Advances in Materials Research, *Vol. 12, No. 2 (2023) 161-177*
https://doi.org/10.12989/amr.2023.12.2.161

Free vibration of functionally graded carbon nanotubes reinforced composite nanobeams

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(Received December 29, 2022, Revised March 2, 2023, Accepted March 3, 2023)

Abstract. This paper proposes an analytical method to investigate the free vibration behaviour of new functionally graded (FG) carbon nanotubes reinforced composite beams based on a higher-order shear deformation theory. Cosine functions represent the material gradation and material properties via the thickness. The kinematic relations of the beam are proposed according to trigonometric functions. The equilibrium equations are obtained using the virtual work principle and solved using Navier's method. A comparative evaluation of results against predictions from literature demonstrates the accuracy of the proposed analytical model. Moreover, a detailed parametric analysis checks for the sensitivity of the vibration response of FG nanobeams to nonlocal length scale, strain gradient microstructure-scale, material distribution and geometry.

Keywords: higher order nonlocal strain gradient theory; nanobeams; Navier's solution; novel trigonometric FGM; vibration

1. Introduction

Nowadays, carbon nanotubes (CNTs) are being proposed as the most candidates for composite reinforcement material due to their remarkable mechanical, electrical, and thermal properties. These properties include a high tensile modulus, high strength, low density, good conductivity, and

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http://www.techno-press.org/?journal=amr&subpage=5 ISSN: 2234-0912 (Print), 2234-179X (Online)

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