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Elastic-plastic analysis of the *J* integral for repaired cracks in plates

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Abstract. In this paper, three-dimensional finite element method is used to analyze the *J* integral for repaired cracks in plates with bonded composite patch and stiffeners. For elastic the effect of cracks, the thickness of the patch (e_r) and properties of the patch are presented for calculating the *J* integral. For elastic-plastic a several calculations have been realized to extract the plasticized elements around the crack tip of repaired and un-repaired crack. The obtained results show that the presence of the composite patch and stiffener reduces considerably the size of the plastic zone ahead of the crack. The effects of crack size and the inter-distance of repaired cracks were analysed.

Keywords: composite; finite element method; crack; *J* integral; fracture mechanic; elastic-plastic

1. Introduction

The cracks can occur in many structural components of plates form They are the cause of premature damage in structures such as bolts, pins and reinforcements of aircraft. Most of the previously performed studies on bonded composite patch repair have involved plane aluminum panels without considering stiffeners.

The stress intensity factor can be taken as an essential parameter to analyze crack (Shen *et al*. 2010, Shouetsu 2009, Sapora *et al*. 2014). To estimate repair performance, SIF can also be utilized as an important measuring parameter (Gu *et al.* 2011, Lam *et al*. 2010).

A considerable amount of research has been conducted on extending the service life of cracked engineering structures with the emphasis on the life extension of aging aircraft structures using various repair techniques (Oudad *et al*. 2009). Analytical models have been developed for analysis of adhesively bonded joints, numerical solutions have been used in the past to predict the strength and stress distributions of adhesively bonded composite joints (Costa Mattas *et al.* 2012, Sayman *et al.* 2013). Bonded composite repair has been recognized as an efficient and economical method to extend the fatigue life of cracked aluminium components, several authors (Liu *et al.* 2009, Shen *et al*. 2011, Hosseini-Toudeshky *et al*. 2011) analysed fatigue crack growth rate of cracked

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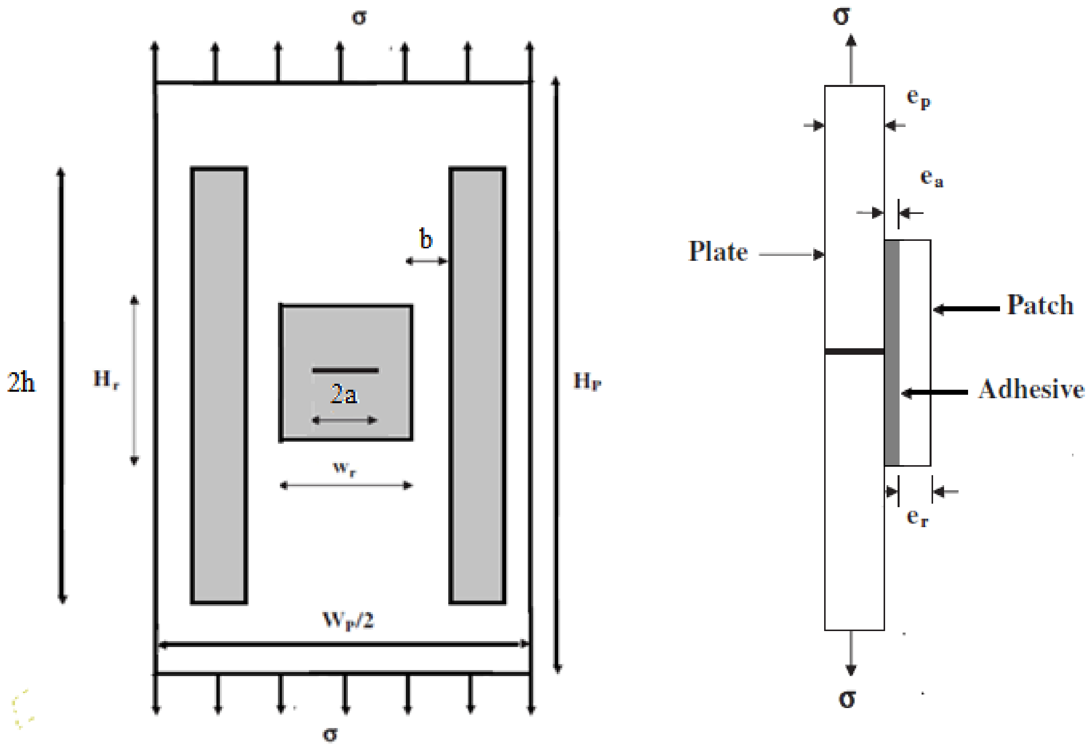

