpha = 0.5$,
- (iii) The green line with triangle symbol relates to hall current for m = 0.6 and $\alpha = 0.5$,
- (iv) The blue line with r symbol relates to hall current for m = 0.9 and $\alpha = 0.5$.

Fig. 1 shows the variations of transverse displacement component u with distance x. It is noticed that the values are oscillatory corresponding to all the cases for the whole range and as x increases, amplitude of oscillation decreases. Also amplitude of oscillation decreases as the hall parameter m increases. Fig. 2 shows the variations of normal displacement component w with distance x. It is noticed that for m = 0.0 and m = 0.3 the normal displacement component w first increases for $0 \le x \le 3$ and then shows oscillations whose amplitude decreases as x increases while for m = 0.6 and m = 0.9 the value of normal displacement component w first decreases for the range $0 \le x \le 3$ and then remains same for rest of the value of x. Fig. 3 shows the variations



Fig. 1 Variations of displacement component u with distance x



Fig. 2 Variations of displacement component w with distance x



Fig. 3 Variations of temperature T with distance x



Fig. 4 Variations of Tangenial stress t_{zx} with distance x



Fig. 5 Variations of transverse current density J_1 distance x



Fig. 6 Variations of normal current density J_3 distance x

of temperature with distance x. in all the cases it shows an oscillatory behavior with decrease in amplitude as x increases. Also with increase in value of hall parameter m except for m = 0.0 the amplitude of oscillation decreases. Fig. 4 shows the variations of tangential stress t_{zx} with distance x. In all the cases it shows an oscillatory behavior with decrease in amplitude as x increases. Also the amplitude of oscillation is higher for m = 0.3 then decrease for m = 0.0, then more less form = 0.9 and further smaller for m = 0.6 Fig. 5 shows the variations of transverse current density J_1 with distance x and Fig. 6 shows the variations of normal current density J_3 with distance x. In all the cases it shows an oscillatory behavior with decrease in amplitude as x increases. Also with increase in value of hall parameter m, the amplitude of oscillation decreases.

A comparison of the dimensionless form of the field variables displacement components, tangential stress t_{zx} , temperature *T*, current density components J_1 and J_3 for a transversely isotropic medium with hall current and fractional order parameter is demonstrated graphically as:

- (i) The black line with square symbol relates to hall current for m = 0.5 and $\alpha = 0.3$,
- (ii) The red line with circle symbol relates to hall current for m = 0.5 and $\alpha = 0.5$,
- (iii) The blue line with triangle symbol relates to hall current for m = 0.5 and $\alpha = 0.7$

Fig. 1 shows the variations of transverse displacement component u with distance x. It is noticed that the values are oscillatory corresponding to all the cases for the whole range and as fractional order parameter α increases, amplitude of oscillation decreases. Fig. 2 shows the variations of normal displacement component w with distance x. It is noticed that the value of normal displacement component w decreases as the value of fractional order parameter α increases. Fig. 3 shows the variations of temperature with distance x. For $\alpha = 0.3$ and $\alpha = 0.5$ temperature increase with increase in distance whereas for $\alpha = 0.7$ temperature T decrease with increase in distance x. For $\alpha = 0.3$ and $\alpha = 0.5$ tangential stress t_{zx} decrease with increase in distance whereas for $\alpha = 0.3$ and $\alpha = 0.5$ tangential stress t_{zx} decrease with increase in distance whereas for $\alpha = 0.3$ and $\alpha = 0.5$ tangential stress t_{zx} decrease with increase in distance whereas for



Fig. 7 Variations of displacement component u with distance x



Fig. 8 Variations of displacement component w with distance x



Fig. 9 Variations of temperature T with distance x



Fig. 10 Variations of Tangential stress t_{zx} with distance x



Fig. 11 Variations of transverse current density J_1 distance x



Fig. 12 Variations of normal current density J_3 distance x

 $\alpha = 0.7$ tangential stress t_{zx} increases with increase in distance x and follows oscillatory pattern. Fig. 5 shows the variations of transeverse current density J_1 with distance x and Fig. 6 shows the variations of normal current density J_3 with distance x. In all the cases it shows an oscillatory behavior with decrease in amplitude as x increases. Also with increase in value of fractional order parameter, the amplitude of oscillation increases.

