Advances in Materials Research, *Vol. 7 No. 2 (2018) 119-136*
DOI: https://doi.org/10.12989/amr.2018.7.2.119

Dynamic analysis for anti-symmetric cross-ply and angle-ply laminates for simply supported thick hybrid rectangular plates

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(*Received January 10, 2019, Revised March 7, 2019, Accepted March 13, 2019*)

Abstract. In this paper, static and vibration analysis for anti-symmetric cross-ply and angle- ply carbon/glass hybrid laminates rectangular composite plate are presented. In this analysis, the equations of motion for simply supported thick laminated hybrid rectangular plates are derived and obtained through the use of Hamilton's principle. The closed-form solutions of anti-symmetric cross-ply and angle- ply laminates are obtained using Navier solution. The effects of side-to-thickness ratio, aspect ratio, and lamination schemes on the fundamental frequencies loads are investigated. The study concludes that shear deformation laminate theories accurately predict the behavior of composite laminates, whereas the classical laminate theory over predicts natural frequencies. The excellent accuracy of the present analytical solution is confirmed by making some comparisons of the present results with those available in the literature. It can be concluded that the proposed theory is accurate and simple in solving the free vibration behaviors of anti-symmetric cross-ply and angle- ply hybrid laminated composite plates.

Keywords: hybrid laminated composite plate; higher-order shear deformation theory; free vibration; Navier's solution

1. Introduction

Laminated composite plates are widely used in different engineering and industrial domains, thanks to their light weight and sustainability. In order to be able to use this kind of plate in the field, Countless studies are done by researchers to study the composition and the behavior of composite plate in order to determinate the major factors that impacts this revolutionary structure. The classical laminated plate theory (CLPT) was the first hypothesis that seen light by Kirchhoff Love. This theory was quickly exceeded due to the negligence of transverse shear effects, which limits its appliance to thin plates only. In the case of plates relatively thick, A lot of shear deformation theories that takes into account the transverse shear effects have been proposed to beat the shortcomings of the classical laminated plate theory, where the first-order shear deformation theory is the first alternative proposed by Mindlin (1951) that takes into consideration the transverse shear effects in determination of deformation and stresses. Such The first theory for laminated isotropic plates was apparently. This theory was generalized to laminated anisotropic

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http://www.techno-press.org/?journal=amr&subpage=5 ISSN: 2234-0912 (Print), 2234-179X (Online)

plates in Stavski (1965). It was shown in Srnivas (1970) and Whitney (1973), the first-order shear deformation theory doesn't satisfy equilibrium conditions at the top and bottom faces of the plate, shear correction factors are required to rectify the unrealistic variation of the shear strain/stress through the thickness. In order to overcome the limitations of first order shear deformation, higher-order shear deformation theories are proposed. Since it involve higher-order terms in Taylor's expansions of the displacements in the thickness coordinate, were developed by Reddy (1984), Bouadi (2018), Bouhadra (2018), Hassaine Daouadji (2016b), Benferhat (2016c), Tahar (2013b), Bousahla (2016), Chikh (2016), Zenkour (2006), Oktem (2008), Tounsi (2013), Hassaine Daouadji (2013), Benferhat (2014), Youcef (2018), Younsi (2018), Zine (2018), Abdelhak (2014), Mantari (2012), Abdelaziz (2017), Adim (2016a), Abunalour (2018), Attia (2018), Hassaine Daouadji (2016c), Belabed (2018), Beldjillali (2016), Bellifa (2017), Draiche (2016), Elhania (2017), Fourn (2018), Kaci (2018), Karami (2018), Sayyad (2014), Swaminathan (2007), Witheney (1972), Yazid (2018), Khalifa (2018), Benhenni (2018), Adim (2018), Hassaine Daouadji (2017), Zaoui (2019), Tahar (2016b), Hassaine Daouadji (2016a), Bakhada (2018), Bourada (2018), Bourad 2019, Zenkour 2004 and Ren (1990), Reddy (1984), Kant and Pandya (1988), and Mohan (1994). A good review of these theories for the analysis of laminated composite plates is available in the work of Tounsi (2013), Mantari (2012), Hassaine Daouadji (2012) and Benferhat (2016a). A two variable refined plate theory using only two unknown functions was developed by Shimpi (2002), Hassaine Daouadji (2012), Benachour (2011), Tahar (2012), Lazreg (2014), Hadji (2015a), Tlidji (2014), Mazari (2015), Zoubida (2016), Sallai (2015), Menasria (2017), Mokhtar (2018), Tahar (2016a), Abdelhak (2016), Tahar (2017), Adim (2016c), Chedad (2017), Lazreg (2016), Tayeb (2018), Bensatallah (2018), Rabahi (2018), Benferhat (2018), Rabahi (2017), Hadji (2015b), Lazreg (2015), Tahar (2013a), Oktem (2007), Benferhat (2016b), Khelifa (2016) and Tlidji (2014) for isotropic plates, and was extended by Shimpi and Patel (2006) for orthotropic plates. The most interesting feature of this theory is that it does not require shear correction factor and has strong similarities with the classical plate theory in some aspects such as governing equation, boundary conditions and moment expressions.

In addition of the matrix, most composite structures are made of one type of fibers which mean that the composite properties depend on this particular type of fibers. If this fiber presents a handicap like fragility or low strength, the all structure will be vulnerable to damage or failure. Hybrid composite plates are made of more than one type of fibers, generally two types; this feature provides various features like reducing manufacturing cost or improving a specific quality of one of the fibers such as wear resistance, vibration damping, toughness, strength, …etc. In this paper, a refined and simple shear deformation theory of plates is presented and applied to the investigation of free vibration behavior of Carbon/Glass hybrid laminated composite plates.

