Finite element based modeling and thermal dynamic analysis of functionally graded graphene reinforced beams

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Abstract. In the present research, dynamic analysis of functionally graded (FG) graphene-reinforced beams under thermal loading has been carried out based on finite element approach. The presented formulation is based on a higher order refined beam element accounting for shear deformations. The graphene-reinforced beam is exposed to transverse periodic mechanical loading. Graphene platelets have three types of dispersion within the structure including uniform-type, linear-type and nonlinear-type. Convergences and validation studies of derived results from finite element approach are also presented. This research shows that the resonance behavior of a nanocomposite beam can be controlled by the GPL content and dispersions. Therefore, it is showed that the dynamical deflections are notably influenced by GPL weight fractions, types of GPL distributions, temperature changes, elastic foundation and harmonic load excitation frequency.

Keywords: finite element method; thermal load; refined beam element; graphene platelet; dynamic loading

1. Introduction

In recent decades, several carbon based structures containing carbon nanotube or carbon fiber have been widely utilized in composites for enhancing their mechanics and thermal specifications (Zhang 2017, Keleshterri et al. 2016). A 273% enhancement of elastic modulus is obtained by Ahankari et al. (2010) for carbon reinforced composites in comparison to conventional composites. Likewise, Gojny et al. (2004) mentioned that structural stiffness of carbon based composites may be enhanced with incorporation of carbon nanotube within material. Impacts of configuration and scale of carbon nanotubes on rigidity growth of material composites having metallic matrices are studied by Esawi et al. (2011). Because of possessing above mentioned properties, beam and plate structures having carbon based fillers are researched to understand their static or dynamical status (Yang et al. 2017). There are also some investigations on composite or functionally graded materials and interested readers are refaced to new investigations on materials (Barati and Zenkour 2018, She et al. 2018, 2019). Furthermore, the graphene based composite material has been recently gained enormous attentions because of having easy producing procedure and high rigidity growth. Nieto et al. (2017) presented a review paper based on several graphene based composite material possessing ceramic or metallic matrices. The multi-scale study

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of mechanical attributes for graphene based composite material has been provided by Lin et al. (2018) utilizing finite elements approach.

Until now, many of researches in the fields of nano-composites have been interested in production and materials characteristics recognition of graphene based composites and structural components containing slight percentages of graphene fillers. For instance, it is mentioned by Rafiee et al. (2009) that some material characteristics of graphene based composites may be enhanced via placing 0.1% volume of graphene filler. However, achieving to this level of reinforcement employing nanotubes required 1% of their volume. Graphene based composites containing epoxy matrix were created by King et al. (2013) by placing 6% weight fraction of graphene fillers to polymeric phases. It was stated that Young modulus of the composite has been increased from 2.72 GPa to 3.36 GPa. Next, 57% increment for Young modulus has been achieved by Fang et al. (2009) based on a sample of graphene based composite.

Moreover, many studies in the fields of nano-mechanic are associated with vibrational and stability investigation of various structural elements containing beam or plate reinforced via diverse graphene dispersions. For instance, vibrational properties of a laminated graphene based plate have been explored by Song et al. (2017) assuming simply support edge condition. They assumed that the plate is constructed from particular numbers of layers each containing a sensible content of graphene. Selecting a perturbation approach, static deflections and bucking loads of graphene based plates have been derived by Shen et al. (2017). In above papers, each material property has discontinuous variation across the thickness of beam or plate. Also, geometrically nonlinear vibration frequencies of graphene based beams having embedded graphene have been explored by Feng et al. (2017) selecting first-order beam theory. Moreover, vibration frequencies of graphene based beams having porosities have been explored by Kitipornchai et al. (2017).

This paper is devoted to analyze mechanical-thermal dynamic behavior of a macro-size beam reinforced with graphene platelets (GPLs) based on finite element approach. Graphene platelets have three types of dispersion within the structure including uniform-type, linear-type and nonlinear-type. The presented formulation is based on a higher order refined beam element accounting for shear deformations. So, it is useful for thick beams. The graphene-reinforced beam is exposed to transverse periodic mechanical loading. Via finite element procedure, forced vibration path of the beam has been derived. It will be demonstrated that dynamic characteristics of the graphene-reinforced beam are dependent on geometric amplitude, thermal loading, graphene distribution and amount.