7. Conclusions

The consideration of Hall effect makes the research more exciting for research perspective. From the analysis of the graphs, it is clear there is a significant influence of Hall Effect parameter m and fractional order parameter α on the deformation of various displacement components, temperature, tangential stress components, and current density components J_1 and J_3 of transversely isotropic magneto thermoelastic medium. The fraction order derivatives are used to find viscoelasticity of such materials with a high precision. As distance x, varied from the point of use of the normal force, the variations of displacement components, temperature and tangential stress components undergoes sudden changes, causing an inconsistent patterns of curves and shows an oscillatory pattern. The shape of curves shows the impact of Hall Effect parameter m on the body and fulfils the purpose of the study. The outcomes of this research are extremely helpful in the 2-D problem in various domains of science and technology, like damping of acoustic waves in a magnetic field, geophysics for understanding the effect of the Earth's magnetic field on seismic waves, emissions at electromagnetic radiation from nuclear devices, optics and plasma physics.

References

Abbas, I. (2018), "A study on fractional order theory in thermoelastic half-space under thermal loading", *Physical Mesomech.*, **21**(2), 150-156.

Bachher, M. and Sarkar, N. (2016), "Fractional order magneto-thermoelasticity in a rotating media with one relaxation time", *Math. Model. Eng.*, **2**(1), 56-68.