2. Material properties

In this study a hybrid laminated carbon/glass epoxy hybrid composite plate is considered, the longitudinal and transversal Young modulus are given in Vaseliev (2001) by

$$E_1 = E_f^1 V_f^1 + E_f^2 V_f^2 + E_m V_m \tag{1}$$

Where E_1 is longitudinal Young's modulus. E_f^1 , E_f^2 and E_m are the Young's moduli of the first type of fibers, the second type of fibers and the matrix respectively. V_f^1 , V_f^2 and V_m are the volume fraction of the first type of fibers, the second type of fibers and the matrix, respectively, where:

$$V_f^1 + V_f^2 + V_m = 1 \text{ and } V_f^1 + V_f^2 = \mathrm{V}_f \tag{2}$$

Assuming that:

$$w_f = \frac{V_f^1}{Vf} \tag{3}$$

Where w_f is the first fiber percentage over the total fiber's volume fraction.
By replacing Eq (3) into Eq (1) we obtain:

$$E_1 = V_f\left[E_f^1 w_f + E_f^2\left(1 - w_f\right)\right] + E_m V_m \tag{4}$$

Using the same approach, the Poisson's coefficient can be calculated by

$$v_{12} = V_f\left[v_f^1 w_f + v_f^2\left(1 - w_f\right)\right] + v_m V_m \tag{5}$$

The shear modulus of the fibers and the matrix are expressed by

$$\mathrm{G}_f^1 = \frac{E_f^1}{2(1 + \mathrm{v}_f^1)}, \qquad \mathrm{G}_f^2 = \frac{E_f^2}{2(1 + v_f^2)}, \qquad \mathrm{G}_m = \frac{E_m}{2(1 + v_m)} \tag{6}$$

Where G_f^1, G_f^2 and G_m, are the shear modulus of the first type of fibers, the second type of fibers and the matrix, respectively, also the total shear modulus of fibers is given by

$$\mathrm{G}_f = \mathrm{G}_f^1 \mathrm{w}_f + G_f^2(1 - w_f) \tag{7}$$

The compressibility modulus of the fibers and the matrix are given as

$$k_f = \frac{E_f^1 w_f}{3(1 - 2\mathrm{v}_f^1)} + \frac{E_f^2 (1 - w_f)}{3(1 - 2v_f^2)}, k_m = \frac{E_m}{3(1 - 2v_m)} \tag{8}$$

The lateral compressibility modulus of the fibers and the matrix are given as

$$K_f = k_f + \frac{G_f}{3},\ K_m = k_m + \frac{G_m}{3} \tag{9}$$

The shear moduli of the plate are (G_{23} is different from $G_{12} = G_{13}$)

$$G_{23} = G_m\left(1 + \frac{V_f}{\frac{G_m}{G_f - G_m} + V_m \frac{k_m + 7G_m/3}{2k_m + 8G_m/3}}\right) \tag{10a}$$

$$G_{12} = G_m \frac{G_f\left(1 + V_f\right) + G_m(1 - V_f)}{G_f\left(1 - V_f\right) + G_m(1 + V_f)}, \qquad G_{13} = G_{12} \tag{10b}$$

The lateral compressibility modulus of the plate is given as

$$K_L = K_m + \frac{V_f}{\frac{1}{k_f - k_m + (G_f - G_m)/3} + \frac{1 - V_f}{k_m + (4/3)G_m}} \tag{11}$$

Using equations from (4) to (11), the transversal Young's modulus is given as follow

$$E_2 = \frac{2}{\frac{1}{2K_L} + \frac{1}{2G_{23}} + \frac{2(v_{12})^2}{E_1}} \tag{12}$$

3. Refined plate theory for laminated composite plates

3.1 Kinematics

Consider a rectangular plate of total thickness h composed of Np orthotropic layers with the coordinate system as shown in Figure 1.

The displacement field can be obtained according to Adim (2016a) as follows:

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w_b}{\partial x} - (z - ze^{-2\frac{z^2}{h^2}})\frac{\partial w_s}{\partial x} \\ v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_b}{\partial y} - (z - ze^{-2\frac{z^2}{h^2}})\frac{\partial w_s}{\partial y} \\ w(x,y,z,t) &= w_b(x,y,t) + w_s(x,y,t) \\ f(z) &= z - ze^{\left(-2\frac{z^2}{h^2}\right)} \end{aligned} \tag{13}$$

where u_0 and v_0 are the mid-plane displacements of the plate in the x and y direction, respectively; w_b and w_s are the bending and shear components of transverse displacement, respectively, while *f(z)* represents shape functions determining the distribution of the transverse shear strains and stresses along the thickness, This function ensures zero transverse shear stresses at the top and bottom surfaces of the plate. The parabolic distributions of transverse shear stresses across the plate thickness are taken into account in the analysis by means of present function of the

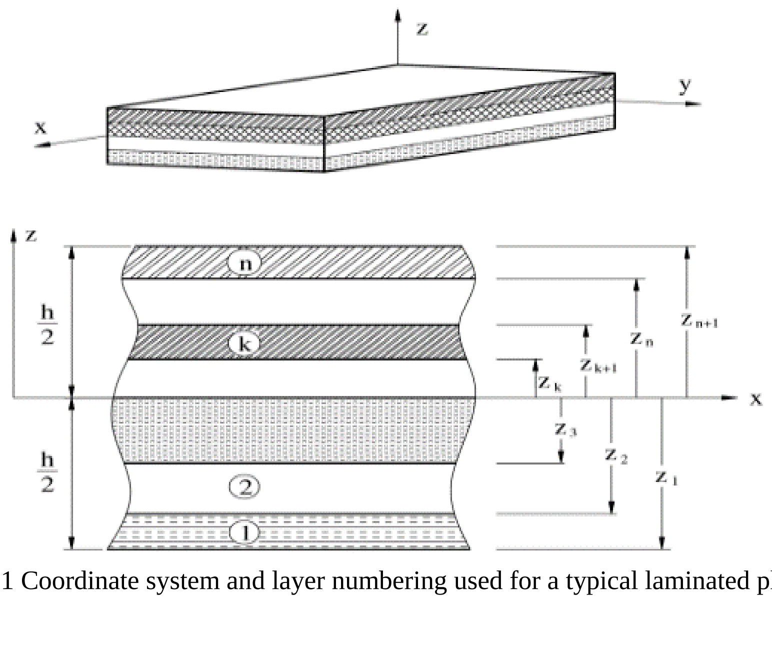

Fig. 1 Coordinate system and layer numbering used for a typical laminated plate

displacement field assumed. The strains and stresses associated with the displacements can be found with details in Adim *et al*. (2016a).