### 2. Graphene based composites

According to Fig.1, it is assumed that graphene platelets have three types of dispersion within the structure including uniform-type, linear-type and nonlinear-type. According to Fig.2, a graphene reinforced composite micro-scale beam is illustrated. Micro-mechanic theory of such composite materials (Liew et al. 2015) introduces the below relationship between graphene platelets weight fraction ($W_{GPL}$) and their volume fraction ($V_{GPL}$) by:

\[
V_{GPL} = \frac{W_{GPL}}{W_{GPL} + \frac{\rho_{GPL}}{\rho_M} - \frac{\rho_{GPL}}{\rho_M} W_{GPL}}
\]

(1)
Finite element based modeling and thermal dynamic analysis of functionally graded graphene...

where $\rho_{\text{GPL}}$ and $\rho_{\text{M}}$ define the mass densities of graphene and polymeric matrices, respectively. Next, the elastic modulus of a graphene based composite might be represented based upon matrix elastic modulus ($E_{\text{M}}$) by:
\[
E_1 = \frac{3}{8} \left( 1 + \frac{\zeta^G_{PL}}{\eta^G_{PL} V^G_{PL}} \right) E_M + \frac{5}{8} \left( 1 + \frac{\zeta^G_{W}}{\eta^G_{W} V^G_{PL}} \right) E_M
\] (2)

so that \( \zeta^G_{PL} \) and \( \zeta^G_{W} \) define two geometrical factors indicating the impacts of graphene configuration and scales as:

\[
\zeta^G_{PL} = \frac{2l^G_{PL}}{t^G_{PL}} \quad (3a)
\]

\[
\eta^G_{PL} = \frac{\left( \frac{E^G_{PL}}{E_M} \right)^{-1}}{\left( \frac{E^G_{PL}}{E_M} \right) + \zeta^G_{PL}} \quad (3b)
\]

\[
\zeta^G_{W} = \frac{2w^G_{PL}}{t^G_{PL}} \quad (3c)
\]

\[
\eta^G_{W} = \frac{\left( \frac{E^G_{PL}}{E_M} \right)^{-1}}{\left( \frac{E^G_{PL}}{E_M} \right) + \zeta^G_{W}} \quad (3d)
\]

so that \( w^G_{PL} \), \( l^G_{PL} \), and \( t^G_{PL} \) define platelets average widths, length, and thickness, respectively. Furthermore, Poisson’s ratio for graphene based composite might be defined based upon Poisson’s ratio of the two constituents in the form:

\[
V_1 = V^G_{PL} V^G_{PL} + V^G_M V^G_M \quad (4)
\]

\[
\alpha_1 = \alpha^G_{PL} V^G_{PL} + \alpha^G_M V^G_M
\]

in which \( V^G_M = 1 - V^G_{PL} \) expresses the volume fractions of matrix component. Herein, three dispersions of the platelets have been assumed as:

Uniform:

\[
W^G_{PL} = \lambda_1 W^0_{GPL} \quad (5a)
\]

Linear:

\[
W^G_{PL} = \lambda_2 W^0_{GPL} \left( \frac{z}{h} + \frac{1}{2} \right) \quad (5b)
\]

Nonlinear:

\[
W^G_{PL} = \frac{\lambda_3 W^0_{GPL} z^2}{s^2 h^2 (4s^2 - h^2)} \left[ 4h^2 z^2 - h^4 + \frac{16s^2}{n} (s^2 - z^2) \right] \quad (5c)
\]

where \( W^0_{GPL} = 1\% \) expresses a particular weight fraction for graphene platelets.
3. Beam modeling via refined theory

With the employment of refined beam theory, a displacement field having following forms might be expressed to start mathematical modeling of the beam (Barati 2017, Mouffoki et al. 2017, Fenjan et al. 2019, Ahmed et al. 2019, Zemri et al. 2015, Bounouara et al. 2016):

\[
\begin{align*}
    u_x(x, z, t) &= u_0(x, t) - z \frac{\partial w_b}{\partial x} - f(z) \frac{\partial w_s}{\partial x} \\
    u_y(x, z, t) &= 0 \\
    u_z(x, z, t) &= w_b(x, t) + w_s(x, t)
\end{align*}
\]

(6a)\hspace{1cm} (6b)\hspace{1cm} (6c)

Here \( u; \ w_b \) and \( w_s \) express the axial and transverse field coefficients.