Fig. 1 Geometrical model

aluminum plates repaired with composite patch. Bachir Bouiadjra *et al*. (2012) analyzed the effect of the adhesive disband for inclined cracks repaired with boron/ epoxy patch. It was concluded that the booth mode I and mode II stress intensity factors are negatively affected by the presence of the adhesive disband. Cetisli and Kaman (2014), Mhamdia *et al*. (2012), these researches have shown that, after repair, the stress intensity factor exhibits an asymptotic behavior as the crack length increases. This behavior is due to the fact that there is a stress transfer from the cracked aluminium plate to the composite patch throughout the adhesive layer.

This study presents three-dimensional finite element method analyses of the *J* integral for repaired cracks in plates with bonded composite patch and stiffeners. For elastic the effect of cracks, the thickness of the patch (e_r) and properties of the patch are presented for calculating the *J* integral. For elastic-plastic a several calculations have been realized to extract the plasticized elements around the crack tip of repaired and un- repaired crack. The effects of crack size and the inter-distance of repaired cracks were analysed.

2. Geometrical model

The basic geometry of the cracked structure considered in this study is shown in Fig. 1. Consider a plate with the following dimensions: height H_p=600 mm, width w_p=300 mm, thickness e_p=4 mm. The plate is subjected to uniaxial tensile load giving a remote stress state of σ=50 MPa

Table 1 Elastic properties of different materials

	Aluminium alloy T3	Boron/epoxy	Graphite/epoxy	Adhésive (FM 73)
E_1 (GPa)	72	200	127.5	2.55
E_2 (GPa)		19.6	9.00	
E_3 (GPa)		19.6	4.80	
υ_{12}	0.3	0.3	0.342	0.32
υ_{13}		0.28	0.342	
υ_{23}		0.28	0.38	
G_{12} (GPa)		7.2	4.8	
G_{13} (GPa)		5.5	4.8	
G_{23} (GPa)		5.5	2.55	

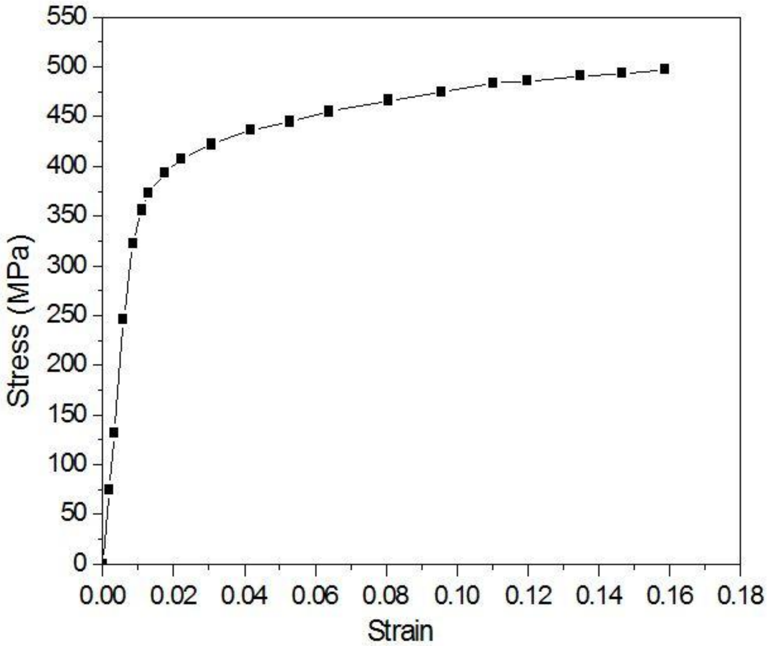

Fig. 2 Stress-strain curves for aluminum 2024-T3

for elastic analysis. A central crack of length 2*a* perpendicular to the loading axis is supposed to exist in the plate. This crack is repaired with unidirectional Boron/Epoxy composites patches. The ply orientation is parallel to the loading axis. The initial dimensions of the patch are: height H_r=100 mm, width w_r=100 mm and thickness e_r=2 mm, 3 mm. The adhesive are used to bond the patch on cracked plate: FM 73, Epoxy adhesive. The adhesive thickness (e_a) is taken equal to 0.2 mm.