3.2 Governing equations

The strain energy of the plate can be written as

$$U = \frac{1}{2}\int_V \sigma_{ij}\varepsilon_{ij} dV = \frac{1}{2}\int_V (\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_{xy}\gamma_{xy} + \sigma_{yz}\gamma_{yz} + \sigma_{xz}\gamma_{xz}) dV \tag{14}$$

Substituting Eqs. (13) into Eq. (14) and integrating through the thickness of the plate, the strain energy of the plate can be rewritten as

$$\begin{aligned} U = \frac{1}{2}\int_A \{ & N_x \varepsilon_x^0 + N_y \varepsilon_y^0 + N_{xy}\gamma_{xy}^0 + M_x^b k_x^b + M_y^b k_y^b + M_{xy}^b k_{xy}^b \\ & + M_x^s k_x^s + M_y^s k_y^s + M_{xy}^s k_{xy}^s + Q_{yz}^s \gamma_{yz}^s + Q_{xz}^s \gamma_{xz}^s \} dxdy \end{aligned} \tag{15}$$

Where the stress resultants N, M, and Q are defined by

$$\begin{aligned} (N_x, N_y, N_{xy}) &= \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \sigma_{xy}) dz = \sum_{k=1}^{N_p} \int_{z_k}^{z_{k+1}} (\sigma_x, \sigma_y, \sigma_{xy}) dz \\ (M_x^b, M_y^b, M_{xy}^b) &= \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \sigma_{xy}) z dz = \sum_{k=1}^{N_p} \int_{z_k}^{z_{k+1}} (\sigma_x, \sigma_y, \sigma_{xy}) z dz \\ (M_x^s, M_y^s, M_{xy}^s) &= \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \sigma_{xy}) f dz = \sum_{k=1}^{N_p} \int_{z_k}^{z_{k+1}} (\sigma_x, \sigma_y, \sigma_{xy}) f dz \\ (Q_{xz}^s, Q_{yz}^s) &= \int_{-h/2}^{h/2} (\sigma_{xz}, \sigma_{yz}) g dz = \sum_{k=1}^{N_p} \int_{z_k}^{z_{k+1}} (\sigma_{xz}, \sigma_{yz}) g dz \end{aligned} \tag{16}$$

The kinetic energy of the plate can be written as

$$\begin{aligned} T = \frac{1}{2}\int_V \rho \ddot{u}_{ii} dV = \frac{1}{2}\int_A \Bigg\{ & \delta u_0 \left(I_1 \ddot{u}_0 - I_2 \frac{\partial \ddot{w}_b}{\partial x} - I_4 \frac{\partial \ddot{w}_s}{\partial x} \right) + \delta v_0 \left(I_1 \ddot{v}_0 - I_2 \frac{\partial \ddot{w}_b}{\partial y} - I_4 \frac{\partial \ddot{w}_s}{\partial y} \right) \\ & + \delta w_b \left[I_1 (\ddot{w}_b + \ddot{w}_s) + I_2 \left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y} \right) - I_3 \left(\frac{\partial^2 \ddot{w}_b}{\partial x^2} + \frac{\partial^2 \ddot{w}_b}{\partial y^2} \right) - I_5 \left(\frac{\partial^2 \ddot{w}_s}{\partial x^2} + \frac{\partial^2 \ddot{w}_s}{\partial y^2} \right) \right] \\ & + \delta w_s \left[I_1 (\ddot{w}_b + \ddot{w}_s) + I_4 \left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y} \right) - I_5 \left(\frac{\partial^2 \ddot{w}_b}{\partial x^2} + \frac{\partial^2 \ddot{w}_b}{\partial y^2} \right) - I_6 \left(\frac{\partial^2 \ddot{w}_s}{\partial x^2} + \frac{\partial^2 \ddot{w}_s}{\partial y^2} \right) \right] \Bigg\} dxdy \end{aligned} \tag{17}$$

Where ρ is mass of density of the plate and I_i are the inertias defined by

$$(I_1, I_2, I_3, I_4, I_5, I_6) = \int_{-h/2}^{h/2} \rho (1, z, z^2, f(z), zf(z), [f(z)]^2) dz \tag{18}$$

Hamilton's principle is used here in order to derive the equations of motion appropriate to the displacement field and the constitutive equation. The principle can be stated in analytical form as

$$\int_0^t \delta (U - T) dt = 0 \tag{19}$$

Substituting Eqs. (15) and (17) into Eq. (19) and integrating the equation by parts, collecting the coefficients of δu, δv, δw_b and δw_s, the equations of motion for the laminate plate are obtained as follows:

$$\delta u_0 : \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_1 \ddot{u}_0 - I_2 \frac{\partial \ddot{w}_b}{\partial x} - I_4 \frac{\partial \ddot{w}_s}{\partial x}$$

$$\delta v_0 : \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_1 \ddot{v}_0 - I_2 \frac{\partial \ddot{w}_b}{\partial y} - I_4 \frac{\partial \ddot{w}_s}{\partial y}$$

$$\begin{aligned} \delta w_b &: \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} \\ &= I_1(\ddot{w}_b + \ddot{w}_s) + I_2\left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y}\right) - I_3\left(\frac{\partial^2 \ddot{w}_b}{\partial x^2} + \frac{\partial^2 \ddot{w}_b}{\partial y^2}\right) - I_5\left(\frac{\partial^2 \ddot{w}_s}{\partial x^2} + \frac{\partial^2 \ddot{w}_s}{\partial y^2}\right) \end{aligned} \tag{20}$$

$$\begin{aligned} \delta w_s &: \frac{\partial^2 M_x^s}{\partial x^2} + 2\frac{\partial^2 M_{xy}^s}{\partial x \partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial Q_{xz}^s}{\partial x} + \frac{\partial Q_{yz}^s}{\partial y} \\ &= I_1(\ddot{w}_b + \ddot{w}_s) + I_4\left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y}\right) - I_5\left(\frac{\partial^2 \ddot{w}_b}{\partial x^2} + \frac{\partial^2 \ddot{w}_b}{\partial y^2}\right) - I_6\left(\frac{\partial^2 \ddot{w}_s}{\partial x^2} + \frac{\partial^2 \ddot{w}_s}{\partial y^2}\right) \end{aligned}$$