For the refined beam model, the strain field might be expressed by (Ahmed et al. 2019):

\[
\begin{align*}
    \epsilon_x &= \frac{\partial u_0}{\partial x} - z \frac{\partial^2 w_b}{\partial x^2} - f(z) \frac{\partial^2 w_s}{\partial x^2} \\
    \epsilon_y &= \epsilon_z = \gamma_{xy} = \gamma_{yz} = 0 \\
    \gamma_{xz} &= 2 \epsilon_{xz} = g(z) \frac{\partial w_s}{\partial x}
\end{align*}
\]

(7a)\hspace{1cm} (7b)\hspace{1cm} (7c)

where \( f(z) = z - h \sinh \left( \frac{z}{h} \right) + z \cosh \left( \frac{1}{2} \right) \) and \( g(z) = 1 - f'(z) \).

To derive weak formulation required for finite element method, first it is necessary to express the Hamiltonian of a dynamic system as:

\[
\begin{align*}
    H &= U - V - K
\end{align*}
\]

Here \( U \) is strain energy and \( K \) is kinetic energy. The strain energy can be stated as (Faleh et al. 2018, She et al. 2018):

\[
\begin{align*}
    U &= 0.5 \int_0^L \int_0^h \left( \sigma_{ij} \delta \epsilon_{ij} \right) dz dx \\
    &= \int_0^L \int_0^h \left( \sigma_x \delta \epsilon_x + \tau_{xz} \delta \gamma_{xz} \right) dz dx = \int_0^L \left( A_{11} \frac{du_0}{dx} \frac{du_0}{dx} - 2B_{11} \frac{du_0}{dx} \frac{d^2 w_b}{dx^2} ight. \\
    &+ D_{11} \frac{d^2 w_b}{dx^2} \frac{d^2 w_b}{dx^2} + 2B_{11}^s \frac{du_0}{dx} \frac{d^2 w_s}{dx^2} - 2D_{11}^s \frac{du_0}{dx} \frac{d^2 w_s}{dx^2} - (2D_{11}^s + 0.5 \cdot (A^s)) \frac{d^2 w_b}{dx^2} \frac{d^2 w_s}{dx^2} \\
    &+ H_{11}^s \right) dx
\end{align*}
\]

(8)

where \( A_{11}, B_{11}^s, \) etc., are the beam stiffness, defined by:
\begin{equation}
(A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s) = \int_{-h/2}^{h/2} E(z) (1, z, z^2, f, zf, f^2) dz
\end{equation}

\begin{equation}
A_s = \int_{-h/2}^{h/2} G(z) g^2 dz
\end{equation}

The kinetic energy of beam is written as:

\begin{equation}
K = \int_0^L \int_{-h/2}^{h/2} \rho(z) [\ddot{u}_x \dot{u}_x + \ddot{u}_y \dot{u}_y] \, dz \, dx
\end{equation}

\begin{equation}
= \int_0^L \left\{ I_0 [\ddot{u}_0 \dot{u}_0 + (\ddot{w}_b + \dddot{w}_b)(\dot{w}_b + \dddot{w}_b)] - 2J_1 \left( \frac{d\dddot{w}_b}{dx} \right) + 2J_2 \left( \frac{d\dddot{w}_s}{dx} \right) \right\} \, dx
\end{equation}

in which dot-superscript denote the differentiation with respect to the time variable \( t \); and the mass inertias are expressed as:

\begin{equation}
(I_0, I_1, I_1, I_2, J_1, J_2) = \int_{-h/2}^{h/2} (1, z, z^2, zf, f^2) \rho(z) \, dz
\end{equation}