3. Material properties

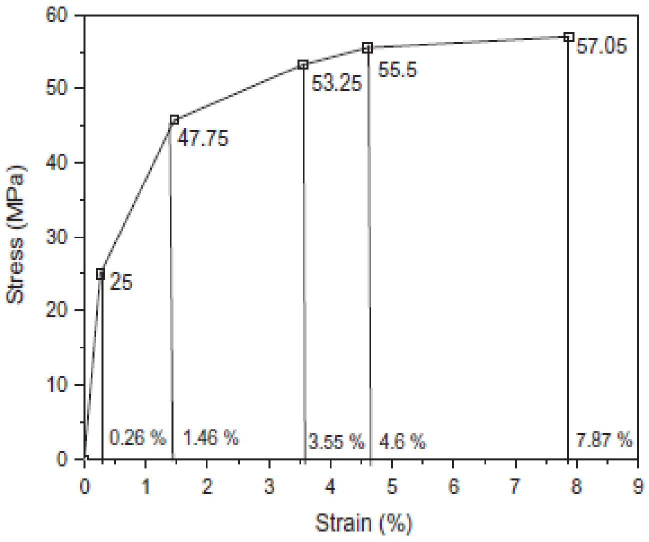

Fig. 3 Multi-linear stress-strain curve of the FM 73 epoxy adhesive (Ban *et al.* 2008)

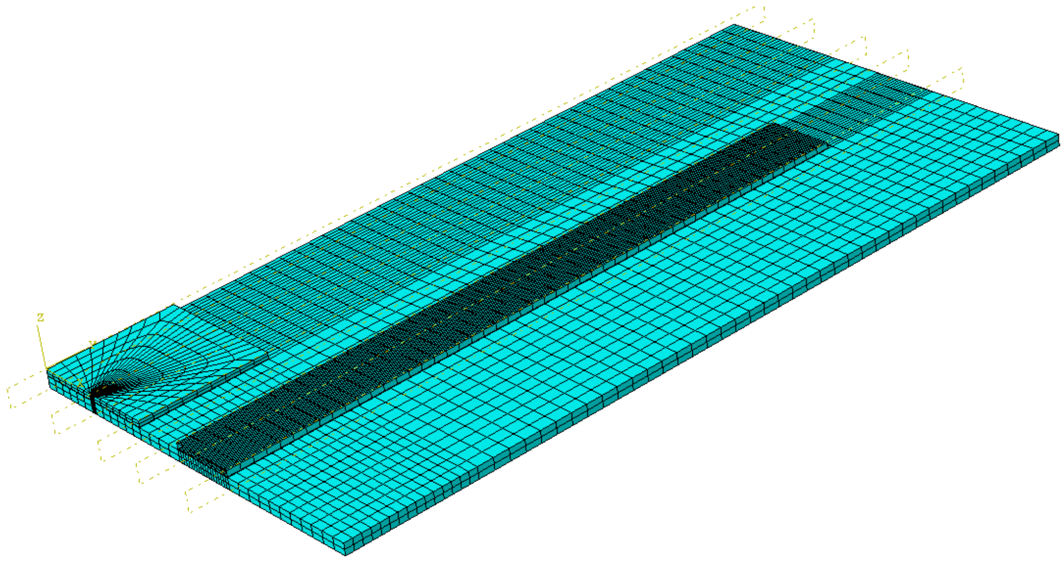

Fig. 4 Typical mesh model of the quarter of the structure

4. Finite element modeling

The three-dimensional finite element analysis is carried out using the commercial finite element code ABAQUS (2007). The finite element model consists of three subsections to model the cracked plate, the adhesive, and the composite patch. Due to loading symmetry, only the half of the repaired plate was considered. To generate crack front some brick elements are replaced by ‘‘crack block’’. These crack-blocks are meshes of brick elements which are mapped into the original element space and merged with surrounding mesh. Boundary conditions and loads are transferred to the crack-block elements. The finite element mesh was generated using brick elements with 20 nodes. Fig. 4 shows the overall mesh of the specimen and mesh refinement in the crack tip region.