Equation (20) can be expressed in terms of displacements (u, v, w_b, w_s) by substituting for the stress resultants from Eq. (16). For homogeneous laminates, the equations of motion (20) take the form

$$\begin{aligned} & A_{11}\frac{\partial^2 u_0}{\partial x^2} + 2A_{16}\frac{\partial^2 u_0}{\partial x \partial y} + A_{66}\frac{\partial^2 u_0}{\partial y^2} + A_{16}\frac{\partial^2 v_0}{\partial x^2} + (A_{12} + A_{66})\frac{\partial^2 v_0}{\partial x \partial y} + A_{26}\frac{\partial^2 v_0}{\partial y^2} \\ & - B_{11}\frac{\partial^3 w_b}{\partial x^3} - 3B_{16}\frac{\partial^3 w_b}{\partial x^2 \partial y} - (B_{12} + 2B_{66})\frac{\partial^3 w_b}{\partial x \partial y^2} - B_{26}\frac{\partial^3 w_b}{\partial y^3} \\ & - B_{11}^s\frac{\partial^3 w_s}{\partial x^3} - 3B_{16}^s\frac{\partial^3 w_s}{\partial x^2 \partial y} - (B_{12}^s + 2B_{66}^s)\frac{\partial^3 w_s}{\partial x \partial y^2} - B_{26}^s\frac{\partial^3 w_s}{\partial y^3} = I_1\ddot{u}_0 - I_2\frac{\partial \ddot{w}_b}{\partial x} - I_4\frac{\partial \ddot{w}_s}{\partial x} \end{aligned} \tag{21a}$$

$$\begin{aligned} & A_{16}\frac{\partial^2 u_0}{\partial x^2} + (A_{12} + A_{66})\frac{\partial^2 u_0}{\partial x \partial y} + A_{26}\frac{\partial^2 u_0}{\partial y^2} + A_{66}\frac{\partial^2 v_0}{\partial x^2} + 2A_{26}\frac{\partial^2 v_0}{\partial x \partial y} + A_{22}\frac{\partial^2 v_0}{\partial y^2} \\ & - B_{16}\frac{\partial^3 w_b}{\partial x^3} - (B_{12} + 2B_{66})\frac{\partial^3 w_b}{\partial x^2 \partial y} - 3B_{26}\frac{\partial^3 w_b}{\partial x \partial y^2} - B_{22}\frac{\partial^3 w_b}{\partial y^3} \\ & - B_{16}^s\frac{\partial^3 w_s}{\partial x^3} - (B_{12}^s + 2B_{66}^s)\frac{\partial^3 w_s}{\partial x^2 \partial y} - 3B_{26}^s\frac{\partial^3 w_s}{\partial x \partial y^2} - B_{22}^s\frac{\partial^3 w_s}{\partial y^3} = I_1\ddot{v}_0 - I_2\frac{\partial \ddot{w}_b}{\partial y} - I_4\frac{\partial \ddot{w}_s}{\partial y} \end{aligned} \tag{21b}$$

$$\begin{aligned} & B_{11}\frac{\partial^3 u_0}{\partial x^3} + 3B_{16}\frac{\partial^3 u_0}{\partial x^2 \partial y} + (B_{12} + 2B_{66})\frac{\partial^3 u_0}{\partial x \partial y^2} + B_{26}\frac{\partial^3 u_0}{\partial y^3} \\ & + B_{16}\frac{\partial^3 v_0}{\partial x^3} + (B_{12} + 2B_{66})\frac{\partial^3 v_0}{\partial x^2 \partial y} + 3B_{26}\frac{\partial^3 v_0}{\partial x \partial y^2} + B_{22}\frac{\partial^3 v_0}{\partial y^3} \\ & - D_{11}\frac{\partial^4 w_b}{\partial x^4} - 4D_{16}\frac{\partial^4 w_b}{\partial x^3 \partial y} - 2(D_{12} + 2D_{66})\frac{\partial^4 w_b}{\partial x^2 \partial y^2} - 4D_{26}\frac{\partial^4 w_b}{\partial x \partial y^3} - D_{22}\frac{\partial^4 w_b}{\partial y^4} \\ & - D_{11}^s\frac{\partial^4 w_s}{\partial x^4} - 4D_{16}^s\frac{\partial^4 w_s}{\partial x^3 \partial y} - 2(D_{12}^s + 2D_{66}^s)\frac{\partial^4 w_s}{\partial x^2 \partial y^2} - 4D_{26}^s\frac{\partial^4 w_s}{\partial x \partial y^3} - D_{22}^s\frac{\partial^4 w_s}{\partial y^4} \\ & = I_1(\ddot{w}_b + \ddot{w}_s) + I_2\left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y}\right) - I_3\left(\frac{\partial^2 \ddot{w}_b}{\partial x^2} + \frac{\partial^2 \ddot{w}_b}{\partial y^2}\right) - I_5\left(\frac{\partial^2 \ddot{w}_s}{\partial x^2} + \frac{\partial^2 \ddot{w}_s}{\partial y^2}\right) \end{aligned} \tag{21c}$$

$$
\begin{aligned}
& B_{11}^{s}\frac{\partial^3 u_0}{\partial x^3}+3B_{16}^{s}\frac{\partial^3 u_0}{\partial x^2\partial y}+(B_{12}^{s}+2B_{66}^{s})\frac{\partial^3 u_0}{\partial x\partial y^2}+B_{26}^{s}\frac{\partial^3 u_0}{\partial y^3}\\
& +B_{16}^{s}\frac{\partial^3 v_0}{\partial x^3}+(B_{12}^{s}+2B_{66}^{s})\frac{\partial^3 v_0}{\partial x^2\partial y}+3B_{26}^{s}\frac{\partial^3 v_0}{\partial x\partial y^2}+B_{22}^{s}\frac{\partial^3 v_0}{\partial y^3}\\
& -D_{11}^{s}\frac{\partial^4 w_b}{\partial x^4}-4D_{16}^{s}\frac{\partial^4 w_b}{\partial x^3\partial y}-2(D_{12}^{s}+2D_{66}^{s})\frac{\partial^4 w_b}{\partial x^2\partial y^2}-4D_{26}^{s}\frac{\partial^4 w_b}{\partial x\partial y^3}-D_{22}^{s}\frac{\partial^4 w_b}{\partial y^4}\\
& -H_{11}^{s}\frac{\partial^4 w_s}{\partial x^4}-4H_{16}^{s}\frac{\partial^4 w_s}{\partial x^3\partial y}-2(H_{12}^{s}+2H_{66}^{s})\frac{\partial^4 w_s}{\partial x^2\partial y^2}-4H_{26}^{s}\frac{\partial^4 w_s}{\partial x\partial y^3}-H_{22}^{s}\frac{\partial^4 w_s}{\partial y^4}\\
& +A_{55}^{s}\frac{\partial^2 w_s}{\partial x^2}+A_{44}^{s}\frac{\partial^2 w_s}{\partial y^2}+2A_{45}^{s}\frac{\partial^2 w_s}{\partial x\partial y}\\
& =I_1(\ddot{w}_b+\ddot{w}_s)+I_4\left(\frac{\partial \ddot{u}_0}{\partial x}+\frac{\partial \ddot{v}_0}{\partial y}\right)-I_5\left(\frac{\partial^2 \ddot{w}_b}{\partial x^2}+\frac{\partial^2 \ddot{w}_b}{\partial y^2}\right)-I_6\left(\frac{\partial^2 \ddot{w}_s}{\partial x^2}+\frac{\partial^2 \ddot{w}_s}{\partial y^2}\right)
\end{aligned}
\tag{21d}
$$