Also, the work done by the external applied forces can be expressed as:

\begin{equation}
V = 0.5 \int_0^L \left[ (k_p - N^T) \frac{\partial(w_b + w_s)}{\partial x} \frac{\partial(w_b + w_s)}{\partial x} - k_w (w_b + w_s) + q(w_b + w_s) \right] \, dx
\end{equation}

where \( k_w \) and \( k_p \) are Winkler and Pasternak constants; \( q_{\text{dynamic}} \) is applied transverse dynamic load and \( N^T \) is in-plane thermal loading: \( N^T = \frac{\partial}{\partial z} \left( \frac{\partial}{\partial z} \right) \left( E \alpha_i (T - T_0) \right) \, dz \); \( T = \Delta T \left( \frac{z}{h} + 0.5 \right) + T_0 \) and \( \Delta T \) is temperature rise. In the present paper, the linear temperature rise has been adopted.

4. Method of finite elements

Through the present section, the method of finite elements has been selected for solving the dynamical problem of a graphene based beam having S-S and C-C edges. For this goal, the refined beam element has been used with ten degrees of freedom indicated in Fig.3. Herein, a shape
Finite element based modeling and thermal dynamic analysis of functionally graded graphene...

Fig. 3 Degrees of freedom for the refined master element

function has been introduced for axial field component, and also Hermit shape function have been introduced for lateral field components which are:

\[ u_0(x,t) = \sum_{i=1}^{2} U_i(t) N_i(x) = N_1 U_1 + N_2 U_2 \] (14)

\[ w_b(x,t) = \sum_{i=1}^{4} W_{m_i}(t) \tilde{N}_1(x) = \tilde{N}_1 W_{b1} + \tilde{N}_2 W_{b1}' + \tilde{N}_3 W_{b2} + \tilde{N}_4 W_{b2}' \] (15)

\[ w_s(x,t) = \sum_{i=1}^{4} W_{s_i}(t) \tilde{N}_1(x) = \tilde{N}_1 W_{s1} + \tilde{N}_2 W_{s1}' + \tilde{N}_3 W_{s2} + \tilde{N}_4 W_{s2}' \] (16)

So that \( U_i \), \( W_{b1} \) and \( W_{s1} \) are field coefficients and shape functions are:

\[ N_1 = 1 - \frac{x}{L_e} \] (17)

\[ N_2 = \frac{x}{L_e} \] (18)

\[ \tilde{N}_1 = \frac{1}{L_e^3} \left( 2x^3 - 3x^2 L_e + L_e^2 \right) \] (19)

\[ \tilde{N}_2 = \frac{1}{L_e^3} \left( x^3 L_e - 2x^2 L_e^2 + xL_e^3 \right) \] (20)

\[ \tilde{N}_3 = \frac{1}{L_e^3} \left( -2x^3 + 3x^2 L_e \right) \] (21)

\[ \tilde{N}_4 = \frac{1}{L_e^3} \left( x^3 L_e - x^2 L_e^2 \right) \] (22)

so that \( L_e \) defines the length for master element.
Placing Eqs. (8)-(13) in Hamiltonian and minimizing it to field coefficients $U_i$, $W_{bi}$, and $W_{si}$ (Rezaiee-Pajand et al. 2018, Al-Maliki et al. 2019) results in below relation containing simultaneous algebraic equations:

$$\frac{\partial H}{\partial U_i} = \frac{\partial H}{\partial W_{bi}} = \frac{\partial H}{\partial W_{si}} = 0$$  \hspace{1cm} (23)

Next, the solution trend yields:

$$\begin{bmatrix} [k_{i1}] & [k_{i2}] & [k_{i3}] \\ [k_{21}] & [k_{22}] & [k_{23}] \\ [k_{31}] & [k_{32}] & [k_{33}] \end{bmatrix}_{10 \times 10} + \omega_{ex}^2 \begin{bmatrix} [m_{i1}] & [m_{i2}] & [m_{i3}] \\ [m_{21}] & [m_{22}] & [m_{23}] \\ [m_{31}] & [m_{32}] & [m_{33}] \end{bmatrix}_{10 \times 10} \begin{bmatrix} U_i \\ W_{bi} \\ W_{si} \end{bmatrix} = \begin{bmatrix} 0 \\ F_i \\ F_i \end{bmatrix}$$  \hspace{1cm} (24)

so that $k_{ij}$ and $m_{ij}$ define stiffness and mass matrices for master element, respectively; $F_i$ defines exerted dynamical loading to master element. Obtained system may be solved for deriving transverse center deflections of a beam $W = W_b + W_s$ as a function of external frequency $\omega_{ex}$. Based on adequate numbers of beam element, it is feasible to reach the exact solutions.