5. Results and discussion

5.1 Validation of the numerical model

The elastic of the *J*-integral can be found

$$J_e = \frac{K_I^2}{E} \tag{1}$$

Where J_e is the energy release rate for mode I
Several authors showed that the mode I stress intensity factor of repaired crack, exhibits an asymptotic behavior as the crack length increases (Mhamdia *et al*. 2012, Ouddad *et al*. 2009). The asymptotic value of the SIF was approximated by Rose (1981) by the formulas

$$K_\infty = Y\sigma_0\sqrt{\pi\lambda} \tag{2}$$

Where

$$\sigma_0 = \frac{\sigma E_P.e_P}{(E_P.e_P + E_r.e_r)} \tag{3}$$

$$\pi\lambda = \sqrt{\frac{E_P.e_P}{\beta(1+\frac{E_P.e_P}{E_r.e_r})}} \tag{4}$$

and

$$\beta = \frac{\frac{e_a}{G_a} + \frac{e_r}{3G_r} + \frac{e_P}{3G_r}}{\left(\frac{e_a}{G_a} + \frac{3e_r}{8G_r} + \frac{3e_P}{8G_P}\right)^2} \tag{5}$$

Where
E: is the Young Modulus
e: is the thickness
G: is the shear modulus
The index *p*, *r* and *a* represent respectively the plate, the composite patch and the adhesive.

In order to validate our numerical results, we have compared in Table 2 the computed values of the *J* integral of repaired crack with those calculated with Rose’s model for crack length of 35 mm and for different values of the applied stress. It can be seen from Table 2 that, whatever the value of the applied stress, the relative difference between the numerical and the analytical *J* integral does not exceed 6.5% witch confirm that the numerical calculations are valid.

Table 2 Comparison between analytical and numerical J integral

σ ap (MPa)	J_e Analytical (MPa.m)	J_e FEM (MPa.m)	Relative difference (%)
20	0.0076	0.0072	5.26
40	0.0305	0.0285	6.5
50	0.0476	0.0447	6.1
60	0.0685	0.0657	4.1
70	0.0932	0.0873	6.3
80	0.1218	0.1165	4.35

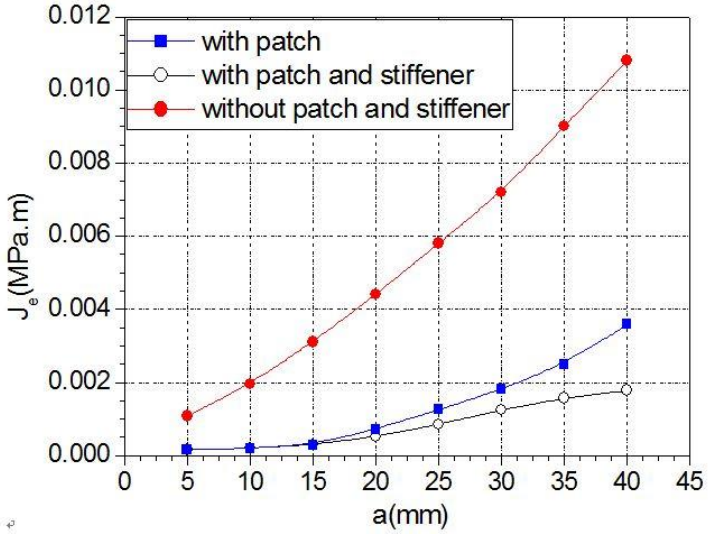

Fig. 5 Variation of the J integral according to the crack length for the cases with and without patch and stiffener

5.2 Elastic analysis

5.2.1 Effect of the patch

Fig. 5 presents the variation of the integral *J* according to the crack length for the cases with and without the patch, the value of applied stress is σ=50 MPa for elastic analysis. The adhesive used for calculation is the FM 73 and the thicknesses of the different materials are: plate (e_p=4 mm), patch (e_r=2 mm), and adhesive (e_a=0.2 mm).

It can be seen according to Fig. 5 that the presence of the patch and stiffener has a considerable effect on the of the *J* integral variation at the crack tip. Indeed, the values of the *J* integral of the structure repaired with boron-epoxy patch are highly decreased. The maximum relative variation of the *J* integral is about 80%.