3.3 Analytical solutions for an antisymmetric cross-ply hybrid laminates

The Navier solutions can be developed for rectangular laminates with two sets of simply supported boundary conditions. For antisymmetric cross-ply and angle-play laminates, the following plate stiffnesses are identically zero:

$$
\begin{gathered}
A_{16}=A_{26}=D_{16}=D_{26}=D_{16}^{s}=D_{26}^{s}=H_{16}^{s}=H_{26}^{s}=0\\
B_{12}=B_{26}=B_{16}=B_{66}=B_{12}^{s}=B_{16}^{s}=B_{26}^{s}=B_{66}^{s}=A_{45}^{s}=0\\
B_{22}=-B_{11},\quad B_{22}^{s}=-B_{11}^{s}
\end{gathered}
\tag{22}
$$

The following boundary conditions for antisymmetric cross-ply and angle-play laminates can be written as

$$
\begin{gathered}
v_0(0,y)=w_b(0,y)=w_s(0,y)=\frac{\partial w_b}{\partial y}(0,y)=\frac{\partial w_s}{\partial y}(0,y)=0\\
v_0(a,y)=w_b(a,y)=w_s(a,y)=\frac{\partial w_b}{\partial y}(a,y)=\frac{\partial w_s}{\partial y}(a,y)=0\\
N_x(0,y)=M_x^b(0,y)=M_x^s(0,y)=N_x(a,y)=M_x^b(a,y)=M_x^s(a,y)=0\\
u_0(x,0)=w_b(x,0)=w_s(x,0)=\frac{\partial w_b}{\partial x}(x,0)=\frac{\partial w_s}{\partial x}(x,0)=0\\
u_0(x,b)=w_b(x,b)=w_s(x,b)=\frac{\partial w_b}{\partial x}(x,b)=\frac{\partial w_s}{\partial x}(x,b)=0\\
N_y(x,0)=M_y^b(x,0)=M_y^s(x,0)=N_y(x,b)=M_y^b(x,b)=M_y^s(x,b)=0
\end{gathered}
\tag{23}
$$

The boundary conditions in Eq. (23) are satisfied by the following expansions

$$
\begin{aligned}
u_0&=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}U_{mn}\,e^{i\omega t}\cos(\alpha\,x)\sin(\beta\,y)\\
v_0&=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}V_{mn}\,e^{i\omega t}\sin(\alpha\,x)\cos(\beta\,y)\\
w_b&=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{bmn}\,e^{i\omega t}\sin(\alpha\,x)\sin(\beta\,y)\\
w_s&=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{smn}\,e^{i\omega t}\sin(\alpha\,x)\sin(\beta\,y)
\end{aligned}
\tag{24}
$$

Where U_{mn}, V_{mn}, W_{bmn} *and* W_{smn} unknown parameters must be determined, ω is the Eigen frequency associated with (*m*, *n)* the Eigen-mode, and $\alpha = \frac{m\pi}{a}$ and $\beta = \frac{n\pi}{b}$.

Substituting Eqs. (22) and (24) into Eq. (21), the Navier solution of antisymmetric cross-ply laminates can be determined from equations

$$\begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{12} & s_{22} & s_{23} & s_{24} \\ s_{13} & s_{23} & s_{33} & s_{34} \\ s_{14} & s_{24} & s_{34} & s_{44} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{bmn} \\ W_{smn} \end{Bmatrix} + \begin{bmatrix} m_{11} & 0 & 0 & 0 \\ 0 & m_{22} & 0 & 0 \\ 0 & 0 & m_{33} & m_{34} \\ 0 & 0 & m_{34} & ms_{44} \end{bmatrix} \begin{Bmatrix} \ddot{U}_{mn} \\ \ddot{V}_{mn} \\ \ddot{W}_{bmn} \\ \ddot{W}_{smn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{25}$$

Where

$$s_{11} = A_{11}\alpha^2 + A_{66}\beta^2, \quad s_{12} = \alpha\beta\left(A_{12} + A_{66}\right), \quad s_{13} = -B_{11}\alpha^3, \quad s_{14} = -B_{11}^s\alpha^3$$

$$s_{22} = A_{66}\alpha^2 + A_{22}\beta^2, \quad s_{23} = B_{11}\beta^3, \quad s_{24} = B_{11}^s\beta^3$$

$$s_{33} = D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4$$

$$s_{34} = D_{11}^s\alpha^4 + 2(D_{12}^s + 2D_{66}^s)\alpha^2\,\beta^2 + D_{22}^s\,\beta^4 \tag{26}$$

$$s_{44} = H_{11}^s\alpha^4 + 2(H_{12}^s + 2H_{66}^s)\alpha^2\beta^2 + H_{22}^s\beta^4 + A_{55}^s\alpha^2 + A_{44}^s\beta^2$$

$$m_{11} = m_{22} = I_1, \quad m_{33} = I_1 + I_3(\alpha^2 + \beta^2)$$

$$m_{34} = I_1 + I_5(\alpha^2 + \beta^2), \quad m_{44} = I_1 + I_6(\alpha^2 + \beta^2)$$