Next, normalized factors may be selected by:

$$\Omega = \omega_{ex} L \sqrt{\frac{\rho M}{E_m I}}, \quad K_w = \frac{k_w L^2}{E_m I}, \quad K_p = \frac{k_p L^2}{E_m I}$$  \hspace{1cm} (25)

All components for mass and stiffness matrices together with load vector achieved by Eq.(23) have been introduced as:

$$k_{11} = A_1 \int_0^{t_e} \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} \, dx$$

$$k_{12} = -B_1 \int_0^{t_e} \frac{\partial N_i}{\partial x} \frac{\partial^2 \tilde{N}_j}{\partial x^2} \, dx, \quad k_{21} = -B_1 \int_0^{t_e} \frac{\partial N_j}{\partial x} \frac{\partial^2 \tilde{N}_i}{\partial x^2} \, dx$$

$$k_{22} = D_1 \int_0^{t_e} \frac{\partial^2 \tilde{N}_i}{\partial x^2} \frac{\partial^2 \tilde{N}_j}{\partial x^2} \, dx - k_w \int_0^{t_e} \tilde{N}_i \tilde{N}_j \, dx + (k_p - N^T) \int_0^{t_e} \frac{\partial \tilde{N}_i}{\partial x} \frac{\partial \tilde{N}_j}{\partial x} \, dx$$

$$k_{13} = -B_1' \int_0^{t_e} \frac{\partial N_i}{\partial x} \frac{\partial^2 \tilde{N}_j}{\partial x^2} \, dx, \quad k_{31} = -B_1' \int_0^{t_e} \frac{\partial N_j}{\partial x} \frac{\partial^2 \tilde{N}_i}{\partial x^2} \, dx$$

$$k_{23} = -D_1' \int_0^{t_e} \frac{\partial^2 \tilde{N}_i}{\partial x^2} \frac{\partial^2 \tilde{N}_j}{\partial x^2} \, dx - k_w \int_0^{t_e} \tilde{N}_i \tilde{N}_j \, dx + (k_p - N^T) \int_0^{t_e} \frac{\partial \tilde{N}_i}{\partial x} \frac{\partial \tilde{N}_j}{\partial x} \, dx$$

$$k_{33} = H_1' \int_0^{t_e} \frac{\partial \tilde{N}_i}{\partial x} \frac{\partial^2 \tilde{N}_j}{\partial x^2} \, dx + A' \int_0^{t_e} \frac{\partial \tilde{N}_i}{\partial x} \frac{\partial^2 \tilde{N}_j}{\partial x^2} \, dx$$

$$- k_w \int_0^{t_e} \tilde{N}_i \tilde{N}_j \, dx + (k_p - N^T) \int_0^{t_e} \frac{\partial \tilde{N}_i}{\partial x} \frac{\partial \tilde{N}_j}{\partial x} \, dx$$

...
\[ m_{11} = I_0 \int_0^{L_e} N_i N_j \, dx \]
\[ m_{22} = I_0 \int_0^{L_e} \tilde{N}_i \tilde{N}_j \, dx + I_2 \int_0^{L_e} \frac{\partial \tilde{N}_i}{\partial x} \frac{\partial \tilde{N}_j}{\partial x} \, dx \]
\[ m_{12} = -I_1 \int_0^{L_e} N_i \frac{\partial \tilde{N}_j}{\partial x} \, dx, \quad m_{21} = -I_1 \int_0^{L_e} N_j \frac{\partial \tilde{N}_i}{\partial x} \, dx \]
\[ m_{13} = -J_1 \int_0^{L_e} N_i \frac{\partial \tilde{N}_j}{\partial x} \, dx, \quad m_{31} = -J_1 \int_0^{L_e} N_j \frac{\partial \tilde{N}_i}{\partial x} \, dx \]
\[ m_{23} = J_2 \int_0^{L_e} \frac{\partial \tilde{N}_i}{\partial x} \frac{\partial \tilde{N}_j}{\partial x} \, dx \]
\[ m_{33} = I_0 \int_0^{L_e} \tilde{N}_i \tilde{N}_j \, dx + K_2 \int_0^{L_e} \frac{\partial \tilde{N}_i}{\partial x} \frac{\partial \tilde{N}_j}{\partial x} \, dx \]
\[ F_i = \int_0^{L_e} q_{\text{dynamic}} \tilde{N}_i \, dx \]