5.2.2 Effect of the patch thickness

Several authors (Bachir Bouiadjra *et al*. 2012, Achour *et al*. 2003) showed the importance of the effect of the patch thickness on the repair performance in damaged aircraft structures. Bachir

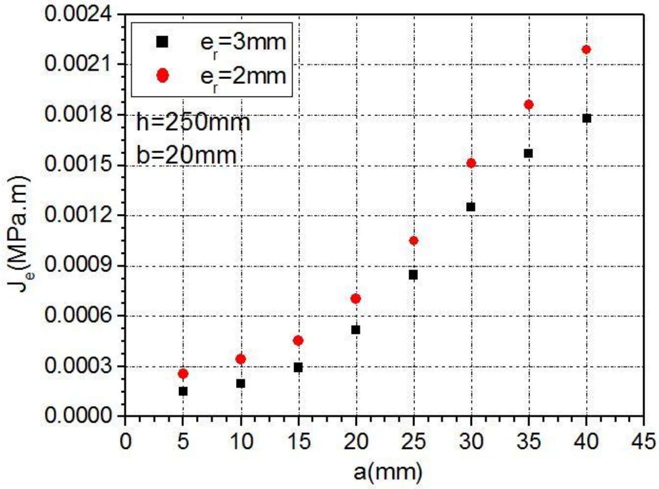

Fig. 6 Variation of the *J* integral according to the patch thickness for different cracks

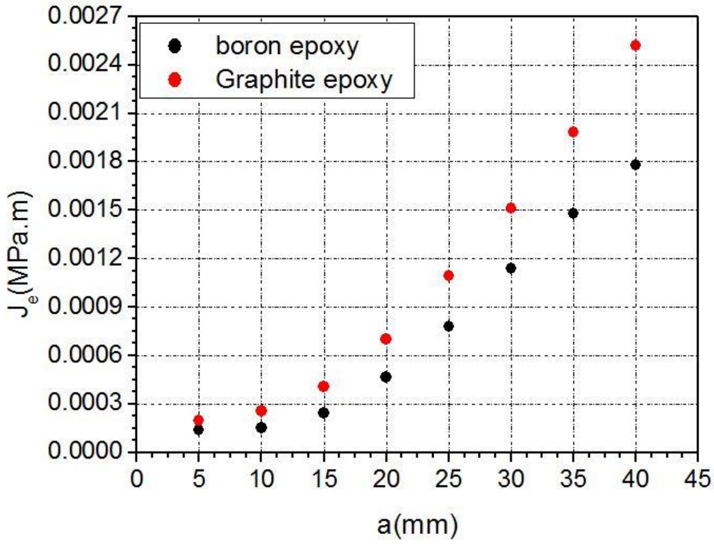

Fig. 7 the variation of the *J* integral according to the cracks for two different properties of the patch

Bouiadjra *et al*. (2002) showed that, under pure mechanical loading, the increase of the patch thickness with 50% decreases the SIF at the crack tip in the same order. Obviously, the *J* integral is inversely proportional to the patch thickness. The *J* integral decreases lightly when the patch thickness is higher than 3 mm. The reduction ratio of *J* integral is about 22.22% for a patch thickness of 3 mm compared with the patch thickness of 2 mm. Optimizing these parameters improves the repair efficiency of structure damaged.

5.2.3 Effect of the patch properties

Jones and Chiu (1999) give a value of 60%, for boron/epoxy patch. Bachir Bouiadjra *et al.*

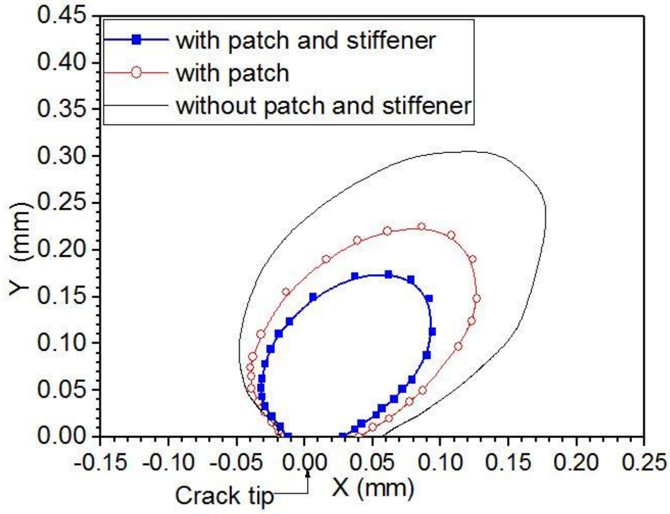

Fig. 8 Contour of the plastic zone for the cases with and without patch

(2002) give a value 80% for a central cracks repaired by graphite/epoxy patch. Fig. 7 presents the variation of the *J* integral according to the cracks for two different properties of the patch. It can be seen that, the value of *J* integral increases with the crack length. The Boron/epoxy compared to Graphite/epoxy give a good performance of the reparation of structure damaged, it can be seen that the patch properties are very important role in the repair process with the bonded composite patch

5.3 Elastic-Plastic analyses

5.3.1 Effect of the repair on the plastic zone size

Oudad *et al*. (2009), investigated the influence of the patch parameters on the size of the plastic zone at the tip of repaired cracks. They showed that the presence of the composite patch reduces considerably the size of the plastic zone ahead of the crack. This reduction is very important so that the concepts of linear fracture mechanics can be applied for repaired cracks.