4. Numerical results and discussion

In this study, a free vibration analysis of an anti-symmetrically cross-ply and angle-ply laminated Carbon/Glass Epoxy hybrid composite plates is examined using the present refined shear deformation theory. The Navier solution is used in order to determine the natural frequencies of laminated hybrid composite plates by solving Eigen value equations. For the verification purpose, the results obtained by present model are compared with those of the Reddy, Belkacem and exact solution of three-dimensional elasticity. The material properties used in this study are cited in Table 1. For convenience, the following dimensionless parameter is used in presenting the numerical results in graphical and tabular forms:

Table 1 The materials properties used in the present research

Material 1 (Noor 1975)	$E_1 = 40E_2$, $G_{12} = G_{13} = 0{,}6E_2$, $G_{23} = 0{,}5E_2$, $\nu_{12} = 0{,}25$
Material 2 (Noor 1973)	$E_1 = 40E_2$, $G_{12} = G_{13} = 0{,}5E_2$, $G_{23} = 0{,}6E_2$, $\nu_{12} = 0{,}25$
Carbon's fiber (Berthelot 1992)	$E_f = 380$, $\nu_f = 0.33$
Glass's fiber (Berthelot 1992)	$E_f = 86$, $\nu_f = 0.22$
Epoxy's matrix (Berthelot 1992)	$E_m = 3.45$, $\nu_m = 0.3$

Table 2 Dimensionless fundamental frequencies of antisymmetric cross-ply square laminated plate under various degrees of orthotropie (a/h=5, Material 1)

N° of layers	Theory	E_1/E_2				
		3	10	20	30	40
$(0°/90°)_1$	Exact (Noor 1973)	6.2578	6.9845	7.6745	8.1763	8.5625
	Belkacem (2016)	6.2168	6.9881	7.8198	8.5028	9.0841
	Reddy (1984)	6.2169	6.9887	7.8210	8.5050	9.0871
	Present	6.2188	6.9964	7.8379	8.5316	9.1236
$(0°/90°)_2$	Exact (Noor 1973)	6.5455	8.1445	9.4055	10.1650	10.6790
	Belkacem (2016)	6.5009	8.1958	9.6273	10.5359	11.1728
	Reddy (1984)	6.5008	8.1954	9.6265	10.5348	11.1716
	Present	6.5012	8.1929	9.6205	10.5268	11.1628
$(0°/90°)_3$	Exact (Noor 1973)	6.6100	8.4143	9.8398	10.6950	11.2720
	Belkacem (2016)	6.5558	8.4053	9.9182	10.8546	11.5009
	Reddy (1984)	6.5558	8.4052	9.9181	10.8547	11.5012
	Present	6.5567	8.4065	9.9210	10.8603	11.5102
$(0°/90°)_5$	Exact (Noor 1973)	6.6458	8.5625	10.0843	11.0027	11.6245
	Belkacem (2016)	6.5842	8.5126	10.0671	11.0191	11.6721
	Reddy (1984)	6.5842	8.5126	10.0674	11.0197	11.6730
	Present	6.5854	8.5156	10.0740	11.0309	11.6893
$(0°/90°)_8$	Present	6.5952	8.5529	10.1263	11.0894	11.7509
$(0°/90°)_{16}$	Present	6.6000	8.5708	10.1515	11.1175	11.7806
$(0°/90°)_{32}$	Present	6.6012	8.5753	10.1577	11.1245	11.7880

Table 3 The volume fraction Vf effect on the natural frequencies variation of a cross-ply antisymmetric $(0°/90°)_2$ hybrid square laminated composite plate

w_f(%)		V_f								
Carbon	Glass	0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7
0	100	7.3744	7.4649	7.5199	7.5423	7.5341	7.4967	7.4308	7.3367	7.2138
10	90	7.8672	7.9785	8.0454	8.0717	8.0600	8.0121	7.9287	7.8098	7.6545
20	80	8.3196	8.4489	8.5262	8.5563	8.5423	8.4862	8.3887	8.2496	8.0675
30	70	8.7373	8.8822	8.9685	9.0020	8.9862	8.9233	8.8139	8.6577	8.4526
40	60	9.1254	9.2838	9.3781	9.4146	9.3974	9.3287	9.2092	9.0381	8.8129
50	50	9.4879	9.6582	9.7595	9.7988	9.7804	9.7068	9.5785	9.3943	9.1514
60	40	9.8280	10.0089	10.1164	10.1582	10.1389	10.0610	9.9250	9.7293	9.4706
70	30	10.1482	10.3386	10.4518	10.4958	10.4758	10.3943	10.2514	10.0456	9.7728
80	20	10.4509	10.6498	10.7679	10.8141	10.7936	10.7088	10.5600	10.3451	10.0597
90	10	10.7377	10.9442	11.0669	11.1151	11.0941	11.0066	10.8525	10.6296	10.3328
100	0	11.0101	11.2236	11.3504	11.4004	11.3791	11.2892	11.1305	10.9004	10.5933

Unless cited otherwise the following parameters are fixed as: $a/h=10$, $a/b=1$, $V_f=0.5$.

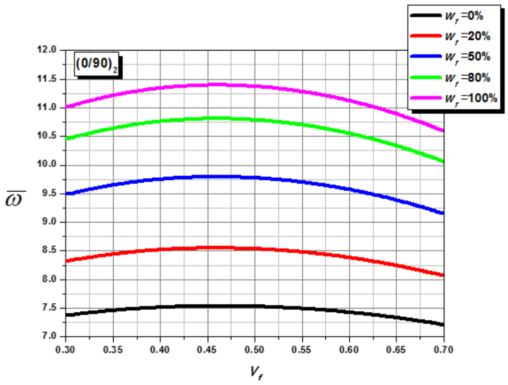

Fig. 2 The volume fraction V_f effect on the natural frequencies variation of a cross-ply antisymmetric $(0°/90°)_2$ hybrid square laminated composite plate

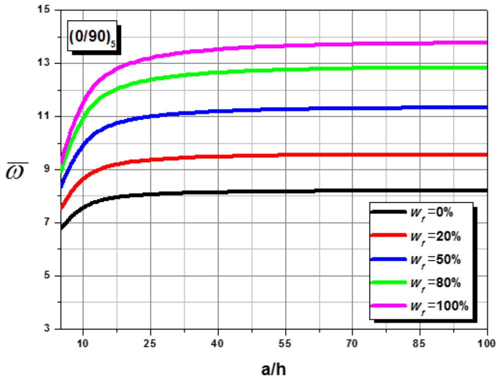

Fig. 3 Dimensionless natural frequency of an antisymmetric cross-ply $(0/90)_5$ hybrid square laminated composite plate under various a/h ratios