Table 1: Gradient index effects on total content of graphene

<table>
<thead>
<tr>
<th>Uniform ((\lambda_1))</th>
<th>Linear ((\lambda_2))</th>
<th>Nonlinear ((\lambda_3))</th>
<th>(%W_{\text{GPL}}^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.33</td>
<td>0.67</td>
<td>0.43</td>
<td>0.33</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1.29</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2: Material and geometrical factors for graphene based beams

<table>
<thead>
<tr>
<th>GPLs</th>
<th>Matrix (Epoxy resin)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E_{\text{GPL}}=1.01) TPa</td>
<td>(E_M=2.85) GPa</td>
</tr>
<tr>
<td>(\rho_{\text{GPL}}=1062.5) kg/m(^3)</td>
<td>(\rho_M=1200) kg/m(^3)</td>
</tr>
<tr>
<td>(v_{\text{GPL}}=0.006)</td>
<td>(v_M=0.34)</td>
</tr>
<tr>
<td>(a_{\text{GPL}}=2.35 \times 10^{-5}/K)</td>
<td>(a_M=8.2 \times 10^{-5}/K)</td>
</tr>
<tr>
<td>(t_{\text{GPL}}=1.5) nm</td>
<td>-</td>
</tr>
<tr>
<td>(w_{\text{GPL}}=1.5) (\mu)m</td>
<td>-</td>
</tr>
<tr>
<td>(l_{\text{GPL}}=2.5) (\mu)m</td>
<td>-</td>
</tr>
</tbody>
</table>

5. Graphical results and discussions

In the present section, dynamic analysis of functionally graded (FG) graphene-reinforced beams under thermal loading has been carried out based on finite element approach. The presented...
formulation was based on a higher order refined beam element accounting for shear deformations. The graphene-reinforced beam is exposed to transverse periodic mechanical loading. Graphene platelets have three types of dispersion within the structure including uniform-type, linear-type and nonlinear-type. Tables 1 and 2 provide excellent information about material properties.

The convergences and verifications of proposed approach have been respectively represented by Table 3 and Fig.4. Based upon Table 3, one can deduce that vibrational frequencies are converged via selecting fifteen elements for S-S beams and thirty elements for C-C beams. The number of elements have been adopted for presented investigation in this article. Furthermore, Fig.4 depicts that achieved frequency for graphene based beams are identical to those represented by Kitipornchai et al. (2016) based upon different graphene weight fractions.

**Table 3 Convergence of the vibrational frequency of uniformly graphene based beam (L/h=10, W_{GPL}=1%)**

<table>
<thead>
<tr>
<th>Elements number</th>
<th>S-S</th>
<th>C-C</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>20.8610</td>
<td>47.5977</td>
</tr>
<tr>
<td>3</td>
<td>20.7961</td>
<td>47.0170</td>
</tr>
<tr>
<td>4</td>
<td>20.7846</td>
<td>46.8766</td>
</tr>
<tr>
<td>5</td>
<td>20.7814</td>
<td>46.8328</td>
</tr>
<tr>
<td>10</td>
<td>20.7794</td>
<td>46.7934</td>
</tr>
<tr>
<td>12</td>
<td>20.7793</td>
<td>46.7901</td>
</tr>
<tr>
<td>15</td>
<td>20.7792</td>
<td>46.7872</td>
</tr>
<tr>
<td>20</td>
<td>20.7792</td>
<td>46.7848</td>
</tr>
<tr>
<td>25</td>
<td>20.7792</td>
<td>46.7836</td>
</tr>
<tr>
<td>30</td>
<td>20.7792</td>
<td>46.7830</td>
</tr>
<tr>
<td>35</td>
<td>20.7792</td>
<td>46.7830</td>
</tr>
</tbody>
</table>

**Fig. 4 Validation of the dimensionless vibration frequency of GPL-reinforced beams (L/h=20)**
Finite element based modeling and thermal dynamic analysis of functionally graded graphene...