In order to estimate the performances of the patch repair, the shapes of the plastic zone for repairs and un-repaired cracks were computed for crack length *a*=30 mm in Fig. 8. The obtained results show. The presence of the composite patch and stiffener reduces considerably the size of the plastic zone ahead of the crack. This reduction is very important so that the concepts of fracture mechanics for repairing cracks.

$$J = J_e + J_P \tag{6}$$

5.3.2 Effect of the crack length and inter-distance «b»

Fig. 9 presents the variation of *J* integral of repairing cracks for different crack lengths for various values of the stress. It can be seen that the variation of the crack length is affected by the variation of stress and increases of the *J* integral as the crack length increases.

Fig. 10 illustrated the effect of inter-distance (b) of the repair performances of the patch and stiffener repair for various values of the stress. It can be noted, that the decrease of the inter-distance (b) involves the reduction of the J integral, what can improve the repair performances.

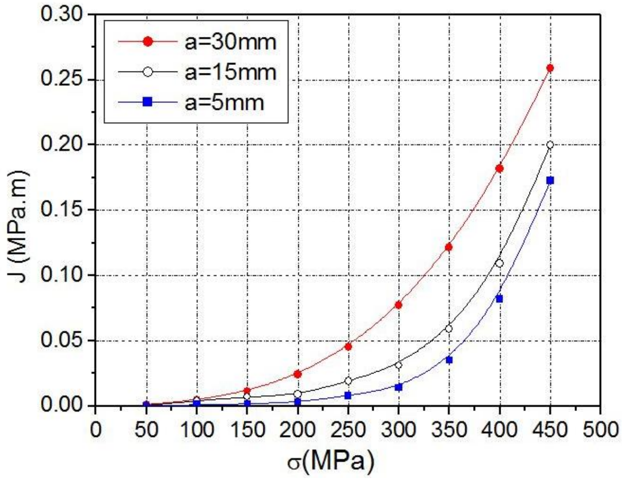

Fig. 9 Variation of the *J* integral according to the stress for different crack length, *b*=20 mm, *h*=250 mm

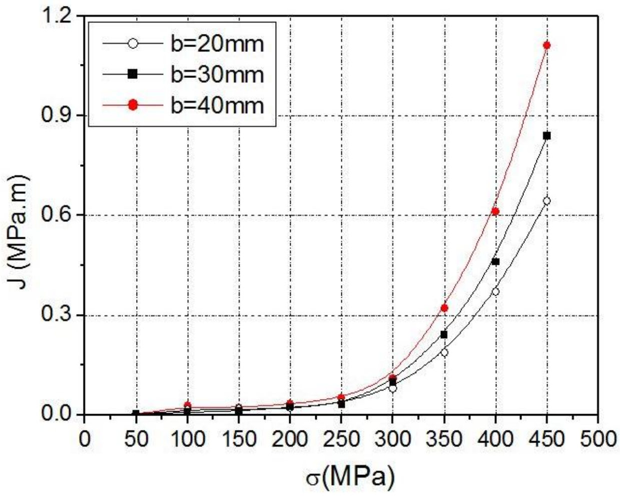

Fig. 10 Variation of the *J* integral according to the stress for different value of inter-distance (b), *h*=250 mm, *a*=45 m

6. Conclusions

In this paper, three-dimensional finite element method is used to analyze the *J* integral for repaired cracks in plates with bonded composite patch and stiffeners. The obtained results allow us to deduce the following conclusions:

- For elastic case, the thickness and the properties of the composite patch (e_r) reduces considerably the value of the *J* integral and increase the performance of the reparation of structure damaged.

-For elastic-plastic, the obtained results show that the presence of the composite patch and stiffener reduces considerably the size of the plastic zone of the crack.

-The decrease of the inter-distance (b) reduces considerably the value of the *J* integral.

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