Table 4 Dimensionless natural frequency of an antisymmetric cross-ply $(0/90)_5$ hybrid square laminated composite plate under various a/h ratios

$w_f(\%)$		a/h				
Carbon	Glass	5	10	20	50	100
0	100	6.7741	7.7531	8.0850	8.1872	8.2022
10	90	7.1787	8.3525	8.7642	8.8926	8.9115
20	80	7.5315	8.8984	9.3935	9.5498	9.5729
30	70	7.8406	9.3978	9.9793	10.1650	10.1925
40	60	8.1138	9.8577	10.5280	10.7447	10.7769
50	50	8.3572	10.2838	11.0451	11.2940	11.3311
60	40	8.5759	10.6807	11.5347	11.8172	11.8594
70	30	8.7734	11.0519	12.0003	12.3175	12.3651
80	20	8.9531	11.4005	12.4446	12.7976	12.8507
90	10	9.1172	11.7290	12.8697	13.2596	13.3185
100	0	9.2679	12.0394	13.2777	13.7054	13.7703

The dimensionless fundamental frequencies of anti-symmetrically laminated cross-ply $(0/90)_n$ plates obtained by using different shear deformation theories are shown in Table 2 for various values of modules ratios E_1/E_2. It can be seen that, in general, the present model gives more accurate results in predicting the natural frequencies than the Belkacem (2016), Reddy (1984) and the three-dimensional elasticity solution given in Noor (1973). It should be noted that the unknown functions in present and Belkacem (2016) models are four; while there are five unknown functions in the Reddy's model (1984). It can be concluded that the present model is not only accurate, but also simple in predicting the natural frequencies of laminated composite plates.

The Table 3 and Figure 2 presents the dimensionless fundamental frequency variation of anti-symmetric cross-ply (0/90)2 Carbon/Glass hybrid laminated composite plate using the present high-order shear deformation theory. It is clear that the natural frequencies increase with augmentation of the volume fraction V_f until reaching its peak (at V_f = 0.45), this point represents the optimum percentage of fibers needed to reach the maximum natural frequencies. Also, the fibers mixture vary gradually from full Glass (w_f =0%) to full Carbon (w_f =100%), where the combination of this two types of fibers gives rise to a hybrid composite plate that respond to different criteria's (resistance, strength, economic…).

Table 5 Dimensionless natural frequency of an antisymmetric cross-ply $(0/90)_2$ hybrid rectangular laminated composite plate under aspect ratio a/b

a/h	w_f(%)		a/b						
	Carbon	Glass	0.2	0.6	0.8	1	1.2	1.6	2
10	0	100	4.6874	5.4482	6.2990	7.5341	9.1481	13.3668	18.6252
	10	90	5.1321	5.8800	6.7574	8.0600	9.7819	14.2970	19.8894
	20	80	5.5299	6.2738	7.1778	8.5423	10.3596	15.1278	20.9927
	30	70	5.8902	6.6352	7.5654	8.9862	10.8884	15.8748	21.9644
	40	60	6.2200	6.9696	7.9249	9.3974	11.3757	16.5518	22.8286
	50	50	6.5244	7.2810	8.2603	9.7804	11.8273	17.1698	23.6041
	60	40	6.8073	7.5725	8.5748	10.1389	12.2481	17.7373	24.3049
	70	30	7.0715	7.8465	8.8709	10.4758	12.6416	18.2611	24.9422
	80	20	7.3194	8.1052	9.1507	10.7936	13.0112	18.7466	25.5250
	90	10	7.5530	8.3501	9.4158	11.0941	13.3593	19.1985	26.0606
	100	0	7.7737	8.5827	9.6677	11.3791	13.6881	19.6205	26.5547
20	0	100	4.8262	5.6194	6.5214	7.8490	9.6137	14.3796	20.6250
	10	90	5.3138	6.0943	7.0307	8.4443	10.3501	15.5404	22.3488
	20	80	5.7570	6.5331	7.5041	8.9983	11.0336	16.6065	23.9133
	30	70	6.1648	6.9414	7.9465	9.5163	11.6711	17.5921	25.3452
	40	60	6.5440	7.3242	8.3625	10.0035	12.2694	18.5101	26.6667
	50	50	6.8994	7.6854	8.7560	10.4643	12.8341	19.3705	27.8949
	60	40	7.2347	8.0280	9.1298	10.9020	13.3696	20.1811	29.0429
	70	30	7.5525	8.3543	9.4864	11.3194	13.8793	20.9479	30.1208
	80	20	7.8551	8.6661	9.8277	11.7187	14.3661	21.6761	31.1371
	90	10	8.1422	8.9652	10.1552	12.1019	14.8324	22.3697	32.0985
	100	0	8.4213	9.2526	10.4704	12.4704	15.2801	23.0320	33.0105

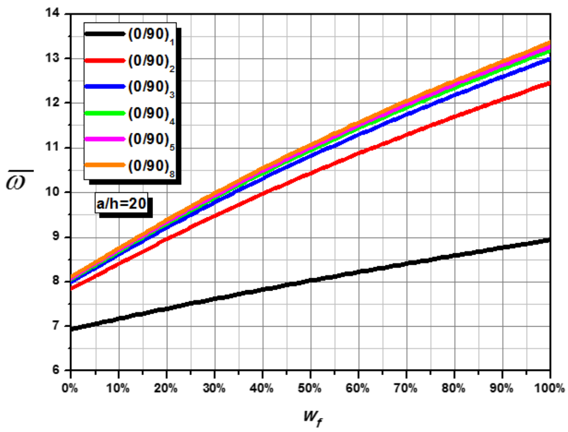

Fig. 4 The stacking sequence effect on the dimensionless natural frequency of an antisymmetric cross-ply $(0/90)_n$ hybrid square laminated composite plate