![Graphene beam deflections](image)

Fig. 5 Dynamical deflections of graphene based beams versus external frequency for different graphene distributions and temperature changes ($L/h=10$, $K_w=0$, $K_p=0$, $W^*_{GPL}=1\%$)

Dynamical deflections of graphene based beams with respect to external frequency based on different graphene dispersions and temperatures have been depicted in Fig.5 when $L=10h$ and $W^*_{GPL}=1\%$. The graphene based beam has been assumed as simply-supported at left and right.
edges. Firstly, it is better to express that this graph illustrates normalized deflection as a function of external frequency of exerted harmonic mechanical load. This means that at a particular value of the external frequency the resonance phenomena together with large deflections may be observed. It is well known that the beam structural rigidity will be reduced via linear temperature rise. Accordingly, the forced vibration curves will move to the left highlighting smaller resonance
frequency. Furthermore, linear temperature rise impacts are dependent on the kinds of graphene dispersions. One may note that nonlinear graphene type results in greater resonance frequency compared to linear/uniform types. Therefore, dynamic behavior of a graphene based beam under thermal loads may be controlled employing a suitable distribution of graphene.

In Figs. 6 and 7, forced vibration curves for graphene based beams have been depicted based on uniform and linear types of graphene dispersion. Linear temperature rise across the thickness has been assumed as $\Delta T = 50$. Increase of graphene weight fractions may lead to greater resonance frequency together with lower deflection before resonances zone. It means that the amplitude-frequency curves may move toward higher frequencies with enlargement of graphene amount. This highlights that the graphene based beams become more rigid and represent superior mechanical performances under dynamical loading. Accordingly, the resonances may be delayed via increase of graphene content. According to the graphs, one may find that nonlinear graphene type may lead to greatest resonance frequency than uniformly or linearly types. These observations are owing to highest structural rigidity of graphene based beams provided by nonlinear dispersion type. Therefore, dynamic behavior of a graphene based beam under mechanical-thermal loads may be controlled employing a suitable distribution for graphene.

Fig. 8 depicts 1st mode shape for clamped graphene based beams based upon different graphene dispersion types at $L=10h$. Linear temperature rise across the thickness has been assumed as $\Delta T = 50$ and exerted harmonic loading has the external frequency as $\Omega=5$. There are two important results in the presented graph. Firstly, nonlinear and linear graphene types respectively represent lowest and greatest dynamical amplitudes. Secondly, one may understand that dynamical amplitude based on all dispersion kinds is remarkably decreased via increase of graphene weight fractions.
Fig. 9 Dynamic response of S-S nano-composite beam with uniform GPLs for various foundation parameters and excitation frequencies ($L/h=10$, $W^{*}_{GPL}=1\%$, $\Delta T=50$)
6. Conclusions

In the present article, dynamic analysis of functionally graded (FG) graphene-reinforced beams under thermal loading was carried out based on finite element approach. The presented formulation was based on a higher order refined beam element accounting for shear deformations. The graphene-reinforced beam was exposed to transverse periodic mechanical loading. Graphene platelets had three types of dispersion within the structure including uniform-type, linear-type and nonlinear-type.

- Increase of graphene weight fractions led to greater resonance frequency together with lower deflection before resonances zone.
- It is well known that the beam structural rigidity will reduced via linear temperature rise. Accordingly, the forced vibration curves will move to the left highlighting smaller resonance frequency.
- Linear temperature rise impacts were dependent on the kinds of graphene dispersions.
- Nonlinear graphene type results in greater resonance frequency compared to linear/uniform types.

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References