- Chen, P.J. and Gurtin, E.M. (1968), "On a theory of heat conduction involving two temperatures", *Zeitschrift für Angewandte Mathematik und Physik*, **19**(4), 614-627.
- Chen, P.J., Gurtin, M.E. and Williams, W.O. (1968), "A note on non-simple heat conduction", Zeitschrift für Angewandte Mathematik und Physik ZAMP, 19(4), 969-970.
- Chen, P.J., Gurtin, E.M. and Williams, O.W. (1969), "On the thermodynamics of non-simple elastic materials with two temperatures", *Zeitschrift für angewandte Mathematik und Physik*, **20**(1), 107-112.
- Chauthale, S. and Khobragade, N.W. (2017), "Thermoelastic Response of a Thick Circular Plate due to Heat Generation and its Thermal Stresses", *Global J. Pure Appl. Math.*, **13**(10), 7505-7527.
- Dhaliwal, R.S. and Sherief, H.H. (1980), "Generalized thermoelasticity for anisotropic media", *Quarterly Appl. Math.*, **38**(1), 1-8.
- Ezzat, M. and AI-Bary, A. (2016), "Magneto-thermoelectric viscoelastic materials with memory dependent derivatives involving two temperature", *Int. J. Appl. Electromagnet. Mech.*, 50(4), 549-567.
- Ezzat, M. and AI-Bary, A. (2017), "Fractional magneto-thermoelastic materials with phase lag Green-Naghdi theories", Steel Compos. Struct., Int. J., 24(3), 297-307.
- Ezzat, M.A., Karamany, A.S. and Fayik, M.A. (2011), "Fractional order theory in thermoelastic solid with three-phase lag heat transfer", Arch. Appl. Mech., 82, 557-572.
- Ezzat, M.A., El-Karamany, A.S. and Ezzat, S.M. (2012), "Two-temperature theory in magnetothermoelasticity with fractional order dual-phase-lag heat transfer", *Nuclear Eng. Des.*, 252, 267-277.
- Ezzat, M., El-Karamany, A. and El-Bary, A. (2015), "Thermo-viscoelastic materials with fractional relaxation operators", *Appl. Math. Model.*, **39**(23), 7499-7512.
- Ezzat, M., El-Karamany, A. and El-Bary, A. (2016), "Generalized thermoelasticity with memory-dependent derivatives involving two temperatures", *Mech. Adv. Mater. Struct.*, 23(5), 545-553.
- Ezzat, M.A., El-Karamany, A.S. and El-Bary, A.A. (2017), "Two-temperature theory in Green–Naghdi thermoelasticity with fractional phase-lag heat transfer", *Microsyst. Technol.*, 24(2), 951-961.
- Green, A. and Nagdhi, P. (1991), "A re-examination of the basic postulates of thermomechanics", *Proceedings of Royal Society of London*, pp. 171-194.
- Green, A. and Naghdi, P. (1992), "On undamped heat waves in an elastic solid", J. Thermal Stress., 15(2), 253-264.
- Green, A., & Naghdi, P. (1993). "Thermoelasticity without energy dissipation". Journal of Elasticity: The Physical and Mathematical Science of Solids, 31(3), 189-208.
- Honig, G. and Hirdes, U. (1984), "A method for the numerical inversion of Laplace transform", *J. Computat. Appl. Math.*, **10**, 113-132.
- Kumar, R., Sharma, N. and Lata, P. (2016), "Thermomechanical interactions in transversely isotropic magnetothermoelastic medium with vacuum and with and without energy dissipation with combined effects of rotation, vacuum and two temperatures", *Appl. Math. Model.*, **40**(13-14), 6560-6575.
- Kumar, R., Sharma, N. and Lata, P. (2017a), "Effects of Hall current and two temperatures intransversely isotropic magnetothermoelastic with and without energy dissipation due to ramp-type heat", *Mech. Adv. Mater. Struct.*, 24(8), 625-635.
- Kumar, R., Sharma, N., Lata, P. and Abo-Dahab, S.M. (2017b), "Rayleigh waves in anisotropic magnetothermoelastic medium", *Coupl. Syst. Mech.*, *Int. J.*, 6(3), 317-333.
- Kumar, R., Kaushal, P. and Sharma, R. (2018), "Transversely Isotropic Magneto-Visco Thermoelastic Medium with Vacuum and without Energy Dissipation", J. Solid Mech., 10(2), 416-434.
- Lata, P. and Kaur, I. (2019a), "Transversely isotropic thick plate with two temperature and GN type-III in frequency domain", *Coupl. Syst. Mech., Int. J.*, 8(1), 55-70.
- Lata, P. and Kaur, I. (2019b), "Study of Transversely Isotropic Thick circular Plate due to Ring Load with Two Temperature & Green Nagdhi Theory of Type-I, II and III", Proceedings of International Conference on Sustainable Computing in Science, Technology & Management (SUSCOM-2019), Amity University Rajasthan, Jaipur, India, pp. 1753-1767.
- Lata, P. and Kaur, I. (2019c), "Thermomechanical Interactions in transversely isotropic thick circular plate with axisymmetric heat supply", *Struct. Eng. Mech.*, *Int. J.*, **69**(6), 607-614.
- Lata, P. and Kaur, I. (2019d), "Transversely isotropic magneto thermoelastic solid with two temperature and

without energy dissipation in generalized thermoelasticity due to inclined load", *SN Appl. Sci.*, **1**(5), p. 426. DOI: https://doi.org/10.1007/s42452-019-0438-z