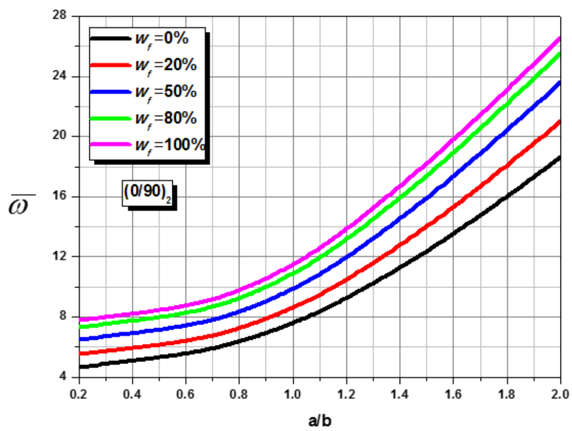

Fig. 5 Dimensionless natural frequency of an antisymmetric cross-ply $(0/90)_2$ hybrid rectangular laminated composite plate under aspect ratio a/b

Table 6 The stacking sequence effect on the dimensionless natural frequency of an antisymmetric cross-ply (0/90)n hybrid square laminated composite plate

Number of layers	a/h	100 % Glass	25 % Carbon 75 % Glass	50 % Carbon 50 % Glass	75 % Carbon 25 % Glass	100 % Carbon
$(0/90)_1$	5	6.0083	6.4154	6.7702	7.0883	7.3779
	10	6.7136	7.2455	7.7114	8.1359	8.5298
	20	6.9443	7.5251	8.0356	8.5039	8.9417
	50	7.0144	7.6110	8.1360	8.6188	9.0713
	100	7.0246	7.6235	8.1508	8.6357	9.0904
$(0/90)_2$	5	6.6014	7.4144	8.0208	8.4923	8.8714
	10	7.5341	8.7687	9.7804	10.6370	11.3791
	20	7.8486	9.2604	10.4624	11.5184	12.4665
	50	7.9453	9.4158	10.6836	11.8115	12.8370
	100	7.9595	9.4387	10.7165	11.8554	12.8929
$(0/90)_3$	5	6.7154	7.5972	8.2433	8.7389	9.1333
	10	7.6794	9.0247	10.1158	11.0317	11.8198
	20	8.0057	9.5476	10.8512	11.9904	13.0088
	50	8.1061	9.7133	11.0911	12.3116	13.4177
	100	8.1208	9.7378	11.1268	12.3598	13.4796
$(0/90)_4$	5	6.7555	7.6613	8.3212	8.8253	9.2253
	10	7.7298	9.1129	10.2309	11.1668	11.9703
	20	8.0600	9.6461	10.9841	12.1512	13.1932
	50	8.1616	9.8154	11.2302	12.4819	13.6150
	100	8.1765	9.8404	11.2669	12.5315	13.6790
$(0/90)_5$	5	6.7741	7.6910	8.3572	8.8653	9.2679
	10	7.7531	9.1535	10.2838	11.2289	12.0394
	20	8.0850	9.6914	11.0451	12.2250	13.2777
	50	8.1872	9.8622	11.2940	12.5599	13.7054
	100	8.2022	9.8875	11.3311	12.6102	13.7703
$(0/90)_8$	5	6.7943	7.7231	8.3963	8.9087	9.3142
	10	7.7782	9.1973	10.3409	11.2957	12.1139
	20	8.1120	9.7402	11.1108	12.3044	13.3687
	50	8.2149	9.9128	11.3628	12.6439	13.8027
	100	8.2299	9.9383	11.4003	13.6949	13.8685
$(0/90)_{16}$	5	6.8040	7.7386	8.4151	8.9296	9.3364
	10	7.7903	9.2183	10.3682	11.3278	12.1495
	20	8.1250	9.7636	11.1422	12.3424	13.4122
	50	8.2281	9.9369	11.3957	12.6841	13.8492
	100	8.2432	9.9626	11.4335	12.7355	13.9155

The thickness variation a/h and the aspect ratio a/b effects on the dimensionless fundamental frequency are shown in Tables (4 and 5) and Figures (3 and 5), where the increase of this two ratios conduct to the direct increase of the fundamental frequencies for all types of composite plates (full carbon, full glass and hybrid plate), it means that the plate geometry has a very important impact on the stability of the hybrid composite plate.

The Table 6 and Figures 4 show the variation if dimensionless fundamental frequency of an anti-symmetric cross-ply (0/90)n hybrid composite laminated plate for different values of thickness ratio a/h, fibers mixture w_f and number of layers. Where the fundamental frequency increase as the number of layers used increases, which is logic because the increase of number of layers conduct to increasing the rigidity of the plate and by consequence the frequencies rises.

In the last example (Table 7) a different combinations of fibers are used in hybrid composite plate, the fundamental frequency are minimum for the case of Kevlar/Glass and maximal for the case of Kevlar/Carbon, which is logic since this last combination gives the better features of the three combination, when the carbon fibers gives rigidity to the plate and the Kevlar fibers assure the vibration damping.

5. Conclusions

In this study, a refined shear deformation theory has been successfully used for the free vibration of simply supported antisymmetric cross-ply hybrid laminated composite plates. The present theory allows for parabolic variation in terms of the transverse shear strains across the plate thickness and satisfies the zero shear stress on the top and bottom surfaces of the plate without needing shear correction factors. The equations of motion were developed using Hamilton's principle. Where the accuracy and efficiency of the present theory has been demonstrated for free vibration stability of anti-symmetric cross-ply hybrid laminated composite plate.

From this research, we conclude as follows:

• The natural frequencies predicted by the present theory using just four unknowns are almost identical to those found by the shear deformation theories of five unknowns and to the three-dimensional elasticity solution.

• The present theory is applicable for different combinations of materials in terms of predicting the natural frequencies.

• The material combinations affect significantly the fundamental frequencies, where the mixture of Carbon and Kevlar gives the maximum frequencies.

Finally, it is up to the researchers and manufacturer to choose wisely the material combinations that gives rise to a hybrid composite plate that offers rigidity, strength and most of all less greedy in terms of cost.

Acknowledgments

This research was supported by the French Ministry of Foreign Affairs and International Development (MAEDI) and Ministry of National Education, Higher Education and Research (MENESR) and by the Algerian Ministry of Higher Education and Scientific Research under Grant No. PHC Tassili 17MDU992. Their support is greatly appreciated.

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