- Lata, P. and Kaur, I. (2019e), "Effect of rotation and inclined load on transversely isotropic magneto thermoelastic solid", *Struct. Eng. Mech.*, *Int. J.*, **70**(2), 245-255.
- Lata, P., Kumar, R. and Sharma, N. (2016), "Thermomechanical interactions in transversely isotropic magnetothermoelastic medium with vacuum and with and without energy dissipation with combined effects of rotation, vacuum and two temperatures", *Appl. Math. Model.E*, **40**(13-14), 6560-6575.
- Marin, M. (1996), "Generalized solutions in elasticity of micropolar bodies with voids", *Revista de la Academia Canaria de Ciencias*, **8**(1), 101-106.
- Marin, M. (1997a), "Cesaro means in thermoelasticity of dipolar bodies", Acta Mechanica, 122(1-4), 155-168.
- Marin, M. (1997b), "On weak solutions in elasticity of dipolar bodies with voids", J. Computat. Appl. Math., **82**(1-2), 291-297.
- Marin, M. (1998), "Contributions on uniqueness in thermoelastodynamics on bodies with voids", *Revista Ciencias Matematicas*(Havana), 16(2), 101-109.
- Marin, M. (2008), "Weak solutions in elasticity of dipolar porous materials", Math. Problems Eng., 1-8.
- Marin, M. (2009), "On the minimum principle for dipolar materials with stretch", Nonlinear Anal. Real World Appl., 10(3), 1572-1578.
- Marin, M. (2010), "A partition of energy in thermoelasticity of microstretch bodies", *Nonlinear Anal.: Real World Appl.*, **11**(4), 2436-2447.
- Marin, M. and Baleanu, D. (2016), "On vibrations in thermoelasticity without energy dissipation for micropolar bodies", *Boundary Value Problems*, 111.
- Marin, M. and Nicaise, S. (2016), "Existence and stability results for thermoelastic dipolar bodies with double porosity", *Continuum Mech. Thermodyn.*, 28(6), 1645-1657.
- Marin, M. and Öchsner, A. (2017), "The effect of a dipolar structure on the Hölder stability in Green-Naghdi thermoelasticity", *Continuum Mech. Thermodyn.*, **29**, 1365-1374.
- Marin, M. and Stan, G. (2013), "Weak solutions in Elasticity of dipolar bodies with stretch", *Carpathian J. Math.*, **29**(1), 33-40.
- Marin, M., Ellahi, R. and Chirilă, A. (2017), "On solutions of Saint-Venant's problem for elastic dipolar bodies with voids", *Carpathian J. Math.*, 33(2), 219-232.
- Moreno-Navarro, P., Ibrahimbegovic, A. and Pérez-Aparicio, J.L. (2018), "Linear elastic mechanical system interacting with coupled thermo-electro-magnetic fields", *Coupl. Syst. Mech.*, *Int. J.*, 7(1), 5-25.
- Othman, M.I. and Marin, M. (2017), "Effect of thermal loading due to laser pulse on thermoelastic porous medium under G-N theory", *Results Phys.*, **7**, 3863-3872.
- Press, W.H., Teukolsky, S.A., Vetterling, W.T. and Flannery, B.P. (1986), FORTRAN numerical recipes, Cambridge University Press Cambridge.
- Salama, M.M., Kozae, A., Elsafty, M.A. and Abelaziz, S. (2015), "A half-space problem in the theory of fractional order thermoelasticity with diffusion", *Int. J. Sci. Eng. Res.*, 6(1), 358-371.
- Schoenberg, M. and Censor, D. (1973), "Elastic Waves In Rotating Media", *Quarterly Appl. Math.*, 31, 115-125.
- Sheoran, S.S. and Kundu, P. (2016), "Fractional order generalized thermoelasticity theories: A review", Int. J. Adv. Appl. Math. Mech., 3(4), 76-81.
- Slaughter, W.S. (2002), "The Linearized Theory of Elasticity, Birkhäuser.
- Tripathi, J.J., Deshmukh, K. and Verma, J. (2017), "Fractional order generalized thermoelastic problem in a thick circular plate with periodically varying heat source", *Int. J. Thermodyn.*, 20(3), 132-138.
- Youssef, H.M. and Abbas, I. (2014), "Fractional order generalized thermoelasticity with variable thermal conductivity", J. Vibroeng., 16(8), 4077-4087.
- Zakaria, M. (2012), "Effects of Hall current and rotation on magneto-micropolar generalized thermoelasticity due to ramp-type heating", *Int. J. Electromagnet. Appl.*, **2**(3), 24-32.